QCD sum rule and its application to hadron physics

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Outline

❖ Introduction to QCD sum rules

❖ Hadronic spectroscopy

✓ Calculation of mass and residue in vacuum

✓ Calculation of mass and residue at finite temperature/density

❖ Decay properties of some hadrons:

✓ Electromagnetic properties and radiative decays

✓ Semileptonic decays

✓ Strong and hadronic decays

✓ Conclusion

Introduction to QCD sum rules

This is one of the most attractive and applicable non-perturbative phenomenological tools to Hadron physics.

QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \sum_q \bar{\psi}_q(i\not{D} - m_q)\psi_q;$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu \,,$$

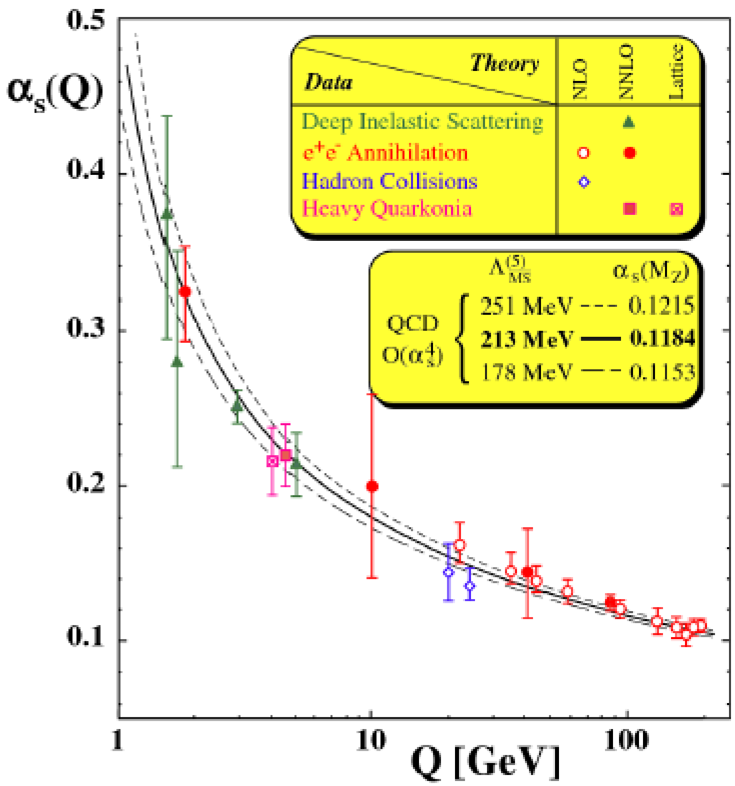
$$\not{D} = \mathcal{D}_\mu \gamma^\mu$$

$$\mathcal{D}_\mu = \partial_\mu + i\frac{g}{2}\lambda^a G^a_\mu$$

In principle, besides the dinamics of quarks and gluons, this lagrangian should be responsible for determination of hadronic properties. Unfortunately, it is valid only in a limited region.

➢In very high energies, due to "asymptotic freedom" we can use this Lagrangian and perturbation theory. However, when energy is decreased the coupling constant between quarks and gluons becomes large and perturbation theory fails.

S. Bethke, J Phys G 26, R27 (2000)



- Hadrons are formed in low energies very far from the "asymptotic freedom" and perturbative region.

→ To investigate their properties, we need some non-perturbative approaches.

Some non-perturbative methods:

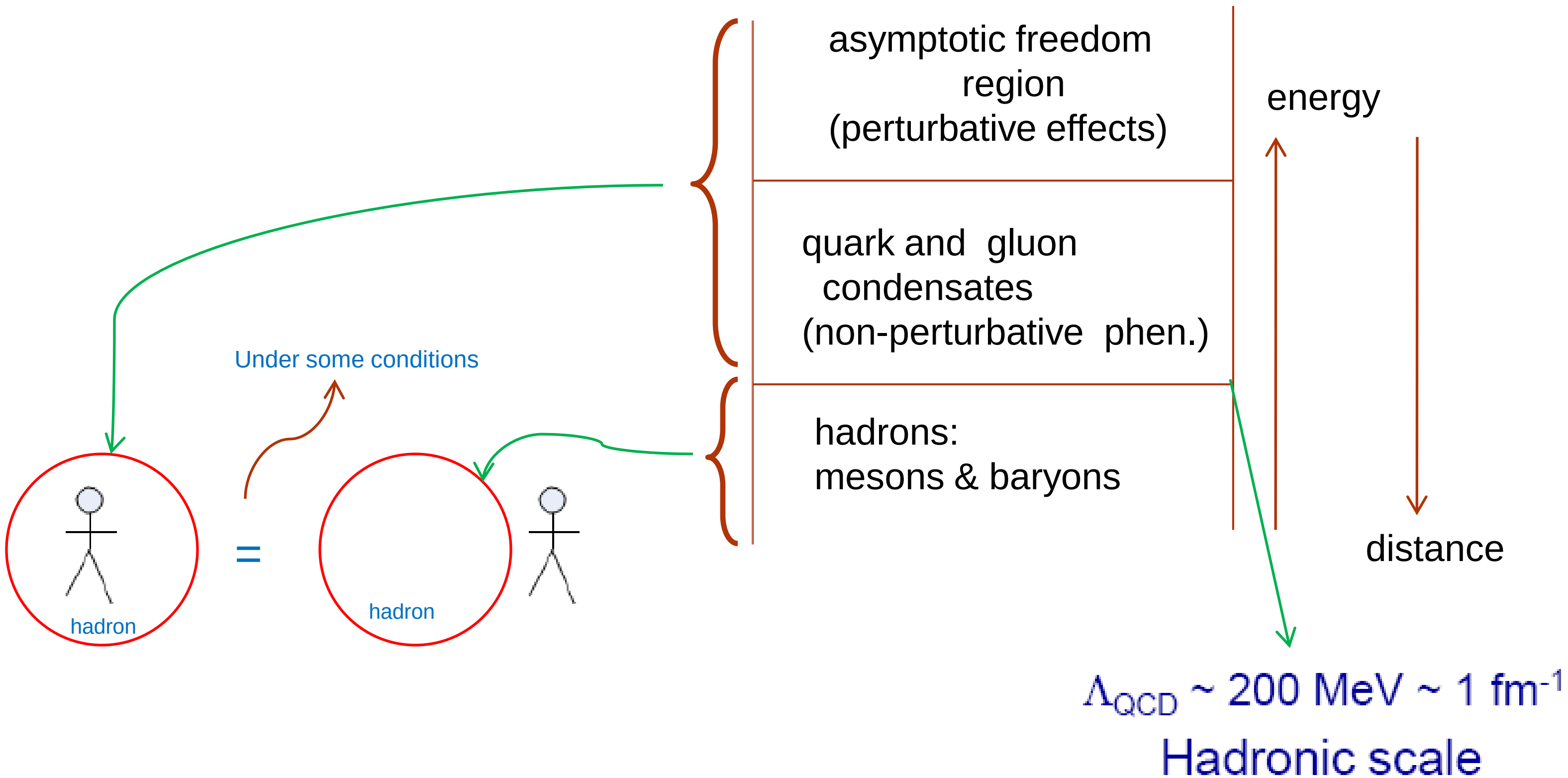
- ✓ Different "relativistic" and "non-relativistic" quark models
- ✓ HQET
- ✓ Nambu–Jona-Lasinio mode
- ✓ Lattice QCD
- ✓ ....
- ✓ QCD sum rules and its extension: light cone QCDSR.

One of the most → Applicable tools to Hadron physics

- does not include any free parameter
- is based on QCD Lagrangianin
- gives results in a good consistency with existing EXP. data
- its results agree with Lattice predi.
- can be expanded to thermal QCD

....

➢Formation of hadrons and QCDSR



QCD sum rules in technique language

✓ In this method, hadrons are represented by their interpolating quark currents.

✓ The main object in this approach is the so called correlation function expressed in terms of these interpolating currents.

- Types of the corr. func.:
  1) two point

$$T = i\int d^4x e^{ipx}\langle 0 \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0\rangle,$$

  we obtain: mass, residue (lep. decay. cons.)
  2) three point

$$T = i\int d^4x d^4y e^{ipx} e^{-ipy}\langle 0 \mid T\{\eta_1(x)\eta^{tr}(0)\bar{\eta}_2(y)\} \mid 0\rangle,$$

  we calculate: form factors used in decay rates, branchin ratio ...

3) light cone:

$$T = i \int d^4 x e^{ipx} \langle \gamma \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0 \rangle,$$

In this version, the OPE is done in terms of distribution amplitudes of the on-shell particles with different twists. This can be used in all electromagnetic, weak and strong decays.

These correlation functions are calculated in two different ways:

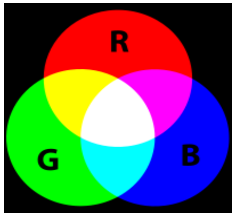
- ✓ Phenomenological or physical side: in terms of hadronic parameters
- ✓ QCD side: in terms of QCD degrees of freedom

The QCD sum rules for physical observables:

Phenomenological = QCD side

Hadrons: subatomic colorless particles made of quarks

Hadrons
- Mesons: one quark and one anti-quark (color+anticolor=white)
- Baryons: 3 quarks



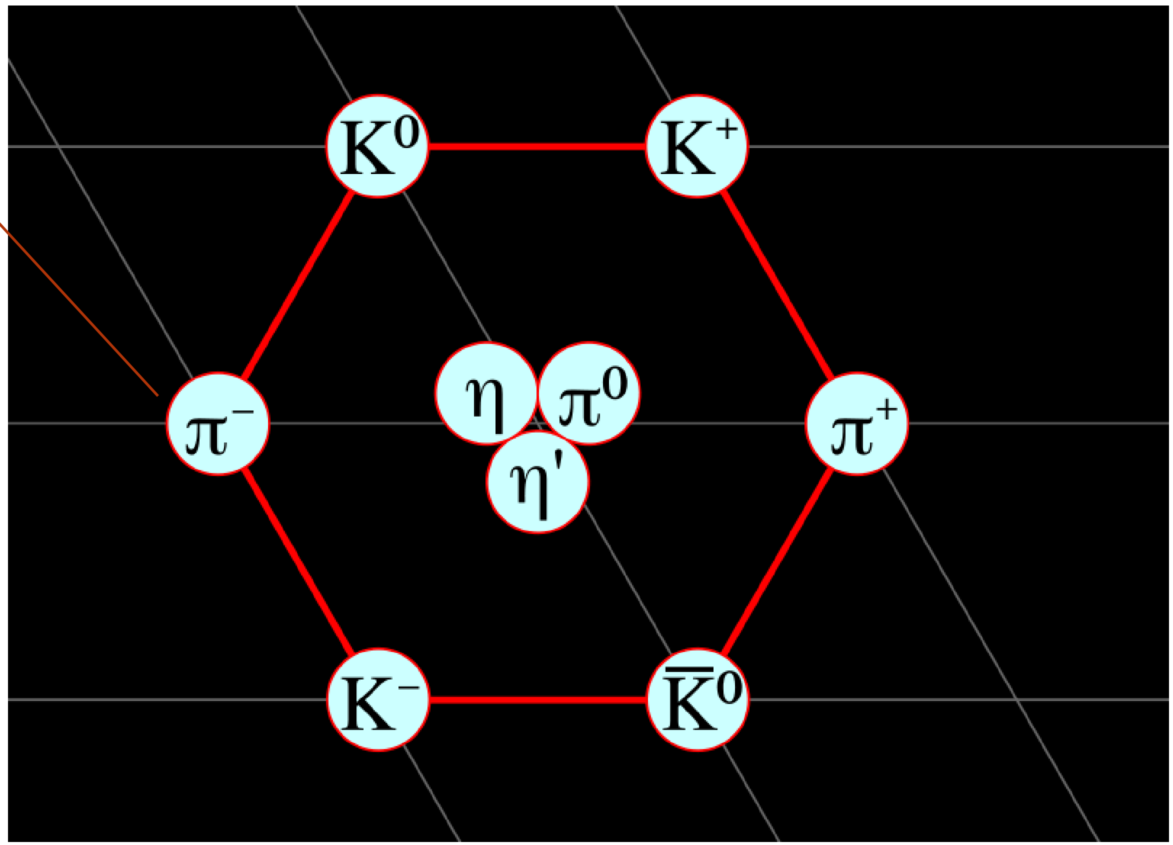
Exotic particles (experimentally not seen): Gluballs, tetraquark (dimeson), pentaquarks, hexaquarks (dibaryon), octaquarks,...

Types of mesons

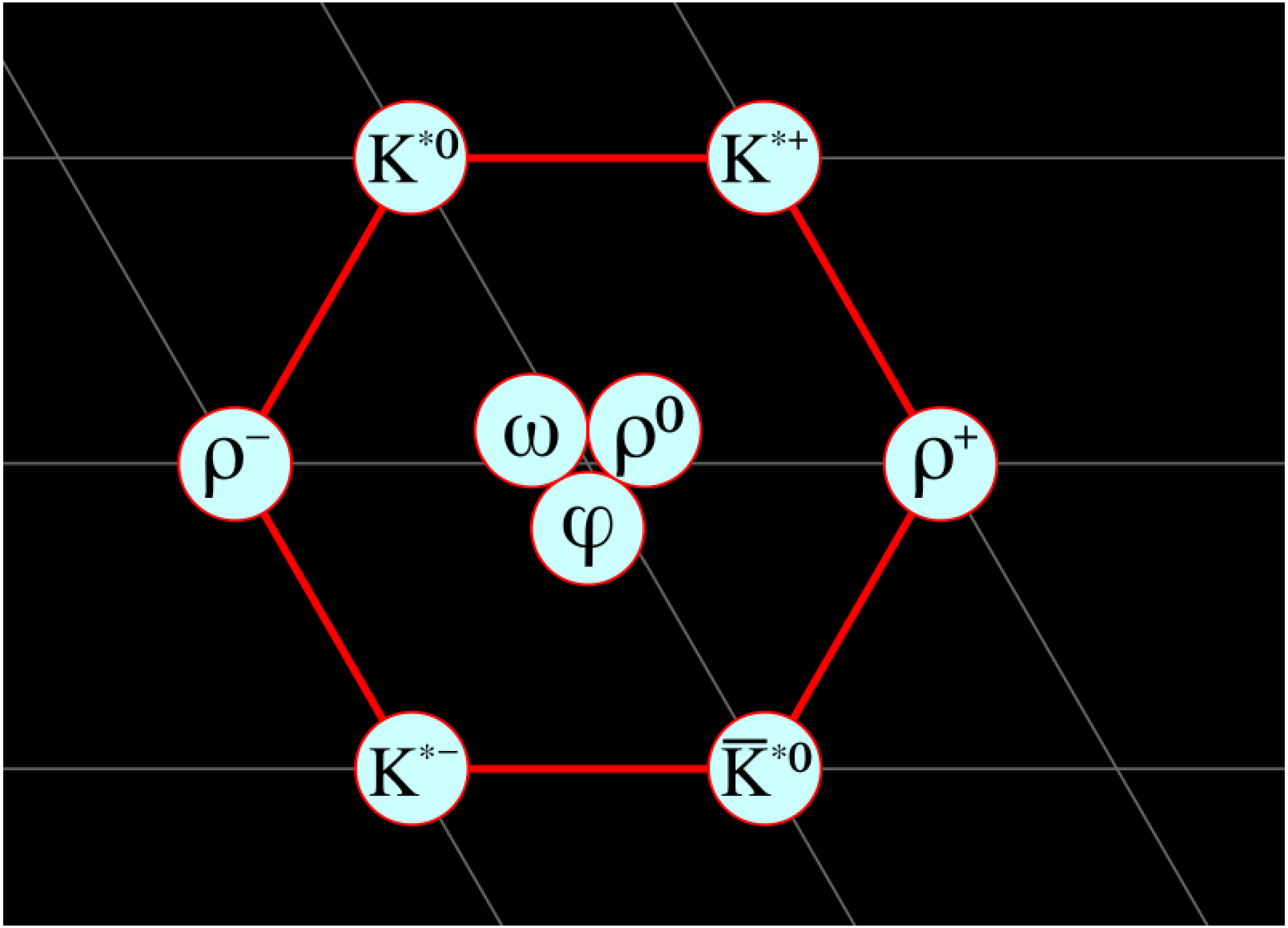
	S	L	P	J	J^P
Scalar	1	1	+	0	0^+
Pseudoscalar	0	0	−	0	0^-
Vector	1	0	−	1	1^-
Axial-vector	0	1	+	1	1^+
Tensor	1	1	+	2	2^+
Pseudotensor	1	1	-	2	2^-

PS Mesons (light)

anti u-d



Vector Mesons (light)



Light Baryons

To construct the light baryons, we consider the SU(3) flavor symmetry with quarks $q^1 = u$, $q^2 = d$ and $q^3 = s$.

$$\boxed{3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1}$$

$$\begin{aligned}
q^i \times q^j \times q^k &= \frac{1}{6}(q^i q^j q^k + q^j q^i q^k + q^i q^k q^j + q^j q^k q^i + q^k q^j q^i + q^k q^i q^j) \\
&+ \frac{1}{6}(2q^i q^j q^k + 2q^j q^i q^k - q^i q^k q^j - q^j q^k q^i - q^k q^j q^i - q^k q^i q^j) \\
&+ \frac{1}{6}(2q^i q^j q^k - 2q^j q^i q^k + q^i q^k q^j - q^j q^k q^i + q^k q^j q^i - q^k q^i q^j) \\
&+ \frac{1}{6}(q^i q^j q^k - q^j q^i q^k - q^i q^k q^j + q^j q^k q^i - q^k q^j q^i + q^k q^i q^j) \\
&= T^{\{ijk\}} + T^{\{ij\}k} + T^{[ij]k} + T^{[ijk]},
\end{aligned}$$

i, j and k goes from 1 to 3.

