

# Mass and residue of $\Lambda(1405)$ as a hybrid baryon

K. Azizi,<sup>1</sup> B. Barsbay,<sup>2</sup> and H. Sundu<sup>2</sup>

<sup>1</sup>*Department of Physics, Doğuş University, Acibadem-Kadıköy, 34722 Istanbul, Turkey*

<sup>2</sup>*Department of Physics, Kocaeli University, 41380 Izmit, Turkey*

(ΩDated: May 31, 2017)

The nature of the  $\Lambda(1405)$  has been a puzzle for decades, whether it is a standard three-quark baryon, a hybrid baryon or a baryon-meson molecule. More information on the decay channels of this particle and its strong, weak and electromagnetic interactions with other hadrons is needed to clarify its internal organization. The residue of this particle is one of the main inputs in investigation of its decay properties in many approaches. We calculate the mass and residue of  $\Lambda(1405)$  state in the context of QCD sum rules considering it as a hybrid baryon with three-quark—one gluon content and the quantum numbers  $I(J^P) = 0(1/2^-)$ . In computations the quark, gluon and mixed vacuum condensates up to ten dimensions are taken into account. The obtained result for the mass of this state is in a nice agreement with the average experimental value given in PDG.

## I. INTRODUCTION

It was already suggested that, in addition to the standard particles, there might exist hadrons with different quark-gluon structures, which can not be included into the ordinary  $q\bar{q}$  and  $qqq$  schemes. Due to their unconventional nature these states were included into a group of the hadrons known as exotic particles [1]. The discoveries of the exotic hadrons by various collaborations, and collected experimental information on their mass, width and decay channels have made investigation of these states one of the central topics in high energy physics. Starting from the first observation of the  $X(3872)$  resonance in 2003 by the Belle Collaboration [2], numerous experimental groups planned to search for and detect resonances with unusual properties, and in fact, measured their mass and width; and determined their quantum numbers. The hadrons with unusual internal structures, i.e. exotic resonances were classified as  $XYZ$  particles [3], glueballs [4], hybrids [5, 6], meson molecules [7], tetraquarks [8], pentaquarks [9] and dibaryons [10]. Like other groups of exotic particles, an identification and classification of hybrid hadrons and calculation of their spectroscopic parameters are important for both revealing their inner organization and gaining new information on quantum chromodynamics (QCD).

The existence of hybrid mesons was first suggested by Jaffe and Johnson in 1976 [11]. The main ingredients of hybrid mesons ( $\bar{q}gq$ ) are a color-octet quark-antiquark pair and an excited gluonic field. A system with these constituents may have all  $J^{PC}$  quantum numbers, implying that one of the fruitful ways to search for hybrids is to study these states with exotic quantum numbers, which are forbidden for the  $q\bar{q}$  states. The light hybrid mesons were studied in the framework of different theoretical methods, such as the Bag model, flux tube model, lattice QCD and QCD sum rules (see for instance [12–16] and references therein). Unfortunately, the predictions for the masses of the hybrids obtained within these approaches differ considerably from each others. The properties of the heavy quarkonium hybrids were also calcu-

lated using various methods. Thus, relevant explorations were carried out in the constituent gluon model, the flux tube model, QCD sum rules, nonrelativistic QCD and lattice (for instance see Refs. [17–23] and references therein).

Hybrid baryons can be defined in two different ways: as particles containing three valence quarks and a gluon; and three quarks moving in an excited adiabatic potential. Despite clear theoretical definitions, the experimental identification of the hybrid baryons is a more difficult task compared to the hybrid mesons. Since for the baryons, unfortunately there are not  $J^{PC}$  exotics, so one must use other features of these particles to determine whether or not they are hybrids (for more information see Ref.[24]).

One of the candidates to hybrid baryons is the  $\Lambda(1405)$  baryon, which for many decades has attracted interest of physicists. More than forty years ago experiments showed that there was a state with the spin  $1/2$  [25], which was predicted to be a  $\bar{K} - N$  resonance with quantum numbers  $I(J^P) = 0(1/2^-)$  [26] using a  $SU(3)$  meson-baryon potential. The  $\Lambda(1405)$  was experimentally observed in the low energy exclusive reactions, where, as usual, the kaon and pion beams were used [27, 28]. Recently, many high-statistics data are available by the LEPS, CLAS and HADES Collaborations [29–31]. The spin-parity  $I(J^P) = 0(1/2^-)$  was then experimentally confirmed for this particle by the CLAS collaboration [32] (for a detailed information, see for instance Ref.[33]).

The mass and different decay properties of  $\Lambda(1405)$  were studied using different theoretical methods including lattice QCD [34–72]. Despite a lot of experimental and theoretical studies on the properties of  $\Lambda(1405)$  state, unfortunately, there remain important questions about its nature and internal quark organization whether it is a standard three-quark baryon, a hybrid baryon or a baryon-meson molecule with one or two-pole structure. Hence, more experimental and theoretical studies are needed to clarify the physical properties of  $\Lambda(1405)$ .

