Magnetic and quadrupole moments of the $Z_c(3900)$

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The electromagnetic properties of the tetraquark state $Z_c(3900)$ are investigated in the diquark-antidiquark picture and its magnetic and quadrupole moments are extracted. To this end, the light-cone QCD sum rule in electromagnetic background field is used. The magnetic and quadrupole moments encode the spatial distributions of the charge and magnetization in the particle. The result obtained for the magnetic moment is quite large and can be measured in future experiments. We obtain a nonzero but small value for the quadrupole moment of $Z_c(3900)$ indicating a nonspherical charge distribution.

Keywords: Tetraquarks, Electromagnetic form factors, Multipole moments

I. INTRODUCTION

In the conventional quark model, the predicted particles are mesons($q\bar{q}$), baryons(qqq) and antibaryons($\bar{q}\bar{q}\bar{q}$). Hundreds of meson and baryon resonances have been observed till now. However, the quark model as well as QCD as theory of strong interaction does not exclude the existence of nonconventional particles. Hence, physicists have thought that there may be particles in different structures [1–3]. Particles having different quark and gluon contents such as tetraquarks, pentaquarks, hybrids, glueballs and so on are called exotic states. To explore the underlying structures of these states, many exotic structures have been suggested [for instance, see [4–10]]. Although predicted in the 1970s, there was not significant experimental evidence of their existence until recently. Experimentally, the adventure of exotic states began when X(3872) was discovered by the Belle Collaboration [11] and continued with the discovery of the Y(4260) by the BABAr Collaboration [12]. At present, more than twenty exotic states have been discovered in many experiments, most of which have been classified as the XYZ family (for details, see [13]). The XYZ family has some decay channels that severely violate the isospin symmetry and negatively affect the identification of conventional charmonium/bottomonium states. Because of that these newly observed XYZ states provide a good platform for studying the nonperturbative behavior of QCD. The study of the properties of these particles is one of the most active and interesting branches of particle physics.

One of the most prominent particles among the exotic states is the charged $Z_c(3900)$ tetraquark. The $Z_c^{\pm}(3900)$ state discovered by BESIII in the process $e^+e^- \to \pi^{\pm} J/\psi$ [14] with a mass $3899.0 \pm 3.6 \pm 4.9$ MeV and width $\Gamma = 46 \pm 10 \pm 20 MeV$. Almost at the same time this state was confirmed by the Belle Collaboration [15], with a mass $3894.5 \pm 6.6 \pm 4.5$ MeV and width $\Gamma = 63 \pm 24 \pm 26$ MeV. Its existence was also confirmed in Ref. [16] on the basis of the CLEO-c data analysis, with mass $3886.0 \pm 4.0 \pm 2.0$ MeV and width $\Gamma = 37 \pm 4 \pm 8$ MeV. The decays into $\pi^{\pm} J/\psi$, reveal that $Z_c^{\pm}(3900)$ must be a tetraquark state with constituents $c\bar{c}u\bar{d}$ or $c\bar{c}d\bar{u}$ [17]. Since the mass of $Z_c^{\pm}(3900)$ is very close to $X(3872)$, it can be advised as the charged partner of the $X(3872)$ in a tetraquark scenario. The properties of the $Z_c^{\pm}(3900)$ particle have been investigated with different theoretical models and approaches [18–31]. Although the spectroscopic properties of these particles have been studied adequately, the internal structure and nature of the $X(3872)$ and $Z_c^{\pm}(3900)$ particles have not been fully understood yet. For this reason, it is important to study their decay properties as well as their interactions with other particles. In this context, examining the interaction of these particles with the photon can play an important role in understanding of their nature and internal structure.

A detailed study of the electromagnetic structures, such as electromagnetic multipole moments and electromagnetic form factors, of hadrons not only provides important information about the nonperturbative nature of QCD but also the multipole moments of the hadrons are important tools for understanding their internal structures in terms of quarks and gluons as well as their geometric shape. The electromagnetic multipole moments encode the spatial distributions of charge and magnetization in the particle. In hadrons, quarks are the carriers of the charge, and thus these observables are directly connected to the spatial distribution of quarks in hadrons, as well as a probe of the underlying dynamics. The examination of the spatial distri butions of the charge and magnetism carried by nuclei

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started in the 1950s. The electromagnetic properties of the nucleon have been studied in the past extensively from unpolarized electron scattering experiments-for reviews on experimental progress, see for instance Refs. [32–36].

There are many studies in the literature devoted to investigation of the multipole moments of the standard hadrons. However, unfortunately, almost nothing is known about the multipole moments of exotic particles and more detailed analyses are needed in this regard. Since direct experimental information on the electromagnetic multipole moments of the exotic particles is very limited, theoretical studies can play an important role in this respect. In this study, the tetraquark state $Z_c(3900)$ is investigated in the diquark-antidiquark picture and its magnetic and quadrupole moments are extracted. This is the first theoretical attempt to calculate the electromagnetic multipole moments of the hidden-charm tetraquark states. To study the electromagnetic multipole moments, a nonperturbative method is needed. The light-cone QCD sum rule (LCSR) is one of the nonperturbative methods that has been successfully applied to study many nonperturbative properties of hadrons for decades [37–39]. In the LCSR, the features of the particles under study are described in terms of the vacuum condensates and the light-cone distribution amplitudes (DAs). Hence, any uncertainty in these parameters affects the estimations on the magnetic and quadrupole moments.

The rest of the paper is organized as follows: In Sec. II, the LCSR for the magnetic and quadrupole moments of the $Z_c(3900)$ are derived. Section III is devoted to the numerical analysis of the obtained sum rules. Section IV includes our concluding remarks. The explicit expressions of the photon distribution amplitudes, magnetic and quadrupole moments as well as some details about calculations are moved to Appendixes A-C.

II. FORMALISM

In order to calculate the magnetic and quadrupole moments of the $Z_c(3900)$ state in the framework of LCSR, we start from the correlation function

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{ip\cdot x}\langle 0|\mathcal{T}\{J^{Z_c}_\mu(x) J^{Z_c\dagger}_\nu(0)\}|0\rangle_\gamma, \tag{1}$$

where γ is the external electromagnetic field and J_μ is the interpolating current of the $Z_c(3900)$ state with quantum numbers $J^{PC} = 1^{+-}$ in the diquark-antidiquark picture. It is given as

$$J^{Z_c}_\mu(x) \;=\; \frac{i\epsilon\tilde{\epsilon}}{\sqrt{2}}\left\{\left[u^T_a(x)C\gamma_5 c_b(x)\right]\left[\overline{d}_d(x)\gamma_\mu C\overline{c}^T_e(x)\right] - \left[u^T_a(x)C\gamma_\mu c_b(x)\right]\left[\overline{d}_d(x)\gamma_5 C\overline{c}^T_e(x)\right]\right\}, \tag{2}$$

where $\epsilon = \epsilon_{abc}$, $\tilde{\epsilon} = \epsilon_{dec}$, C is the charge conjugation matrix and a, b, c, d, e are color indices.

We start to calculate the correlation function in terms of the hadronic parameters called the hadronic side. To this end, we insert complete sets of intermediate states having the same quantum numbers as the interpolating current of $Z_c(3900)$ into the correlation function, and isolate the contribution of the ground state. As a result the following expression is obtained:

$$\Pi^{Had}_{\mu\nu}(p,q) = \frac{\langle 0 \mid J^{Z_c}_\mu \mid Z_c(p)\rangle}{p^2 - m^2_{Z_c}}\langle Z_c(p) \mid Z_c(p+q)\rangle_\gamma \frac{\langle Z_c(p+q) \mid J^{\dagger Z_c}_\nu \mid 0\rangle}{(p+q)^2 - m^2_{Z_c}} + \cdots, \tag{3}$$

where dots represent the contributions coming from the higher states and continuum and q is the momentum of the photon. The matrix element $\langle 0 \mid J^{Z_c}_\mu \mid Z_c\rangle$ is parametrized as

$$\langle 0 \mid J^{Z_c}_\mu \mid Z_c\rangle = \lambda_{Z_c}\varepsilon^\theta_\mu\,, \tag{4}$$

with λ_{Z_c} being the current coupling constant or residue of the $Z_c(3900)$ state.

In the presence of the electromagnetic background field, the vertex of the two axial vector mesons can be written in terms of form factors as follows [40]:

$$\begin{aligned}\langle Z_c(p,\varepsilon^\theta) \mid Z_c(p+q,\varepsilon^\delta)\rangle_\gamma = -\varepsilon^\tau(\varepsilon^\theta)^\alpha(\varepsilon^\delta)^\beta \Bigg[& G_1(Q^2)\;(2p+q)_\tau\; g_{\alpha\beta} + G_2(Q^2)\;(g_{\tau\beta}\; q_\alpha - g_{\tau\alpha}\; q_\beta)\\ & - \frac{1}{2m^2_{Z_c}} G_3(Q^2)\;(2p+q)_\tau\; q_\alpha q_\beta\Bigg],\end{aligned} \tag{5}$$

where ε^δ and ε^θ are the polarization vectors of the initial and final $Z_c(3900)$ mesons and ε^τ is the polarization vector of the photon. The form factors $G_1(Q^2)$, $G_2(Q^2)$ and $G_3(Q^2)$ can be written in terms of the charge $F_C(Q^2)$, magnetic

$F_M(Q^2)$ and quadrupole $F_{\mathcal{D}}(Q^2)$ form factors in the following way:

$$
\begin{aligned}
F_C(Q^2) &= G_1(Q^2) + \frac{2}{3}\lambda F_{\mathcal{D}}(Q^2)\,,\\
F_M(Q^2) &= G_2(Q^2)\,,\\
F_{\mathcal{D}}(Q^2) &= G_1(Q^2) - G_2(Q^2) + (1+\lambda)G_3(Q^2)\,,
\end{aligned}
\tag{6}
$$

where $\lambda = Q^2/4m_{Z_c}^2$ with $Q^2 = -q^2$. At $Q^2 = 0$, the form factors $F_C(Q^2=0)$, $F_M(Q^2=0)$, and $F_{\mathcal{D}}(Q^2=0)$ are related to the electric charge, magnetic moment μ, and quadrupole moment $\mathcal{D}$ in the following way:

$$
\begin{aligned}
eF_C(0) &= e\,,\\
eF_M(0) &= 2m_{Z_c}\mu\,,\\
eF_{\mathcal{D}}(0) &= m_{Z_c}^2\mathcal{D}\,.
\end{aligned}
\tag{7}
$$

Using Eqs. (3)-(5) and imposing the condition, $q\cdot\varepsilon = 0$, and performing summation over polarization vectors, the correlation function takes the form,

$$
\begin{aligned}
\Pi_{\mu\nu}^{Had} &= \lambda_{Z_c}^2\frac{\varepsilon^\tau}{[m_{Z_c}^2-(p+q)^2][m_{Z_c}^2-p^2]}\Bigg[2p_\tau F_C(0)\Bigg(g_{\mu\nu} - \frac{p_\mu q_\nu - p_\nu q_\mu}{m_{Z_c}^2}\Bigg)\\
&+ F_M(0)\Bigg(q_\mu g_{\nu\tau} - q_\nu g_{\mu\tau} + \frac{1}{m_{Z_c}^2}p_\tau(p_\mu q_\nu - p_\nu q_\mu)\Bigg) - \Bigg(F_C(0) + F_{\mathcal{D}}(0)\Bigg)\frac{p_\tau}{m_{Z_c}^2}q_\mu q_\nu\Bigg]\,.
\end{aligned}
\tag{8}
$$

The next step is to calculate the correlation function in Eq. (1) in terms of quarks and gluon properties in the deep Euclidean region called the QCD side. For this aim, the interpolating currents are inserted into the correlation function and after the contracting of quark pairs using the Wick theorem the following result is obtained:

$$
\begin{aligned}
\Pi_{\mu\nu}^{\rm QCD}(q) &= -i\frac{\epsilon\tilde{\epsilon}\epsilon'\tilde{\epsilon}'}{2}\int d^4x e^{ipx}\langle 0|\Bigg\{\mathrm{Tr}\Big[\gamma_5\widetilde{S}_u^{aa'}(x)\gamma_5 S_c^{bb'}(x)\Big]\mathrm{Tr}\Big[\gamma_\mu\widetilde{S}_c^{e'e}(-x)\gamma_\nu S_d^{d'd}(-x)\Big]\\
&-\mathrm{Tr}\Big[\gamma_\mu\widetilde{S}_c^{e'e}(-x)\gamma_5 S_d^{d'd}(-x)\Big]\mathrm{Tr}\Big[\gamma_\nu\widetilde{S}_u^{aa'}(x)\gamma_5 S_c^{bb'}(x)]\\
&-\mathrm{Tr}\Big[\gamma_5\widetilde{S}_u^{a'a}(x)\gamma_\mu S_c^{b'b}(x)\Big]\mathrm{Tr}\Big[\gamma_5\widetilde{S}_c^{e'e}(-x)\gamma_\nu S_d^{d'd}(-x)\Big]\\
&+\mathrm{Tr}\Big[\gamma_\nu\widetilde{S}_u^{aa'}(x)\gamma_\mu S_c^{bb'}(x)\Big]\mathrm{Tr}\Big[\gamma_5\widetilde{S}_c^{e'e}(-x)\gamma_5 S_d^{d'd}(-x)\Big]\Bigg\}|0\rangle_\gamma,
\end{aligned}
\tag{9}
$$

where

$$
\widetilde{S}_{c(q)}^{ij}(x) = CS_{c(q)}^{ij\mathrm{T}}(x)C,
$$

with $S_{q(c)}(x)$ being the quark propagators. In the x-space for the light quark propagator we use in the $m_q \to 0$ limit

$$
S_q(x) \;=\; i\frac{\not x}{2\pi^2x^4} - \frac{\bar{q}q}{12} - \frac{\bar{q}q}{192}m_0^2x^2 - \frac{ig_s}{16\pi^2x^2}\int_0^1 dv\; G^{\mu\nu}(vx)\Big[\not x\sigma_{\mu\nu} + \sigma_{\mu\nu}\not x\Big]. \tag{10}
$$