Light decuplet baryons: (Spin 3/2)

$$\begin{array}{lccccccc}
s=0 & \Delta^{-} & & \Delta^{0} & & \Delta^{+} & & \Delta^{++} \\
s=-1 & & \Sigma^{*-} & & \Sigma^{*0} & & \Sigma^{*0} & \\
s=-2 & & & \Xi^{*-} & & \Xi^{*0} & & \\
s=-3 & & & & \Omega^{*-} & & & \\
I_3 = & -\frac{3}{2} & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{3}{2}
\end{array}$$

quark contents for decuplet baryons are

$$\begin{array}{c}
ddd \quad udd \quad uud \quad uuu \\
sdd \quad sud \quad suu \\
ssd \quad ssu \\
sss
\end{array}$$

Light octet baryons: (spin 1/2)

$$
\begin{array}{lccccc}
s=0 & & n & & p & \\
s=-1 & \Sigma^- & & (\Sigma^0,\Lambda) & & \Sigma^+ \\
s=-2 & & \Xi^- & & \Xi^0 & \\
 & I_3=-1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1
\end{array}
$$

or in terms of the quark contents:

$$
\begin{array}{ccc}
 & udd \quad uud & \\
sdd & sud & suu \\
 & ssd \quad ssu &
\end{array}
$$

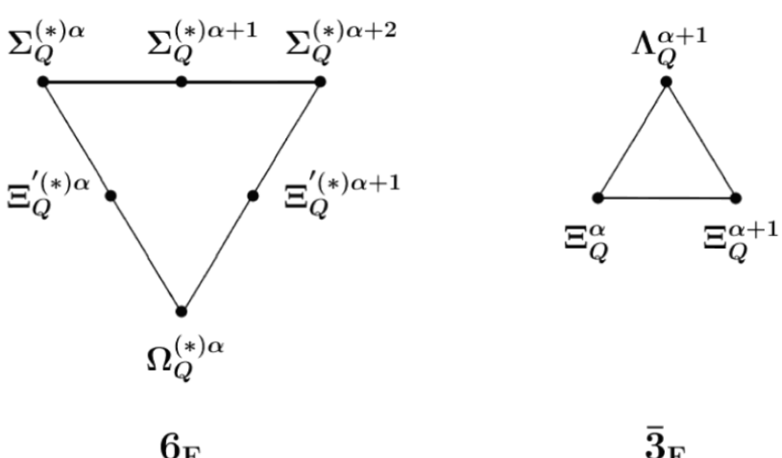
Light singlet (spin 1/2): $\Lambda(uds)$

Heavy Baryons: contain one-two or three heavy "b" or "c" quarks

The baryons containing single heavy quark can be classified according to the spin of the light degrees of freedom in the heavy quark limit, $m_Q \to \infty$.

$$\boxed{3 \otimes 3 = \bar{3}_F \oplus 6_F}$$

The spin of the light diquark is either $S = 1$ for 6_F, or $S = 0$ for $\bar{3}_F$. The ground state will have angular momentum $l = 0$. Therefore, the spin of the ground state is $1/2$ for $\bar{3}_F$, while it can be both $3/2$ or $1/2$ for 6_F



α, $\alpha+1$, $\alpha+2$ determine the charges of baryons ($\alpha = -1$ or 0), and the asterix $(*)$ denote $J^P = \frac{3}{2}^+$ states.

Heavy sextet baryons (Spin 3/2)

	q_1	q_2
$\Sigma_{b(c)}^{*+(++)}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	u	d
$\Sigma_{b(c)}^{*-(0)}$	d	d
$\Xi_{b(c)}^{*0(+)}$	s	u
$\Xi_{b(c)}^{*-(0)}$	s	d
$\Omega_{b(c)}^{*-(0)}$	s	s

Heavy sextet baryons (spin 1/2)

	q_1	q_2
$\Sigma_{b(c)}^{+(++)}$	u	u
$\Sigma_{b(c)}^{0(+)}$	u	d
$\Sigma_{b(c)}^{-(0)}$	d	d
$\Xi_{b(c)}^{0(+)}$	s	u
$\Xi_{b(c)}^{-(0)}$	s	d
$\Lambda_{b(c)}^{0(+)}$	u	d

Doubly Heavy baryons: contain two heavy b or c quarks

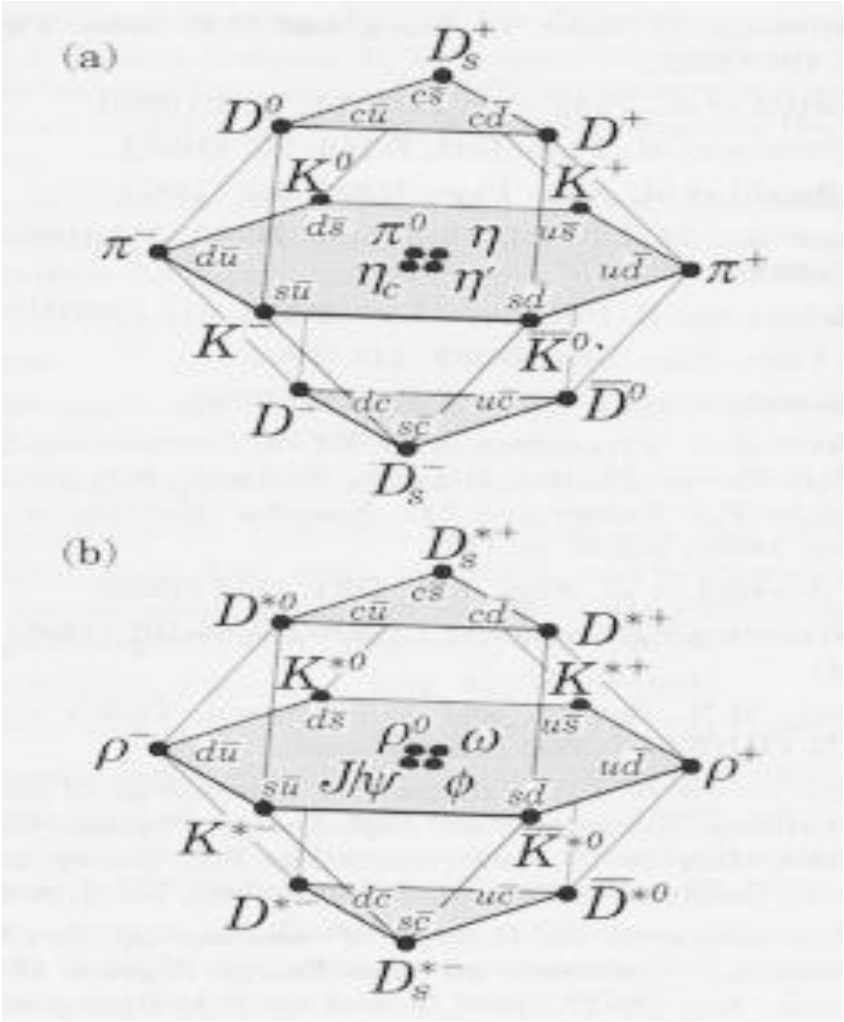
ground state of doubly heavy baryons with the identical heavy quarks $\boxed{\Xi_{QQ}^{(*)} \text{ and } \Omega_{QQ}^{(*)}}$, the pair of heavy quarks form a diquark with total spin 1. Adding then the spin–1/2 of light quark, we have two states with total spin–1/2 or spin–3/2. The baryons with (without) star stand for spin 3/2 (1/2) doubly heavy baryons. For these states, the wave functions should be symmetric with respect to the exchange of two identical heavy quarks. For the states containing two different heavy quarks, in addition to the previous case, i.e., total spin of diquark equal to one, the diquark can also have total spin zero, which leads to the total spin 1/2 of these states. Obviously, the wave functions of these states (usually these states are denoted by $\boxed{\Xi'_{bc} \text{ and } \Omega'_{bc})}$ are anti-symmetric with respect to the replacement $b \leftrightarrow c$. In the present work, we deal only with spin–1/2 doubly heavy baryons.

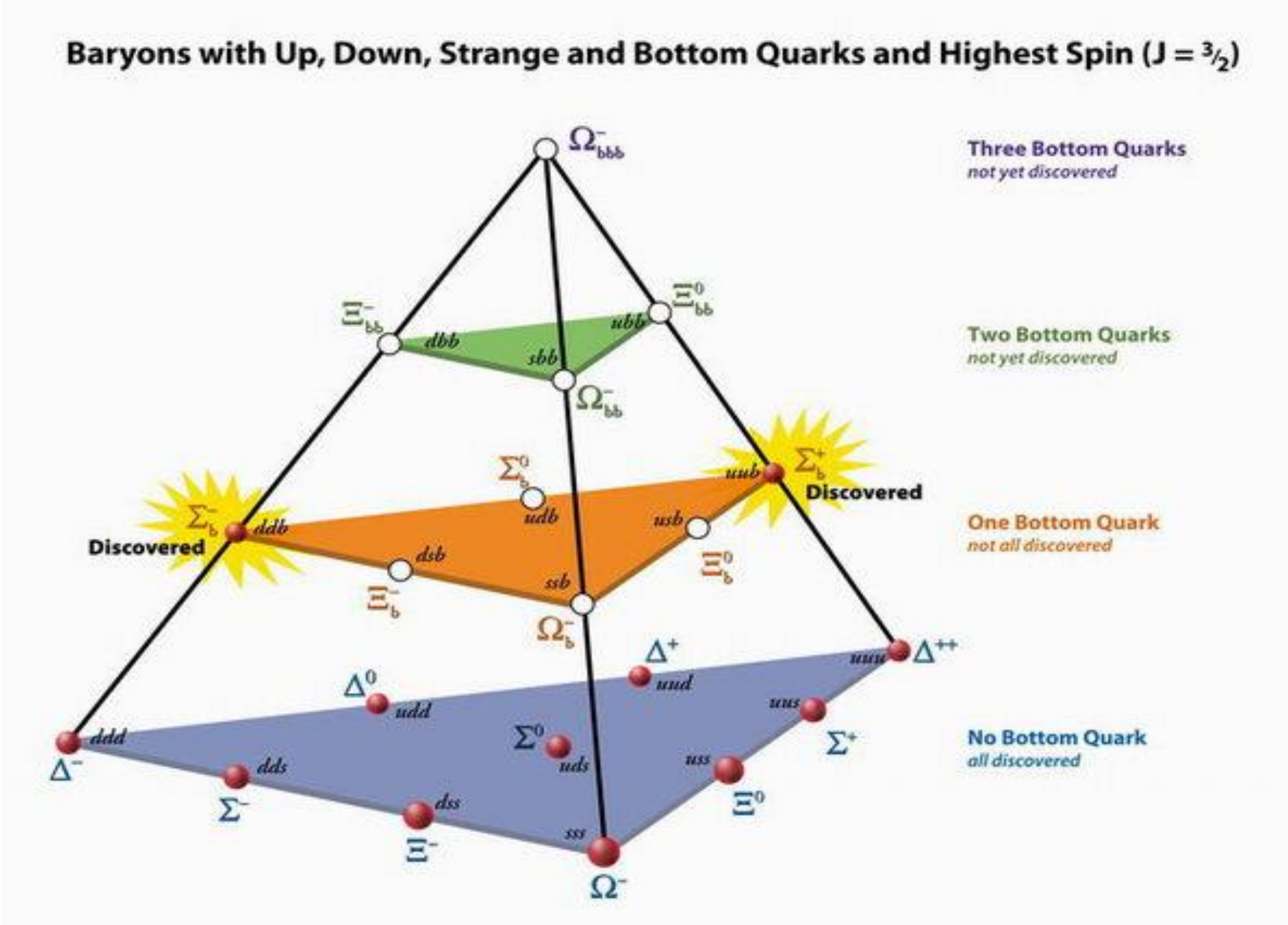
Asnti-symmetric wavefunctions
with respect to exchange of
two heavy quarks

Symmetric wavefunctions
With respect to exchange of
Two heavy quarks

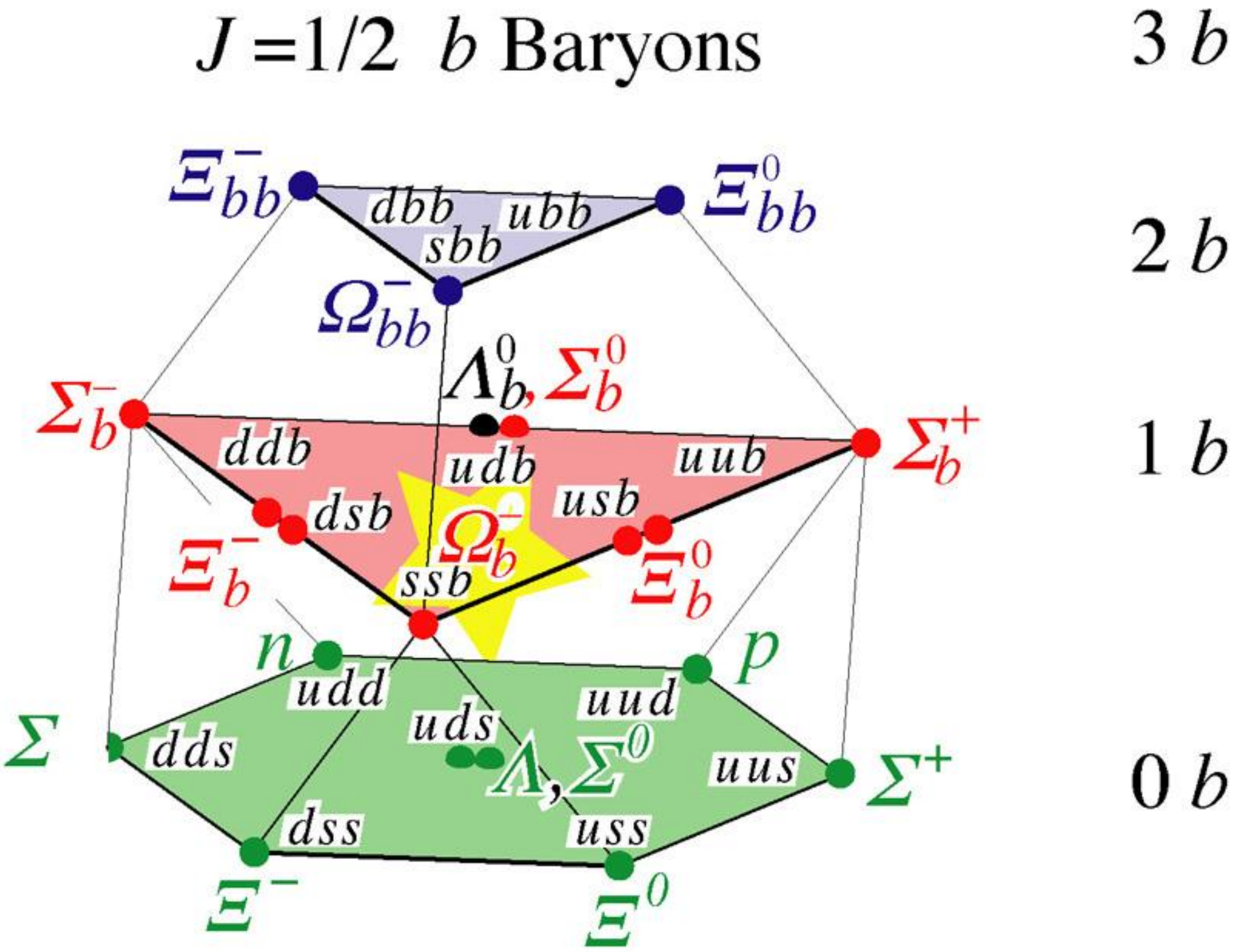
==

Thriply Heavy baryons: contain three heavy b or c quarks: \Omega_{QQQ}


(a)
D_s^+
$c\bar{s}$
D^0
$c\bar{u}$
$c\bar{d}$
D^+
K^0
K^+
$d\bar{s}$
$u\bar{s}$
π^-
π^0
η
$d\bar{u}$
$u\bar{d}$
π^+
η_c
η'
$s\bar{u}$
$s\bar{d}$
K^-
$\bar{K}^0$
$d\bar{c}$
$u\bar{c}$
D^-
$\bar{D}^0$
$s\bar{c}$
D_s^-
(b)
D_s^{*+}
$c\bar{s}$
D^{*0}
$c\bar{u}$
$c\bar{d}$
D^{*+}
K^{*0}
K^{*+}
$d\bar{s}$
$u\bar{s}$
ρ^-
ρ^0
ω
$d\bar{u}$
$u\bar{d}$
ρ^+
J/ψ
ϕ
$s\bar{u}$
$s\bar{d}$
K^{*-}
$\bar{K}^{*0}$
$d\bar{c}$
$u\bar{c}$
D^{*-}
$\bar{D}^{*0}$
$s\bar{c}$
D_s^{*-}


Baryons with Up, Down, Strange and Bottom Quarks and Highest Spin (J = 3/2)
Ω^-_{bbb}
Three Bottom Quarks
not yet discovered
Ξ^-_{bb}
Ξ^0_{bb}
dbb
ubb
sbb
Ω^-_{bb}
Two Bottom Quarks
not yet discovered
Σ^0_b
udb
Σ^+_b
uub
Discovered
Σ^-_b
ddb
Discovered
usb
Ξ^0_b
dsb
Ξ^-_b
ssb
Ω^-_b
One Bottom Quark
not all discovered
Δ^{++}
uuu
Δ^+
uud
Δ^0
udd
uus
Σ^+
ddd
Δ^-
Σ^0
uds
dds
Σ^-
uss
Ξ^0
dss
Ξ^-
sss
Ω^-
No Bottom Quark
all discovered

Only symmetric part



All hadrons except protons are unstable and they decay. Neutrons are stable only inside the atomic nuclei.

In most of non-perturbative approaches, hadrons are represented by their interpolating currents.

İnterpolating currents→ correspond to the wavefunctions in quark model.

interpolating currents of mesons

scalar: $J^S(x)= \bar{q}_1(x) q_2(x)$

Pseudoscalar: $J^{PS}(x)= \bar{q}_1(x)\gamma_5 q_2(x)$

Vector: $J^V_\mu(x)= \bar{q}_1(x)\gamma_\mu q_2(x)$

Axial-Vector: $J^{AV}_\mu(x)= \bar{q}_1(x)\gamma_\mu\gamma_5 q_2(x)$

interpolating current of each particle can create that particle from the vacuum with the same quantum numbers as the interpolating current.