In the present study we are going to calculate the mass and residue of the  $\Lambda(1405)$  considering it as a hybrid baryon with three-quark—one gluon content and the

quantum numbers  $I(J^P) = 0(1/2^-)$  in the framework of QCD sum rule. The mass of this state was already calculated in Ref. [73] using the same method. In Ref. [73] the  $\Lambda(1405)$  was considered in two different pictures: as a hybrid and as a mixed hybrid/3-quark strange baryon. The calculations on the mass of this state and comparison of the obtained result with the experimental data allowed the authors to conclude that this state is consistent with being a strange hybrid baryon than a mixed state. The main aim here is to calculate the residue of this state in the considered picture besides its mass. The residue is one of the main inputs in calculations of many parameters related to the strong, weak and electromagnetic decays of  $\Lambda(1405)$  in many theoretical approaches. Investigations of such decay channels may help us better understand the internal organization of  $\Lambda(1405)$  resonance and hopefully solve the puzzle on its nature.

This work is structured in the following way. In sec. II we derive two-point QCD sum rules for the mass and residue of the  $\Lambda(1405)$  baryon by taking into account the vacuum condensates up to ten dimensions. In sec. III we perform numerical computations and extract values of  $m_\Lambda$  and  $\lambda_\Lambda$ . Section IV is reserved for our concluding remarks.

## II. $\Lambda(1405)$ AS A HYBRID BARYON

To calculate the mass and residue of the  $\Lambda(1405)$  state in the framework of two-point QCD sum rule, we start with the correlation function

$$\Pi_{\Lambda_H}(q) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \eta_{\Lambda_H}(x) \bar{\eta}_{\Lambda_H}(0) | 0 \rangle, \quad (1)$$

where  $\eta_{\Lambda_H}$  is the interpolating current for the hybrid  $\Lambda(1405)$  baryon. In the three-quark—one gluon picture, one of the acceptable interpolating currents having the quantum numbers  $I(J^P) = 0(1/2^-)$  is

$$\begin{aligned} \eta_{\Lambda_H}(x) = & \frac{1}{\sqrt{2}} \epsilon^{abc} \{ [u^a(x) C \gamma^\mu s^b(x)] \gamma^\alpha [G_{\mu\alpha} d(x)]^c \\ & - [d^a(x) C \gamma^\mu s^b(x)] \gamma^\alpha [G_{\mu\alpha} u(x)]^c \}, \end{aligned} \quad (2)$$

where  $a, b, c$  are color indices,  $C$  is the charge conjugation operator,  $u, d, s$  are light quark fields and

$$G^{\mu\nu} = \sum_{A=1}^8 \frac{\lambda^A}{2} G_A^{\mu\nu}, \quad (3)$$

with  $\lambda_A$  being the generators of the color SU(3) group.

The correlation function  $\Pi_{\Lambda_H}(q)$  can be calculated in two different ways. From phenomenological or physical side it is obtained in terms of hadronic parameters. From theoretical or QCD side, it is evaluated in terms of quark's and gluon's degrees of freedom by the help of the operator product expansion (OPE) in deep Euclidean region. The QCD sum rules for the physical observables such as the mass and residue are obtained equating the

coefficients of the same structure in both representations of the correlation function. Finally, the continuum subtraction and Borel transformation are performed in order to suppress the contribution of the higher states and continuum.

First we calculate the correlation function in terms of the hadronic degrees of freedom. By inserting a complete set of hadronic state into Eq. (1) and performing integral over  $x$ , we get

$$\Pi_{\Lambda_H}^{\text{Had}}(q) = \frac{\langle 0 | \eta_{\Lambda_H} | \Lambda(q) \rangle \langle \Lambda(q) | \bar{\eta}_{\Lambda_H} | 0 \rangle}{m_\Lambda^2 - q^2} + \dots, \quad (4)$$

with  $m_\Lambda$  being the mass of the  $\Lambda(1405)$  state. Here the dots indicate contributions to the correlation function arising from the higher resonances and continuum states. We define the residue  $\lambda_\Lambda$  using the matrix element

$$\langle 0 | \eta_{\Lambda_H} | \Lambda(q) \rangle = \lambda_\Lambda \gamma_5 u_{\Lambda_H}(q, s). \quad (5)$$

Then performing the summation over spins in accordance with

$$\sum_s u_{\Lambda_H}(q, s) \bar{u}_{\Lambda_H}(q, s) = \not{q} + m_\Lambda, \quad (6)$$

for the physical side of the correlation function we get

$$\Pi_{\Lambda_H}^{\text{Had}}(q) = \frac{\lambda_\Lambda^2}{m_\Lambda^2 - q^2} (-\not{q} + m_\Lambda) + \dots. \quad (7)$$