The heavy quark propagator is given, in terms of the second kind Bessel functions $K_\nu(x)$, as

$$
\begin{aligned}
S_c(x) &= \frac{m_c^2}{4\pi^2}\Bigg[\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}} + i\frac{\not x\; K_2(m_c\sqrt{-x^2})}{(\sqrt{-x^2})^2}\Bigg] - \frac{g_s m_c}{16\pi^2}\int_0^1 dv\; G^{\mu\nu}(vx)\Bigg[(\sigma_{\mu\nu}\not x + \not x\sigma_{\mu\nu})\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}}\\
&+2\sigma^{\mu\nu}K_0(m_c\sqrt{-x^2})\Bigg].
\end{aligned}
\tag{11}
$$

The correlation function contains different types of contributions. In the first part, one of the free quark propagators in Eq. (9) is replaced by

$$
S^{free} \to \int d^4y\, S^{free}(x-y)\not{A}(y)\, S^{free}(y)\,, \tag{12}
$$

with S^{free} representing the first term of the light or heavy quark propagators and the remaining three propagators with the full quark propagators. In the calculations the Fock-Schwinger gauge, $x_\mu A^\mu = 0$, is used.

In the second case one of the light quark propagators in Eq. (9) is replaced by

$$S^{ab}_{\alpha\beta} \to -\frac{1}{4}(\bar{q}^a \Gamma_i q^b)(\Gamma_i)_{\alpha\beta}, \tag{13}$$

and the remaining propagators with the full quark propagators. Here, Γ_i are the full set of Dirac matrices. Once Eq. (13) is plugged into Eq. (9), there appear matrix elements such as $\langle\gamma(q)\,|\bar{q}(x)\Gamma_i q(0)|\,0\rangle$ and $\langle\gamma(q)\,|\bar{q}(x)\Gamma_i G_{\alpha\beta} q(0)|\,0\rangle$, representing the nonperturbative contributions. These matrix elements can be expressed in terms of photon wave functions with definite twists. Additionally, in principle, nonlocal operators such as $\bar{q}G^2q$ and $\bar{q}q\bar{q}q$ are anticipated to appear. In this study, we take into account operators with only one gluon field and contributions coming from three particle nonlocal operators and neglect terms with two gluons $\bar{q}G^2q$, and four quarks $\bar{q}q\bar{q}q$. The matrix elements $\langle\gamma(q)\,|\bar{q}(x)\Gamma_i q(0)|\,0\rangle$ and $\langle\gamma(q)\,|\bar{q}(x)\Gamma_i G_{\alpha\beta} q(0)|\,0\rangle$ are expressed in terms of the photon distribution amplitudes whose expressions are given in Appendix A. The QCD side of the correlation function can be obtained in terms of quarks and gluon properties using Eqs. (9)-(13) and after performing the Fourier transformation to transfer the calculations to the momentum space.

The sum rules are obtained by matching the expression of the correlation function in terms of quark-gluon properties to its expression in terms of the hadron properties, using their spectral representation. In order to eliminate the subtraction terms in the spectral representation of the correlation function, the Borel transformation with respect to the variables p^2 and $(p+q)^2$ is carried out. After the transformation, contributions from the excited and continuum states are also exponentially suppressed. Finally, we choose the structures $q_\mu\varepsilon_\nu$ and $(\varepsilon.p)q_\mu q_\nu$, respectively for the magnetic and quadrupole moments and obtain

$$\mu = \frac{e^{m^2_{Z_c}/M^2}}{\lambda^2_{Z_c}}\Bigg[\Pi_1 + \Pi_2\Bigg],$$
$$\mathcal{D} = m^2_{Z_c}\frac{e^{m^2_{Z_c}/M^2}}{\lambda^2_{Z_c}}\Bigg[\Pi_3 + \Pi_4\Bigg], \tag{14}$$

where the functions Π_1 and Π_3 indicate that one of the quark propagators enters the perturbative interaction with the photon and the remaining three propagators are taken as full propagators. The functions Π_2 and Π_4 show that one of the light quark propagators enters the nonperturbative interaction with the photon and the remaining three propagators are taken as full propagators. Explicit expressions of the Π_1, Π_2, Π_3 and Π_4 are given in Appendix B. As an example we show some details of the calculations i.e., Fourier and Borel transformations as well as the continuum subtraction, for a specific term in Appendix C.

III. NUMERICAL ANALYSIS

In this section, we numerically analyze the results of calculations for magnetic and quadrupole moments. We use $m_{Z_c} = 3899 \pm 8.5\ MeV$, $f_{3\gamma} = -0.0039\ GeV^2$ [41], $\overline{m}_c(m_c) = (1.275 \pm 0.025)\,GeV$, $\langle\bar{u}u\rangle(1\,GeV) = \langle\bar{d}d\rangle(1\,GeV) = (-0.24 \pm 0.01)^3\,GeV^3$ [42], $m_0^2 = 0.8 \pm 0.1\ GeV^2$, $\langle g_s^2 G^2\rangle = 0.88\ GeV^4$ [4] and $\lambda_{Z_c} = m_{Z_c} f_{Z_c} = (1.79 \pm 0.12) \times 10^{-2}\ GeV^5$ [30, 31]. We also need the value of the magnetic susceptibility which is obtained in different studies as $\chi(1\,GeV) = -2.85 \pm 0.5\ GeV^{-2}$ [43], $\chi(1\,GeV) = -3.15 \pm 0.3\ GeV^{-2}$ [41] and $\chi(1\,GeV) = -4.4\ GeV^{-2}$ [44]. The parameters used in the photon distribution amplitudes are also given in Appendix A.

The predictions for the magnetic and quadrupole moments depend on two auxiliary parameters; the Borel mass parameter M^2 and continuum threshold s_0. According to the standard prescriptions in the method used the predictions should weakly depend on these helping parameters. The continuum threshold represents the scale at which, the excited states and continuum start to contribute to the correlation function. Our analyses show that the results depend very weakly on s_0 in the interval $(m_{Z_c}+0.3)^2\ GeV^2 \leq s_0 \leq (m_{Z_c}+0.7)^2\ GeV^2$. The working region for M^2 is determined requiring that the contributions of the higher states and continuum are effectively suppressed. In technique language, the upper bound on M^2 is found demanding the maximum pole contribution. The lower bound is obtained demanding that the contribution of the perturbative part exceeds the nonperturbative one and series of the operator product expansion in the obtained sum rules converge. The above requirements restrict the working region of the Borel parameter to $5\ GeV^2 \leq M^2 \leq 7\ GeV^2$. It is worth nothing that with these intervals of s_0 and M^2 we receive a $(85-93)\%$ pole contribution, which nicely satisfies the requirements of the QCD sum rule approach.

In Fig. 1, we plot the dependencies of the magnetic and quadrupole moments on M^2 at several fixed values of the continuum threshold s_0. As is seen, the variation of the results with respect to the Borel parameters is considerable,

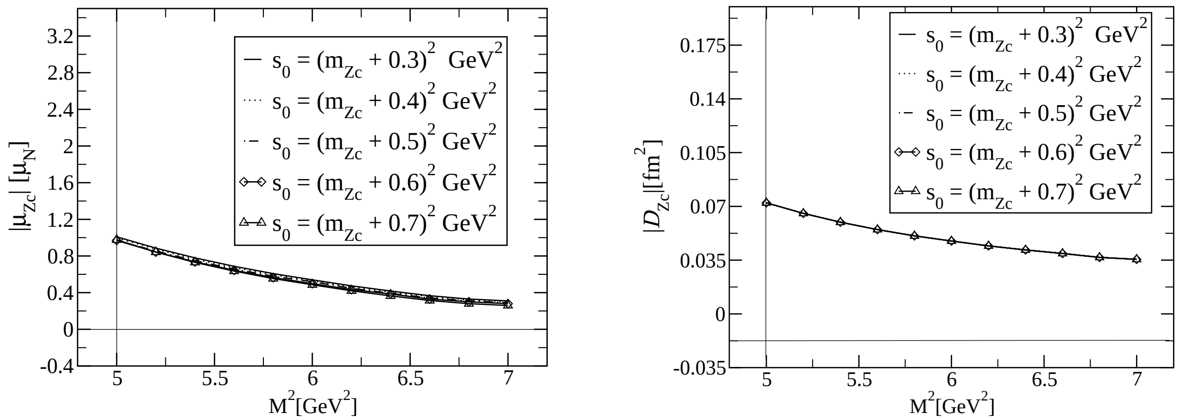

FIG. 1: The dependence of the magnetic and quadrupole moments; on the Borel parameter squared M^2 at different fixed values of the continuum threshold.

but there is much less dependence of the quantities under consideration on the continuum threshold in its working interval. In Fig. 2, we show the contributions of Π_1, Π_2, Π_3 and Π_4 functions to the results obtained at the average value of s_0 with respect to the Borel mass parameter. In the case of the magnetic moment, we see that the contribution of Π_1 is the dominant contribution. Π_1 corresponds to roughly 65% of the result in average, while the remaining 35% belongs to Π_2. In the case of quadrupole moment, we see that all contributions come from Π_4 and the contribution of Π_3 is 0.

Our final results for the magnetic and quadrupole moments are

$$
\begin{aligned}
|\mu_{Z_c}| &= 0.67 \pm 0.32\ \mu_N \\
|\mathcal{D}_{Z_c}| &= 0.054 \pm 0.018\ fm^2,
\end{aligned}
\tag{15}
$$

where the errors in the results come from the variations in the calculations of the working regions of M^2 and s_0 as well as the uncertainties in the values of the input parameters and the photon DAs. We remark that the main source of uncertainties is the variations with respect to M^2 and the results very weakly depend on the choices of the continuum threshold.

IV. DISCUSSION AND CONCLUDING REMARKS

We calculated the magnetic and quadrupole moments of the $Z_c(3900)$ state within the framework of the LCSR method. We obtained a measurable value for the magnetic dipole moment but a small value for the quadrupole moment indicating a nonspherical charge distribution. It is useful to note that the values of the magnetic and quadrupole moments do not depend on the values of the magnetic susceptibility χ presented in the previous section. It is worth mentioning also that there are different Lorentz structures to calculate the magnetic moment in the correlation function, but our result is almost independent of these structures. Any experimental measurements of the electromagnetic multipole moments of the $Z_c(3900)$ state and comparison of the obtained results with the predictions of the present study may serve as valuable knowledge on the internal structure of the tetraquark states as well as the nonperturbative nature of the QCD. A comparison of our results on the electromagnetic multipole moments of the $Z_c(3900)$ state with those that can be obtained via considering different internal structures and interpolating currents, such as a molecular type one, would be very helpful in the determination of the internal structure of this multiquark state. A comparison of the results obtained with the predictions of other approaches, such as lattice QCD, chiral perturbation theory, quark model, etc., would also be interesting.

V. ACKNOWLEDGEMENTS

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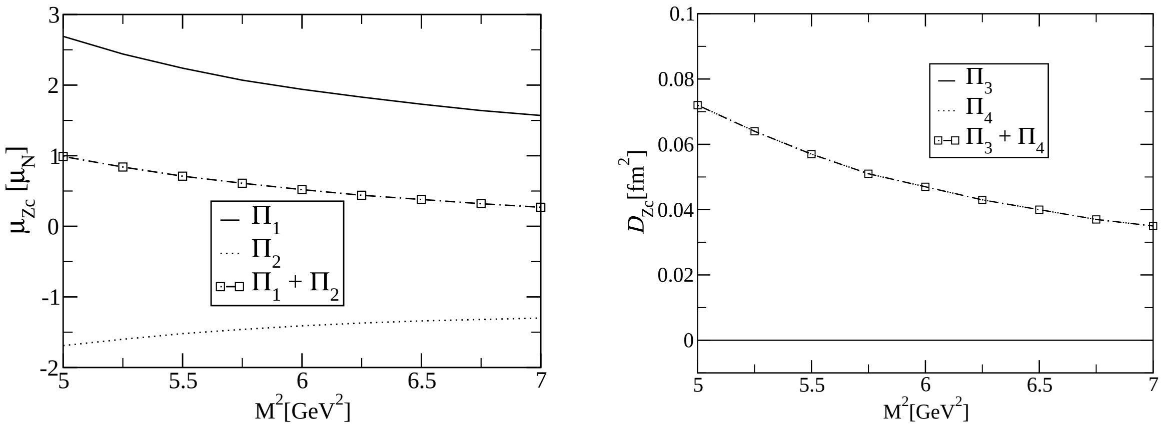

FIG. 2: Comparison of the contributions to the magnetic and quadrupole moments with respect to M^2 at average value of s_0.