Interpolating currents for baryons

We start to derive the interpolating field for nucleon (proton). The proton consists of two "u" quarks and one "d" quark and has the quantum numbers I=1/2, I_3=1/2 and $J^P=(1/2)^+$. The simplest way to obtain the approciate isospin is to consider the proton to be an up-down "diquark" with I=0 with an up quark (I_3=1/2) attached. Therefore, one must first consider the construction of an interpolating field for diquark.

The interpolating current for a diquark is expected to be similar to that of a meson.

$$J_{meson} = \bar{q}\Gamma q$$

Γ → a Dirac matrix: $\{I, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$

How does one construct the diquark? It is possible by replacing antiquark with its charge-conjugation analog:

$$q = C\bar{q}^T \quad \rightarrow \quad \bar{q} = q^T C$$

$C = \gamma_0\gamma_2$ $\qquad C^T = C^{-1} = C^+ = -C \qquad \gamma_0 C \gamma_0 = -C$

$J_{diquark}=q^T C\Gamma q$

Now add the other "u" quark. The current must be color singlet so,

$$\eta \sim \varepsilon_{abc}(u^{aT} C\Gamma d^b)\Gamma' u^c$$

$\{I, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$

one should determine Γ and Γ'. The form of Γcan be constrained through isospin considerations. Since the "u" quark quark attached to the "ud" diquark has I=1/2 and I_3=1/2, one can most readily guarantee that the proton has these same quantum numbers by insisting that the diquark has I=0.

Consider an infinitesimal isospin rotation. Under this transformation, the "u" quark obtaines a small "d" quark component and vice versa. If the diquark has vanishing isospin, then its contribution to the nucleon interpolating field must remain invariant under this transformation, i.e., one must have

$\varepsilon_{abc} u^{aT} C\Gamma u^b=0$ (1) and analogous relation for "d" quark

A simple way te determine those values of Γ that satisfy the Eq. (1) is to consider the transpose of both sides of this equation. One must first develop a simple theorem. If A=BC, where A, B and C are matrices whose elements are Grassmann numbers (**anticommuting numbers** or **anticommuting c-numbers**), then $A^T=-C^TB^T$

Proof:

$$(A^T)_{ki} = A_{ik} = B_{ij}\, C_{jk} = -C_{jk}\, B_{ij} = -(C^T)_{kj}\, (B^T)_{ji} = -(C^TB^T)_{ki}$$

Now consider the transpose of the left-hand side of Eq. (1):

$$(\varepsilon_{abc} u^{aT} C\Gamma u^b)^T = -\varepsilon_{abc} u^{bT}\, \Gamma^T C^T u^a$$

Using $C^T=C^{-1}=C^+=-C$ one obtains

$$(\varepsilon_{abc} u^{aT} C\Gamma u^b)^T = \varepsilon_{abc} u^{bT}\, C(C\Gamma^T C^{-1})\, u^a$$

Where,

$$C\Gamma^T C^{-1} = \begin{cases} \Gamma \quad \text{for } \Gamma = I,\ \gamma_5,\ \gamma_5\gamma_\mu \\ -\Gamma \quad \text{for } \Gamma = \gamma_\mu,\ \sigma_{\mu\nu} \end{cases}$$

Switching color dummy indicies, one obtains

$$(\varepsilon_{abc} u^{aT} C\Gamma u^b)^T = \begin{cases} -\varepsilon_{abc} u^{aT} C\Gamma u^b & \text{for } \Gamma = I,\ \gamma_5,\ \gamma_5\gamma_\mu \\ \varepsilon_{abc} u^{aT} C\Gamma u^b & \text{for } \Gamma = \gamma_\mu,\ \sigma_{\mu\nu} \end{cases}$$

The transpose of a 1 bye1 matrix, such as $\varepsilon_{abc} u^{aT} C\Gamma u^b$ İs equal to itself. Therefor,

$$\varepsilon_{abc} u^{aT} C\Gamma u^b = 0 \quad \text{for } \Gamma = I,\ \gamma_5,\ \gamma_5\gamma_\mu$$

Thus isospin considerations impose the constraint $\Gamma = I$, γ_5 or $\gamma_5\gamma_\mu$. Now consider the spin. The "u" quark attached to the diquark has J=1/2 and $J_3 = \pm$ ½. In constrac-ting the interpolating field for the proton, it is simplest to take the proton to have the same total spin and spin projection as the "u" quark. Thus the spin of the diquark is zero . This implies $\Gamma = I$, γ_5; therefore, the two possible forms of the proton interpolating field can be written as:

$$\begin{cases} \eta_1=\varepsilon_{abc}(u^{aT}Cd^b)\Gamma_1'u^c \\ \eta_2=\varepsilon_{abc}(u^{aT}C\,\gamma_5\,d^b)\Gamma_2'u^c \end{cases}$$

The values of Γ_1' and Γ_2' are determined through considerations involving Lorentz structure and parity. Since η_1 and η_2 are Lorentz scalars, one must have

$$\Gamma_1'\,,\ \Gamma_2' = I \text{ or } \gamma_5$$

Under the parity transformation the spinor $\psi(x)$ becomes

$$\Psi'(x') = \gamma_0\ \psi(x)$$

Applying this transformation to the individual quark fields in η_1 and η_2 one obtains

$$\begin{cases} \eta'_1=\varepsilon_{abc}[(\gamma_0u^a)^TC\gamma_0d^b]\Gamma_1'\,\gamma_0u^c=-\,\varepsilon_{abc}(u^{aT}Cd^b)\Gamma_1'\,\gamma_0u^c \\ \eta'_2=\varepsilon_{abc}[(\gamma_0u^a)^TC\,\gamma_5\,\gamma_0d^b]\Gamma_2'\,\gamma_0u^c=\varepsilon_{abc}(u^{aT}C\,\gamma_5\,d^b)\Gamma_2'\,\gamma_0\,u^c \end{cases}$$

On the other hand, applying the parity transformation to the interpolating fields as a whole gives

$$\begin{cases} \eta'_1=\varepsilon_{abc}\ (u^{aT}Cd^b)\ \gamma_0\Gamma_1' u^c \\ \eta'_2=\varepsilon_{abc}\ (u^{aT}C\ \gamma_5\ d^b)\ \gamma_0\Gamma_2' u^c \end{cases}$$

Thus $-\gamma_0\Gamma_1'=\Gamma_1'\ \gamma_0$ and $\Gamma_2'\gamma_0=\gamma_0\Gamma_2'$, which implies

$\Gamma_1'=\gamma_5$ $\quad\quad$ $\Gamma_2'=I$

Therefore, the two possible interpolating fields are given by

$$\begin{cases} \eta_1=\varepsilon_{abc}\ (u^{aT}Cd^b)\ \gamma_5\ u^c \\ \eta_2=\varepsilon_{abc}\ (u^{aT}C\ \gamma_5\ d^b)\ u^c \end{cases}$$

These two fields can be combined in an arbitrary linear combination. A useful form is

$\eta(t)=2\ \varepsilon_{abc}\,[(u^{aT}Cd^b)\ \gamma_5\ u^c+\ \beta\ (u^{aT}C\ \gamma_5\ d^b)\ u^c\ ]$ $\quad$ β is an arbitrary real parameters.

$\beta=-1$ corresponds to the famous Ioffe current

From similar manner one can obtain the following interpolating currents for the light and heavy baryons.

light spin ½ (octet) baryons:

$$\eta = A\varepsilon^{abc}\left\{(q_1^{aT}Cq_2^b)\gamma_5 q_3^c - (q_2^{aT}Cq_3^b)\gamma_5 q_1^c + \beta(q_1^{aT}C\gamma_5 q_2^b)q_3^c - \beta(q_2^{aT}C\gamma_5 q_3^b)q_1^c\right\}$$

	A	q_1	q_2	q_3
Σ^0	$-\sqrt{1/2}$	u	s	d
Σ^+	$1/2$	u	s	u
Σ^-	$1/2$	d	s	d
p	$-1/2$	u	d	u
n	$-1/2$	d	u	d
Ξ^0	$1/2$	s	u	s
Ξ^-	$1/2$	s	d	s

$$2\eta^{\Sigma^0}(d\to s) + \eta^{\Sigma^0} = -\sqrt{3}\eta^{\Lambda} \; ,$$
$$2\eta^{\Sigma^0}(u\to s) + \eta^{\Sigma^0} = \sqrt{3}\eta^{\Lambda} \; .$$

light spin 3/2 (Decuplet) baryons

$$\eta_\mu = A'\varepsilon^{abc}\left\{(q_1^{aT}C\gamma_\mu q_2^b)q_3^c + (q_2^{aT}C\gamma_\mu q_3^b)q_1^c + (q_3^{aT}C\gamma_\mu q_1^b)q_2^c\right\}$$

	A'	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$\sqrt{1/3}$	u	u	s
Σ^{*-}	$\sqrt{1/3}$	d	d	s
Δ^{++}	$1/3$	u	u	u
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	$1/3$	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	$1/3$	s	s	s

Heavy spin ½ baryons

$$\eta_Q^{(s)} = -\frac{1}{\sqrt{2}}\epsilon^{abc}\Big\{\Big(q_1^{aT}CQ^b\Big)\gamma_5 q_2^c + \beta\Big(q_1^{aT}C\gamma_5 Q^b\Big)q_2^c - \Big[\Big(Q^{aT}Cq_2^b\Big)\gamma_5 q_1^c + \beta\Big(Q^{aT}C\gamma_5 q_2^b\Big)q_1^c\Big]\Big\} ,$$

$$\eta_Q^{(anti-t)} = \frac{1}{\sqrt{6}}\epsilon^{abc}\Big\{2\Big(q_1^{aT}Cq_2^b\Big)\gamma_5 Q^c + 2\beta\Big(q_1^{aT}C\gamma_5 q_2^b\Big)Q^c + \Big(q_1^{aT}CQ^b\Big)\gamma_5 q_2^c + \beta\Big(q_1^{aT}C\gamma_5 Q^b\Big)q_2^c$$
$$+ \Big(Q^{aT}Cq_2^b\Big)\gamma_5 q_1^c + \beta\Big(Q^{aT}C\gamma_5 q_2^b\Big)q_1^c\Big\}$$

Q=b or c

	$\Sigma_{b(c)}^{+(++)}$	$\Sigma_{b(c)}^{0(+)}$	$\Sigma_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{-(0)\prime}$	$\Xi_{b(c)}^{0(+)\prime}$	$\Omega_{b(c)}^{-(0)}$	$\Lambda_{b(c)}^{0(+)}$	$\Xi_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{0(+)}$
q_1	u	u	d	d	u	s	u	d	u
q_2	u	d	d	s	s	s	d	s	s

Heavy spin 3/2 baryons

$$\eta_\mu = A\epsilon^{abc}\Big\{(q_1^a C\gamma_\mu q_2^b)Q^c + (q_2^a C\gamma_\mu Q^b)q_1^c + (Q^a C\gamma_\mu q_1^b)q_2^c\Big\}$$

	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*-(0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*-(0)}$	$\Omega_{b(c)}^{*-(0)}$
q_1	u	u	d	u	d	s
q_2	u	d	d	s	s	s
A	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$

Interpolating currents for doubly heavy baryons

✓Spin-1/2

$$\eta^S = \frac{1}{\sqrt{2}}\epsilon_{abc}\left\{(Q^{aT}Cq^b)\gamma_5 Q'^c + (Q'^{aT}Cq^b)\gamma_5 Q^c + \beta(Q^{aT}C\gamma_5 q^b)Q'^c + \beta(Q'^{aT}C\gamma_5 q^b)Q^c\right\},$$

$$\eta^A = \frac{1}{\sqrt{6}}\epsilon_{abc}\left\{2(Q^{aT}CQ'^b)\gamma_5 q^c + (Q^{aT}Cq^b)\gamma_5 Q'^c - (Q'^{aT}Cq^b)\gamma_5 Q^c + 2\beta(Q^{aT}C\gamma_5 Q'^b)q^c\right.$$

$$\left. + \quad \beta(Q^{aT}C\gamma_5 q^b)Q'^c - \beta(Q'^{aT}C\gamma_5 q^b)Q^c\right\},$$

T. M. Aliev, K. Azizi, M. Savci, **Nucl.Phys. A895 (2012) 59-70; Phys.Lett. B715 (2012) 149-151**

✓Spin-3/2

$$\eta_\mu = \frac{1}{\sqrt{3}}\epsilon^{abc}\left\{(q^{aT}C\gamma_\mu Q^b)Q'^c + (q^{aT}C\gamma_\mu Q'^b)Q^c + (Q^{aT}C\gamma_\mu Q'^b)q^c\right\}$$

T. M. Aliev, K. Azizi, M. Savci, **J.Phys. G40 (2013) 065003**

Interpolating currents for triply heavy baryons

✓Spin-1/2

$$\eta_{QQQ'} = 2\epsilon_{ijk}\left\{\left(Q^{iT}CQ'^{j}\right)\gamma_5 Q^k + \beta\left(Q^{iT}C\gamma_5 Q'^{j}\right)Q^k\right\}$$

Baryon	Q	Q'
Ω_{bbc}	b	c
Ω_{ccb}	c	b

T. M. Aliev, K. Azizi, M. Savci, **JHEP 1304 (2013) 042**

✓Spin-3/2

$$\eta_\mu = \frac{1}{\sqrt{3}}\epsilon^{abc}\left\{2(Q^{aT}C\gamma_\mu Q'^{b})Q^c + (Q^{aT}C\gamma_\mu Q^{b})Q'^{c}\right\},$$

Baryon	Q	Q'
Ω^*_{bbc}	b	c
Ω^*_{ccb}	c	b
Ω^*_{bbb}	b	b
Ω^*_{ccc}	c	c

T. M. Aliev, K. Azizi, M. Savci, **J.Phys. G41 (2014) 065003**

Correlation Function

The main advantage of the QCD sum rules as a nonperturbative method is that, it is based on the fundamental QCD Lagrangian.

As we previously said a direct use of this Lagrangian is possible only within the limited framework of the perturbation theory. The condition that guarantees the smallness of the corresponding effective quark-gluon coupling $\alpha_s = g_s^2/4\pi$ and adequacy of the perturbative expansion is that, at least some of the quarks or gluons in a hadronic process have to be highly virtual.

Mainly, high virtuality is obtained in a scattering of hadrons at large momentum transfer. However, even for these hard scattering processes, a perturbative calculation of the quark-gluon Feynman diagrams is not sufficient, since the quarks participating in the hard scattering are confined inside hadrons. Therefore, one should combine the perturbative QCD result with certain wave functions or momentum distributions of quarks in hadrons.

We consider the two-point correlation function

$$\Pi(q^2) = i \int d^4x\, e^{iq\cdot x} \langle 0|\, T\{j(x)\bar{j}(0)\}\, |0\rangle,$$

where *q is the momentum of the quarks, j(x) is the quark current that injects quarks* into the QCD vacuum at point x.

Phenomenological Side

In this part, we show how the correlation function is related to the physically observed hadrons. The invariant amplitude $\Pi(q^2)$ is an analytic function of q^2 defined at both negative (spacelike) and positive (timelike) values of q^2. When q^2 is shifted from large and negative to positive values, the correlation function starts receiving contributions from long-distance quark-gluon interactions. In this regime, the quarks start to form hadrons. Inserting a complete set of intermediate hadronic states with the same quantum number as interpolating currents

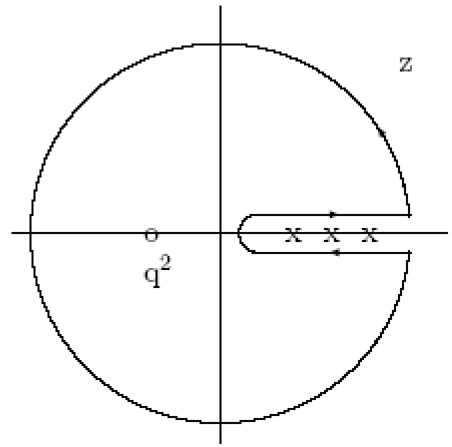
$$1 = |0\rangle\langle 0| + \sum_h \int \frac{d^4 p_h}{(2\pi)^4} 2\pi\delta(p_h^2 - m_h^2)|h(p_h)\rangle\langle h(p_h)| + \text{higher Fock states},$$

we get the physical or phenomenological representation of the correlation function as:

$$
\begin{aligned}
2\mathrm{Im}\,\Pi(q^2) &= \sum_h \int \langle 0|\, j\, |h\rangle\langle h|\, \bar{j}\, |0\rangle\, d\tau_h (2\pi)^4 \delta^{(4)}(q - p_h) \\
&= 2\pi f_h^2 \delta(q^2 - m_h^2) + 2\pi\rho^h(q^2)\,,
\end{aligned}
$$

where, the summation goes over all possible hadronic states $|h\rangle$ created by the quark current j. The $\rho^h(q^2)$ represents the contributions of the higher states and continuum and $f_h = \langle 0 \mid j(0) \mid h\rangle$ is the leptonic decay constant (for mesons) or residue (for baryons) of the ground state hadron (hadron with the lowest mass) and $d\tau_h$ denotes the integration measure over the phase space volume of these states.

Now, to link the values of the $\Pi(q^2)$ for positive values of the q^2 to the negative values, we use the Cauchy formula for the analytic function $\Pi(q^2)$ and derive the dispersion relation for the correlation function. Using the Cauchy formula for analytic functions, for the contour shown



The contour in the plane of the complex variable $z = q^2$. The open point represents the $q^2 < 0$ reference point of the QCD calculation and the crosses indicate positions of the hadronic thresholds at $q^2 > 0$.

$$\Pi(q^2) = \frac{1}{2\pi i}\oint_C dz \frac{\Pi(z)}{z-q^2} = \frac{1}{2\pi i}\oint_{|z|=R} dz \frac{\Pi(z)}{z-q^2}$$

$$+\frac{1}{2\pi i}\int_{t_{min}}^{R} dz \frac{\Pi(z+i\epsilon)-\Pi(z-i\epsilon)}{z-q^2} .$$

where, t_{min} is the threshold for creation of real states, the radius R of the circular part of the contour is to be sent to infinity.

