Introductory Course on Black Hole Physics and AdS/CFT Duality

Lecturer: M.M. Sheikh-Jabbari

This is a PhD level course, designed for second year PhD students in Theoretical High Energy Physics (HEP-TH) area and assumes a background knowledge of Quantum Field Theory at the level of Peskin-Schroeder book and General Relativity, basic familiarity with Schwarzchild solution and Einstein equations.

The course will have two essentially separate parts. First part will be about physics of black holes and the second will be on the AdS/CFT correspondence and how it is formulated.

Part I: Black hole physics

Our black hole discussions consists of two parts,

First: *classical aspects of black holes*, and **Second:** *quantum aspects of black holes*.

Main references for this part of the course are:

Modeling Black Hole Evaporation By *A. Fabbri & J. Navarro-Salas*, Imperial College Press, 2005.

Part I of **An Introduction to Black holes, Information and the string theory revolution** By *L. Susskind & J. Lindesay*, World Scientific, 2005.

Black Holes Lecture notes by *P.K. Townsend*, DAMTP, University of Cambridge, UK, *arXiv:gr-qc/9707012*.

My estimate for this part is 21-23 sessions. Here are more detailed description of the course topics.

I.1 Classical aspects of black holes

Opening and basics of Sch'd solution, *Number of estimated sessions*: 4

- Quick review of Ein.GR; conventions; Ein.Eq. and Ein.Hil. action.
- General spherical symmetric vacuum solution; Birkhoff & Israel theorems. Interplay of being 1) Sph.Sym., 2) Vacuum sol'n, 3) static.
- Sch'd sol'n in Sch'd coord.; some discussions about horizon and singularity, curvature invariants.
- Killing vectors and symmetries of Sch'd sol'n.
- Geodesic motion around the black hole; surface gravity.
- Near horizon metric.
- Physical Observers, Frefos and Fidos.

- Redshift factors for different observers; Null surfaces; Killing horizon, surface gravity and Rindler space; degenerate Killing horizons.
- Other coordinate systems: Eddington-Finkelstein coord.; Kruskal-Szekeres coord.
- Maximal Analytic Extension; Einstein-Rosen bridge, white hole and worm hole.
- Penrose diagram and Causal structure; Rindler and flat space as other examples.
- Acceleration horizons and more on Rindler space.
- Asymptopia and horizons; event and apparent horizons; trapped surfaces. (section 2.8 of Navaro-Salas).
- Black holes vs. naked singularities, cosmic censorship conjecture.

Black hole formation, gravitational collapse, *Number of estimated sessions*: 2

- Spherical symmetric pressure-free collapse onto a black hole.
- Gravitational collapse, Chandrasekhar limit.
- Oppenheimer-Volkof equation.
- Oppenheimer-Synder model.
- Brief discussion on non-spherical collapse and black hole formation

Charged black holes, *Number of estimated sessions*: 2

- Einstein-Maxwell theory and its e.o.m.
- RN black holes, their isometries.
- The three cases $M \lessgtr Q$.
- Can electron be a charged black hole?!
- Inner and outer horizons, Killing horizons of RN black hole, the Cauchy horizon.
- Penrose diagrams of the three cases.
- Collapse onto a charged black hole.
- AdS_2 throat of extremal RN.
- Multi-center extremal black hole.

Stationary, rotating black holes, *Number of estimated sessions*: 3

- Uniqueness theorems for axisymmetric spacetimes.
- Hawking-Wald theorem: vacuum Einstein eq.+ stationary $\Longrightarrow$ Kerr geometry.
- Kerr metric, its isometries, asymptopia.
- The three cases $M \gtreqless a$.
- Horizons, ergosphere and Singularity structure.
- Angular velocity of horizon(s).
- Causal structure of the three cases.
- Penrose process and extracting energy from the hole.
- Axisymmetric Charged black holes, Kerr-Newman solution.

Charge and Energy Momentum of black holes, *Number of estimated sessions*: 2
In this section we will mainly follow section 6 of Townsend lecture notes.

- Covariant formula for (electric) charge.
- Energy Momentum tensor and ADM mass and angular momentum.
- Komar integrals.
- Energy conditions

I.2 Quantum aspects of black holes

Quantum fields on black hole background, *Number of estimated sessions*: 3

- Scalar field on Sch'd background, (section 2 of Susskind).
- The near horizon limit of the field equation.
- Quantum fields on Rindler space, the *Unruh effect*.
- The proper temperature.
- Unruh density matrix and entanglement.
- Quantum fields in curved space: Bogolubov transformations, particle production.
- Hawking radiation.

The Entropy, *Number of estimated sessions*: 2

- Entropy of quantum field in Rindler background.
- Bekenstein argument, black hole entropy,
- the Area Law.

Black hole Thermodynamics, *Number of estimated sessions*: 3

- Zeroth law, the temperature.
- The Smarr formula, and the First law.
- The second Law.
- Black hole evaporation.
- The third Law?!
- Black hole complementarity and its possible loopholes.

Part II: AdS/CFT

The celebrated AdS/CFT correspondence, since its conception about 15 years ago, has dominated the whole HEP-TH area and beyond. Having a basic knowledge of the correspondence and its formulation has now become an essential part of background knowledge a theoretical physicist is expected to be equipped with. In this part, we present basic ideas, statement and a formulation of the correspondence. We do not, however, intend to provide a detailed string theory motivations and analysis led to the duality.

Main references for this part of the course are:

Part II of **An Introduction to Black holes, Information and the string theory revolution** By *L. Susskind & J. Lindesay*, World Scientific, 2005.

arXiv: 1010.6134 by **J. Polchinski**,

arXiv: hep-th/0209067 by **K. Skenderis** and,

Chapters 4, 5 of *arXiv: 1101.0618* by **Hong Liu et al**.

This part is estimated to be 12-14 sessions. Topics which will be covered here include

Some fact about the AdS space, *Number of estimated sessions*: 4

- Anti de Sitter (AdS) space as solutions to gravity theories in various dimensions
- Causal and geodesic structure of AdS space
- Asymptotically AdS spaces
- Field theory on AdS space

Short review on Conformal Field theories, *Number of estimated sessions*: 4

- The conformal group in various dimensions
- Conformal field theories in 4 dimensions
- Correlation functions and conformal symmetry
- 't Hooft expansion of the large N Yang-Mills theories
- Conformal (trace) anomaly.
- SuperConformal groups and algenras

The statement of AdS/CFT correspondence, *Number of estimated sessions*: 2

- Statement of the AdS/CFT duality,
- AdS/CFT as a tool to study strongly coupled field theories
- AdS/CFT as a tool to study non-zero temperature field theories

Holographic renormalization, *Number of estimated sessions*: 2

Date and time of lectures

Place: **Lectures will be in Farmanieh Bldg of IPM.**

Time: **Saturday and Monday morning, 9-11am.**

Starting date: **1st of Mehr, 22nd of Sept.**

Tutorials: Dr M. Ali-Akbari and Dr H. Ebrahim will be my assistants in this course and there will be tutorial classes, once per week. The date and timing of that will be fixed later.