Appendix A: Photon distribution amplitudes

In this appendix, we present the definitions of the matrix elements of the form $\langle\gamma(q)\,|\bar{q}(x)\Gamma_i q(0)|\,0\rangle$ and $\langle\gamma(q)\,|\bar{q}(x)\Gamma_i G_{\mu\nu} q(0)|\,0\rangle$ in terms of the photon DAs, and the explicit expressions of the photon distribution amplitudes [41],

$$
\begin{aligned}
&\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = e_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx}\right)\int_0^1 du e^{i\bar{u}qx}\psi^v(u) \\
&\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -\frac{1}{4}e_q f_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx}\psi^a(u) \\
&\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu} q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 du e^{i\bar{u}qx}\left(\chi\varphi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right) \\
&-\frac{i}{2(qx)}e_q\bar{q}q\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du e^{i\bar{u}qx}h_\gamma(u) \\
&\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle\left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu\right)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{S}(\alpha_i) \\
&\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)i\gamma_5 q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle\left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu\right)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\tilde{\mathcal{S}}(\alpha_i) \\
&\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}(\alpha_i) \\
&\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}(\alpha_i) \\
&\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_s G_{\mu\nu}(vx)q(0)|0\rangle = e_q\langle\bar{q}q\rangle\left\{\left[\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\nu} - \frac{1}{qx}(q_\alpha x_\nu + q_\nu x_\alpha)\right)q_\beta\right.\right. \\
&-\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\nu} - \frac{1}{qx}(q_\beta x_\nu + q_\nu x_\beta)\right)q_\alpha - \left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\mu} - \frac{1}{qx}(q_\alpha x_\mu + q_\mu x_\alpha)\right)q_\beta \\
&\left.+\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{q.x}\right)\left(g_{\beta\mu} - \frac{1}{qx}(q_\beta x_\mu + q_\mu x_\beta)\right)q_\alpha\right]\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1(\alpha_i) \\
&+\left[\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu)\right)q_\nu\right. \\
&-\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\nu\beta} - \frac{1}{qx}(q_\nu x_\beta + q_\beta x_\nu)\right)q_\mu \\
&-\left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu)\right)q_\nu \\
&\left.+\left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\nu\alpha} - \frac{1}{qx}(q_\nu x_\alpha + q_\alpha x_\nu)\right)q_\mu\right]\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_2(\alpha_i) \\
&+\frac{1}{qx}(q_\mu x_\nu - q_\nu x_\mu)(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_3(\alpha_i) \\
&\left.+\frac{1}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_4(\alpha_i)\right\} ,
\end{aligned}
$$

where $\varphi_\gamma(u)$ is the leading twist-2, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}(\alpha_i)$ and $\mathcal{V}(\alpha_i)$, are the twist-3, and $h_\gamma(u)$, $\mathbb{A}(u)$, $\mathcal{S}(\alpha_i)$, $\tilde{\mathcal{S}}(\alpha_i)$, $\mathcal{T}_1(\alpha_i)$, $\mathcal{T}_2(\alpha_i)$, $\mathcal{T}_3(\alpha_i)$ and $\mathcal{T}_4(\alpha_i)$ are the twist-4 photon DAs. The measure $\mathcal{D}\alpha_i$ is defined as

$$
\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}}\int_0^1 d\alpha_q\int_0^1 d\alpha_g\delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g) .
$$

The expressions of the DAs entering into the above matrix elements are defined as:

$$
\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u}\left(1+\varphi_2(\mu)C_2^{\frac{3}{2}}(u-\bar{u})\right), \\
\psi^v(u) &= 3\left(3(2u-1)^2-1\right)+\frac{3}{64}\left(15w_\gamma^V-5w_\gamma^A\right)\left(3-30(2u-1)^2+35(2u-1)^4\right), \\
\psi^a(u) &= \left(1-(2u-1)^2\right)\left(5(2u-1)^2-1\right)\frac{5}{2}\left(1+\frac{9}{16}w_\gamma^V-\frac{3}{16}w_\gamma^A\right), \\
h_\gamma(u) &= -10\left(1+2\kappa^+\right)C_2^{\frac{1}{2}}(u-\bar{u}), \\
\mathbb{A}(u) &= 40u^2\bar{u}^2\left(3\kappa-\kappa^++1\right)+8(\zeta_2^+-3\zeta_2)\left[u\bar{u}(2+13u\bar{u})\right. \\
&\quad \left.+\,2u^3(10-15u+6u^2)\ln(u)+2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln(\bar{u})\right], \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2\left(1+w_\gamma^A\frac{1}{2}(7\alpha_g-3)\right), \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q-\alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2, \\
\mathcal{T}_1(\alpha_i) &= -120(3\zeta_2+\zeta_2^+)(\alpha_{\bar{q}}-\alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}}-\alpha_q)\left((\kappa-\kappa^+)+(\zeta_1-\zeta_1^+)(1-2\alpha_g)+\zeta_2(3-4\alpha_g)\right), \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2-\zeta_2^+)(\alpha_{\bar{q}}-\alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}}-\alpha_q)\left((\kappa+\kappa^+)+(\zeta_1+\zeta_1^+)(1-2\alpha_g)+\zeta_2(3-4\alpha_g)\right), \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa+\kappa^+)(1-\alpha_g)+(\zeta_1+\zeta_1^+)(1-\alpha_g)(1-2\alpha_g)+\zeta_2[3(\alpha_{\bar{q}}-\alpha_q)^2-\alpha_g(1-\alpha_g)]\}, \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa-\kappa^+)(1-\alpha_g)+(\zeta_1-\zeta_1^+)(1-\alpha_g)(1-2\alpha_g)+\zeta_2[3(\alpha_{\bar{q}}-\alpha_q)^2-\alpha_g(1-\alpha_g)]\}.
\end{aligned}
$$

Numerical values of parameters used in DAs; $\varphi_2(1\ GeV)=0$, $w_\gamma^V=3.8\pm1.8$, $w_\gamma^A=-2.1\pm1.0$, $\kappa=0.2$, $\kappa^+=0$, $\zeta_1=0.4$, $\zeta_2=0.3$, $\zeta_1^+=0$, and $\zeta_2^+=0$.

Appendix B:

In this appendix, we present the explicit expressions for the functions, Π_1, Π_2, Π_3 and Π_4:

$$
\begin{aligned}
\Pi_1 &= \frac{3m_c^4M^2}{64\pi^6}\Bigg[2(3e_u+4e_d-2e_c)N[3,3,0]-3m_c(e_u+e_d-e_c)N[3,4,1]-e_c\Big(8N[4,2,0]-m_cN[5,2,1]\Big)\Bigg] \\
&- \frac{m_c^3M^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{12288\pi^4}\Big(3e_u+3e_d-2e_c\Big)N[1,2,1] \\
&+ \frac{m_c^4M^2\langle g_s^2G^2\rangle}{147456\pi^6}\Big(2e_u+2e_d-e_c\Big)\Big(N[1,3,1]+N[2,2,1]\Big) \\
&- \frac{m_c^3M^2\langle g_s^2G^2\rangle}{18432\pi^6}\Big(2e_u+2e_d-e_c\Big)N[1,2,0] \\
&- \frac{m_c^2M^2\langle g_s^2G^2\rangle}{1536\pi^6}\Big(e_u+e_d-7e_c\Big)N[2,2,0] \\
&+ \frac{m_c^3M^2}{24576\pi^6}\Bigg[-(17e_u+17e_d-31e_c)\langle g_s^2G^2\rangle+576(3e_u+3e_d-4e_c)\pi^2m_c\langle\bar{q}q\rangle\Bigg]N[2,3,1] \\
&+ \frac{m_c^2}{13824M^6\pi^6}\Bigg[(e_u+e_d+e_c)\langle g_s^2G^2\rangle M^2-(e_u+e_d)\;36\pi^2m_c\langle\bar{q}q\rangle(3m_0^2+16M^2)\Bigg]\Bigg(64\;m_c^6FlP[-3,4,0] \\
&- 48m_c^4FlP[-2,4,0]+12\;m_c^2FlP[-1,4,0]-FlP[0,4,0]\Bigg) \\
&- \frac{m_c\langle\bar{q}q\rangle}{110592M^8\pi^4}\Bigg[3(e_u+e_d)\langle g_s^2G^2\rangle M^2(3m_0^2+16M^2)+2e_c\Big(\langle g_s^2G^2\rangle M^2(3m_0^2-4M^2)+18\pi^2m_c\langle\bar{q}q\rangle \\
&(3m_0^4-128M^4)\Big)\Bigg]\Big(16\;m_c^4FlP[-1,2,0]-8m_c^2\;FlP[0,2,0]+FlP[1,2,0]\Big) \\
&+ \frac{e_cm_cm_0^2\langle\bar{q}q\rangle^2}{221184M^6\pi^4}\Big(5\langle g_s^2G^2\rangle+1152\pi^2m_c\langle\bar{q}q\rangle\Big)\Bigg[16\;m_c^4FlP[1,2,1]-8m_c^2FlP[2,2,1]-FlP[3,2,1]\Bigg] \\
&+ \frac{e_cm_c^3m_0^2\langle\bar{q}q\rangle}{48M^4\pi^4}\Bigg[64m_c^4FlP[-2,3,0]+28m_c^2FlP[0,3,0]-5FlP[1,3,0]\Bigg] \\
&+ \frac{e_cm_c^2m_0^4\langle\bar{q}q\rangle^2}{6144M^8\pi^4}\Bigg[16\;m_c^4FlP[3,2,2]-8\;m_c^2FlP[4,2,2]+FlP[5,2,2]\Bigg]. \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_2 = &\frac{m_c^4\langle\bar{q}q\rangle}{128\pi^4}\Bigg[e_u\Big(WFD[\mathcal{S},\bar{v}]-2WF[\mathcal{S},\bar{v}]\Big)+e_d\Big(WFD[\mathcal{S},v]-2WF[\mathcal{S},v]\Big)\Bigg]\Big(4N[2,3,0]-M^2N[2,3,1]\Big)\\
&+\frac{m_c^4\langle\bar{q}q\rangle}{32\pi^4}\Bigg[e_u\Big(-2WF[\mathcal{T}_1,\bar{v}]-2WF[\mathcal{T}_2,\bar{v}]+2WF[\tilde{S},\bar{v}]+WFD[\mathcal{T}_1,\bar{v}]+WFD[\mathcal{T}_2,\bar{v}]-WFD[\tilde{S},\bar{v}]\Big)\\
&+e_d\Big(-8WF[\mathcal{T}_1,v]-2WF[\mathcal{T}_2,v]+2WF[\mathcal{S},v]+4WFD[\mathcal{T}_2,v]-WFD[\mathcal{S},v]-WFD[\tilde{S},v]\Big)\Bigg]N[1,4,0]\\
&+\frac{m_c^4M^2\langle\bar{q}q\rangle}{256\pi^4}\Bigg[e_u\Big(4WF[\mathcal{T}_1,\bar{v}]+4WF[\mathcal{T}_2,\bar{v}]-2WF[\tilde{S},\bar{v}]-2WFD[\mathcal{T}_1,\bar{v}]-2WFD[\mathcal{T}_2,\bar{v}]+2WFD[\tilde{S},v]\Big)\\
&+e_d\Big(13WF[\mathcal{T}_1,v]+7WF[\mathcal{T}_2,v]-WF[\tilde{S},v]-8WFD[\mathcal{T}_1,v]-2WFD[\mathcal{T}_2,v]+2WFD[\tilde{S},v]\Big)\Bigg]N[1,4,1]\\
&+\frac{f_{3\gamma}m_c^4}{64\pi^4}\Bigg[16(e_u-e_d)WFD[\psi^a,u]+e_uWFD[\mathcal{V},\bar{v}]+e_dWFD[\mathcal{V},v]\Bigg]N[3,3,0]\\
&+\frac{m_c^3M^2}{512\pi^4}\Bigg[m_c\langle\bar{q}q\rangle\Bigg\{e_u\Big(-2WF[\mathcal{S},\bar{v}]+6WF[\mathcal{T}_1,\bar{v}]+6WF[\mathcal{T}_2,\bar{v}]-2WF[\tilde{S},\bar{v}]+3WFD[\mathcal{S},\bar{v}]\\
&-3WFD[\mathcal{T}_1,\bar{v}]-3WFD[\mathcal{T}_2,\bar{v}]+3WFD[\tilde{S},\bar{v}]\Big)+e_d\Big(-2WF[\mathcal{S},v]+18WF[\mathcal{T}_1,v]+12WF[\mathcal{T}_2,v]+4WF[\tilde{S},v]\\
&+3WFD[\mathcal{S},v]-12WFD[\mathcal{T}_1,v]-3WFD[\mathcal{T}_2,v]+3WFD[\tilde{S},v]\Big)\Bigg\}+2e_uf_{3\gamma}M^2\Big(2WFD[\mathcal{A},\bar{v}]+WFD[\mathcal{V},\bar{v}]\Big)\\
&+2e_df_{3\gamma}M^2\Big(2WFD[\mathcal{A},v]+3WFD[\mathcal{V},v]\Big)\Bigg]N[2,3,1]\\
&+\frac{m_c^2M^2f_{3\gamma}\langle g_s^2G^2\rangle}{110592\pi^4}\Big[-10(e_u+e_d)\psi^a(u_0)+2(e_u-4e_d)\varphi_\gamma(u_0)-5(e_u-e_d)WFD[\psi^a,u]+4(e_u-e_d)WFD[\psi^\nu,u]\Big]\\
&N[1,1,0]\\
&+\frac{\langle\bar{q}q\rangle m_c^4M^4}{2048\pi^4}\Bigg[e_u\Big(-2WF[\mathcal{S},\bar{v}]+2WF[\mathcal{T}_1,\bar{v}]+2WF[\mathcal{T}_2,v]-2WF[\tilde{S},v]+WFD[\mathcal{S},\bar{v}]-WFD[\mathcal{T}_1,\bar{v}]\\
&-WFD[\mathcal{T}_2,\bar{v}]+WFD[\tilde{S},\bar{v}]\Big)+e_d\Big(-2WF[\mathcal{S},v]+8WF[\mathcal{T}_1,v]+2WF[\mathcal{T}_2,v]-2WF[\tilde{S},v]+WFD[\mathcal{S},v]-\\
&4WFD[\mathcal{T}_1,v]-WFD[\mathcal{T}_2,v]+WFD[\tilde{S},v]\Big)\Bigg]N[2,3,2]\\
&-\frac{f_{3\gamma}m_c^4}{32\pi^4}\Bigg[e_uWFD[\mathcal{V},\bar{v}]+e_dWFD[\mathcal{V},v]\Bigg]N[2,4,0]\\
&+\frac{m_c^3}{128\pi^4}\Bigg[m_c\langle\bar{q}q\rangle\Bigg\{e_u\Big(2WF[\mathcal{S},\bar{v}]+2WF[\mathcal{T}_1,\bar{v}]-2WF[\mathcal{T}_2,\bar{v}]+2WF[\tilde{S},\bar{v}]-WFD[\mathcal{S},\bar{v}]+WFD[\mathcal{T}_1,\bar{v}]\\
&+WFD[\mathcal{T}_2,\bar{v}]-WFD[\tilde{S},\bar{v}]\Big)+e_d\Big(2WF[\mathcal{S},v]-8WF[\mathcal{T}_1,v]-2WF[\mathcal{T}_2,v]+2WF[\tilde{S},v]-WFD[\mathcal{S},v]\\
&+4WFD[\mathcal{T}_1,v]+2WFD[\mathcal{T}_2,v]-WFD[\tilde{S},v]\Big)\Bigg\}+f_{3\gamma}M^2\Big(e_uWFD[\mathcal{V},\bar{v}]+e_dWFD[\mathcal{V},v]\Big)\Bigg]N[2,3,0]\\
&-\frac{m_c^4M^2f_{3\gamma}}{128\pi^4}\Bigg(e_uWFD[\mathcal{V},\bar{v}]+e_dWFD[\mathcal{V},v]\Bigg)N[2,4,1]\\
&+\frac{m_c^2M^4\langle g_s^2G^2\rangle}{1769472\pi^4}\Bigg[-4(4e_u-e_d)\chi m_c\langle\bar{q}q\rangle WFD[\varphi_\gamma,u]-16(e_u-e_d)f_{3\gamma}WFD[\psi^a,u]-11e_uf_{3\gamma}WFD[\mathcal{A},\bar{v}]\\
&-11e_df_{3\gamma}WFD[\mathcal{A},v]\Bigg]N[2,2,2]
\end{aligned}
$$