At $|z| \to \infty$, if the function $\Pi(z)$ vanishes fast enough, the first term in the above equation vanishes. If it does not vanish, we can expand the denominator in terms of q^2/z, and eventually, at some order n, $\Pi(z)/z^n$ would vanish sufficiently fast and the remaining terms will not contribute. Therefore, at this limit the first term reduces to a polynomial in q^2 called subtraction terms. Considering the fact that the $\Pi(q^2)$ function is real in the region $q^2 < t_{min}$, Schwartz reflection principle states that $\Pi(z + i\epsilon) - \Pi(z - i\epsilon) = 2iIm\Pi(q^2)$ at $q^2 > t_{min}$. Hence

the following dispersion relation

$$\Pi(q^2) = \frac{1}{\pi} \int_{t_{min}}^{\infty} ds \frac{Im\Pi(s)}{s - q^2 - i\epsilon} + \text{subtraction terms}$$

is obtained. The $Im\Pi(s)/\pi = \rho(s)$ is the so called the spectral density

Combining Eqs. one gets the following relation for the hadronic side of the correlation function:

$$\Pi(q^2) = \frac{f_h^2}{m_h^2 - q^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s - q^2} + \text{subtraction terms},$$

where s_0^h is the threshold for the creation of higher mass states.

Theoretical or QCD Side

From the QCD side, on the other hand, the correlation function can also be calculated in the quark-gluon language with the help of the operator product expansion in large Euclidean region $q^2 \ll -\Lambda^2_{QCD}$

The OPE states that for small x, the time order product of two currents at different points in the correlation function can be expanded in terms of local operators with space time dependent coefficients as:

$$T\{j(x)\bar{j}(0)\} = \sum_d C_d(x^2) O_d,$$

where, $C_d(x^2)$ are Wilson coefficients which can be calculated using perturbation theory and O_d are a set of local operators ordered according to their dimensions. The lowest-dimension operator with $d = 0$ is the unit operator associated with the perturbative contribution. In QCD, there are no colorless operators with lower dimensions, $d = 1, 2$. The operators with 3, 4, 5 and 6 dimensions are:

$$\begin{aligned}
O_3 &= \bar{\psi}\psi, \\
O_4 &= G^a_{\mu\nu} G^{a\mu\nu}, \\
O_5 &= \bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2} G^{a\mu\nu}\psi \,, \\
O_6^{\psi} &= (\bar{\psi}\Gamma_r\psi)(\bar{\psi}\Gamma_s\psi) \,, \\
O_6^{G} &= f_{abc} G^a_{\mu\nu} G^{b\,\nu}_{\sigma} G^{c\sigma\mu},
\end{aligned}$$

and so on. Here, operators with Lorentz indices are ignored because they do not contribute to the vacuum-vacuum correlation function. The $\Gamma_{r,s}$ denote various combinations of Lorentz and color matrices. The correlation function can be written in terms of the OPE as:
$$\Pi(q^2) = \sum_d C_d(x^2)\langle O_d\rangle.$$

The $\langle O_d\rangle$ form a set of vacuum condensates which parameterize the non-perturbative effects. The vacuum averages of O_3 and O_4 are so called the quark and gluon condensates, respectively, and the vacuum averages of the remaining operators are, correspondingly the quark-gluon, four-quark and three-gluon condensates. Once the correlation function is calculated using OPE, the spectral density is given as $\rho^{OPE}(q^2) = \frac{1}{\pi}Im\Pi^{OPE}(q^2)$ and

$$\Pi(q^2) = \int_0^\infty ds\frac{\rho^{OPE}(s)}{s-q^2} + \text{subtraction terms}.$$

QCD Sum Rules for Physical Quantities

Equating two representations of the correlation function, we obtain

$$
\frac{f_h^2}{m_h^2 - q^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s - q^2} + \text{subtraction terms} \;=\; \int_0^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2} + \text{subtraction terms.}
$$

To eliminate the subtraction terms, the Borel transformation with respect to the $Q^2 = -q^2$ is applied on both sides. This also suppresses the contributions of the higher states and continuum. The Borel transformation is defined as

$$
\mathcal{B}_{M^2} f(q^2) = lim_{\substack{Q^2, n \to \infty \\ \frac{Q^2}{n} = M^2}} \frac{(-q^2)^n}{(n-1)!} \left(\frac{d}{dq^2}\right)^{n-1} f(q^2),
$$

where, it leads to the following three important examples:

$$
\begin{aligned}
\mathcal{B}_{M^2}(q^2)^k &= 0, \qquad k \geq 0 \\
\mathcal{B}_{M^2}\Big(\frac{1}{(m^2[s]-q^2)^k}\Big) &= \frac{1}{(k-1)!}\frac{e^{-m^2[s]/M^2}}{(M^2)^{(}k-1)}, \\
\mathcal{B}_{M^2}(e^{-\alpha q^2}) &= \delta\Big(\frac{1}{M^2}-\alpha\Big).
\end{aligned}
$$

Applying the Borel transformation

$$
f_h^2 e^{-m_h^2/M^2} + \int_{s_0^h}^{\infty} ds\rho^h(s)e^{-s/M^2} = \int_0^{\infty} ds\rho^{OPE}(s)e^{-s/M^2}
$$

where, not only the subtraction terms has been eliminated, but the contributions of the higher states and continuum are exponentially suppressed.

Quark-Hadron Duality Assumption

At $Q^2 \to \infty$ limit, the correlation function in QCD side is completely given in terms of the perturbative part. The matching between two representations of the correlation function and writing the QCD side in terms of two integrals, we get:

$$f_h^2 e^{-m_h^2/M^2} + \int_{s_0^h}^{\infty} ds\rho^h(s)e^{-s/M^2} = \int_0^{s_0} ds\rho^{OPE}(s)e^{-s/M^2} + \int_{s_0}^{\infty} ds\rho^{OPE}(s)e^{-s/M^2}.$$

To parameterize the contribution of the higher states and the continuum the following approximation is considered:

$$\int_{s_0^h}^{\infty} ds\rho^h(s)e^{-s/M^2} = \int_{s_0}^{\infty} ds\rho^{OPE}(s)e^{-s/M^2},$$

which is called the local quark hadron duality assumption. The s_0 in the above equation is called the continuum threshold. This threshold is not completely arbitrary and it is related to the energy of the exited states. The result of the physical quantities should be stable with respect to the small variations of this parameter. Applying the quark hadron duality approximation, the final sum rules is obtained as:

$$f_h^2 e^{-m_h^2/M^2} = \int_0^{s_0} ds \rho^{OPE}(s) e^{-s/M^2},$$

where, the Borel mass squared, M^2, is an auxiliary unphysical parameter in this sum rules, hence the physical quantities should be independent of it. However, in sum rules method the operator product expansion (OPE) is truncated and as a result the dependency of the predictions of physical quantities on the auxiliary parameter M^2 appears. For this reason one should look for a region of M^2 such that the predictions for the physical quantities do not vary with respect to the Borel mass parameter. This region is the so called the "working region" and within this region the truncation is reasonable and meaningful. The upper limit of M^2 is determined from condition that the continuum and higher states contributions should be small than the total dispersion integral. The lower limit is determined by demanding that in the truncated OPE the condensate term with highest dimension remains small than sum of all terms, i.e., convergence of OPE should be under control.

✓ Hadronic spectroscopy in vacuum:

Heavy baryons with a single heavy quark

- Heavy sextet baryons (Spin 3/2)

	q_1	q_2
$\Sigma_{b(c)}^{*+(++)}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	u	d
$\Sigma_{b(c)}^{*-(0)}$	d	d
$\Xi_{b(c)}^{*0(+)}$	s	u
$\Xi_{b(c)}^{*-(0)}$	s	d
$\Omega_{b(c)}^{*-(0)}$	s	s

Table: The quark fields q_1 and q_2 for the heavy decuplet baryons.

İnterpolating current:

$$\eta_\mu = A\epsilon_{abc}\left\{(q_1^{aT}C\gamma_\mu q_2^b)Q^c + (q_2^{aT}C\gamma_\mu Q^b)q_1^c + (Q^{aT}C\gamma_\mu q_1^b)q_2^c\right\}$$

where C is the charge conjugation operator and a, b and c are color indices.

	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*-(0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*-(0)}$	$\Omega_{b(c)}^{*-(0)}$
A	$1/\sqrt{3}$	$\sqrt{2/3}$	$1/\sqrt{3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$1/\sqrt{3}$

Table: The value of A for the corresponding baryons.

$$T_{\mu\nu} = i \int d^4x e^{ipx} \langle 0 \mid T\{\eta_\mu(x)\bar{\eta}_\nu(0)\} \mid 0\rangle$$

Phenomenological side

- Inserting the complete set of states between the interpolating currents in $T_{\mu\nu}$ with quantum numbers of heavy baryons.

$$T_{\mu\nu} = \frac{\langle 0 \mid \eta_\mu \mid B(p)\rangle\langle B(p) \mid \bar{\eta}_\nu \mid 0\rangle}{p^2 - m_B^2} \cdots$$

- The vacuum to baryon matrix element of the interpolating current is defined as

$$\langle 0 \mid \eta_\mu(0) \mid B(p,s)\rangle = \lambda_B u_\mu(p,s),$$

where $\lambda_B \rightarrow$ residue & $u_\mu(p,s) \rightarrow$ Rarita-Schwinger spinor.

- perform summation over spins of the spin 3/2 particles

$$\sum_s u_\mu(p,s)\bar{u}_\nu(p,s) = \frac{(\not{p}+m)}{2m}\{-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3m^2} - \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m}\}.$$

- Putting all together

$$T_{\mu\nu} = \frac{\lambda_B^2}{p^2 - m_B^2}[-\not{p}g_{\mu\nu} + \ldots],$$

QCD or theoretical side:

- On QCD side, after contracting out the quark pairs in $T_{\mu\nu}$ using the Wick's theorem, we get the following expression for the correlation function in terms of quark propagators

$$
\begin{aligned}
& T_{\mu\nu} \\
= \; & -iA^2 \epsilon_{abc}\epsilon_{a'b'c'} \int d^4x e^{ipx} \langle 0 | \{ S_Q^{ca'} \gamma_\nu S'^{bb'}_{q_2} \gamma_\mu S^{ac'}_{q_1} \\
+ \; & S_Q^{cb'} \gamma_\nu S'^{aa'}_{q_1} \gamma_\mu S^{bc'}_{q_2} + S_{q_2}^{ca'} \gamma_\nu S'^{bb'}_{q_1} \gamma_\mu S^{ac'}_{Q} + S_{q_2}^{cb'} \gamma_\nu S'^{aa'}_{Q} \gamma_\mu S^{bc'}_{q_1} \\
+ \; & S_{q_1}^{cb'} \gamma_\nu S'^{aa'}_{q_2} \gamma_\mu S^{bc'}_{Q} + S_{q_1}^{ca'} \gamma_\nu S'^{bb'}_{Q} \gamma_\mu S^{ac'}_{q_2} + Tr(\gamma_\mu S^{ab'}_{q_1} \gamma_\nu S'^{ba'}_{q_2}) \\
\times \; & S_Q^{cc'} + Tr(\gamma_\mu S^{ab'}_{Q} \gamma_\nu S'^{ba'}_{q_1}) S^{cc'}_{q_2} + Tr(\gamma_\mu S^{ab'}_{q_2} \gamma_\nu S'^{ba'}_{Q}) S^{cc'}_{q_1} \} | 0 \rangle,
\end{aligned}
$$

where $S' = CS^TC$ and $S_Q(S_q)$ is the full heavy (light) quark propagator.

Wick's Theorem

$$T\phi_I(x)\phi_I(y) \underset{x^0>y^0}{=} \phi_I^+(x)\phi_I^+(y) + \phi_I^+(x)\phi_I^-(y) + \phi_I^-(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y)$$

$$= \phi_I^+(x)\phi_I^+(y) + \phi_I^-(y)\phi_I^+(x) + \phi_I^-(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y)$$

$$+ \left[\phi_I^+(x), \phi_I^-(y)\right].$$

$$\phi_I(x) = \phi_I^+(x) + \phi_I^-(x)$$

$$\phi_I^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} a_{\mathbf{p}} e^{-ip\cdot x}; \qquad \phi_I^-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} a_{\mathbf{p}}^\dagger e^{+ip\cdot x}.$$

$$\phi_I^+(x)\left|0\right\rangle = 0 \qquad \text{and} \qquad \left\langle 0\right|\phi_I^-(x) = 0.$$

the *contraction* of two fields

$$\phi(x)\phi(y) \equiv \begin{cases} \left[\phi^+(x), \phi^-(y)\right] & \text{for } x^0 > y^0; \\ \left[\phi^+(y), \phi^-(x)\right] & \text{for } y^0 > x^0. \end{cases}$$

This quantity is exactly the Feynman propagator:

$$\wick{\c\phi(x)\c\phi(y)} = D_F(x-y).$$

$$T\{\phi(x_1)\phi(x_2)\cdots\phi(x_m)\}$$
$$= N\{\phi(x_1)\phi(x_2)\cdots\phi(x_m) + \text{all possible contractions}\}.$$

normal ordering

for two fields:

$$T\{\phi(x)\phi(y)\} = N\{\phi(x)\phi(y) + \wick{\c\phi(x)\c\phi(y)}\}.$$

$$\begin{aligned} T\{\phi_1\phi_2\phi_3\phi_4\} = N\{&\phi_1\phi_2\phi_3\phi_4 + \wick{\c\phi_1\c\phi_2}\phi_3\phi_4 + \wick{\c\phi_1\phi_2\c\phi_3}\phi_4 + \wick{\c\phi_1\phi_2\phi_3\c\phi_4} \\ &+ \phi_1\wick{\c\phi_2\c\phi_3}\phi_4 + \phi_1\wick{\c\phi_2\phi_3\c\phi_4} + \phi_1\phi_2\wick{\c\phi_3\c\phi_4} \\ &+ \wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4} + \wick{\c1\phi_1\c2\phi_2\c1\phi_3\c2\phi_4} + \wick{\c1\phi_1\c2\phi_2\c2\phi_3\c1\phi_4}\}. \end{aligned}$$

$$N\{\contraction{}{\phi}{{}_1\phi_2}{\phi}\phi_1\phi_2\phi_3\phi_4\} \quad \text{means} \quad D_F(x_1 - x_3)\cdot N\{\phi_2\phi_4\}$$

Feynman propagator

$$\contraction{}{\phi}{(x)}{\phi}\phi(x)\phi(y) = D_F(x-y).$$

$$\langle 0|\, N(\text{any operator})\,|0\rangle = 0$$

since

$$N\left(a_{\mathbf{p}}a^\dagger_{\mathbf{k}}a_{\mathbf{q}}\right) \equiv a^\dagger_{\mathbf{k}}a_{\mathbf{p}}a_{\mathbf{q}}$$

$$\begin{aligned}\langle 0|\, T\{\phi_1\phi_2\phi_3\phi_4\}\,|0\rangle = {} & D_F(x_1-x_2)D_F(x_3-x_4) \\ & + D_F(x_1-x_3)D_F(x_2-x_4) \\ & + D_F(x_1-x_4)D_F(x_2-x_3).\end{aligned}$$

So the time ordering product between vacuum states=all possible contractions

$$
\begin{aligned}
S_Q(x) &= S_Q^{free}(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \\
&\quad \left[\frac{\not{k} + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} vx_\mu G^{\mu\nu}\gamma_\nu \right], \\
S_q(x) &= S_q^{free}(x) - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i\frac{m_q}{4} \not{x} \right) \\
&- \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i\frac{m_q}{6} \not{x} \right) - ig_s \int_0^1 du \\
&\quad \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux)\sigma_{\mu\nu} - ux^\mu G_{\mu\nu}(ux)\gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
&\quad \left. - i\frac{m_q}{32\pi^2} G_{\mu\nu}\sigma^{\mu\nu} \left(\ln\left(\frac{-x^2\Lambda^2}{4} \right) + 2\gamma_E \right) \right].
\end{aligned}
$$

- the free part of the propagators are:

$$S_q^{free} = \frac{i \not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2},$$

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q\sqrt{-x^2}),$$

where K_i are Bessel functions.

Integral representation of bessel functions:

$$\frac{K_\nu(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^\nu} = \frac{1}{2}\int_0^\infty \frac{dt}{t^{\nu+1}} e^{-m_Q\left(t - \frac{x^2}{t}\right)}$$

We use these expressions for the propagators then perform integrate over x (Fourier transformation).

To suppress the contribution of higher states and continuum, we apply also the Borel transformation and continuum subtraction.

These transformations bring two auxiliary parameters:
Borel mass parameter, M^2 and continuum subtraction s_0.

We should find their working regions such that the physical quantities are independent of them.

- we obtain the following result for λ_B^2:

$$\lambda_B^2 e^{\frac{-m_{B_Q}^2}{M^2}} = A^2 \left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2)\right],$$

-

$$\begin{aligned}
\Pi' &= \int_{m_Q^2}^{s_0} ds\, e^{-s/M^2} \Bigg\{ \\
& m_0^2 < q_1 q_1 > \left[\frac{(m_{q_1} - 6m_Q)(\psi_{22} + 2\psi_{12} - 1)}{192 m_Q^2 \pi^2}\right] \\
- & < q_1 q_1 > \left[\frac{1}{32\pi^2}[2(\psi_{02} + 2\psi_{10} - 2\psi_{21} - 1)m_Q \right. \\
+ & \left. (\psi_{02} - 1)(3m_{q_1} - 2m_{q_2})]\right] \ldots\ldots \Bigg\}.
\end{aligned}$$

$$m_{B_Q}^2 = \frac{-\frac{d}{d(1/M^2)}\left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2)\right]}{\left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2)\right]}.$$

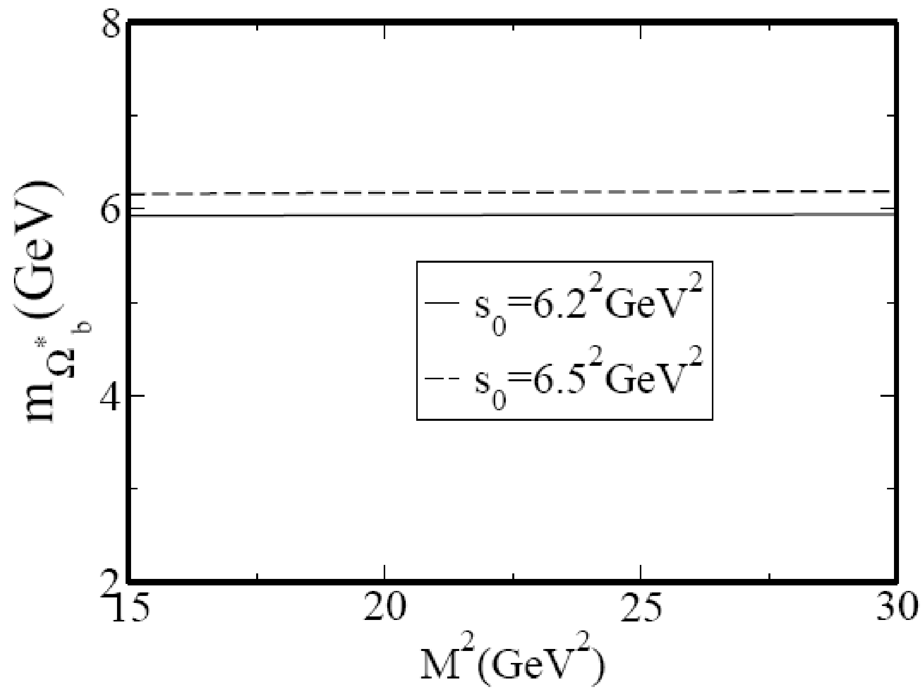


Figure: The dependence of mass of the Ω_b^* on the Borel parameter M^2 for two fixed values of continuum threshold s_0.