The Borel transformation with respect to  $q^2$  applied to  $\Pi_{\Lambda_H}^{\text{Had}}(q)$  leads to the final form of the hadronic representation:

$$\widehat{\mathcal{B}}_{q^2} \Pi_{\Lambda_H}^{\text{Had}}(q) = \lambda_\Lambda^2 e^{-\frac{m_\Lambda^2}{M^2}} (-\not{q} + m_\Lambda) + \dots. \quad (8)$$

The OPE side of the correlation function is calculated at large space-like region, where  $q^2 \ll 0$  in terms of quark-gluon degrees of freedom. For this end, we substitute the interpolating current given by Eq. (2) into Eq. (1), and contract the relevant quark fields. As a result, we get

$$\begin{aligned} \Pi_{\Lambda_H}^{\text{OPE}}(q) = & \frac{i}{4} \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{iqx} \langle 0 | G_{\mu\alpha}(x) G_{\nu\alpha'}(0) | 0 \rangle \\ & \times \left\{ (\gamma_\alpha S_d^{cc'}(x) \gamma_{\alpha'}) Tr \left[ \gamma_\nu \tilde{S}_u^{aa'}(x) \gamma_\mu S_s^{bb'}(x) \right] \right. \\ & + (\gamma_\alpha S_u^{cc'}(x) \gamma_{\alpha'}) Tr \left[ \gamma_\nu \tilde{S}_d^{aa'}(x) \gamma_\mu S_s^{bb'}(x) \right] \\ & - \gamma_\alpha S_d^{ca'}(x) \gamma_\nu \tilde{S}_s^{bb'}(x) \gamma_\mu S_u^{ac'}(x) \gamma_{\alpha'} \\ & \left. - \gamma_\alpha S_u^{ca'}(x) \gamma_\nu \tilde{S}_s^{bb'}(x) \gamma_\mu S_d^{ac'}(x) \gamma_{\alpha'} \right\}. \end{aligned} \quad (9)$$

where  $Tr[\lambda^A \lambda^B] = 2\delta^{AB}$  has been used. In Eq. (9)  $S_{s,u,d}^{ab}(x)$  are the light quarks' propagators and we have used the notation

$$\tilde{S}_{s,u,d}(x) = C S_{s,u,d}^T(x) C. \quad (10)$$

We work with the light quark propagator  $S_q^{ab}(x)$  defined in the form

$$\begin{aligned}
S_q^{ab}(x) &= i\delta_{ab}\frac{\not{x}}{2\pi^2x^4} - \delta_{ab}\frac{m_q}{4\pi^2x^2} - \delta_{ab}\frac{\langle\bar{q}q\rangle}{12} \\
&+ i\delta_{ab}\frac{\not{x}m_q\langle\bar{q}q\rangle}{48} - \delta_{ab}\frac{x^2}{192}\langle\bar{q}g_s\sigma Gq\rangle + i\delta_{ab}\frac{x^2\not{x}m_q}{1152} \\
&\times\langle\bar{q}g_s\sigma Gq\rangle - i\frac{g_sG_{ab}^{\alpha\beta}}{32\pi^2x^2}[\not{x}\sigma_{\alpha\beta} + \sigma_{\alpha\beta}\not{x}] \\
&- i\delta_{ab}\frac{x^2\not{x}g_s^2\langle\bar{q}q\rangle^2}{7776} - \delta_{ab}\frac{x^4\langle\bar{q}q\rangle\langle g_s^2G^2\rangle}{27648} + \dots \quad (11)
\end{aligned}$$

Let us emphasize that in calculations we set the light quark masses  $m_u$  and  $m_d$  equal to zero, preserving at the same time dependence of the propagator  $S_s^{ab}(x)$  on the  $m_s$ .

We will treat with  $\langle 0|G_{\mu\alpha}(x)G_{\nu\alpha'}(0)|0\rangle$  in Eq. (9) in two different ways: first we will replace it by the gluon full-propagator in space representation, i.e.,

$$\begin{aligned}
\langle 0|G_{\mu\alpha}(x)G_{\nu\alpha'}(0)|0\rangle &= \frac{1}{2\pi^2x^4}[g_{\alpha\alpha'}(g_{\mu\nu} - \frac{4x_\mu x_\nu}{x^2}) \\
&+ (\alpha, \alpha') \leftrightarrow (\mu, \nu) - \alpha \leftrightarrow \mu - \alpha' \leftrightarrow \nu], \quad (12)
\end{aligned}$$

and do all calculations. Such calculations are equivalent to the diagrams with valence-gluon as a full propagator. Secondly, we will write it in terms of gluon condensate

using

$$\langle 0|G_{\mu\alpha}(x)G_{\nu\alpha'}(0)|0\rangle = \frac{\langle g_s^2G^2\rangle}{96}[g_{\mu\nu}g_{\alpha\alpha'} - g_{\mu\alpha'}g_{\nu\alpha}], \quad (13)$$

which represents the diagrams containing the gluon interacting with the QCD vacuum.

The correlation function  $\Pi_{\Lambda_H}^{\text{OPE}}(q)$  can be decomposed over the Lorentz structures  $\sim \not{q}$  and  $\sim I$ . In calculations, we choose the terms  $\sim \not{q}$ .