$$+\frac{m_c^3}{6912\pi^4}\Bigg[-54M^2\langle\bar{q}q\rangle\Bigg\{e_u\Big(2WF[\mathcal{T}_1,\bar{v}]+2WF[\mathcal{T}_2,\bar{v}]+3WF[\tilde{S},\bar{v}]-WFD[\mathcal{T}_1,\bar{v}]-WFD[\mathcal{T}_2,\bar{v}]+WFD[\tilde{S},\bar{v}]\Big)$$

$$+e_d\Big(5WF[\mathcal{T}_1,v]+5WF[\mathcal{T}_2,v]+WF[\tilde{S},v]-4WFD[\mathcal{T}_1,v]-WFD[\mathcal{T}_2,v]+WFD[\tilde{S},v]\Big)\Bigg\}$$

$$-54m_cM^2f_{3\gamma}\Big(e_uWFD[\mathcal{A},\bar{v}]+e_dWFD[\mathcal{A},v]\Big)+(6e_u+7e_d)\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle WF[\varphi_\gamma,u]\Bigg]N[1,3,0]$$

$$+\frac{m_c^3M^2\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{55296\pi^4}\Bigg[-13(e_u-e_d)\varphi_\gamma(u_0)-2(7e_u-6e_d)WFD[\varphi_\gamma,u]\Bigg]N[1,3,1]$$

$$-\frac{m_c^3M^4\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{110592\pi^4}\Bigg[(e_u-e_d)WFD[\varphi_\gamma,u]\Bigg]N[1,3,2]$$

$$+\frac{m_c^2\langle g_s^2G^2\rangle}{110592\pi^4}\Bigg[52(e_u-e_d)M^2\chi\langle\bar{q}q\rangle\varphi_\gamma(u_0)-4(8e_u+5e_d)M^2\chi\langle\bar{q}q\rangle WFD[\varphi_\gamma,u]-30(e_u-e_d)m_cf_{3\gamma}WFD[\psi^a,u]$$

$$+11\langle\bar{q}q\rangle\Bigg\{e_u\Big(6WF[\mathcal{S},\bar{v}]-4WF[\mathcal{T}_1,\bar{v}]-10WF[\mathcal{T}_2,\bar{v}]+2WF[\mathcal{T}_3,\bar{v}]-WF[\mathcal{T}_4,\bar{v}]+WF[\tilde{S},\bar{v}]-3WFD[\mathcal{S},\bar{v}]$$

$$+2WFD[\mathcal{T}_1,\bar{v}]+5WFD[\mathcal{T}_2,\bar{v}]-WFD[\mathcal{T}_3,\bar{v}]+WFD[\mathcal{T}_4,\bar{v}]-2WFD[\tilde{S},\bar{v}]\Big)$$

$$+e_d\Big(6WF[\mathcal{S},v]-4WF[\mathcal{T}_1,v]-10WF[\mathcal{T}_2,v]+2WF[\mathcal{T}_3,v]-WF[\mathcal{T}_4,\bar{v}]+WF[\tilde{S},\bar{v}]-3WFD[\mathcal{S},\bar{v}]$$

$$+2WFD[\mathcal{T}_1,v]+5WFD[\mathcal{T}_2,v]-WFD[\mathcal{T}_3,v]+WFD[\mathcal{T}_4,v]-2WFD[\tilde{S},v]\Big)\Bigg]N[1,2,0]$$

$$+\frac{f_{3\gamma}m_c^4M^2}{512\pi^4}\Bigg[3e_uWFD[\mathcal{V},\bar{v}]+3e_dWFD[\mathcal{V},v]+16(e_u-e_d)WFD[\psi^a,u]\Bigg]N[3,3,1]$$

$$+\frac{m_c^2M^2\langle g_s^2G^2\rangle}{884736\pi^4}\Bigg[40m_cf_{3\gamma}(e_u-e_d)\psi^a(u_0)-8f_{3\gamma}m_c(e_u-e_d)\psi^\nu(u_0)-16m_cf_{3\gamma}(4e_u-e_d)WF[\psi^\gamma,u]$$

$$+30f_{3\gamma}m_c(e_u-e_d)WFD[\psi^a,u]-2(8e_u-5e_d)\langle\bar{q}q\rangle WFD[A,u]+11\langle\bar{q}q\rangle\Bigg\{e_u\langle\bar{q}q\rangle\Big(-6WF[\mathcal{S},\bar{v}]+4WF[\mathcal{T}_1,\bar{v}]$$

$$+10WF[\mathcal{T}_2,\bar{v}]-2WF[\mathcal{T}_3,\bar{v}]+4WF[\mathcal{T}_4,\bar{v}]-4WF[\tilde{S},\bar{v}]+3WFD[\mathcal{S},\bar{v}]-2WFD[\mathcal{T}_1,\bar{v}]-5WFD[\mathcal{T}_2,\bar{v}]$$

$$+WFD[\mathcal{T}_3,\bar{v}]-2WFD[\mathcal{T}_4,\bar{v}]+2WFD[\tilde{S},\bar{v}]\Big)+e_d\Big(-6WF[\mathcal{S},v]+4WF[\mathcal{T}_1,v]+10WF[\mathcal{T}_2,v]-2WF[\mathcal{T}_3,v]$$

$$+4WF[\mathcal{T}_4,v]-4WF[\tilde{S},v]+3WFD[\mathcal{S},v]-2WFD[\mathcal{T}_1,v]-5WFD[\mathcal{T}_2,v]+WFD[\mathcal{T}_3,v]-2WFD[\mathcal{T}_4,v]$$

$$+2WFD[\tilde{S},v]\Big)\Bigg\}\Bigg]N[1,2,1]$$

$$+\frac{m_c^2\langle g_s^2G^2\rangle}{110592\pi^4}\Bigg[-104(e_u-e_d)m_c\chi\langle g_s^2G^2\rangle\varphi_\gamma(u_0)+e_uf_{3\gamma}(11\langle g_s^2G^2\rangle+432m_c^2M^2)WFD[\mathcal{A},\bar{v}]$$

$$+e_df_{3\gamma}(11\langle g_s^2G^2\rangle+432m_c^2M^2)WFD[\mathcal{A},v]+4(28e_u-19e_d)m_c\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle WFD[\varphi_\gamma,u]$$

$$+16(e_u-e_d)f_{3\gamma}\langle g_s^2G^2\rangle WFD[\psi^a,u]\Bigg]N[2,2,1]$$

$$+\frac{\langle g_s^2G^2\rangle m_c^2M^4}{28311552\pi^4}\Bigg[11e_df_{3\gamma}WFD[\mathcal{A},v]-11f_{3\gamma}e_uWFD[\mathcal{A},\bar{v}]+4\chi m_c\langle\bar{q}q\rangle(e_d-4e_u)WFD[\varphi_\gamma,u]$$

$$-16f_{3\gamma}(e_u-e_d)WFD[\psi^a,u]\Bigg]N[2,2,2]$$

$$-\frac{m_c^2M^4f_{3\gamma}}{2048\pi^4}\Bigg[e_uWFD[\mathcal{V},\bar{v}]+e_dWFD[\mathcal{V},v]-16(e_u-e_d)WFD[\psi^a,u]\Bigg]N[3,3,2]$$