T.M. Aliev, K. Azizi, A. Ozpineci, ***Nucl. Phys. B 808 (2009) 137.***

	$m_{\Omega_b^*}$	$m_{\Omega_c^*}$	$m_{\Sigma_b^*}$	$m_{\Sigma_c^*}$	$m_{\Xi_b^*}$	$m_{\Xi_c^*}$
this work	6.08 ± 0.40	2.72 ± 0.20	5.85 ± 0.35	2.51 ± 0.15	5.97 ± 0.40	2.66 ± 0.18
[7]	$6.063^{+0.083}_{-0.082}$	$2.790^{+0.109}_{-0.105}$	$5.835^{+0.082}_{-0.077}$	$2.534^{+0.096}_{-0.081}$	$5.929^{+0.088}_{-0.079}$	$2.634^{+0.102}_{-0.094}$
[8]	6.088	2.768	5.834	2.518	5.963	2.654
[9]	-	-	5.805	2.495	-	-
[10]	6.090	2.770	5.850	2.520	5.980	2.650
[11]	-	2.768	-	2.518	-	-
[12]	6.083	2.760	5.840	-	5.966	-
[13]	6.060	2.752	5.871	2.5388	5.959	2.680
Exp[14]	-	2.770	5.836	2.520	-	2.645

PDG

Table: Comparison of mass of the heavy flavored baryons in *GeV* from present work and other approaches and with experiment.

✓Masses and residues of the doubly and triply heavy baryons

Brief introduction to calculations: spin ½ triply heavy

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T}\{\eta_{QQQ'}(x)\bar{\eta}_{QQQ'}(0)\}| 0 \rangle \ ,$$

Hadronic side

$$\Pi(q) = \frac{\langle 0|\eta_{QQQ'}(0)|B(q)\rangle\langle B(q)|\bar{\eta}_{QQQ'}(0)|0\rangle}{q^2 - m_B^2} + \cdots \ ,$$

$$\langle 0|\eta_{QQQ'}(0)|B(q,s)\rangle = \lambda_B u(q,s) \ ,$$

$$\Pi(q) = \frac{\lambda_B^2(\not{q} + m_B)}{q^2 - m_B^2} + \cdots \ ,$$

OPE side

$$\begin{aligned}\Pi(q) &= 4i\epsilon_{ijk}\epsilon_{lmn}\int d^4x e^{iqx}\left\langle 0\left|\left\{-\gamma_5 S_Q^{nj} S_{Q'}^{\prime mi} S_Q^{lk}\gamma_5 + \gamma_5 S_Q^{nk}\gamma_5 Tr\left[S_Q^{lj} S_{Q'}^{\prime mi}\right]\right.\right.\right.\\ &+ \beta\Big(-\gamma_5 S_Q^{nj}\gamma_5 S_{Q'}^{\prime mi} S_Q^{lk} - S_Q^{nj} S_{Q'}^{\prime mi}\gamma_5 S_Q^{lk}\gamma_5 + \gamma_5 S_Q^{nk} Tr\left[S_Q^{lj}\gamma_5 S_{Q'}^{\prime mi}\right]\\ &+ \left.\left.\left. S_Q^{nk}\gamma_5 Tr\left[S_Q^{lj} S_{Q'}^{\prime mi}\gamma_5\right]\Big) + \beta^2\Big(-S_Q^{nj}\gamma_5 S_{Q'}^{\prime mi}\gamma_5 S_Q^{lk} + S_Q^{nk} Tr\left[S_{Q'}^{mi}\gamma_5 S_Q^{\prime lj}\gamma_5\right]\Big)\right\}\right|0\right\rangle ,\end{aligned}$$

where $S' = CS^TC$

$$\begin{aligned}S_Q(x) &= \frac{m_Q^2}{4\pi^2}\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2 \not{x}}{4\pi^2 x^2}K_2(m_Q\sqrt{-x^2})\\ &- ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 du\left[\frac{\not{k}+m_Q}{2(m_Q^2-k^2)^2}G^{\mu\nu}(ux)\sigma_{\mu\nu} + \frac{u}{m_Q^2-k^2}x_\mu G^{\mu\nu}\gamma_\nu\right] + \cdots ,\end{aligned}$$

where K_1 and K_2 are the modified Bessel functions of the second kind

$$\Pi_i(q) = \int\frac{\rho_i(s)}{s-q^2}ds ,$$

$i = 1$ and 2 correspond to the structures $\not{q}$ and I, respectively.

$$
\begin{aligned}
\rho_1(s) &= \frac{1}{64\pi^4}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}} d\psi d\eta \Bigg\{-3\mu_{QQQ'}\Bigg[-12(-1+\eta)m_Q m_{Q'}(-1+\beta)^2 \\
&+ \psi^2\eta(3\mu_{QQQ'}-2s)\Big[5+\beta(2+5\beta)\Big] + \psi\Bigg(2m_Q^2(-1+\beta)^2 - 12m_Q m_{Q'}(-1+\beta^2) \\
&+ (-1+\eta)\eta(3\mu_{QQQ'}-2s)\Big[5+\beta(2+5\beta)\Big]\Bigg)\Bigg]\Bigg\} \\
&+ \frac{\langle g_s^2 GG\rangle}{256\pi^4 m_Q m_{Q'}}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}} d\psi d\eta \Bigg\{6(-3+4\psi)(-1+\psi+\eta)m_Q^2(-1+\beta^2) \\
&+ 6(-3+4\eta)(-1+\psi+\eta)m_{Q'}^2(-1+\beta^2) + m_Q m_{Q'}\Bigg[48\psi^2(1+\beta^2)+\psi\Big[-63 \\
&+ 68\eta - 30\beta + 8\eta\beta + (-63+68\eta)\beta^2\Big] + 2(-1+\eta)\Bigg(-3\Big[3+\beta(2+3\beta)\Big] \\
&+ 2\eta\Big[5+\beta(2+5\beta)\Big]\Bigg)\Bigg]\Bigg\},
\end{aligned}
$$

$$\mu_{QQQ'} = \frac{m_Q^2}{1-\psi-\eta} + \frac{m_Q^2}{\eta} + \frac{m_{Q'}^2}{\psi} - s \, ,$$

$$\eta_{min} = \frac{1}{2}\left[1-\psi-\sqrt{(1-\psi)\left(1-\psi-\frac{4\psi m_Q^2}{\psi s - m_{Q'}^2}\right)}\;\right] ,$$

$$\eta_{max} = \frac{1}{2}\left[1-\psi+\sqrt{(1-\psi)\left(1-\psi-\frac{4\psi m_Q^2}{\psi s - m_{Q'}^2}\right)}\;\right] ,$$

$$\psi_{min} = \frac{1}{2s}\left[s+m_{Q'}^2-4m_Q^2-\sqrt{(s+m_{Q'}^2-4m_Q^2)^2-4m_{Q'}^2 s}\;\right] ,$$

$$\psi_{max} = \frac{1}{2s}\left[s+m_{Q'}^2-4m_Q^2+\sqrt{(s+m_{Q'}^2-4m_Q^2)^2-4m_{Q'}^2 s}\;\right] .$$

$$\lambda_B^2 e^{-m_B^2/M^2} = \int_{s_{min}}^{s_0} ds \rho_1(s) e^{-s/M^2} ,$$

$$s_{min} = (2m_Q + m_{Q'})^2$$

Borel mass parameter

$$m_B^2 = \frac{\int_{s_{min}}^{s_0} ds \; s\rho_i(s) e^{-s/M^2}}{\int_{s_{min}}^{s_0} ds \; \rho_i(s) e^{-s/M^2}} , \qquad i = 1 \text{ or } 2 \, ,$$

Continuum threshold

Doubly heavy spin--1/2 baryons :

T.M. Aliev, K. Azizi, M. Savci, Nucl.Phys. A895 (2012) 59

Some non-perturbative approaches

Baryon	M^2	$\sqrt{s_0}$	This work	[11]	[12]	[13]	[6]	[19]	Exp [20]
Ξ_{bb}	11.0	10.9	9.96(0.90)	9.78(0.07)	10.17(0.14)	9.94(0.91)	10.202	–	–
Ω_{bb}	11.0	10.9	9.97(0.90)	9.85(0.07)	10.32(0.14)	9.99(0.91)	10.359	–	–
Ξ_{bc}	8.0	7.5	6.72(0.20)	6.75(0.05)	–	6.86	6.933	7.053	–
Ω_{bc}	8.0	7.5	6.75(0.30)	7.02(0.08)	–	6.864	7.088	7.148	–
Ξ_{cc}	5.0	4.6	3.72(0.20)	4.26(0.19)	3.57(0.14)	3.52(0.06)	3.620	3.676	3.5189(0.0009)
Ω_{cc}	5.0	4.6	3.73(0.20)	4.25(0.20)	3.71(0.14)	3.53(0.06)	3.778	3.787	–
Ξ'_{bc}	8.0	7.5	6.79(0.20)	6.95(0.08)	–	–	6.963	7.062	–
Ω'_{bc}	8.0	7.5	6.80(0.30)	7.02(0.08)	–	–	7.116	7.151	–

Table 1: The mass of the doubly heavy spin–1/2 baryons (in units of GeV) at $\beta = \pm 2$.

SELEX Collaboration

M. Mattson et.al, SELEX Collaboration, Phys. Rev. Lett. 89, 112001 (2002).

Doubly heavy spin--3/2 baryons :

	Our Work		[10] and [15]	[11]	[6]	[14]
	Structure $\not{q} g_{\mu\nu}$	Structure $g_{\mu\nu}$				
Ξ_{cc}^*	3.69±0.16	3.72±0.18	3.58±0.05	3.90±0.10	3.727	3.61±0.18
Ω_{cc}^*	3.78±0.16	3.78±0.16	3.67±0.05	3.81±0.06	3.872	3.76±0.17
Ξ_{bb}^*	10.4±1.0	10.3±0.2	10.33±1.09	10.35±0.08	10.237	10.22±0.15
Ξ_{bc}^*	7.25±0.20	7.2±0.2	—	8.00±0.26	6.98	—
Ω_{bc}^*	7.3±0.2	7.35±0.25	—	7.54±0.08	7.13	—
Ω_{bb}^*	10.5±0.2	10.4±0.2	10.38±1.10	10.28±0.05	10.389	10.38±0.14

Table 3: The mass spectra of the spin–3/2 doubly heavy baryons in units of GeV.

T.M. Aliev, K. Azizi M. Savci, J.Phys. G40 (2013) 065003

We have no experimental data yet

Triply heavy spin--1/2 baryons

We have no experimental data yet

	This work ($\not q$)	This work (I)	[20]	[12]	[13]	[19]	[14]
Ω_{bbc}	11.73 ± 0.16	11.71 ± 0.16	11.50 ± 0.11	11.139	11.280	10.30 ± 0.10	11.535
Ω_{ccb}	8.50 ± 0.12	8.48 ± 0.12	8.23 ± 0.13	7.984	8.018	7.41 ± 0.13	8.245
$\overline{\Omega}_{bbc}$	10.59 ± 0.14	10.56 ± 0.14	10.47 ± 0.12	-	-	-	-
$\overline{\Omega}_{ccb}$	7.79 ± 0.11	7.74 ± 0.11	7.61 ± 0.13	-	-	-	-

Table 2: The masses of the triply heavy spin–1/2 baryons (in units of GeV). For the baryons with over-line, the $\overline{MS}$ values of the quark masses are used.

	This work ($\not q$)	This work (I)	[20]
Ω_{bbc}	0.53 ± 0.17	0.45 ± 0.15	0.68 ± 0.15
Ω_{ccb}	0.38 ± 0.13	0.30 ± 0.10	0.47 ± 0.10
$\overline{\Omega}_{bbc}$	0.85 ± 0.28	0.65 ± 0.22	0.68 ± 0.15
$\overline{\Omega}_{ccb}$	0.56 ± 0.18	0.38 ± 0.13	0.47 ± 0.10

Table 3: The residues of the triply heavy spin–1/2 baryons (in units of GeV^3). For the baryons with over-line, the $\overline{MS}$ values of the quark masses are used.

T.M. Aliev, K. Azizi, M. Savci, **JHEP 1304 (2013) 042**

Triply heavy spin--3/2 baryons

	$M^2(GeV^2)$	$\sqrt{s_0}(GeV)$	$m(GeV)$	$\lambda(GeV^3)$
$\Omega_{ccc}(\frac{3}{2}^+)$	$4.5-8.0$	5.6 ± 0.2	4.72 ± 0.12	0.09 ± 0.01
$\Omega_{ccc}(\frac{3}{2}^-)$	$4.5-8.0$	5.8 ± 0.2	4.9 ± 0.1	0.11 ± 0.01
$\Omega_{ccb}(\frac{3}{2}^+)$	$6.0-10.0$	8.8 ± 0.2	8.07 ± 0.10	0.06 ± 0.01
$\Omega_{ccb}(\frac{3}{2}^-)$	$6.0-10.0$	9.0 ± 0.2	8.35 ± 0.10	0.07 ± 0.01
$\Omega_{bbc}(\frac{3}{2}^+)$	$8.0-10.5$	12.0 ± 0.2	11.35 ± 0.15	0.08 ± 0.01
$\Omega_{bbc}(\frac{3}{2}^-)$	$8.0-10.5$	12.2 ± 0.2	11.5 ± 0.2	0.09 ± 0.01
$\Omega_{bbb}(\frac{3}{2}^+)$	$12.0-18.0$	15.3 ± 0.2	14.3 ± 0.2	0.14 ± 0.02
$\Omega_{bbb}(\frac{3}{2}^-)$	$12.0-18.0$	15.5 ± 0.2	14.9 ± 0.2	0.20 ± 0.02

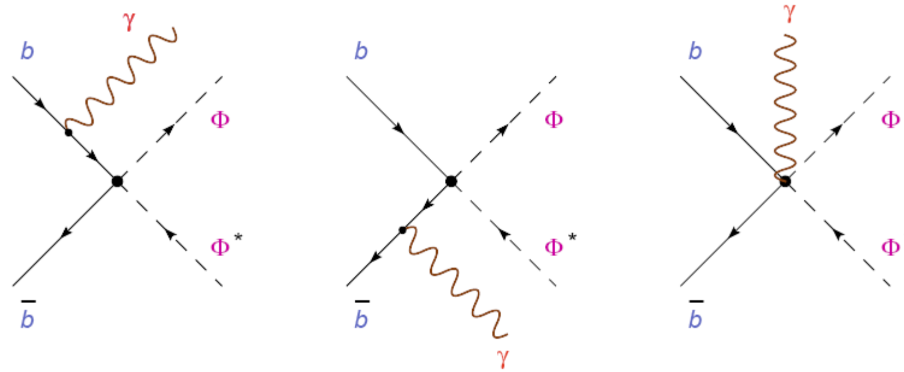
No experimental data yet

$$\langle 0 \left| \eta_\mu \right| B_{(3/2)^+}(q) \rangle = \lambda_{(3/2)^+} u_\mu(q) ,$$
$$\langle 0 \left| \eta_\mu \right| B_{(3/2)^-}(q) \rangle = \lambda_{(3/2)^-} \gamma_5 u_\mu(q) ,$$

T. M. Aliev, K. Azizi, M. Savci, **J.Phys. G41 (2014) 065003**

✓ mass and decay constant of vector \Upsilon meson at finite temperature QCD

Υ Decays into Light Scalar Dark Matter



Thermal correlation function

$$\Pi_{\mu\nu}(q,T) = i\int d^4x e^{iq.x}\langle \mathcal{T}\left(J_\mu(x)J_\nu^\dagger(0)\right)\rangle,$$

$$\langle A\rangle = \frac{Tr(e^{-\beta H}A)}{Tr(e^{-\beta H})}, \qquad J_\mu(x) =: \overline{Q}(x)\gamma_\mu Q(x) : \text{with } Q = b$$

where H is the QCD Hamiltonian, and $\beta = 1/T$

We modify the hadronic and QCD sides considering the medum conditions and use the following thermal propagator:

$$S(k) = (\gamma^\mu k_\mu + m)\Big(\frac{1}{k^2 - m^2 + i\varepsilon} + 2\pi i n(|k_0|)\delta(k^2 - m^2)\Big),$$

where $n(x) = [\exp(\beta x) + 1]^{-1}$ is the Fermi distribution function

$$S^{aa' nonpert}(k) = -\frac{i}{4} g (t^c)^{aa'} G^c_{\kappa\lambda}(0) \frac{1}{(k^2-m^2)^2} \Big[\sigma_{\kappa\lambda}(\not{k}+m) + (\not{k}+m)\sigma_{\kappa\lambda} \Big]$$
$$- \frac{i}{4} g^2 (t^c t^d)^{aa'} G^c_{\alpha\beta}(0) G^d_{\mu\nu}(0) \frac{\not{k}+m}{(k^2-m^2)^5} (f_{\alpha\beta\mu\nu} + f_{\alpha\mu\beta\nu} + f_{\alpha\mu\nu\beta})(\not{k}+m),$$

where,

$$f_{\alpha\beta\mu\nu} = \gamma_\alpha(\not{k}+m)\gamma_\beta(\not{k}+m)\gamma_\mu(\not{k}+m)\gamma_\nu.$$

