The wrapping and internalization of nanoparticles by biomembranes plays a critical role in drug delivery applications and nanomedicine. The wrapping transition of a single spherical particle adsorbed on a vesicle is obtained by solving the shape equations for rotationally symmetric vesicles. These shape equations are based on the Helfrich bending energy of the vesicle membrane. We predict different regimes for partial and full wrapping of the nanoparticle and relate the wrapping transition and its energy barrier to the reduced volume of the vesicle and the relative size of the vesicle and particle. We also study cooperative wrapping of several spherical nanoparticles using simulated annealing Monte Carlo simulations of triangulated vesicles. We report novel tubular membrane structures induced by the nanoparticles, which we obtain from energy minimization. The membrane tubules enclose linear aggregates of particles and protrude into the vesicles. The high stability of the particle-filled tubules implies strongly attractive, membrane-mediated interactions between the particles. The tubular structures may provide a new route to encapsulate nanoparticles reversibly in vesicles.