$$
\begin{aligned}
&+\frac{m_c^2 M^4\langle g_s^2G^2\rangle}{3538944\pi^4}\Bigg[10(e_u-e_d)m_c f_{3\gamma}WFD[\psi^a,u]+2(8e_u+5e_d)\langle\bar{q}q\rangle WFD[A,u]+11e_u\Big(6WF[\mathcal{S},\bar{v}]-4WF[\mathcal{T}_1,\bar{v}]\\
&-10WF[\mathcal{T}_2,\bar{v}]+2WF[\mathcal{T}_3,\bar{v}]-4WF[\mathcal{T}_4,\bar{v}]+4WF[\tilde{S},\bar{v}]-3WFD[\mathcal{S},\bar{v}]+2WFD[\mathcal{T}_1,\bar{v}]+5WFD[\mathcal{T}_2,\bar{v}]\\
&-3WFD[\mathcal{T}_3,\bar{v}]+2WFD[\mathcal{T}_4,\bar{v}]-WFD[\tilde{S},\bar{v}]\Big)+11e_d\Big(6WF[\mathcal{S},v]-4WF[\mathcal{T}_1,v]-10WF[\mathcal{T}_2,v]+2WF[\mathcal{T}_3,v]\\
&-4WF[\mathcal{T}_4,v]+4WF[\tilde{S},v]-3WFD[\mathcal{S},v]+2WFD[\mathcal{T}_1,v]+5WFD[\mathcal{T}_2,v]-3WFD[\mathcal{T}_3,v]+2WFD[\mathcal{T}_4,v]\\
&-WFD[\tilde{S},v]\Big)\Bigg]N[1,2,2]\\
&-\frac{m_c^2}{55296\pi^4}\Bigg[f_{3\gamma}\Big(11\langle g_s^2G^2\rangle+432m_c^2M^2\Big)\Big(e_uWFD[\mathcal{A},\bar{v}]+e_dWFD[\mathcal{A},v]\Big)+12m_c\langle\bar{q}q\rangle\Bigg\{36M^2\Big(e_uWFD[\mathcal{S},\bar{v}]\\
&+e_dWFD[\mathcal{S},v]\Big)-(4e_u-3e_d)\chi\langle g_s^2G^2\rangle WFD[\varphi_\gamma,u]\Bigg\}+16(e_u-e_d)f_{3\gamma}\langle g_s^2G^2\rangle WFD[\psi^a,u]\Bigg]N[2,2,0]\\
&+\frac{m_c\langle\bar{q}q\rangle^2}{1990656M^{10}\pi^2}\Bigg[-(e_u-e_d)\Big(5\langle g_s^2G^2\rangle(23m_0^2-8M^2)-1728m_c^2m_0^2M^2\Big)A(u_0)\\
&+4m_0^2\langle g_s^2G^2\rangle\Big(10(e_u-e_d)M^2\chi\varphi_\gamma(u_0)+(7e_u+2e_d)WF[h_\gamma,u]\Big)\Bigg]\Bigg(16m_c^4FLNP[2,3,2]-8m_c^2FlNP[3,3,2]\\
&-FlNP[4,3,2]\\
&+\frac{m_cm_0^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{165888M^8\pi^2}(e_u-e_d)\Bigg(4m_c^2FlNP[4,1,2]-FlNP[5,1,2]\Bigg)A(u_0)\\
&-\frac{f_{3\gamma}m_0^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{73728M^8\pi^2}(e_u-e_d)\Bigg(16m_c^4FlNP[3,3,2]-8m_c^2FlNP[4,2,2]+FlNP[5,2,2]\Bigg)\psi^a(u_0)\\
&+\frac{5m_cm_0^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2}{995328M^{10}\pi^2}(e_u-e_d)\Bigg(16m_c^4FlNP[4,3,3]-8m_c^2FlNP[5,3,3]+FlNP[6,3,3]\Bigg)A(u_0)\\
&+\frac{m_cm_0^2\langle\bar{q}q\rangle}{9216M^8\pi^2}\Bigg[-4(e_u-e_d)m_c\psi^a(u_0)-\langle\bar{q}q\rangle\Bigg\{e_u\Big(4WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)+e_d\Big(4WF[\mathcal{T}_1,v]+WF[\mathcal{T}_2,v]\Big)\Bigg\}\Bigg]\\
&\Bigg(64m_c^6FlNP[1,4,2]-48m_c^4FlNP[2,4,2]+12m_c^2FlNP[3,4,2]-FlNP[4,4,2]\Bigg)\\
&+\frac{m_c\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2}{82944M^8\pi^2}\Bigg[(e_u+e_d)(3m_0^2-2M^2)A(u_0)-m_0^2\Big(2M^2\chi\varphi_\gamma(u_0)+WF[h_\gamma,u]\Big)\Bigg]\Bigg(4m_c^2FlNP[2,1,1]\\
&-FlNP[3,1,1]\Bigg)\\
&+\frac{m_c^5\langle\bar{q}q\rangle}{124416M^{10}\pi^4}\Bigg[\Bigg\{e_u\Bigg(-3456\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-2M^2)+\langle g_s^2G^2\rangle\Big(-69m_cM^4+20\pi^2\langle\bar{q}q\rangle(16m_0^2-11M^2)\Big)\Bigg)\\
&+e_d\Bigg(3456\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-2M^2)+5\langle g_s^2G^2\rangle\Big(9m_cM^4+4\pi^2\langle\bar{q}q\rangle(-16m_0^2+11M^2)\Big)\Bigg)\Bigg\}A(u_0)\\
&+4\pi^2\langle\bar{q}q\rangle\Bigg\{-(e_u-e_d)M^2\chi\Big(5\langle g_s^2G^2\rangle(11m_0^2-8M^2)-1728m_c^2m_0^2M^2\Big)\varphi_\gamma(u_0)+4e_u\Big(216m_c^2m_0^2M^2\\
&+7\langle g_s^2G^2\rangle(-2m_0^2+M^2)\Big)+4e_d\Big(-216m_c^2m_0^2M^2+\langle g_s^2G^2\rangle(-4m_0^2+2M^2)\Big)\Bigg\}WF[h_\gamma,u]\Bigg]FlNP[0,3,1]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\langle\bar{q}q\rangle}{663552M^8\pi^2}\Bigg[9(e_u-e_d)f_{3\gamma}\langle g_s^2G^2\rangle(5m_0^2-4M^2)\psi^a(u_0)+2(4e_u-e_d)m_0^2f_{3\gamma}\langle g_s^2G^2\rangle WF[\psi^\nu,u]\\
&+(e_u-e_d)m_0^2f_{3\gamma}\langle g_s^2G^2\rangle\psi^\nu(u_0)+m_cm_0^2M^2\langle\bar{q}q\rangle\Bigg\{e_u\Big(3WF[\mathcal{S},\bar{v}]-2WF[\mathcal{T}_1,\bar{v}]-2WF[\mathcal{T}_2,\bar{v}]+2WF[\mathcal{T}_3,\bar{v}]\\
&+2WF[\mathcal{T}_4,\bar{v}]-2WF[\tilde{S},\bar{v}]\Big)+e_d\Big(3WF[\mathcal{S},v]-2WF[\mathcal{T}_1,v]-2WF[\mathcal{T}_2,v]+2WF[\mathcal{T}_3,v]+2WF[\mathcal{T}_4,v]\\
&-2WF[\tilde{S},v]\Big)\Bigg\}\Bigg]\Bigg(16m_c^4FlNP[1,2,1]-8m_c^2FlNP[2,2,1]+FlNP[3,2,1]\Bigg)\\
&+\frac{1}{1327104M^8\pi^4}\Bigg[27(e_u-e_d)M^4\langle g_s^2G^2\rangle\langle g\bar{q}q\rangle A(u_0)+3(e_u-e_d)f_{3\gamma}\langle g_s^2G^2\rangle\Big(5m_cM^4+6\pi^2\langle\bar{q}q\rangle(-m_0^2+4M^2)\Big)\psi^a(u_0)\\
&-\pi^2f_{3\gamma}\langle g_s^2G^2\rangle\langle\bar{q}q\rangle(3m_0^2-4M^2)\Big(2(e_u-4e_d)\psi^\nu(u_0)+4(4e_u-e_d)WF[\psi^\nu,u]\Big)\\
&+\langle\bar{q}q\rangle\Bigg\{\Big(-23M^2\langle g_s^2G^2\rangle-864\pi^2m_c\langle\bar{q}q\rangle(m_0^2-2M^2)\Big)\Big(e_uWF[\mathcal{S},\bar{v}]+e_dWF[\mathcal{S},v]\Big)\\
&+\Big(17M^2\langle g_s^2G^2\rangle+288\pi^2m_c\langle\bar{q}q\rangle(m_0^2-2M^2)\Big)\Big(e_uWF[\mathcal{T}_1,\bar{v}]+e_dWF[\mathcal{T}_1,v]\Big)\\
&+\Big(102M^2\langle g_s^2G^2\rangle+1728\pi^2m_c\langle\bar{q}q\rangle(m_0^2-2M^2)\Big)\Big(e_uWF[\mathcal{T}_2,\bar{v}]+e_dWF[\mathcal{T}_2,v]\Big)\\
&-\Big(36M^2\langle g_s^2G^2\rangle+1728\pi^2m_c\langle\bar{q}q\rangle(m_0^2-2M^2)\Big)\Big(e_uWF[\mathcal{T}_3,\bar{v}]+e_dWF[\mathcal{T}_3,v]\Big)\\
&-\Big(36M^2\langle g_s^2G^2\rangle+1728\pi^2m_c\langle\bar{q}q\rangle(m_0^2-2M^2)\Big)\Big(e_uWF[\tilde{S},\bar{v}]+e_dWF[\tilde{S},v]\Big)\Bigg\}\Bigg]\Bigg(16m_c^4FlNP[1,2,0]\\
&-8m_c^2FlNP[0,2,0]+FlNP[-1,2,0]\Bigg)\\
&+\frac{m_c}{15925248M^{12}\pi^4}\Bigg[10368(e_d-e_u)m_c\langle\bar{q}q\rangle M^8A(u_0)+48(e_d-e_u)f_{3\gamma}M^4\Big(7\langle g_s^2G^2\rangle M^2\\
&+144\ m_c\ \pi^2\langle\bar{q}q\rangle(4M^2-3m_0^2)\Big)\psi^a(u_0)-288(e_u-e_d)f_{3\gamma}M^4\Big(\langle g_s^2G^2\rangle M^2+24\ m_c\ \pi^2\langle\bar{q}q\rangle(4M^2-3m_0^2)\Big)\psi^\nu(u_0)\\
&+2592m_c\ M^8\langle\bar{q}q\rangle\Big(e_dWF[\mathcal{S},v]+e_uWF[\mathcal{S},\bar{v}]\Big)-e_d\langle\bar{q}q\rangle M^4\Big(10368\ m_c\ M^4+33696\pi^2\langle\bar{q}q\rangle m_0^2\\
&+44928\pi^2\langle\bar{q}q\rangle M^2\Big)WF[\mathcal{T}_1,v]-e_u\langle\bar{q}q\rangle M^4\Big(2592\ m_c\ M^4+10368\pi^2\langle\bar{q}q\rangle m_0^2-13824\pi^2\langle\bar{q}q\rangle M^2\Big)WF[\mathcal{T}_1,\bar{v}]\\
&-e_d\langle\bar{q}q\rangle M^4\Big(2592\ m_c\ M^4+18144\pi^2\langle\bar{q}q\rangle m_0^2+24192\pi^2\langle\bar{q}q\rangle M^2\Big)WF[\mathcal{T}_2,v]-e_u\langle\bar{q}q\rangle M^4\Big(2592\ m_c\ M^4\\
&+10368\pi^2\langle\bar{q}q\rangle m_0^2-13824\pi^2\langle\bar{q}q\rangle M^2\Big)WF[\mathcal{T}_2,\bar{v}]-4320e_d\pi^2\langle\bar{q}q\rangle^2M^4(3m_0^2-4M^2)WF[\tilde{S},v]\\
&-1728e_u\pi^2\langle\bar{q}q\rangle^2M^4(3m_0^2-4M^2)WF[\tilde{S},\bar{v}]+576(e_d-e_u)f_{3\gamma}\langle g_s^2G^2\rangle M^6WF[\psi^\nu,u]\\
&+13824(e_u-e_d)\pi^2f_{3\gamma}\langle\bar{q}q\rangle m_c\ M^4(3m_0^2-4M^2)WF[\psi^\nu,u]\\
&+(39e_u-38e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle m_c\ M^4WFD[\mathbb{A},u]+8(4e_u-e_d)\pi^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2(5m_0^2-4M^2)WFD[\mathbb{A},u]\\
&-e_df_{3\gamma}M^4\Big(204\langle g_s^2G^2\rangle M^2-2592\pi^2\langle\bar{q}q\rangle m_c\ m_0^2+3456\pi^2\langle\bar{q}q\rangle m_c\ M^2\Big)WFD[\mathcal{A},v]-e_uf_{3\gamma}M^4\Big(204\langle g_s^2G^2\rangle M^2\\
&-2592\pi^2\langle\bar{q}q\rangle m_c\ m_0^2+3456\pi^2\langle\bar{q}q\rangle m_c\ M^2\Big)WFD[\mathcal{A},\bar{v}]-e_df_{3\gamma}M^4\Big(138\langle g_s^2G^2\rangle M^2+2592\pi^2\langle\bar{q}q\rangle m_c\ m_0^2\\
&-3456\pi^2\langle\bar{q}q\rangle m_c\ M^2\Big)WFD[\mathcal{V},v]-e_uf_{3\gamma}M^4\Big(138\langle g_s^2G^2\rangle M^2+2592\pi^2\langle\bar{q}q\rangle m_c\ m_0^2\\
&-3456\pi^2\langle\bar{q}q\rangle m_c\ M^2\Big)WFD[\mathcal{V},\bar{v}]+32(e_d-e_u)\pi^2\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2M^2(m_0^2-M^2)WFD[\varphi_\gamma,u]\Bigg]\\
&\Big(FlNP[0,4,0]-8m_c^2FlNP[1,4,0]+16m_c^4FlNP[2,4,0]\Big)
\end{aligned}
$$