To go on, we also need to know the expectation value $\langle Tr G_{\alpha\beta} G_{\mu\nu} \rangle$. The Lorentz covariance at finite temperature permits us to write the general structure of this expectation value in the following manner:

$$\langle Tr^c G_{\alpha\beta} G_{\mu\nu} \rangle = \frac{1}{24}(g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}) \langle G^a_{\lambda\sigma} G^{a\lambda\sigma} \rangle$$
$$+ \frac{1}{6} \Big[g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} - 2(u_\alpha u_\mu g_{\beta\nu} - u_\alpha u_\nu g_{\beta\mu} - u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu}) \Big] \langle u^\lambda \Theta^g_{\lambda\sigma} u^\sigma \rangle,$$

where, u^μ as we also previously mentioned is the four-velocity of the heat bath and it is introduced to restore Lorentz invariance formally in the thermal field theory. In the rest frame of the heat bath $u^\mu = (1,0,0,0)$ and $u^2 = 1$. Furthermore, $\Theta^g_{\lambda\sigma}$ is the traceless gluonic part of the stress-tensor of the QCD. Up to terms required for our calculations, the non perturbative part of massive quark propagator at finite temperature is obtained as:

$$S^{aa'nonpert}(k) = -\frac{i}{4} g (t^c)^{aa'} G^c_{\kappa\lambda} \frac{1}{(k^2-m^2)^2} \Big[\sigma_{\kappa\lambda}(\not{k}+m) + (\not{k}+m)\sigma_{\kappa\lambda}\Big]$$
$$+ \frac{i\, g^2\, \delta^{aa'}}{3\,(k^2-m^2)^4} \Big\{ \frac{m(k^2+m\,\not{k})}{4} \langle G^c_{\alpha\beta} G^{c\alpha\beta}\rangle + \frac{1}{3(k^2-m^2)} \Big[m(k^2-m^2)\Big(k^2-4(k\cdot u)^2\Big)$$
$$+ (m^2-k^2)\Big(-m^2+4(k\cdot u)^2\Big)\,\not{k} + 4(k\cdot u)(m^2-k^2)^2\,\not{u}\Big] \langle u^\alpha \Theta^g_{\alpha\beta} u^\beta\rangle \Big\}.$$

$$\langle\Theta\rangle = 2\langle\Theta^g\rangle = 6\times 10^{-6} exp[80(T-0.1)](GeV^4),$$ Lattice QCD

$$\langle\Theta\rangle = \langle\Theta^\mu_\mu\rangle + 3\,p,$$ Chiral perturbation theory

$$\langle\Theta^\mu_\mu\rangle = \frac{\pi^2}{270}\frac{T^8}{F_\pi^4} \ln\Big(\frac{\Lambda_p}{T}\Big),$$

$$p = 3T\Big(\frac{m_\pi\, T}{2\,\pi}\Big)^{\frac{3}{2}}\Big(1+\frac{15\,T}{8\,m_\pi}+\frac{105\,T^2}{128\,m_\pi^2}\Big) exp\Big(-\frac{m_\pi}{T}\Big),$$

where $\Lambda_p = 0.275\,GeV$, $F_\pi = 0.093\,GeV$ and $m_\pi = 0.14\,GeV$.

Continum threshold

$$s_0(T) = s_0\left[1-\left(\frac{T}{T_c^*}\right)^8\right] + 4\ m_Q^2\ \left(\frac{T}{T_c^*}\right)^8,$$

where $T_c^* = 1.1\ T_c = 0.176\ GeV.$

Gluon condensate

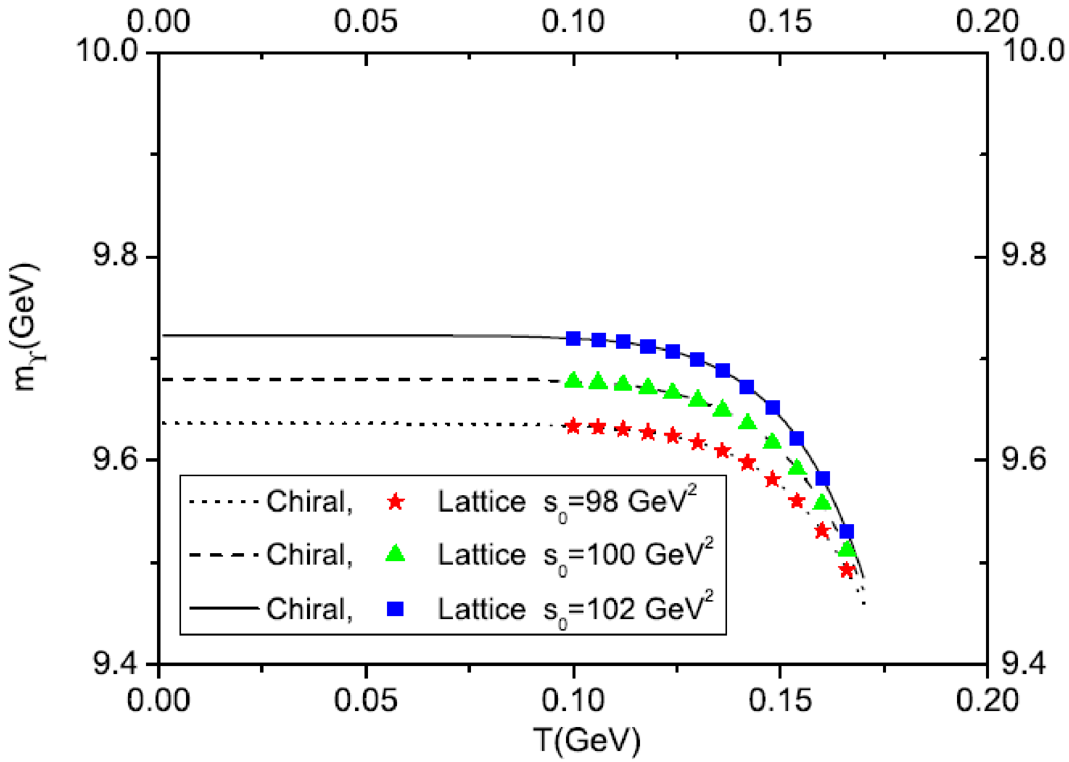
$$\langle G^2\rangle = \frac{\langle 0|G^2|0\rangle}{exp\left[12\left(\frac{T}{T_c}-1.05\right)\right]+1}.$$

Temperature dependent strong coupling constant

$$g^{-2}(T) = \frac{11}{8\pi^2}\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right) + \frac{51}{88\pi^2}\ln\left[2\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right)\right],$$

where, $\Lambda_{\overline{MS}} \simeq T_c/1.14$

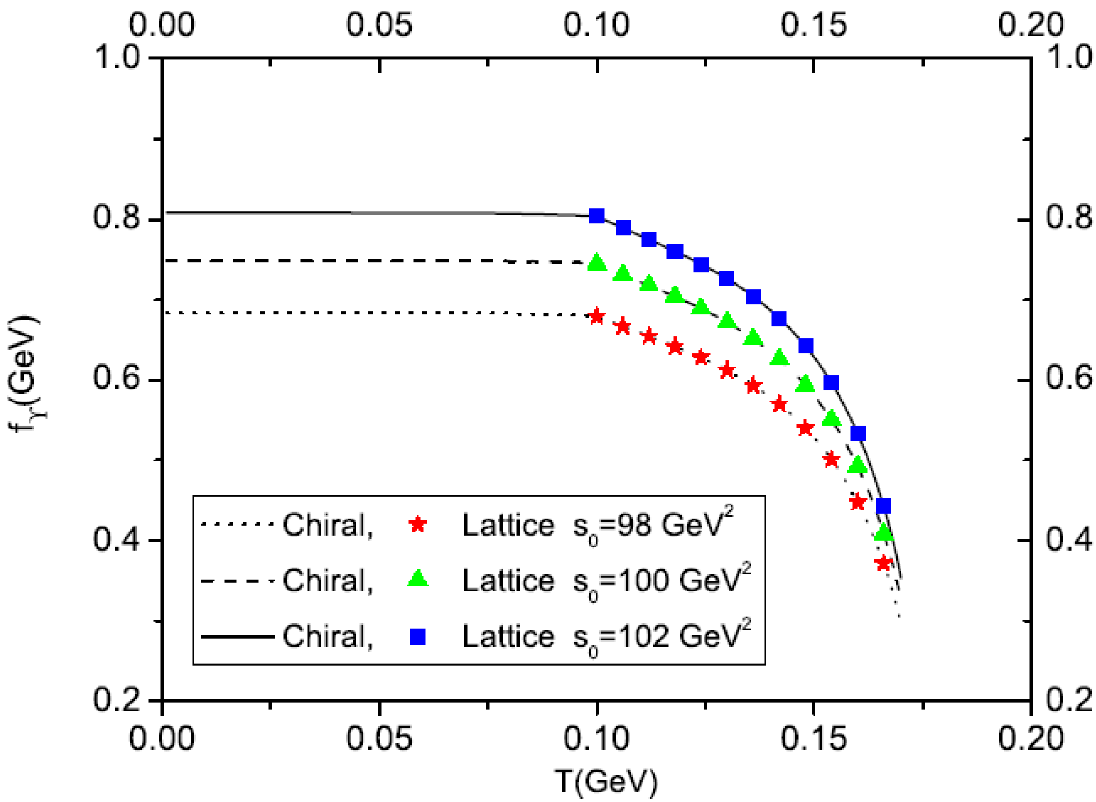
$$\alpha^{pert}(T) = \frac{g^2(T)}{4\pi}$$



	$m_{J/\psi}\ (GeV)$	$m_\Upsilon\ (GeV)$
Present Work	3.05 ± 0.08	9.68 ± 0.25
Experimental [27]	3.096916 ± 0.000011	9.46030 ± 0.00026

PDG

E. Veli Veliev, K. Azizi, H. Sundu, G. Kaya, A. Turkan
Eur.Phys.J. A47 (2011) 110



	$f_{J/\psi}(MeV)$	$f_{\Upsilon}(MeV)$
Present Work	481 ± 36	746 ± 62
Lattice[25, 26]	399 ± 4	—
Experimental [25, 26]	409 ± 15	708 ± 8
Potential Model [25]	400 ± 45	685 ± 30
Nonrelativistic Quark Model [26]	423	716

Mass and decay constant remain unchanged up to T=(0-100) MeV but they start to diminish after this point. At critical or deconfinement temp. The mass and decay constants decrease about %(10-20) , (50-80)%, respectively. This can be considered as a signal of transition to quark-gluon plasma phase.

✓ Nucleons in nuclear matter

K. Azizi, N. Er, Eur. Phys. J. C (2014) 74:2904

$$\Pi(p) = i\int d^4x e^{ip\cdot x}\langle\psi_0|T[J(x)\bar{J}(0)]|\psi_0\rangle,$$

Hadronic side

$$\Pi^{Had}(p) = -\frac{\langle\psi_0|J(x)|N(p,s)\rangle\langle N(p,s)|\bar{J}(0)|\psi_0\rangle}{p^2 - m_N^{*2}} + ...,$$

$$\langle\psi_0|J(x)|N(p,s)\rangle = \lambda_N^* u(p,s),$$

$$\Pi^{Had}(p) = -\frac{\lambda_N^{*2}(\not{p} + m_N^*)}{p^2 - m_N^{*2}} + ... = -\frac{\lambda_N^{*2}}{(\not{p} - m_N^*)} + ...$$

$$\Pi^{Had}(p) = -\frac{\lambda_N^{*2}}{(p^\mu - \Sigma_\nu^\mu)\gamma_\mu - (m_N + \Sigma_S)} + ...,$$

where Σ_ν^μ and Σ_S are vector and scalar self-energies of the nucleon in nuclear matter

$$\Sigma_\nu^\mu = \Sigma_\nu u^\mu + \Sigma_\nu' p^\mu,$$

where Σ_ν and Σ_ν' are constants and u^μ is the four velocity of the nuclear medium.

$$\Pi^{Had}(p) = -\frac{\lambda_N^{*2}}{(\not{p} - \Sigma_\nu \not{u}) - (m_N + \Sigma_S)} + ...,$$

$$\Pi^{Had}(p) = \Pi_p^{Had}(p^2, p_0)\not{p} + \Pi_u^{Had}(p^2, p_0)\not{u} + \Pi_S^{Had}(p^2, p_0)I,$$

$$
\begin{aligned}
\Pi_p^{Had}(p^2, p_0) &= -\lambda_N^{*2} \frac{1}{p^2 - \mu^2}, \\
\Pi_u^{Had}(p^2, p_0) &= +\lambda_N^{*2} \frac{\Sigma_\nu}{p^2 - \mu^2}, \\
\Pi_S^{Had}(p^2, p_0) &= -\lambda_N^{*2} \frac{m_N^*}{p^2 - \mu^2}.
\end{aligned}
$$

Here $m_N^* = m_N + \Sigma_S$ and $\mu^2 = m_N^{*2} - \Sigma_\nu^2 + 2p_0\Sigma_\nu$.

$$
\begin{aligned}
\hat{B}\Pi_p^{Had}(p^2, p_0) &= -\lambda_N^{*2} e^{-\mu^2/M^2}, \\
\hat{B}\Pi_u^{Had}(p^2, p_0) &= +\lambda_N^{*2} \Sigma_\nu e^{-\mu^2/M^2}, \\
\hat{B}\Pi_S^{Had}(p^2, p_0) &= -\lambda_N^{*2} m_N^* e^{-\mu^2/M^2},
\end{aligned}
$$

OPE side

$$\Pi^{OPE}(p) = \Pi_p^{OPE} \not{p} + \Pi_u^{OPE} \not{u} + \Pi_S^{OPE} I.$$

$$\Pi_i^{OPE} = \int \frac{\rho_i(s)}{s-p^2} ds,$$

where $\rho_i(s) = \frac{1}{\pi}\mathrm{Im}[\Pi_i^{OPE}]$ are the spectral densities.

$$\begin{aligned}
\Pi^{OPE}(p) \;=\; & -4i\epsilon_{abc}\epsilon_{a'b'c'} \int d^4x e^{ipx} \Big\langle \psi_0 \Big| \Big\{ \Big(\gamma_5 S_u^{cb'}(x) S_d'^{ba'}(x) S_u^{ac'}(x)\gamma_5 \\
- \; & \gamma_5 S_u^{cc'}(x)\gamma_5 Tr\Big[S_u^{ab'}(x) S_d'^{ba'}(x) \Big] \Big) + \beta \Big(\gamma_5 S_u^{cb'}(x)\gamma_5 S_d'^{ba'}(x) S_u^{ac'}(x) \\
+ \; & S_u^{cb'}(x) S_d'^{ba'}(x)\gamma_5 S_u^{ac'}(x)\gamma_5 - \gamma_5 S_u^{cc'}(x) Tr\Big[S_u^{ab'}(x)\gamma_5 S_d'^{ba'}(x) \Big] \\
- \; & S_u^{cc'}(x)\gamma_5 Tr\Big[S_u^{ab'}(x) S_d'^{ba'}(x)\gamma_5 \Big] \Big) + \beta^2 \Big(S_u^{cb'}(x)\gamma_5 S_d'^{ba'}(x)\gamma_5 S_u^{ac'}(x \\
- \; & S_u^{cc'}(x) Tr\Big[S_d^{ba'}(x)\gamma_5 S_u'^{ab'}(x)\gamma_5 \Big] \Big) \Big\} \Big| \psi_0 \Big\rangle,
\end{aligned}$$

$$
\begin{aligned}
S_q^{ab}(x) &\equiv \langle\psi_0|T[q^a(x)\bar{q}^b(0)]|\psi_0\rangle_{\rho_N} \\
&= \frac{i}{2\pi^2}\delta^{ab}\frac{1}{(x^2)^2}\ \not{x} - \frac{m_q}{4\pi^2}\delta^{ab}\frac{1}{x^2} \\
&+ \chi_q^a(x)\bar{\chi}_q^b(0) - \frac{ig_s}{32\pi^2}F_{\mu\nu}^A(0)t^{ab,A}\frac{1}{x^2}[\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\ \not{x}] + \ldots,
\end{aligned}
$$

$$
\chi_{a\alpha}^q(x)\bar{\chi}_{b\beta}^q(0) = \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N}, \qquad F_{\kappa\lambda}^A F_{\mu\nu}^B = \langle G_{\kappa\lambda}^A G_{\mu\nu}^B\rangle_{\rho_N},
$$

$$
\chi_{a\alpha}^q\bar{\chi}_{b\beta}^q F_{\mu\nu}^A = \langle q_{a\alpha}\bar{q}_{b\beta}G_{\mu\nu}^A\rangle_{\rho_N}, \qquad \chi_{a\alpha}^q\bar{\chi}_{b\beta}^q\chi_{c\gamma}^q\bar{\chi}_{d\delta}^q = \langle q_{a\alpha}\bar{q}_{b\beta}q_{c\gamma}\bar{q}_{d\delta}\rangle_{\rho_N}.
$$

$$
\begin{aligned}
\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N} &= -\frac{\delta_{ab}}{12}\Bigg[\Big(\langle\bar{q}q\rangle_{\rho_N} + x^\mu\langle\bar{q}D_\mu q\rangle_{\rho_N} + \frac{1}{2}x^\mu x^\nu\langle\bar{q}D_\mu D_\nu q\rangle_{\rho_N} + \ldots\Big)\delta_{\alpha\beta} \\
&\quad +\Big(\langle\bar{q}\gamma_\lambda q\rangle_{\rho_N} + x^\mu\langle\bar{q}\gamma_\lambda D_\mu q\rangle_{\rho_N} + \frac{1}{2}x^\mu x^\nu\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} + \ldots\Big)\gamma_{\alpha\beta}^\lambda\Bigg].
\end{aligned}
$$

$$+\langle g_s \bar{q} \not{u} \sigma \cdot G q \rangle_{\rho N} \left[\sigma_{\mu\nu} \not{u} + i(u_\mu \gamma_\nu - u_\nu \gamma_\mu) \right]_{\alpha\beta}$$