The chosen invariant amplitude  $\Pi^{\text{OPE}}(q^2)$  can be written down as the dispersion integral

$$\Pi^{\text{OPE}}(q^2) = \int_{m_s^2}^{\infty} \frac{\rho^{\text{OPE}}(s)}{s - q^2} ds + \dots, \quad (14)$$

where  $\rho^{\text{OPE}}(s)$  is the two-point spectral density obtained after lengthy calculations on OPE side and taking the imaginary part of the obtained result. The spectral density corresponding to the structure  $\not{q}$  is obtained as

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert.}}(s) + \sum_{k=3}^{10} \rho_k(s), \quad (15)$$

where  $\rho^{\text{pert.}}(s)$  is the perturbative part of the obtained result and by  $\rho_k(s)$  we denote the nonperturbative contributions to  $\rho^{\text{OPE}}(s)$ . The perturbative and nonperturbative parts of the spectral density are obtained as:

$$\begin{aligned}
\rho^{\text{pert.}}(s) &= -\frac{g_s^2s^4}{491520\pi^6}, \\
\rho_3(s) &= \frac{g_s^2m_s s^2 [\langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle + \langle\bar{u}u\rangle]}{4096\pi^4}, \\
\rho_4(s) &= 0, \\
\rho_5(s) &= -\frac{g_s^2m_0^2m_s s [3\langle\bar{d}d\rangle - 4\langle\bar{s}s\rangle + 3\langle\bar{u}u\rangle]}{6144\pi^4}, \\
\rho_6(s) &= -\frac{g_s^2s [\langle\bar{d}d\rangle^2g_s^2 + 27\pi^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle + 27\pi^2\langle\bar{d}d\rangle(\langle\bar{s}s\rangle + \langle\bar{u}u\rangle) + g_s^2(\langle\bar{s}s\rangle^2 + \langle\bar{u}u\rangle^2)]}{10348\pi^4}, \\
\rho_7(s) &= g_s^2\langle\alpha_s\frac{G^2}{\pi}\rangle\frac{m_s [\langle\bar{d}d\rangle + \langle\bar{s}s\rangle + \langle\bar{u}u\rangle]}{6144\pi^2} + \frac{1}{256}\langle\alpha_s\frac{G^2}{\pi}\rangle m_s [\langle\bar{d}d\rangle + \langle\bar{u}u\rangle], \\
\rho_8(s) &= \frac{g_s^2m_0^2 [\langle\bar{s}s\rangle\langle\bar{u}u\rangle + \langle\bar{d}d\rangle(\langle\bar{s}s\rangle + \langle\bar{u}u\rangle)]}{256\pi^2}, \\
\rho_9(s) &= 0, \\
\rho_{10}(s) &= 0. \quad (16)
\end{aligned}$$

Now applying the Borel transformation to  $\Pi^{\text{OPE}}(q^2)$ , equating the obtained expression with the relevant part of the function  $\mathcal{B}_{q^2}\Pi_{\Lambda_H}^{\text{Had}}(q)$ , and subtracting the continuum contribution we get the required sum rules. Thus the mass of the  $\Lambda$  state can be evaluated from the sum

rule

$$m_\Lambda^2 = \frac{\int_{m_s^2}^{s_0} ds s \rho^{\text{OPE}}(s) e^{-s/M^2}}{\int_{m_s^2}^{s_0} ds \rho(s) e^{-s/M^2}}. \quad (17)$$

Parameters	Values
$m_s$	$96_{-4}^{+8}$ MeV
$\langle \bar{q}q \rangle$	$(-0.24 \pm 0.01)^3$ GeV <sup>3</sup>
$\langle \bar{s}s \rangle$	$0.8 \langle \bar{q}q \rangle$
$m_0^2$	$(0.8 \pm 0.1)$ GeV <sup>2</sup>
$\langle \bar{s}g_s \sigma G s \rangle$	$m_0^2 \langle \bar{s}s \rangle$
$\langle \frac{\alpha_s G^2}{\pi} \rangle$	$(0.012 \pm 0.004)$ GeV <sup>4</sup>

TABLE I: Input parameters.

To extract the residue  $\lambda_\Lambda$  we can employ the sum rule

$$\lambda_\Lambda^2 e^{-m_\Lambda^2/M^2} = \int_{m_s^2}^{s_0} ds \rho^{\text{QCD}}(s) e^{-s/M^2}. \quad (18)$$