$$+\frac{m_c\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{82944M^8\pi^2}\Bigg[(e_u+e_d)\Big((3m_0^2-4M^2)A(u_0)+4M^2\chi(-m_0^2+2M^2)\varphi_\gamma(u_0)+(-3m_0^2+4M^2)WF[h_\gamma,u]\Big)\Bigg]$$
$$\Bigg(-4m_c^2FlNP[-1,1,0]+FlNP[0,1,0]\Bigg)$$
$$+\frac{m_cm_0^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2}{7962624M^{12}\pi^4}\Bigg[-(4e_u-e_d)WFD[\mathcal{A},u]\Bigg]\Bigg(16m_c^4FlNP[2,4,2]-8m_c^2FlNP[3,4,2]+FlNP[4,4,2]\Bigg)$$
$$+\frac{m_c^3f_{3\gamma}}{92160M^8\pi^4}\Bigg[e_uWFD[\mathcal{V},\bar{v}]+e_dWF[\mathcal{V},v]\Bigg]\Bigg(64m_c^6FlNP[4,6,0]-48m_c^4FlNP[3,6,0]+12m_c^2FlNP[2,6,0]$$
$$-FlNP[1,6,0]\Bigg)$$
$$-\frac{f_{3\gamma}m_0^2\langle\bar{q}q\rangle}{73728M^8\pi^2}\Bigg[e_dWFD[\mathcal{V},v]+e_uWFD[\mathcal{V},\bar{v}]\Bigg]FlNP[3,4,1]$$
$$+\frac{m_c}{3981312M^{12}\pi^4}\Bigg[-12(e_d-e_u)f_{3\gamma}M^4\Big(7\langle g_s^2G^2\rangle+144\pi^2\langle\bar{q}q\rangle m_c(4M^2-5m_0^2)\Big)\psi^a(u_0)$$
$$+\langle\bar{q}q\rangle\pi^2\Bigg(1728(e_d-e_u)f_{3\gamma}m_0^2m_c\ M^4\psi^\nu(u_0)+216e_dM^4\langle\bar{q}q\rangle(45m_0^2-32M^2)WF[\mathcal{T}_1,v]$$
$$+864e_uM^4\langle\bar{q}q\rangle(3m_0^2-2M^2)WF[\mathcal{T}_1,\bar{v}]+216e_dM^4\langle\bar{q}q\rangle(15m_0^2-8M^2)WF[\mathcal{T}_2,v]$$
$$+864e_uM^4\langle\bar{q}q\rangle(3m_0^2-2M^2)WF[\mathcal{T}_2,\bar{v}]+216m_0^2864M^4\langle\bar{q}q\rangle\Big(5e_dWF[\tilde{S},v]+2e_uWF[\tilde{S},\bar{v}]\Big)$$
$$+3456(e_d-e_u)f_{3\gamma}m_0^2m_c\ M^4WF[\psi^\nu,u]+(4e_u-e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle(2m_0^2-5M^2)WFD[\mathbb{A},u]$$
$$-216\ e_df_{3\gamma}m_0^2m_c\ M^4\Big(WFD[\mathcal{A},v]-WFD[\mathcal{V},v]\Big)-216\ e_uf_{3\gamma}m_0^2m_c\ M^4\Big(WFD[\mathcal{A},\bar{v}]-WFD[\mathcal{V},\bar{v}]\Big)$$
$$-2(4e_u-e_d)\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle m_0^2M^2WFD[\varphi_\gamma,u]\Bigg)\Bigg]FlNP[2,4,1]$$
$$+\frac{m_c^3}{497664M^{12}\pi^4}\Bigg[18(e_d-e_u)\pi^2f_{3\gamma}\langle\bar{q}q\rangle M^4(4M^2-5m_0^2)\psi^a(u_0)+\langle\bar{q}q\rangle\pi^2\Bigg(2592(e_u-e_d)f_{3\gamma}m_0^2M^4\psi^\nu(u_0)$$
$$-324e_d\langle\bar{q}q\rangle M^4(45m_0^2-32M^2)WF[\mathcal{T}_1,v]-324e_u\langle\bar{q}q\rangle M^4(12m_0^2-8M^2)WF[\mathcal{T}_1,\bar{v}]$$
$$-324e_d\langle\bar{q}q\rangle M^4(15m_0^2-8M^2)WF[\mathcal{T}_2,v]-324e_u\langle\bar{q}q\rangle M^4(12m_0^2-8M^2)WF[\mathcal{T}_2,\bar{v}]$$
$$-324e_u\langle\bar{q}q\rangle M^4\Big(5\ e_dm_0^2WF[\tilde{S},v]-2\ e_u\ M^2WF[\tilde{S},\bar{v}]\Big)+5184(e_u-e_d)f_{3\gamma}m_c\ m_0^2M^4WFD[\psi^\nu,u]$$
$$+(4e_u-e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle(5m_0^2-2M^2)WFD[\mathbb{A},u]+108e_df_{3\gamma}m_0^2M^2m_c\Big(2WFD[\mathbb{A},v]+WFD[\mathcal{V},v]\Big)$$
$$+108e_uf_{3\gamma}m_0^2M^2m_c\Big(2WFD[\mathbb{A},\bar{v}]+WFD[\mathcal{V},\bar{v}]\Big)-2(4e_u-e_d)\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle m_0^2M^2WFD[\varphi_\gamma,u]\Bigg)\Bigg]$$
$$FlNP[1,4,1]$$
$$+\frac{m_c^5}{248832M^{12}\pi^4}\Bigg[-36(e_u-e_d)f_{3\gamma}M^4\Big(7\langle g_s^2G^2\rangle M^2+144\ m_c\ \pi^2\langle\bar{q}q\rangle(4M^2-5m_0^2)\Big)\psi^a(u_0)$$
$$+\pi^2\langle\bar{q}q\rangle\Bigg(5184(e_d-e_u)f_{3\gamma}m_0^2m_c\ M^2\psi^\nu(u_0)+648\ e_d\langle\bar{q}q\rangle M^4(45m_0^2-32M^2)WF[\mathcal{T}_1,v]$$
$$+2592\ e_u\langle\bar{q}q\rangle M^4(3m_0^2-2M^2)WF[\mathcal{T}_1,\bar{v}]+2592\ e_d\langle\bar{q}q\rangle M^4(4m_0^2-2M^2)WF[\mathcal{T}_2,v]$$
$$+2592\ e_u\langle\bar{q}q\rangle M^4(3m_0^2-2M^2)WF[\mathcal{T}_2,\bar{v}]+648\langle\bar{q}q\rangle M^4\Big(5\ e_d\ m_0^2WF[\tilde{S},v]+2\ e_u\ M^2WF[\tilde{S},\bar{v}]\Big)$$
$$+(4e_u-e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle(2M^2-5m_0^2)WFD[\mathbb{A},u]-216e_df_{3\gamma}m_0^2M^4m_c\Big(WFD[\mathbb{A},v]+WFD[\mathcal{V},v]\Big)$$
$$-216e_uf_{3\gamma}m_0^2M^4m_c\Big(WFD[\mathbb{A},\bar{v}]+WFD[\mathcal{V},\bar{v}]\Big)+2(4e_u-e_d)\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle m_0^2M^2WFD[\varphi_\gamma,u]\Bigg)\Bigg]FlNP[0,4,1]$$

$$
\begin{aligned}
&+\frac{\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{1327104M^{10}\pi^4}\Bigg[2(e_u+e_d)M^4A(u_0)+3(e_u-e_d)\pi^2m_0^2f_{3\gamma}WFD[\psi^a,u]\Bigg]FlNP[3,3,1]\\
&+\frac{m_c\langle\bar{q}q\rangle}{1990656M^{10}\pi^4}\Bigg[(e_u-e_d)\Bigg(-3456\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-2M^2)+\langle g_s^2G^2\rangle\Big(-21m_cM^4\\
&+20\pi^2\langle\bar{q}q\rangle(16m_0^2-11M^2)\Big)\Bigg)A(u_0)+4\pi^2\Bigg\{(e_d-e_u)\chi M^2\langle\bar{q}q\rangle\Big(5\langle g_s^2G^2\rangle(11m_0^2-8M^2)+1728m_0^2m_c^2M^2\Big)\varphi_\gamma(u_0)\\
&+4(e_u+e_d)\langle\bar{q}q\rangle\Big(7\langle g_s^2G^2\rangle(2m_0^2-M^2)-216m_c^2m_0^2M^2\Big)WF[\varphi_\gamma,u]+9(e_d-e_u)m_cm_0^2f_{3\gamma}\langle g_s^2G^2\rangle WF[\psi^a,u]\Bigg\}\Bigg]\\
&FlNP[2,3,1]\\
&+\frac{m_c^3\langle\bar{q}q\rangle}{248832M^{10}\pi^4}\Bigg[\Bigg(3(25e_u-13e_d)m_c\ M^4\langle g_s^2G^2\rangle-4(e_u-e_d)\pi^2\langle\bar{q}q\rangle\Big(-864\ \pi^2m_c^2\langle\bar{q}q\rangle(3m_0^2-2M^2)\\
&-\langle g_s^2G^2\rangle(80m_0^2+55M^2)\Big)\Bigg)A(u_0)+4(e_d-e_u)\pi^2\chi\langle\bar{q}q\rangle M^2\Big(5\langle g_s^2G^2\rangle(11m_0^2-8M^2)+1728m_0^2m_c^2M^2\Big)\varphi_\gamma\\
&+4\pi^2\langle\bar{q}q\rangle\Big((2e_d-7e_u)\langle g_s^2G^2\rangle(M^2-2m_0^2)+216(e_d-e_u)m_0^2m_c^2M^2\Big)WF[h_\gamma,u]\\
&+9(e_d-e_u)\pi^2f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_c\ WF[\psi^a,u]\Bigg]FlNP[1,3,1]\\
&+\frac{m_c^5}{82944M^{10}\pi^4}\Bigg[\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(2304\pi^2m_c^2M^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)+\langle g_s^2G^2\rangle\Big(-73m_cM^4+120\pi^2\langle\bar{q}q\rangle(m_0^2-M^2)\Big)\Bigg)\\
&+e_d\Bigg(2304\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-4M^2)+\langle g_s^2G^2\rangle\Big(99m_cM^4+120\pi^2\langle\bar{q}q\rangle(m_0^2M^2)\Big)\Bigg)\Bigg\}A(u_0)\\
&-8M^2\chi\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(1152\pi^2m_c^2M^2\langle\bar{q}q\rangle(m_0^2-2M^2)+\langle g_s^2G^2\rangle\Big(23m_cM^4+5\pi^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)\Big)\Bigg)\\
&+e_d\Bigg(-1152\pi^2m_c^2M^2\langle\bar{q}q\rangle(m_0^2-M^2)+5\langle g_s^2G^2\rangle\Big(-3m_cM^4+\pi^2\langle\bar{q}q\rangle(3m_0^2-4M^2)\Big)\Bigg)\Bigg\}\varphi_\gamma(u_0)\\
&+\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(576\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-4M^2)+\langle g_s^2G^2\rangle\Big(17m_cM^4+56\pi^2\langle\bar{q}q\rangle(-m_0^2+M^2)\Big)\Bigg)\\
&+e_d\Bigg(288\pi^2m_c^2M^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)+\langle g_s^2G^2\rangle\Big(-13m_cM^4+8\pi^2\langle\bar{q}q\rangle(-m_0^2+M^2)\Big)\Bigg)\Bigg\}WF[h_\gamma,u]\\
&+4(e_u-e_d)M^6f_{3\gamma}\langle g_s^2G^2\rangle\psi^a(u_0)\Bigg]FlNP[2,3,0]
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&+\frac{m_c^3}{165888M^{10}\pi^4}\Bigg[3\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(768\pi^2m_c^2M^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)+\langle g_s^2G^2\rangle\Big(23m_cM^4+40\pi^2\langle\bar{q}q\rangle(m_0^2-M^2)\Big)\Bigg)\\
&+e_d\Bigg(768\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-4M^2)+\langle g_s^2G^2\rangle\Big(31m_cM^4+40\pi^2\langle\bar{q}q\rangle(m_0^2-M^2)\Big)\Bigg)\Bigg\}A(u_0)\\
&-8M^2\chi\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(1152\pi^2m_c^2M^2\langle\bar{q}q\rangle(m_0^2-2M^2)+\langle g_s^2G^2\rangle\Big(25m_cM^4+5\pi^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)\Big)\Bigg)\\
&+e_d\Bigg(-1152\pi^2m_c^2M^2\langle\bar{q}q\rangle(m_0^2-M^2)+\langle g_s^2G^2\rangle\Big(-13m_cM^4+5\pi^2\langle\bar{q}q\rangle(3m_0^2-4M^2)\Big)\Bigg)\Bigg\}\varphi_\gamma(u_0)\\
&+4\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(576\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-4M^2)+\langle g_s^2G^2\rangle\Big(17m_cM^4+56\pi^2\langle\bar{q}q\rangle(-m_0^2+M^2)\Big)\Bigg)\\
&+e_d\Bigg(288\pi^2m_c^2M^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)+\langle g_s^2G^2\rangle\Big(-13m_cM^4+8\pi^2\langle\bar{q}q\rangle(-m_0^2+M^2)\Big)\Bigg)\Bigg\}WF[h_\gamma,u]\\
&-32(e_u-e_d)M^6f_{3\gamma}\langle g_s^2G^2\rangle\psi^a(u_0)\Bigg]FlNP[1,3,0]\\
&+\frac{m_c}{1327104M^{10}\pi^4}\Bigg[3\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(768\pi^2m_c^2M^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)+\langle g_s^2G^2\rangle\Big(35m_cM^4+40\pi^2\langle\bar{q}q\rangle(m_0^2-M^2)\Big)\Bigg)\\
&+e_d\Bigg(-768\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-4M^2)+\langle g_s^2G^2\rangle\Big(-19m_cM^4+40\pi^2\langle\bar{q}q\rangle(m_0^2-M^2)\Big)\Bigg)\Bigg\}A(u_0)\\
&-8M^2\chi\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(1152\pi^2m_c^2M^2\langle\bar{q}q\rangle(m_0^2-2M^2)+\langle g_s^2G^2\rangle\Big(31m_cM^4+5\pi^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)\Big)\Bigg)\\
&+e_d\Bigg(-1152\pi^2m_c^2M^2\langle\bar{q}q\rangle(m_0^2-M^2)+\langle g_s^2G^2\rangle\Big(-7m_cM^4+5\pi^2\langle\bar{q}q\rangle(3m_0^2-4M^2)\Big)\Bigg)\Bigg\}\varphi_\gamma(u_0)\\
&+4\langle\bar{q}q\rangle\Bigg\{e_u\Bigg(576\pi^2m_c^2M^2\langle\bar{q}q\rangle(3m_0^2-4M^2)+\langle g_s^2G^2\rangle\Big(17m_cM^4+56\pi^2\langle\bar{q}q\rangle(-m_0^2+M^2)\Big)\Bigg)\\
&+e_d\Bigg(288\pi^2m_c^2M^2\langle\bar{q}q\rangle(-3m_0^2+4M^2)+\langle g_s^2G^2\rangle\Big(-13m_cM^4+8\pi^2\langle\bar{q}q\rangle(-m_0^2+M^2)\Big)\Bigg)\Bigg\}WF[h_\gamma,u]\\
&-24(e_u-e_d)M^6f_{3\gamma}\langle g_s^2G^2\rangle\psi^a(u_0)\Bigg]FlNP[0,3,0]\\
&+\frac{\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{663552M^{10}\pi^4}\Bigg[(e_u+e_d)M^4\Big(3M^2\chi\varphi_\gamma(u_o)-A(u_0)\Big)-6(e_u-e_d)\pi^2f_{3\gamma}(m_0^2-M^2)WFD[\psi^a,u]\Bigg]FlNP[-1,3,0]\\
&+\frac{m_c^3}{3317760M^{10}\pi^4}\Bigg[-3456(e_d-e_u)f_{3\gamma}M^6\Big(FlNP[1,5,0]-12\ m_c^2FlNP[2,5,0]+48\ m_c^4FlNP[3,5,0]\Big)\psi^a(u_0)\\
&+144\pi^2\langle\bar{q}q\rangle^2(m_0^2-M^2)\Bigg\{e_u\Big(-2WF[\mathcal{T}_1,\bar{v}]-2WF[\mathcal{T}_2,\bar{v}]+2WF[\tilde{S},\bar{v}]+WFD[\mathcal{T}_1,\bar{v}]+WFD[\mathcal{T}_2,\bar{v}]+WFD[\tilde{S},\bar{v}]\Big)\\
&+e_d\Big(-8WF[\mathcal{T}_1,v]-2WF[\mathcal{T}_2,v]+2e_dWF[\tilde{S},v]+4WFD[\mathcal{T}_1,v]+WFD[\mathcal{T}_2,v]-e_dWFD[\tilde{S},v]\Big)\Bigg\}\\
&+(e_u-e_d)f_{3\gamma}\Big(7\langle g_s^2G^2\rangle M^2+576\langle\bar{q}q\rangle m_c(M^2-m_0^2)\Big)WFD[\psi^a,u]\Bigg]\Bigg(FlNP[0,5,0]-12\ m_c^2FlNP[1,5,0]\\
&+48\ m_c^4FlNP[2,5,0]-64\ m_c^6FlNP[3,5,0]\Bigg).
\end{aligned}
\tag{18}
$$

