14 Azar 1396 Take-home exam, Due date 18 Azar, morning

## Midterm Exam of String Theory

**Bosonic Open String Theory:** Consider a *D*-dimensional open string theory on flat space with *n* NN modes, *m* DN modes and *d* DD modes. For the DD modes assume the two ends of strings are separated by  $Y^i$ ,  $i = 1, \dots, d$ .

- 1) Write out mode expansion of this open string theory.
- 2) Study the spectrum of this theory using light-cone quantization.
- 3) Fix the worldsheet diffeomorphisms using the appropriate bc-ghost system.
- 4) Using the *bc*-ghost system, show cancellation of the Weyl anomaly leads to D = 26, irrespective of values of n, d.
- 5) Write out the first three low-lying (lightest) states in the spectrum and write the corresponding vertex operators.
- 6) Construct unoriented version of the above mentioned open string theory:
  - 6-1) How does NN, DN and DD modes transform under worldsheet parity  $\Omega$ ?
  - 6-2) Mod out the Hilbert space by  $\Omega$  to find the spectrum of unoriented open string theory.
- 7) Compute one-loop partition function of the theory. In particular, discuss the  $Y^i$  dependence of the partition function. Note that for open string case the one-loop worldsheet is annulus (or cylinder), in contrast with the torus in the closed string case.
- 8) Compute the one-loop partition function of the unoriented theory (constructed in item 6)). Discuss  $Y^i$  dependence of the partition function. For the unoriented case the worldsheet to consider is a Möbius strip.
- 9) Add the most general open string coupling to the worldsheet action. *Hint: this term is a boundary term on the worldsheet.*
- 10) Compute the one-loop  $\beta$ -function for the worldsheet coupling and work out the (target space) action which yields this  $\beta$ -function equation as its equation of motion.

All the best, Sheikh-Jabbari.