Abstract

An interesting conjecture, $Z_{BH} = |Z_{top}|^2$, as recently proposed by Ooguri, Strominger and Vafa, connects the free energy of topological strings with the degeneracy of 4-dimensional BPS black hole states arising in $\mathcal{N}= 2$ compactifications of IIA(B) string theories. In the original proposal the black hole ensemble is a *mixed* ensemble, in the sense that it treats the magnetic charges microcanonically while the electric charges canonically. Consequently, the results for degeneracy of states as obtained from the ensemble inverse-Laplace integration, suffer from prefactors which do not respect the (relevant) *electric-magnetic dualities*. One idea to overcome this deficiency, as claimed recently, is imposing a nontrivial measure for the ensemble sum.

We address this problem and *upon a redefinition of the OSV ensemble* whose variables are as numerous as the electric potentials, show that for restoring the symmetry *no non-Euclidean measure is needful*. In detail, we rewrite the OSV free energy as a function of new variables which are combinations of the electric-potentials and the black hole charges. Subsequently the Legendre transformation which bridges between the entropy and the black hole free energy in terms of these variables, points to a generalized ensemble. In this context we will consider *all* the cases of relevance: *small* and *large* black holes, with or without D_6-brane charge. For the case of vanishing D_6-brane charge, the ensemble is *pure canonical* and the electric-magnetic duality is restored *exactly*, leading to the proper results for the black hole degeneracy of states. For more general cases, the construction still works well *as far as* the violation of the duality by the corresponding OSV result is restricted to a *prefactor*. In a concrete example we shall show that for black holes with non-vanishing D_6-brane charge, there are cases where the duality violation *goes beyond this restriction*, thus imposing non-trivial measures is *incapable* of restoring the duality. This observation signals for a *deeper modification* in the OSV proposal.