$$\Pi_3 = 0. \tag{19}$$

$$\Pi_4 = -\frac{m_c^3 M^2 \chi \langle g_s^2 G^2\rangle \langle \bar{q}q\rangle}{18432\pi^4}(e_u+e_d)\varphi_\gamma(u_0) N[1,3,2]$$
$$+ \frac{5m_c^2 f_{3\gamma}\langle g_s^2 G^2\rangle}{6912\pi^4}(e_u-e_d)WF[\psi^\nu,u]\ N[1,1,0]$$
$$+ \frac{f_{3\gamma}m_c^4}{16\pi^4}WF[\psi^\nu,u]\ N[3,3,1]$$
$$+ \frac{f_{3\gamma}\langle g_s^2 G^2\rangle m_c^2 M^2}{884736\pi^4}WF[\psi^\nu,u]\ N[1,1,1]$$
$$+ \frac{m_c^2}{2304\pi^4}\Bigg[-(e_u+e_d)\chi\langle g_s^2 G^2\rangle\langle\bar{q}q\rangle\varphi_\gamma(u_0) + 18 m_c M^2 f_{3\gamma}\Big(2e_u WF[\mathcal{A},\bar{v}] + 5e_d WF[\mathcal{A},v]\Big)\Bigg]N[1,2,0]$$
$$+ \frac{m_c^3\langle\bar{q}q\rangle}{128\pi^4}\Bigg[e_d\Big(5WF[\mathcal{T}_1,v]+5WF[\mathcal{T}_2,v]+3WF[\tilde{S},v]\Big)+2e_u\Big(WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)\Bigg]$$
$$\Bigg(3\ m_c\ N[2,3,1] - 4\ N[1,3,0] + 4\ m_c\ N[1,4,1]\Bigg)$$
$$+ \frac{m_c^3}{2304\pi^4}\Bigg[(e_u+e_d)\chi\langle g_s^2 G^2\rangle\langle\bar{q}q\rangle\varphi_\gamma(u_0) - 9m_c M^2 f_{3\gamma}\Big((5\ e_d WF[\mathcal{A},v] + 2\ e_u\ WF[\mathcal{A},\bar{v}])\Big)$$
$$+ 9\ M^2\langle\bar{q}q\rangle\Bigg(e_d\Big(5WF[\mathcal{T}_1,v]+5WF[\mathcal{T}_2,v]+3WF[\tilde{S},v]\Big)+2e_u\Big(WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)\Bigg)\Bigg]N[1,3,1]$$
$$- \frac{m_c^4 M^2\langle\bar{q}q\rangle}{1024\pi^4}\Bigg[e_d\Big(5WF[\mathcal{T}_1,v]+5WF[\mathcal{T}_2,v]+3WF[\tilde{S},v]\Big)+2e_u\Big(WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)\Bigg]$$
$$\Bigg(4\ N[1,4,2] + 3\ N[2,3,2]\Bigg)$$
$$- \frac{m_c^2\langle g_s^2 G^2\rangle}{221184\pi^2}\Bigg[\langle\bar{q}q\rangle\Bigg(6(e_u+e_d)\Big(A(u_0)+2\chi M^2\varphi_\gamma(u_0)\Big) + 22e_d\Big(2WF[\mathcal{T}_1,v]+5WF[\mathcal{T}_2,v]-WF[\mathcal{T}_3,v]$$
$$+ 2WF[\mathcal{T}_4,v]\Big) + e_u\Big(3WF[\mathcal{S},\bar{v}]+44WF[\mathcal{T}_1,\bar{v}]+113WF[\mathcal{T}_2,\bar{v}]-25WF[\mathcal{T}_3,\bar{v}]+44WF[\mathcal{T}_4,\bar{v}]\Big)\Bigg)$$
$$- 48(e_u+e_d)\langle\bar{q}q\rangle WF[f_\gamma,u] - 120(e_u-e_d)f_{3\gamma}m_c\ WF[\psi^\nu,u]\Bigg]N[1,2,1]$$
$$- \frac{m_c^2 M^2\langle g_s^2 G^2\rangle}{1769472\pi^2}\Bigg[6(e_u+e_d)\langle\bar{q}q\rangle A(u_0) + 22e_d\langle\bar{q}q\rangle\Big(2WF[\mathcal{T}_1,v]+5WF[\mathcal{T}_2,v]-WF[\mathcal{T}_3,v]$$
$$+ 2WF[\mathcal{T}_4,v]\Big) + e_u\langle\bar{q}q\rangle\Big(3WF[\mathcal{S},\bar{v}]-44WF[\mathcal{T}_1,\bar{v}]+113WF[\mathcal{T}_2,\bar{v}]-25WF[\mathcal{T}_3,\bar{v}]+44WF[\mathcal{T}_4,\bar{v}]\Big)\Bigg)$$
$$- 48(e_u+e_d)\langle\bar{q}q\rangle WF[f_\gamma,u] - 280(e_u-e_d)f_{3\gamma}m_c\ WF[\psi^\nu,u]\Bigg]N[1,2,2]$$
$$+ \frac{11 f_{3\gamma}\langle g_s^2 G^2\rangle m_c^2}{9216 M^2\pi^4}\Bigg[e_u\Big(2WF[\mathcal{A},\bar{v}] - WFD[\mathcal{A},\bar{v}]\Big) + e_d\Big(2WF[\mathcal{A},v] - WFD[\mathcal{A},v]\Big)\Bigg]N[2,2,0]$$
$$+ \frac{m_c^2}{110592\pi^4}\Bigg[36(e_u+e_d)m_c\chi\langle g_s^2 G^2\rangle\langle\bar{q}q\rangle\varphi_\gamma(u_0) + f_{3\gamma}\Bigg(4e_d\Big(11\langle g_s^2 G^2\rangle + 540 m_c^2 M^2\Big)WF[\mathcal{A},v]$$
$$+ e_u\Big(44\langle g_s^2 G^2\rangle + 864 m_c^2 M^2\Big)WF[\mathcal{A},\bar{v}] - 33\langle g_s^2 G^2\rangle\Big(e_d WFD[\mathcal{A},v] + e_u WFD[\mathcal{A},\bar{v}]\Big)\Bigg)\Bigg]N[2,2,1]$$

$$+\frac{m_c^2 M^2\langle g_s^2G^2\rangle}{442368\pi^4}\Bigg[-18(e_u+e_d)m_c\chi\langle\bar{q}q\rangle\varphi_\gamma(u_0)+11f_{3\gamma}\Bigg(e_d\Big(WF[\mathcal{A},v]-WFD[\mathcal{A},v]\Big)+e_u\Big(2WF[\mathcal{A},\bar{v}]$$
$$-WFD[\mathcal{A},\bar{v}]\Big)\Bigg)\Bigg]N[2,2,2]$$
$$+\frac{11m_c^2M^2f_{3\gamma}\langle g_s^2G^2\rangle}{3538944\pi^4}\Bigg[+e_d\Big(2WF[\mathcal{A},v]-WFD[\mathcal{A},v]\Big)+e_u\Big(WF[\mathcal{A},\bar{v}]+-WFD[\mathcal{A},\bar{v}]\Big)\Bigg]N[2,2,3]$$
$$-\frac{m_c m_0^2\langle\bar{q}q\rangle^2}{4608M^{10}\pi^2}\Bigg[5e_d\Big(WF[\mathcal{T}_1,v]+WF[\mathcal{T}_2,v]\Big)+2e_u\Big(WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)\Bigg]\Bigg(64\ m_c^6FlNP[1,4,2]$$
$$-48\ m_c^4FlNP[2,4,2]+12m_c^2FlNP[3,4,3]-FlNP[4,4,2]\Bigg)$$
$$-\frac{m_c m_0^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2}{331776M^{12}\pi^2}(e_u+e_d)\Big(A(u_0)-8WF[h_\gamma,u]\Big)\Bigg(16\ m_c^4FlNP[2,3,2]-8\ m_c^2FlNP[3,3,2]$$
$$+FlNP[4,3,2]\Bigg)$$
$$+\frac{m_c\langle\bar{q}q\rangle}{165888M^{12}\pi^2}\Bigg[(e_u+e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle(5m_0^2-2M^2)A(u_0)+2m_0^2M^2\Bigg((e_u+e_d)\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle\varphi_\gamma(u_0)$$
$$+36m_cM^2f_{3\gamma}\Big(e_dWF[\mathcal{A},v]-2e_uFW[\mathcal{A},\bar{v}]\Big)\Bigg)-8(e_u+e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle(5m_0^2-2M^2)WF[h_\gamma,u]\Bigg]$$
$$\Bigg(16\ m_c^4FlNP[0,3,1]-8\ m_c^2FlNP[1,3,1]+FlNP[2,3,1]\Bigg)$$
$$+\frac{m_c}{663552M^{12}\pi^4}\Bigg[(e_u+e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle\Big(-3m_cM^4-8\pi^2\langle\bar{q}q\rangle(5m_0^2-4M^2)\Big)A(u_0)$$
$$+8\ M^2\Bigg\{4(e_u+e_d)\pi^2\chi\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2(m_0^2-M^2)\varphi_\gamma(u_0)+M^2f_{3\gamma}\Bigg(e_d\Big(17\langle g_s^2G^2\rangle M^2+36\pi^2m_c\langle\bar{q}q\rangle(3m_0^2-4M^2)\Big)$$
$$WF[\mathcal{A},v]+e_u\Big(17\langle g_s^2G^2\rangle M^2-72\pi^2m_c\langle\bar{q}q\rangle(3m_0^2-4M^2)\Big)WF[\mathcal{A},\bar{v}]\Bigg)\Bigg\}+8(e_u+e_d)\langle g_s^2G^2\rangle\langle\bar{q}q\rangle\Big(3\ m_c\ M^4$$
$$+8\pi^2\langle\bar{q}q\rangle(5m_0^2-4M^2)\Big)WF[h_\gamma,u]\Bigg]\Bigg(16\ m_c^4FlNP[2,3,0]-8\ m_c^2FlNP[1,3,0]+FlNP[0,3,0]\Bigg)$$
$$-\frac{m_0^2\langle\bar{q}q\rangle}{165888M^{10}\pi^2}\Bigg[432m_cM^2\langle\bar{q}q\rangle\Bigg(e_d\Big(WF[\mathcal{T}_1,v]+WF[\mathcal{T}_2,v]-WF[\mathcal{T}_3,v]-WF[\mathcal{T}_4,v]\Big)+e_u\Big(WF[\mathcal{T}_1,\bar{v}]$$
$$+WF[\mathcal{T}_2,\bar{v}]-WF[\mathcal{T}_3,\bar{v}]-WF[\mathcal{T}_4,\bar{v}]\Big)-5(e_u-e_d)f_{3\gamma}\langle g_s^2G^2\rangle WF[\psi^\nu,u]\Bigg]\Bigg(16\ m_c^24FlNP[1,2,1]$$
$$+8\ m_c^2FlNP[2,2,1]-FlNP[3,2,1]\Bigg)$$

