Electrostatic properties and stability of virus-like nano-shells are examined in ionic solutions with monovalent and multivalent ions. A theoretical model based on a thin charged spherical shell and multivalent ions within the "dressed multivalention" approximation, yielding their distribution across the shell, is compared with extensive implicit Monte-Carlo simulations. It is found to be accurate for positive or low negative surface charge densities of the shell and for sufficiently high (low) monovalent (multivalent) salt concentrations. Phase diagrams involving electrostatic(osmotic) pressure exhibit positive and negative values, corresponding to an outward and an inward facing force on the shell, respectively. This provides an explanation for the high sensitivity of viral shell stability and self-assembly of viral shells on the ionic environment.