$$-4 \Big(\langle \bar{q} u \cdot D u \cdot D q \rangle_{\rho N} + i m_q \langle \bar{q} \not{u} u \cdot D q \rangle_{\rho N} \Big)$$

$$\times \left[\sigma_{\mu\nu} + 2i(u_\mu \gamma_\nu - u_\nu \gamma_\mu) \not{u} \right]_{\alpha\beta} \Bigg\},$$

$$D_\mu = \tfrac{1}{2}(\gamma_\mu \not{D} + \not{D} \gamma_\mu).$$

$$\langle G^A_{\kappa\lambda} G^B_{\mu\nu} \rangle_{\rho N} = \frac{\delta^{AB}}{96} \left[\langle G^2 \rangle_{\rho N} (g_{\kappa\mu} g_{\lambda\nu} - g_{\kappa\nu} g_{\lambda\mu}) + O(\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle_{\rho N}) \right],$$

$$
\begin{aligned}
\langle \bar{q}\gamma_\mu q\rangle_{\rho_N} &= \langle \bar{q}\,\not{u} q\rangle_{\rho_N} u_\mu, \\
\langle \bar{q}D_\mu q\rangle_{\rho_N} &= \langle \bar{q}u\cdot Dq\rangle_{\rho_N} u_\mu = -im_q\langle \bar{q}\,\not{u} q\rangle_{\rho_N} u_\mu, \\
\langle \bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N} &= \frac{4}{3}\langle \bar{q}\,\not{u} u\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}) + \frac{i}{3}m_q\langle \bar{q}q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}), \\
\langle \bar{q}D_\mu D_\nu q\rangle_{\rho_N} &= \frac{4}{3}\langle \bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}) - \frac{1}{6}\langle g_s\bar{q}\sigma\cdot Gq\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}) \\
\langle \bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} &= 2\langle \bar{q}\,\not{u} u\cdot Du\cdot Dq\rangle_{\rho_N}\Big[u_\lambda u_\mu u_\nu - \frac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})\Big] \\
&\quad -\frac{1}{6}\langle g_s\bar{q}\,\not{u}\sigma\cdot Gq\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}),
\end{aligned}
$$

$$
\Pi_i^{OPE} = \Pi_i^E + p_0\Pi_i^O,
$$

$$
\begin{aligned}
\hat{B}\Pi_p^E &= -\frac{1}{256\pi^4}\int_0^{s_0} ds e^{-s/M^2} s^2\Big[5+\beta(2+5\beta)\Big] \\
&\quad +\frac{1}{72\pi^2}\int_0^{s_0} ds e^{-s/M^2}\Big\{-8\Big[5+\beta(2+5\beta)\Big]m_q\langle\bar{q}q\rangle_{\rho_N} \\
&\quad +9(-1+\beta)\Big[3(1+\beta)m_d + 2m_u + 4\beta m_u\Big]\langle\bar{q}q\rangle_{\rho_N}
\end{aligned}
$$

$$+5\Big[5+\beta(2+5\beta)\Big]\langle q^\dagger iD_0 q\rangle_{\rho_N}\Bigg\}$$

$$-\frac{\langle g_s^2G^2\rangle_{\rho_N}}{1024\pi^4}\int_0^{s_0} ds e^{-s/M^2}(6+\beta+5\beta^2)$$

$$+\frac{1}{192M^2\pi^2}\Bigg\{(-1+\beta)\Bigg[-\Big(40(1+\beta)m_d+(26+43\beta)m_u\Big)M^2$$

$$+8\Big(3(1+\beta)m_d+2m_u+4\beta m_u\Big)p_0^2\Bigg]\Bigg\}\langle \bar{q}g_s\sigma Gq\rangle_{\rho_N}$$

$$-\frac{1}{48M^2\pi^2}\Bigg\{(-1+\beta)\Bigg[(1+5\beta)m_uM^2-32(1+2\beta)m_up_0^2$$

$$-4(1+\beta)m_d(M^2+12p_0^2)\Bigg]\Bigg\}\langle \bar{q}iD_0iD_0q\rangle_{\rho_N}$$

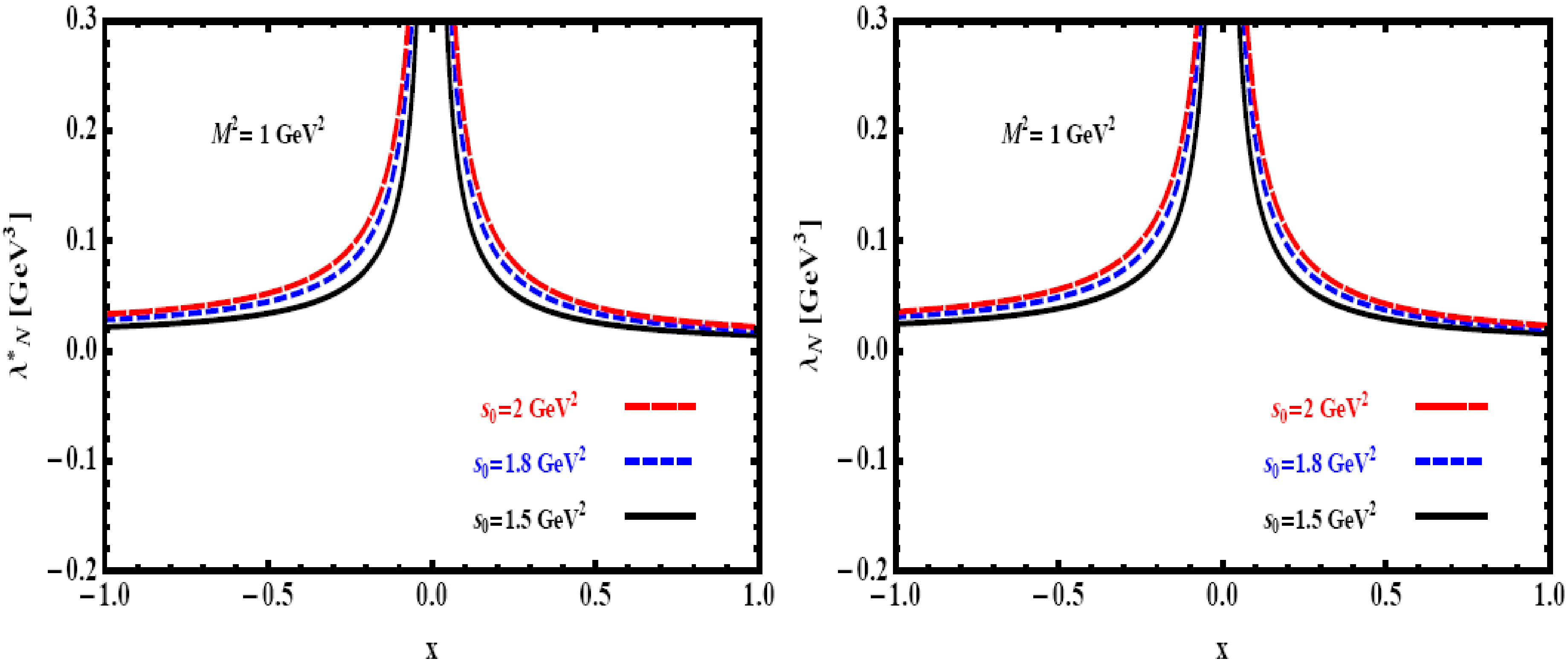
$$-\frac{1}{144\pi^2}\Bigg\{\Bigg[3(\beta-1)m_q\Big(4(1+\beta)m_d-(1+5\beta)m_u\Big)$$

$$+16\Big(5+\beta(2+5\beta)\Big)p_0^2\Bigg]\Bigg\}\langle q^\dagger iD_0q\rangle_{\rho_N}+\frac{1}{36\pi^2}\Bigg\{\Bigg[5+\beta(2+5\beta)\Bigg]m_qp_0^2\Bigg\}$$

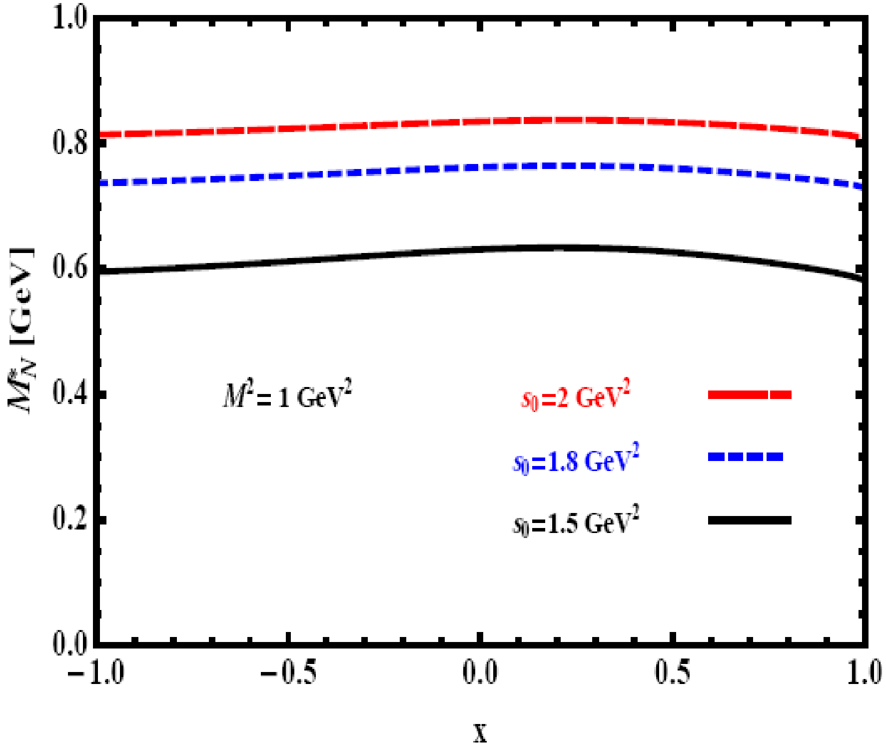
$$-\lambda_N^{*2} e^{-\mu^2/M^2} = \hat{B}\Pi_p^{OPE}.$$

$$\mu^2 = \frac{\frac{\partial}{\partial(-\frac{1}{M^2})}\left(\hat{B}\Pi_p^{OPE}\right)}{\hat{B}\Pi_p^{OPE}}.$$

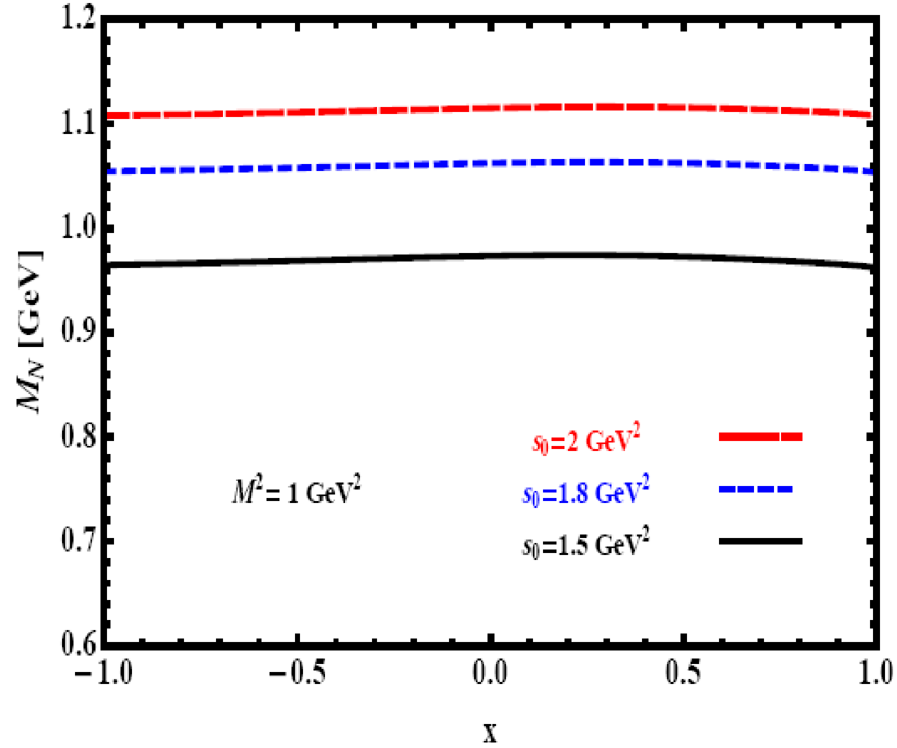
Input Parameters	Values
p_0	$1\ GeV$
m_u	$2.3\ MeV$
m_d	$4.8\ MeV$
ρ_N	$(0.11)^3\ GeV^3$
$\langle q^\dagger q\rangle_{\rho_N}$	$\frac{3}{2}\rho_N$
$\langle\bar{q}q\rangle_0$	$(-0.241)^3\ GeV^3$
m_q	$0.5(m_u+m_d)$
σ_N	0.045 GeV
$\langle\bar{q}q\rangle_{\rho_N}$	$\langle\bar{q}q\rangle_0 + \frac{\sigma_N}{2m_q}\rho_N$
$\langle q^\dagger g_s\sigma Gq\rangle_{\rho_N}$	$-0.33\ GeV^2\rho_N$
$\langle q^\dagger iD_0 q\rangle_{\rho_N}$	$0.18\ GeV\,\rho_N$
$\langle\bar{q}iD_0 q\rangle_{\rho_N}$	$\frac{3}{2}m_q\rho_N \simeq 0$
m_0^2	$0.8\ GeV^2$
$\langle\bar{q}g_s\sigma Gq\rangle_0$	$m_0^2\langle\bar{q}q\rangle_0$
$\langle\bar{q}g_s\sigma Gq\rangle_{\rho_N}$	$\langle\bar{q}g_s\sigma Gq\rangle_0 + 3\ GeV^2\rho_N$
$\langle\bar{q}iD_0iD_0q\rangle_{\rho_N}$	$0.3\ GeV^2\rho_N - \frac{1}{8}\langle\bar{q}g_s\sigma Gq\rangle_{\rho_N}$
$\langle q^\dagger iD_0iD_0q\rangle_{\rho_N}$	$0.031\ GeV^2\rho_N - \frac{1}{12}\langle q^\dagger g_s\sigma Gq\rangle_{\rho_N}$
$\langle\frac{\alpha_s}{\pi}G^2\rangle_0$	$(0.33\pm0.04)^4\ GeV^4$
$\langle\frac{\alpha_s}{\pi}G^2\rangle_{\rho_N}$	$\langle\frac{\alpha_s}{\pi}G^2\rangle_0 - 0.65\ GeV\,\rho_N$



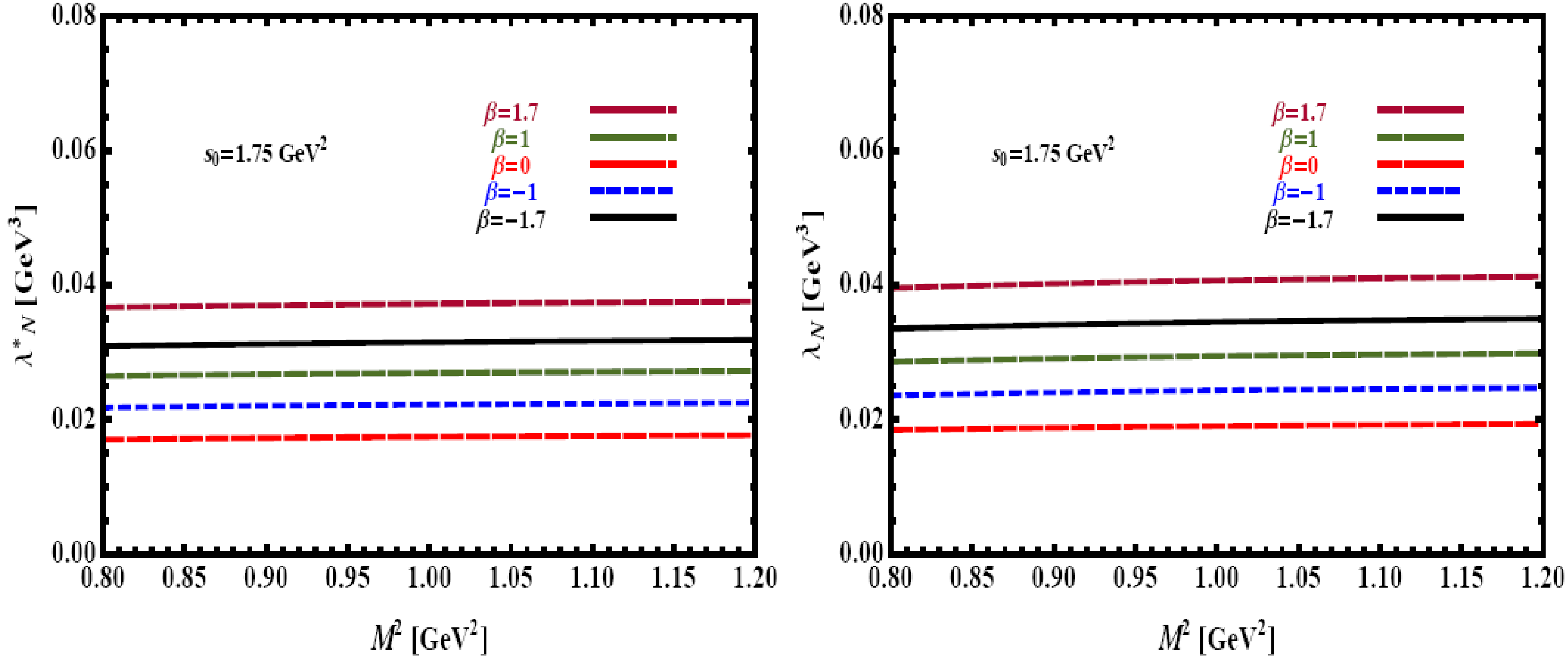
$x = cos\theta$, where $\beta = tan\theta$