$$+\frac{m_c\langle\bar{q}q\rangle}{9216M^{10}\pi^2}\Bigg[\langle\bar{q}q\rangle(17m_0^2-8M^2)\Bigg(5e_d\Big(WF[\mathcal{T}_1,v]+WF[\mathcal{T}_2,v]\Big)+2e_u\Big(WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)\Bigg)$$
$$+3e_d\langle\bar{q}q\rangle m_0^2WF[\tilde{S},v]-32(e_u-e_d)f_{3\gamma}m_c\ m_0^2WF[\psi^\nu,u]\Bigg]\Bigg(64\ m_c^6FlNP[-1,4,1]-48\ m_c^4FlNP[0,4,1]$$
$$-12\ m_c^2FLNP[1,4,1]-FlNP[2,4,1]\Bigg)$$
$$+\frac{m_c}{27648M^{10}\pi^4}\Bigg[\langle\bar{q}q\rangle\Bigg(3\ m_c\ M^4+28\pi^2\langle\bar{q}q\rangle(m_0^2-M^2)\Bigg)\Bigg(15e_d\Big(WF[\mathcal{T}_1,v]+WF[\mathcal{T}_2,v]\Big)$$
$$+6e_u\Big(WF[\mathcal{T}_1,\bar{v}]+WF[\mathcal{T}_2,\bar{v}]\Big)\Bigg)+12e_d\pi^2\langle\bar{q}q\rangle^2(m_0^2-M^2)WF[\tilde{S},v]+4(e_u-e_d)f_{3\gamma}\Big(\langle g_s^2G^2\rangle M^2$$
$$+96\pi^2\langle\bar{q}q\rangle m_c(-m_0^2+M^2)\Big)WF[\psi^\nu,u]\Bigg]\Bigg(-64\ m_c^6FlNP[3,4,0]+48\ m_c^4FlNP[2,4,0]-12\ m_c^2FlNP[1,4,0]$$
$$+FlNP[0,4,0]\Bigg)$$
$$-\frac{\langle\bar{q}q\rangle}{165888M^{10}\pi^4}\Bigg[3e_dM^2\Big(23\langle g_s^2G^2\rangle M^2+288\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_1,v]+6e_u\Big(17\langle g_s^2G^2\rangle M^2$$
$$+144\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_1,\bar{v}]+e_dM^2\Big(69\langle g_s^2G^2\rangle M^2+864\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_2,v]$$
$$+e_u\Big(102\langle g_s^2G^2\rangle M^2+864\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_2,\bar{v}]-e_dM^2\Big(36\langle g_s^2G^2\rangle M^2$$
$$+864\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_3,v]-e_u\Big(36\langle g_s^2G^2\rangle M^2+864\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_3,\bar{v}]$$
$$-e_dM^2\Big(36\langle g_s^2G^2\rangle M^2+864\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_4,v]-e_u\Big(36\langle g_s^2G^2\rangle M^2$$
$$+864\pi^2\langle\bar{q}q\rangle m_c(4M^2-3m_0^2)\Big)WF[\mathcal{T}_4,\bar{v}]-40(e_u-e_d)\pi^2f_{3\gamma}\langle g_s^2G^2\rangle(m_0^2-M^2)WF[\psi^\nu,u]\Bigg]\Bigg(16\ m_c^4FlNP[1,2,0]$$
$$-8\ m_c^2FlNP[0,2,0]+FlNP[-1,2,0]\Bigg). \tag{20}$$

The functions $N[n,m,k]$, $FlP[n,m,k]$, $FlNP[n,m,k]$, $WFD[\mathcal{A},\bar{v}]$, $WFD[\mathcal{A},v]$, $WF[\mathcal{A},\bar{v}]$, $WF[\mathcal{A},v]$, $WFD[\mathcal{A},u]$ and $WF[\mathcal{A},u]$ are defined as:

$$N[n,m,k]=\int_0^\infty dt\int_0^\infty dt'\ \frac{e^{-m_c/2(t+t')}}{t^n\ (\frac{m_c}{t}+\frac{m_c}{t'})^k\ t'^m},$$
$$FlP[n,m,k]=\int_{4m_c^2}^{s_0}ds\int_{4m_c^2}^{s}dl\ \frac{e^{-l^2/\phi}\ l^n\ (l-s)^m}{(4m^2-l)^2\ \phi^k},$$
$$FlNP[n,m,k]=\int_{4m_c^2}^{s_0}ds\int_{4m_c^2}^{s}dl\ \frac{e^{-l^2/\beta}\ l^n\ (l-s)^m}{(l-2m_c^2)\ \beta^k},$$
$$WFD[\mathcal{A},\bar{v}]=\int D_{\alpha_i}\int_0^1dv\ \mathcal{A}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta'(\alpha_q+\bar{v}\alpha_g-u_0),$$
$$WFD[\mathcal{A},v]=\int D_{\alpha_i}\int_0^1dv\ \mathcal{A}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta'(\alpha_{\bar{q}}+v\alpha_g-u_0),$$
$$WF[\mathcal{A},\bar{v}]=\int D_{\alpha_i}\int_0^1dv\ \mathcal{A}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta(\alpha_q+\bar{v}\alpha_g-u_0),$$

$$
\begin{aligned}
WF[\mathcal{A},v] &= \int D_{\alpha_i}\int_0^1 dv\ \mathcal{A}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta(\alpha_{\bar{q}}+v\alpha_g-u_0),\\
WFD[\mathcal{A},u] &= \int_0^1 du\ A(u)\delta'(u-u_0),\\
WF[\mathcal{A},u] &= \int_0^1 du\ A(u),
\end{aligned}
\tag{21}
$$

where

$$
\beta = 4\ l\ M^2 - 16m_c^2M^2, \qquad\qquad \phi = 8\ l\ M^2 - 32m_c^2M^2.
$$

Appendix C:

In this appendix, we give some details on Fourier and Borel transformations as well as continuum subtraction. We take a term in the form

$$
I = \int_0^1 duA(u)\int d^4xe^{i(p+qu)x}\frac{K_\nu(m_Q\sqrt{-x^2})}{\sqrt{-x^2}^{\nu}}\frac{K_\mu(m_Q\sqrt{-x^2})}{\sqrt{-x^2}^{\mu}}, \tag{22}
$$

where K_ν comes from the heavy quark propagator. To proceed we apply the integral representation of the Bessel function of second kind as

$$
\frac{K_\nu\left(m_Q\sqrt{-x^2}\right)}{\left(\sqrt{-x^2}\right)^{v}} = \frac{1}{2}\int_0^\infty \frac{dt}{t^{\nu+1}}\exp\left[-\frac{m_Q}{2}\left(t-\frac{x^2}{t}\right)\right].
$$

As a result, we get

$$
I = \int_0^1 duA(u)\int d^4xe^{i(p+qu)x}\int_0^\infty\frac{dt}{t^{\nu+1}}\exp\left[-\frac{m_Q}{2}\left(t-\frac{x^2}{t}\right)\right]\int_0^\infty\frac{dt'}{t'^{\mu+1}}\exp\left[-\frac{m_Q}{2}\left(t'-\frac{x^2}{t'}\right)\right]. \tag{23}
$$

By applying the Wick rotation we obtain

$$
I = \int_0^1 duA(u)\int_0^\infty\frac{dt}{t^{\nu+1}}\int_0^\infty\frac{dt'}{t'^{\mu+1}}\exp\left[-\frac{m_Q}{2}(t+t')\right]\int d^4x\exp\left[-i(p.x+q.x)-ax^2\right], \tag{24}
$$

where $a = (\frac{m_Q}{t}+\frac{m_Q}{t'})$. Taking the four-dimensional Gaussian integral we get

$$
I = \int_0^1 duA(u)\int_0^\infty\frac{dt}{t^{\nu+1}}\int_0^\infty\frac{dt'}{t'^{\mu+1}}\exp\left[-\frac{m_Q}{2}(t+t')-\frac{(p+qu)^2}{4a}\right]\frac{1}{a^2}. \tag{25}
$$

Now, we apply the Borel transformation over the variables p^2 and $(p+q)^2$, which results in

$$
I = \int_0^1 duA(u)\int_0^\infty\frac{dt}{t^{\nu+1}}\int_0^\infty\frac{dt'}{t'^{\mu+1}}\exp\left[-\frac{m_Q}{2}(t+t')\right]\frac{M^2}{a^2}\delta\Big[\frac{1}{M^2}-\frac{1}{4a}\Big]\ \delta\Big[u-u_0\Big]. \tag{26}
$$

After this step, we take the t integral using the corresponding Dirac delta. To do this, we use the property:

$$
\delta(g(x)) = \frac{\delta(x-x_0)}{|g'(x)|}\theta(x_0), \tag{27}
$$

and replace t by

$$
t \to \left(\frac{2m_Q\ t'}{M^2t'-2m_Q}\Bigg/\left|\frac{2\ m_Q\ t^2}{M^2t'-2\ m_Q}\right|\right)\ \theta\left(\frac{2m_Q\ t'}{M^2t'-2m_Q}\right). \tag{28}
$$

Then, we change the variable $t' \to s$ via

$$t' \to \frac{2\ m_Q}{4\ m_Q^2 M^2}\ s. \tag{29}$$

Meanwhile, for the Borel transformations the following rules are applied:

$$\begin{aligned}
&B_{p^2}B_{(p+q)^2}\exp\left[-\frac{(p+qu)^2}{4a}\right](p+q\ u)^n\ (p.q)^m \to M^2\ (M^2/2)^m\ D\Big[\frac{1}{M^2},n\Big]\delta\Big[\frac{1}{M^2}-\frac{1}{4a}\Big]\ \delta'\Big[u-u_0\Big],\\
&B_{p^2}B_{(p+q)^2}\ \exp\left[-\frac{(p+qu)^2}{4a}\right](p.q)^m \to M^2\ (M^2/2)^m\ \delta\Big[\frac{1}{M^2}-\frac{1}{4a}\Big]\ \delta'\Big[u-u_0\Big],\\
&B_{p^2}B_{(p+q)^2}\ \exp\left[-\frac{(p+qu)^2}{4a}\right](p+q\ u)^n \to M^2\ D\Big[\frac{1}{M^2},n\Big]\delta\Big[\frac{1}{M^2}-\frac{1}{4a}\Big],\\
&B_{p^2}B_{(p+q)^2}\ \exp\left[-\frac{(p+qu)^2}{4a}\right] \to M^2\ \delta\Big[\frac{1}{M^2}-\frac{1}{4a}\Big]\ \delta\Big[u-u_0\Big].
\end{aligned} \tag{30}$$

where, D represents the derivation and

$$M^2=\frac{M_1^2 M_2^2}{M_1^2+M_2^2}, \qquad u_0=\frac{M_1^2}{M_1^2+M_2^2}. \tag{31}$$

The following formula for the continuum subtraction is used

$$\left(M^2\right)^N\int_{4m_Q^2}^{\infty} dse^{-s/M^2}f(s) \to \int_{4m_Q^2}^{s_0} dse^{-s/M^2}F_N(s), \tag{32}$$

where

$$\begin{aligned}
F_N(s) &= \left(\frac{d}{ds}\right)^{-N} f(s), \qquad N\le 0,\\
F_N(s) &= \frac{1}{\Gamma(N)}\int_{4m_Q^2}^{s} dl\ (s-l)^{N-1}f(l), \qquad N>0,
\end{aligned} \tag{33}$$

as a result of which we obtain the following expression:

$$\int_0^1 duA(u)\int_{4m_{Q'}}^{s_0} ds\int_{4m_{Q'}}^{s} dl\ \exp\left[-\frac{l+m_{Q'}\Big(-3-\frac{m_Q}{m_Q^2-m_{Q'}^2}\Big)}{M^2}\right]\frac{(l-s)^3\ \delta\Big[u-u_0\Big]}{3\ m_Q\ m_{Q'}^4\ M^{12}\left|\frac{m_Q\left(-2m_Q+\frac{2m_{Q'}^2}{m_Q}\right)^2}{m_{Q'}^4}\right|}, \tag{34}$$

with m_Q and $m_{Q'}$ being the charm quark mass. Here we face with the well-known problem in the case of doubly heavy hadrons when we take $m_Q = m_{Q'}$. The expression above becomes indeterminate. To get rid of this problem we take the limit of the expression in the integral, i.e.,

$$\int_0^1 duA(u)\int_{4m_{Q'}}^{s_0} ds\int_{4m_{Q'}}^{s} dl\ \lim_{m_{Q'}\to m_Q}\left[\exp\left\{-\frac{l+m_{Q'}\Big(-3-\frac{m_Q}{m_Q^2-m_{Q'}^2}\Big)}{M^2}\right\}\frac{(l-s)^3\ \delta\Big[u-u_0\Big]}{3\ m_Q\ m_{Q'}^4\ M^{12}\left|\frac{m_Q\left(-2m_Q+\frac{2m_{Q'}^2}{m_Q}\right)^2}{m_{Q'}^4}\right|}\right], \tag{35}$$

which gives a finite result.

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