### III. NUMERICAL RESULTS

The QCD sum rules for the mass and residue of the  $\Lambda(1405)$  contain various parameters that should be fixed in accordance with the standard procedures. Thus, for numerical computation of the  $m_\Lambda$  and  $\lambda_\Lambda$  we need values of the quark, gluon and mixed condensates as well as the  $s$  quark mass. The values of these parameters can be found in Table I. The QCD sum rules for the physical quantities under consideration additionally depend

on the continuum threshold  $s_0$  and Borel parameter  $M^2$ . One needs to fix some regions, where physical quantities are practically independent of or demonstrate weak dependence on these auxiliary parameters according to the standard prescription. To find the working window for the Borel parameter, we require the convergence of the operator product expansion as well as adequate suppression of the contributions arising from the higher resonances and continuum. As a result we find the interval

$$1.8 \text{ GeV}^2 \leq M^2 \leq 3.6 \text{ GeV}^2, \quad (19)$$

for the Borel mass parameter. Our analyses show that in the interval

$$2.1 \text{ GeV}^2 \leq s_0 \leq 2.3 \text{ GeV}^2, \quad (20)$$

the results relatively weakly depend on the continuum threshold  $s_0$ . By varying the parameters  $M^2$  and  $s_0$  within the allowed ranges, as well as taking into account the errors coming from other input parameters we estimate uncertainties of the whole calculations. The mass  $m_\Lambda$  and residue  $\lambda_\Lambda$  are depicted as functions of the Borel and threshold parameters in Figs. 1 and 2. From these figures we see that the results demonstrate good stability with respect to the helping parameters  $M^2$  and  $s_0$  in their working windows.

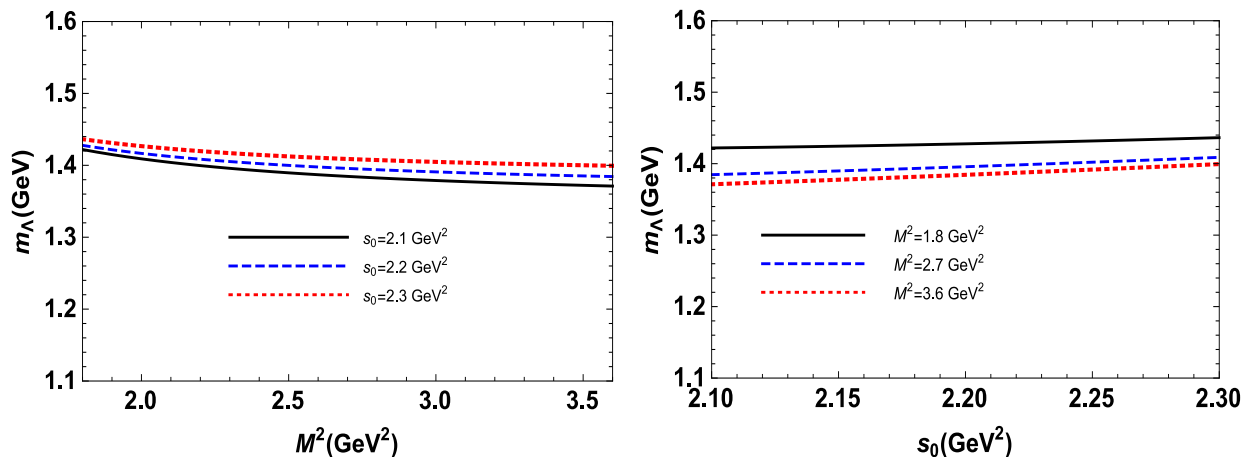


FIG. 1: The mass  $m_\Lambda$  as a function of the Borel parameter  $M^2$  at different fixed values of  $s_0$  (left panel), and as a function of the threshold  $s_0$  at fixed values of  $M^2$  (right panel).

Obtained from our analyses, the average values of the mass and residue for  $\Lambda(1405)$  are depicted in table II. For comparison, we also depict the QCD sum rules prediction on the mass of this state from Ref. [73] and the average experimental value from PDG [74] in the same

table. From this table we see that our result on the mass is nicely consistent with the prediction of [73] and the average experimental value. Our prediction on the residue of  $\Lambda(1405)$  may be checked via different approaches.

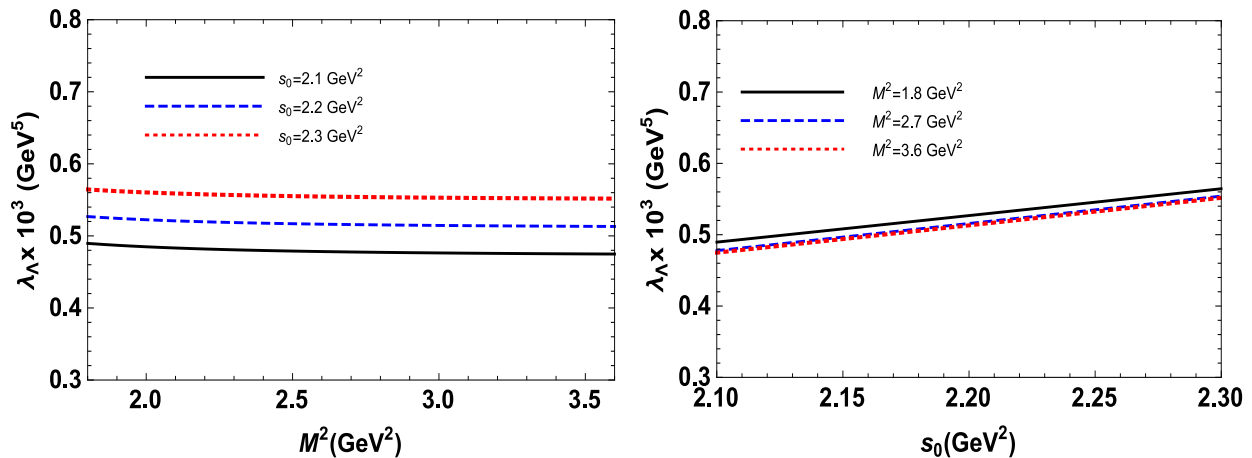


FIG. 2: The residue  $\lambda_\Lambda$  as a function of the Borel parameter  $M^2$  at different fixed values of  $s_0$  (left panel), and as a function of the threshold  $s_0$  at fixed values of  $M^2$  (right panel).

	Present Work	[73]	Experiment [74]
$m_\Lambda$	$1403^{+33}_{-32}$ MeV	1407 MeV	$(1405.1^{+1.3}_{-1.0})$ MeV
$\lambda_\Lambda$	$0.52^{+0.05}_{-0.04} \times 10^{-3}$ GeV <sup>5</sup>	—	—

TABLE II: Values for the mass and residue of the  $\Lambda(1405)$  state.

#### IV. CONCLUDING REMARKS

In this letter we reported the QCD sum rules predictions on the mass and residue of the  $\Lambda(1405)$  considering it as a hybrid baryon with three-quark—one gluon internal organization and the quantum numbers  $I(J^P) = 0(1/2^-)$ . We found the mass of the  $\Lambda(1405)$  baryon, which is in a good agreement with its average experimental value, as well as with other theoretical pre-

dictions. Our prediction for the residue of  $\Lambda(1405)$ , which has been the essential aim of the present investigation, can be used as input information to study its strong, weak and electromagnetic interactions with other particles, and determine widths of its various decays. Such calculations, especially investigation of internal charge distribution of the  $\Lambda(1405)$  baryon and its multipole moments together with the comparison of the obtained predictions on its mass and width with experimental data will allow us with high confidence level to determine whether the  $\Lambda(1405)$  is a hybrid baryon or not.

#### ACKNOWLEDGEMENTS

K. A. and B. B. thank TÜBİTAK for partial financial support provided under the grant no: 115F183.

- 
- [1] S. L. Olsen, *Hyperfine Interact.* **229**, no. 1-3, 7 (2014) [arXiv:1403.1254 [hep-ex]].
- [2] S. K. Choi *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **91**, 262001 (2003) [hep-ex/0309032].
- [3] C. Shen, *AIP Conf. Proc.* **1735**, 060002 (2016).
- [4] A. Pimikov, H. J. Lee, N. Kochelev and P. Zhang, *Phys. Rev. D* **95**, no. 7, 071501 (2017) [arXiv:1611.08698 [hep-ph]].
- [5] J. Ho, D. Harnett and T. G. Steele, arXiv:1609.06750 [hep-ph].
- [6] M. S. Chanowitz and S. R. Sharpe, *Nucl. Phys. B* **222**, 211 (1983) Erratum: [*Nucl. Phys. B* **228**, 588 (1983)].
- [7] S. F. Tuan, *Phys. Rev. D* **15**, 3478 (1977).
- [8] F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij and P. Santorelli, *Phys. Rev. D* **94**, no. 9, 094017 (2016) [arXiv:1608.04656 [hep-ph]].
- [9] H. X. Chen, E. L. Cui, W. Chen, X. Liu, T. G. Steele and S. L. Zhu, *Eur. Phys. J. C* **76**, no. 10, 572 (2016) [arXiv:1602.02433 [hep-ph]].
- [10] A. Gal and H. Garcilazo, *Nucl. Phys. A* **928**, 73 (2014) [arXiv:1402.3171 [nucl-th]].
- [11] R. L. Jaffe and K. Johnson, *Phys. Lett.* **60B**, 201 (1976).
- [12] T. Barnes, F. E. Close, F. de Viron and J. Weyers, *Nucl. Phys. B* **224**, 241 (1983).
- [13] T. Barnes, F. E. Close and E. S. Swanson, *Phys. Rev. D* **52**, 5242 (1995) [hep-ph/9501405].
- [14] F. E. Close and P. R. Page, *Nucl. Phys. B* **443**, 233 (1995) [hep-ph/9411301].
- [15] J. N. Hedditch, W. Kamleh, B. G. Lasscock, D. B. Leinweber, A. G. Williams and J. M. Zanotti, *Phys. Rev. D* **72**, 114507 (2005) [hep-lat/0509106].
- [16] H. Y. Jin, J. G. Korner and T. G. Steele, *Phys. Rev. D* **67**, 014025 (2003) [hep-ph/0211304].
- [17] D. Horn and J. Mandula, *Phys. Rev. D* **17**, 898 (1978).
- [18] J. Govaerts, L. J. Reinders, P. Francken, X. Gonze and J. Weyers, *Nucl. Phys. B* **284**, 674 (1987).

- [19] R. Kleiv, arXiv:1407.2292 [hep-ph].
- [20] W. Chen, T. G. Steele and S. L. Zhu, *J. Phys. G* **41**, 025003 (2014) [arXiv:1306.3486 [hep-ph]].
- [21] W. Chen, T. G. Steele and S. L. Zhu, *The Universe* **2**, 13 (2014) [arXiv:1403.7457 [hep-ph]].
- [22] A. A. Petrov, hep-ph/9808347.
- [23] L. Liu *et al.* [Hadron Spectrum Collaboration], *JHEP* **1207**, 126 (2012) [arXiv:1204.5425 [hep-ph]].
- [24] T. Barnes, nucl-th/0009011.
- [25] A. Engler, H. E. Fisk, R. w. Kraemer, C. M. Meltzer and J. B. Westgard, *Phys. Rev. Lett.* **15**, 224 (1965).
- [26] R. H. Dalitz, T. C. Wong and G. Rajasekaran, *Phys. Rev.* **153**, 1617 (1967).
- [27] D. W. Thomas, A. Engler, H. E. Fisk and R. W. Kraemer, *Nucl. Phys. B* **56**, 15 (1973).
- [28] R. J. Hemingway, *Nucl. Phys. B* **253**, 742 (1985).
- [29] M. Niiyama *et al.*, *Phys. Rev. C* **78**, 035202 (2008) [arXiv:0805.4051 [hep-ex]].
- [30] K. Moriya *et al.* [CLAS Collaboration], *Phys. Rev. C* **88**, 045201 (2013) Addendum: [*Phys. Rev. C* **88**, no. 4, 049902 (2013)] [arXiv:1305.6776 [nucl-ex]].
- [31] G. Agakishiev *et al.* [HADES Collaboration], *Phys. Rev. C* **87**, 025201 (2013) [arXiv:1208.0205 [nucl-ex]].
- [32] K. Moriya *et al.* [CLAS Collaboration], *Phys. Rev. Lett.* **112**, no. 8, 082004 (2014) [arXiv:1402.2296 [hep-ex]].
- [33] S. Cho *et al.* [ExHIC Collaboration], arXiv:1702.00486 [nucl-th].
- [34] D. Jido, E. Oset and T. Sekihara, *Eur. Phys. J. A* **42**, 257 (2009) [arXiv:0904.3410 [nucl-th]].
- [35] T. Hyodo, W. Weise, D. Jido, L. Roca and A. Hosaka, *Mod. Phys. Lett. A* **23**, 2393 (2008) [arXiv:0802.2212 [hep-ph]].
- [36] Y. Akaishi, T. Yamazaki, M. Obu and M. Wada, *Nucl. Phys. A* **835**, 67 (2010) [arXiv:1002.2560 [nucl-th]].
- [37] C. S. An, B. Saghai, S. G. Yuan and J. He, *Phys. Rev. C* **81**, 045203 (2010) [arXiv:1002.4085 [nucl-th]].
- [38] D. Jido, T. Sekihara, Y. Ikeda, T. Hyodo, Y. Kanada-En'yo and E. Oset, *Nucl. Phys. A* **835**, 59 (2010) [arXiv:1003.4560 [nucl-th]].
- [39] A. Martinez Torres and D. Jido, *Phys. Rev. C* **82**, 038202 (2010) [arXiv:1008.0457 [nucl-th]].
- [40] C. An and B. Saghai, *Int. J. Mod. Phys. A* **26**, 619 (2011) [arXiv:1008.1177 [nucl-th]].
- [41] T. T. Takahashi and M. Oka, *Prog. Theor. Phys. Suppl.* **186**, 172 (2010) [arXiv:1009.1790 [hep-lat]].
- [42] T. Sekihara, T. Hyodo and D. Jido, *Phys. Rev. C* **83**, 055202 (2011) [arXiv:1012.3232 [nucl-th]].
- [43] T. Hyodo and D. Jido, *Prog. Part. Nucl. Phys.* **67**, 55 (2012) [arXiv:1104.4474 [nucl-th]].
- [44] T. Sekihara, T. Hyodo and D. Jido, arXiv:1109.0061 [nucl-th].
- [45] B. J. Menadue, W. Kamleh, D. B. Leinweber and M. S. Mahbub, *Phys. Rev. Lett.* **108**, 112001 (2012) [arXiv:1109.6716 [hep-lat]].
- [46] A. Martinez Torres, M. Bayar, D. Jido and E. Oset, *Phys. Rev. C* **86**, 055201 (2012) [arXiv:1202.4297 [hep-lat]].
- [47] J. Revai, *Few Body Syst.* **54**, 1865 (2013) Erratum: [*Few Body Syst.* **54**, 1877 (2013)] [arXiv:1203.1813 [nucl-th]].
- [48] J. A. Oller, *Int. J. Mod. Phys. Conf. Ser.* **26**, 1460096 (2014) [arXiv:1309.2196 [nucl-th]].
- [49] S. X. Nakamura and D. Jido, *PTEP* **2014**, 023D01 (2014) [arXiv:1310.5768 [nucl-th]].
- [50] T. Sekihara and S. Kumano, *Phys. Rev. C* **89**, no. 2, 025202 (2014) [arXiv:1311.4637 [nucl-th]].
- [51] B. J. Menadue, W. Kamleh, D. B. Leinweber, M. Selim Mahbub and B. J. Owen, *PoS LATTICE* **2013**, 280 (2014) [arXiv:1311.5026 [hep-lat]].
- [52] S. X. Nakamura and D. Jido, *PoS Hadron* **2013**, 141 (2013) [arXiv:1312.6768 [nucl-th]].
- [53] A. Dot and T. Myo, *Nucl. Phys. A* **930**, 86 (2014) [arXiv:1406.1540 [nucl-th]].
- [54] S. Ohnishi, Y. Ikeda, T. Hyodo, E. Hiyama and W. Weise, *J. Phys. Conf. Ser.* **569**, no. 1, 012077 (2014) [arXiv:1408.0118 [nucl-th]].
- [55] J. M. M. Hall, W. Kamleh, D. B. Leinweber, B. J. Menadue, B. J. Owen, A. W. Thomas and R. D. Young, *Phys. Rev. Lett.* **114**, no. 13, 132002 (2015) [arXiv:1411.3402 [hep-lat]].
- [56] T. Sekihara and S. Kumano, *JPS Conf. Proc.* **8**, 022006 (2015) [arXiv:1411.3414 [hep-ph]].
- [57] J. M. M. Hall, W. Kamleh, D. B. Leinweber, B. J. Menadue, B. J. Owen, A. W. Thomas and R. D. Young, *PoS LATTICE* **2014**, 094 (2014) [arXiv:1411.3781 [hep-lat]].
- [58] S. i. Nam and H. K. Jo, arXiv:1503.00419 [hep-ph].
- [59] K. Miyahara and T. Hyodo, *Phys. Rev. C* **93**, no. 1, 015201 (2016) [arXiv:1506.05724 [nucl-th]].
- [60] K. Miyahara, T. Hyodo and E. Oset, *Phys. Rev. C* **92**, no. 5, 055204 (2015) [arXiv:1508.04882 [nucl-th]].
- [61] K. Miyahara and T. Hyodo, *JPS Conf. Proc.* **10**, 022011 (2016) [arXiv:1508.07707 [nucl-th]].
- [62] J. He and P. L. Lu, *Int. J. Mod. Phys. E* **24**, no. 11, 1550088 (2015) [arXiv:1510.00580 [nucl-th]].
- [63] S. Ohnishi, Y. Ikeda, T. Hyodo and W. Weise, *Phys. Rev. C* **93**, no. 2, 025207 (2016) [arXiv:1512.00123 [nucl-th]].
- [64] K. Miyahara and T. Hyodo, arXiv:1512.02735 [nucl-th].
- [65] C. Fernandez-Ramirez, I. V. Danilkin, V. Mathieu and A. P. Szczepaniak, *Phys. Rev. D* **93**, no. 7, 074015 (2016) [arXiv:1512.03136 [hep-ph]].
- [66] T. Hyodo, *AIP Conf. Proc.* **1735**, 020012 (2016) [arXiv:1512.04708 [hep-ph]].
- [67] R. Molina and M. Dring, *Phys. Rev. D* **94**, no. 5, 056010 (2016) Addendum: [*Phys. Rev. D* **94**, no. 7, 079901 (2016)] [arXiv:1512.05831 [hep-lat]].
- [68] Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset and W. Weise, *Nucl. Phys. A* **954**, 41 (2016) [arXiv:1602.08852 [hep-ph]].
- [69] Z. W. Liu, J. M. M. Hall, D. B. Leinweber, A. W. Thomas and J. J. Wu, *Phys. Rev. D* **95**, no. 1, 014506 (2017) [arXiv:1607.05856 [nucl-th]].
- [70] F. Y. Dong, B. X. Sun and J. L. Pang, arXiv:1609.08354 [nucl-th].
- [71] J. M. M. Hall, W. Kamleh, D. B. Leinweber, B. J. Menadue, B. J. Owen and A. W. Thomas, *Phys. Rev. D* **95**, no. 5, 054510 (2017) [arXiv:1612.07477 [hep-lat]].
- [72] S. H. Kim, S. i. Nam, D. Jido and H. C. Kim, arXiv:1702.08645 [hep-ph].
- [73] L. S. Kisslinger and E. M. Henley, *Eur. Phys. J. A* **47**, 8 (2011) [arXiv:0911.1179 [hep-ph]].
- [74] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C*, **40**, 100001 (2016).