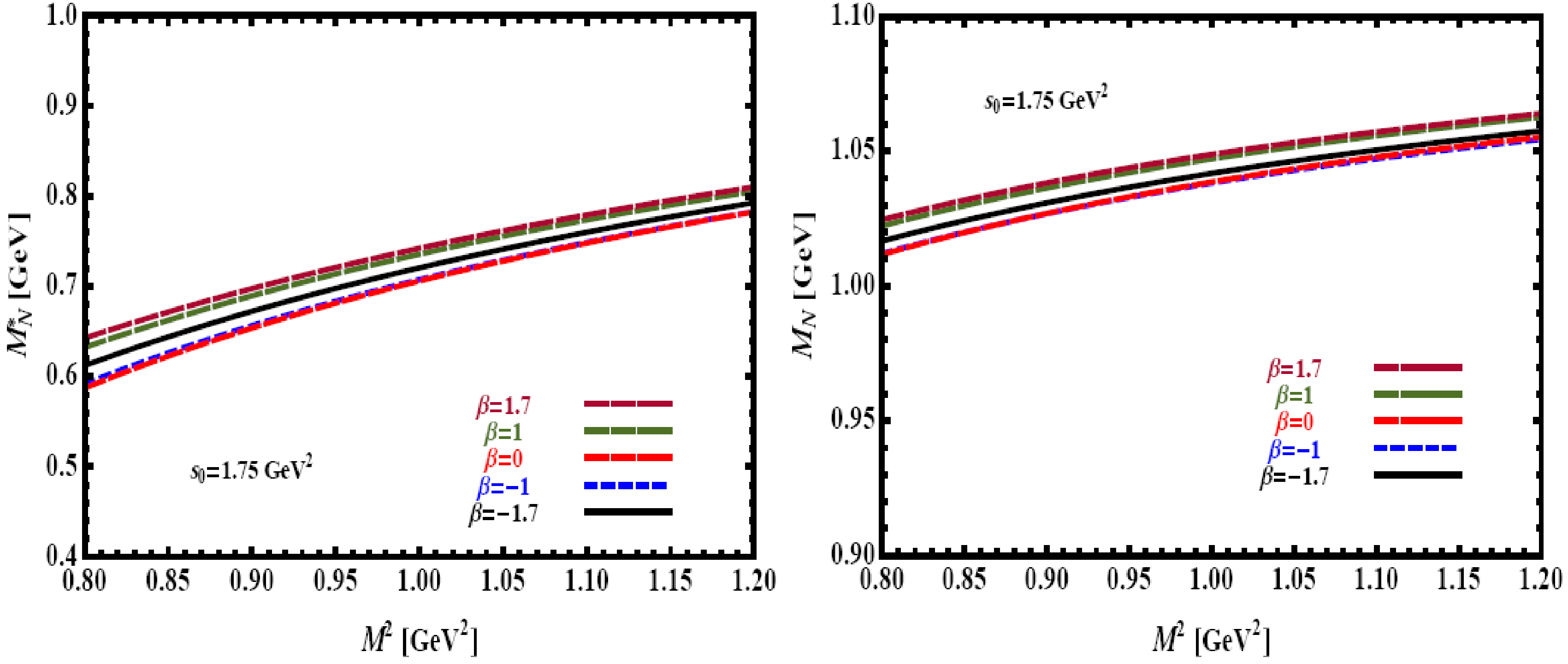
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
x
M*N [GeV]
M² = 1 GeV²
s0=2 GeV²
s0=1.8 GeV²
s0=1.5 GeV²



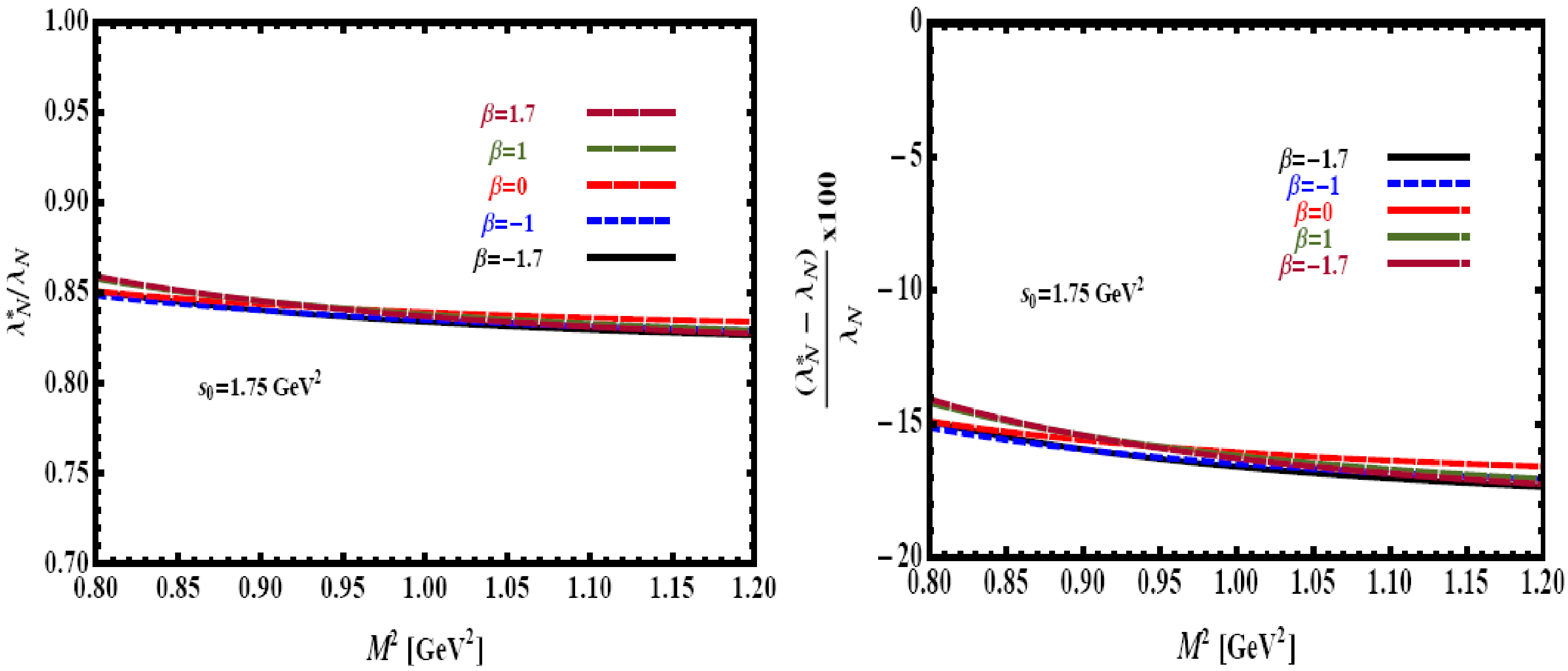
1.2
1.1
1.0
0.9
0.8
0.7
0.6
−1.0
−0.5
0.0
0.5
1.0
x
MN [GeV]
s0=2 GeV²
s0=1.8 GeV²
M² = 1 GeV²
s0=1.5 GeV²



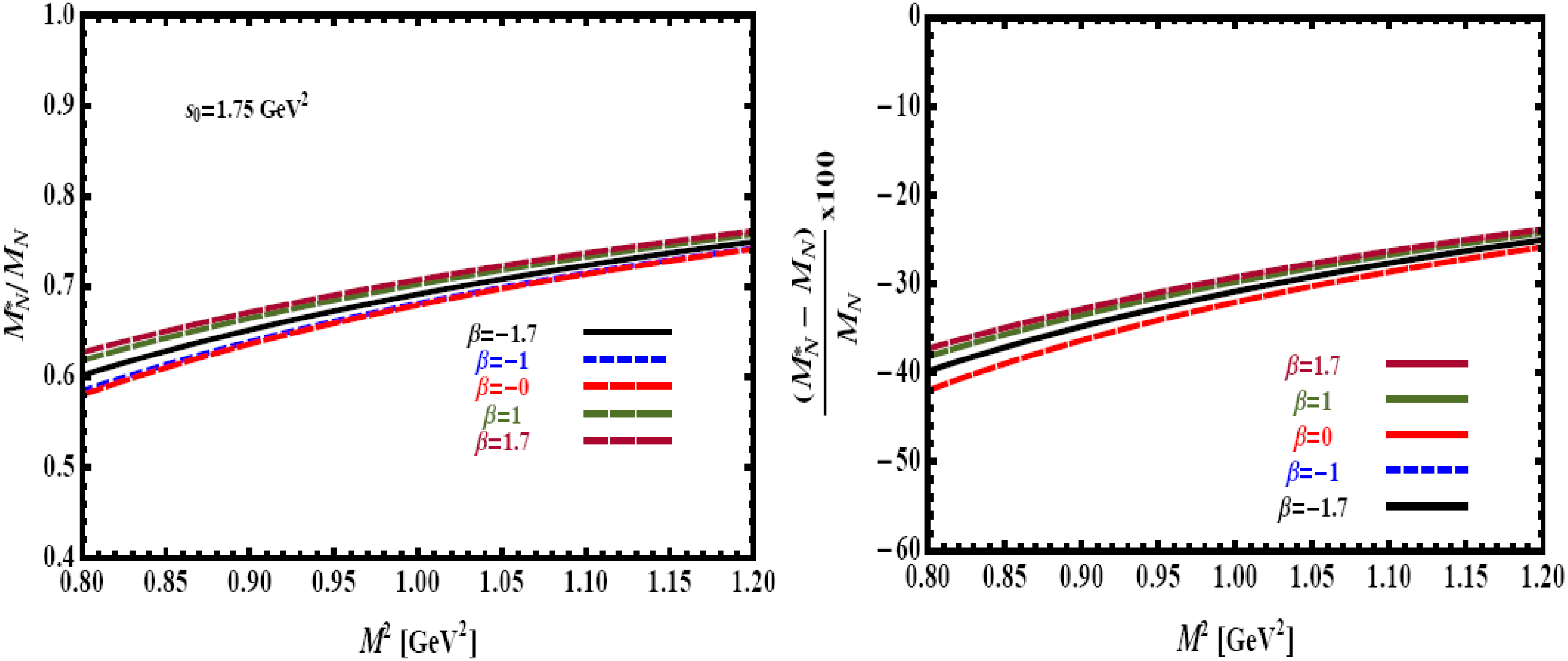
0.08
0.06
0.04
0.02
0.00
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
β=1.7
β=1
β=0
β=−1
β=−1.7
s_0=1.75 GeV^2
$\lambda^*{}_N$ [GeV^3]
λ_N [GeV^3]
M^2 [GeV^2]



1.0
0.9
0.8
0.7
0.6
0.5
0.4
M_N^* [GeV]
β=1.7
β=1
β=0
β=−1
β=−1.7
s_0=1.75 GeV^2
0.80 0.85 0.90 0.95 1.00 1.05 1.10 1.15 1.20
M^2 [GeV^2]
1.10
1.05
1.00
0.95
0.90
M_N [GeV]
s_0=1.75 GeV^2
β=1.7
β=1
β=0
β=−1
β=−1.7
0.80 0.85 0.90 0.95 1.00 1.05 1.10 1.15 1.20
M^2 [GeV^2]



1.00
0.95
0.90
0.85
0.80
0.75
0.70
λ^*_N/λ_N
β=1.7
β=1
β=0
β=−1
β=−1.7
s_0=1.75 GeV^2
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
M^2 [GeV^2]
0
−5
−10
−15
−20
$\frac{(\lambda^*_N - \lambda_N)}{\lambda_N}$x100
β=−1.7
β=−1
β=0
β=1
β=−1.7
s_0=1.75 GeV^2
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
M^2 [GeV^2]



s_0=1.75 GeV^2
M_N^*/M_N
M^2 [GeV^2]
β=−1.7
β=−1
β=−0
β=1
β=1.7
$\frac{(M_N^*-M_N)}{M_N}$x100
β=1.7
β=1
β=0
β=−1
β=−1.7

	m_N^* (GeV)	m_N (GeV)	λ_N^{*2} (GeV^6)	λ_N^2 (GeV^6)
Present work	0.723 ± 0.122	1.045 ± 0.076	0.0009 ± 0.0004	0.0011 ± 0.0005
[40]	-	0.985	-	0.0012 ± 0.0006
[41]	-	0.990 ± 0.050	-	-

Table 2: Average values of the masses and residues squared in nuclear matter and vacuum obtained from sum rules analysis and the comparison of the results with the existing results of the vacuum sum rules for the Ioffe current [40], and value of the mass obtained via Ioffe current in vacuum cosidering the strangness content of the nucleon [41].

From our analysis we obtain the values $\Sigma_S = -(322 \pm 51) MeV$ and $\Sigma_0 = (420 \pm 65) MeV$ for the scalar and time-like vector self-energies of the nucleon in nuclear medium, respectively.

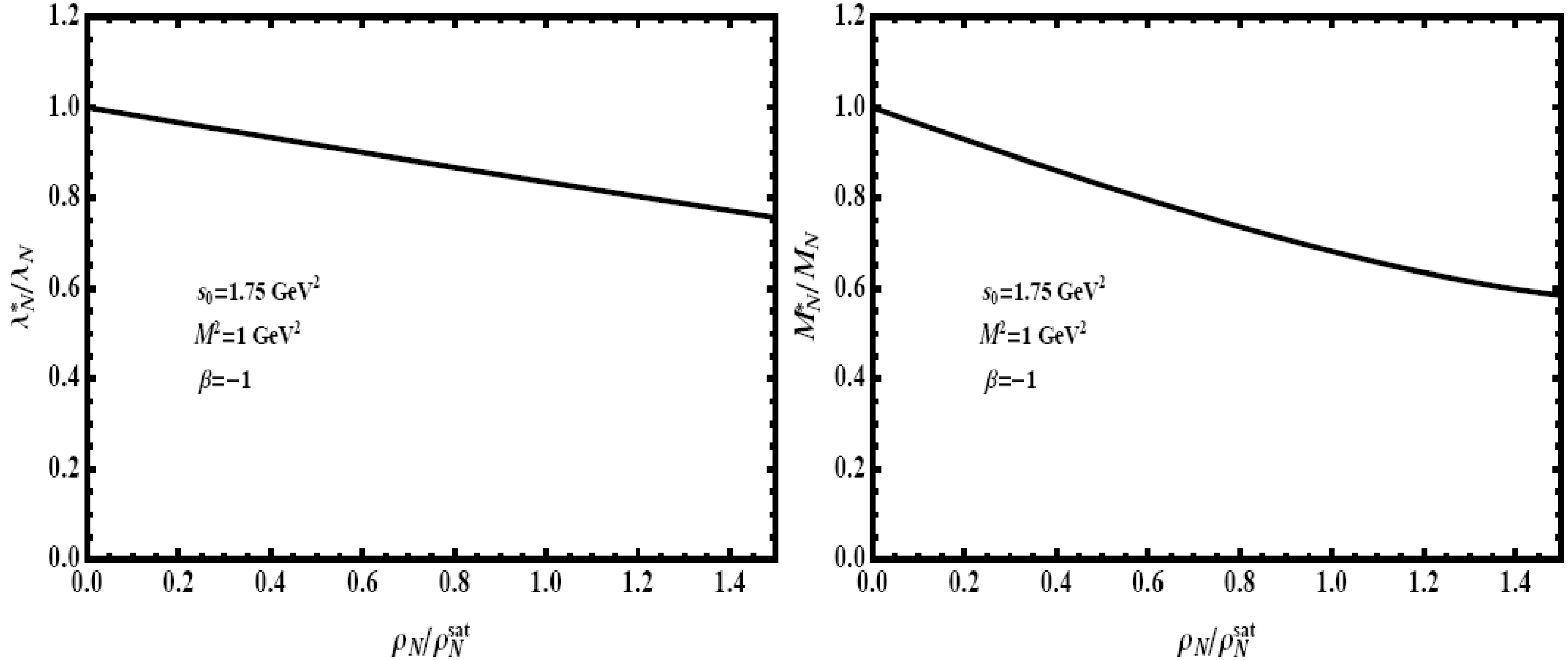


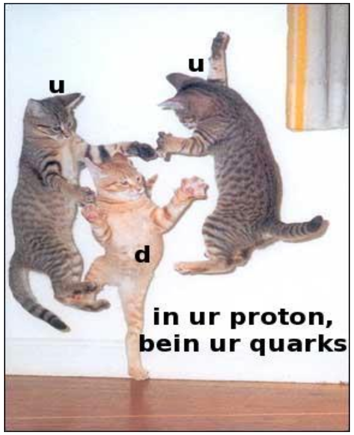
Figure 7: λ^*_N/λ_N versus ρ_N/ρ_N^{sat} (left panel). m^*_N/m_N versus ρ_N/ρ_N^{sat} (right panel).

❖**Decay properties of hadrons:**

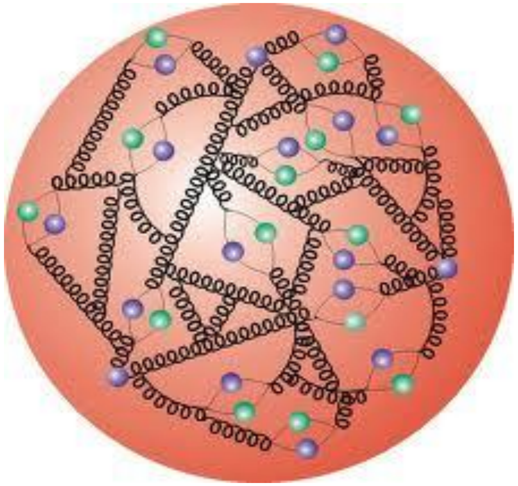
✓ Electromagnetic intractions of hadrons /their electromagnetic properties and radiative decays

➢ using the radiative N→N+photon transition, we investigate the electromagnetic properties of the nucleons

✓ One of the main objectives of QCD is to understand the properties of nucleons (p&n).



Simple picture



Complicated picture

Generalizing the interpolating currents of the nucleon and using distribution amplitudes of it, we calculated the electromagnetic form factors of the nucleons and got comparable results with the experiment. We observed that the Ioffe current sometimes lies out of the reliable region. Using the EM form factors, we calculated the charge and multipole moments of the p &n.

- The electromagnetic form factors of nucleon are defined by the matrix element of the electromagnetic current J_λ^{el} between the initial and final nucleon states

$$\begin{aligned}
&\langle N(p') \mid J_\lambda^{el}(0) \mid N(p)\rangle \\
= \;& \bar{N}(p')\left[\gamma_\lambda F_1(Q^2) - \frac{i}{2m_N}\sigma_{\lambda\nu}q^\nu F_2(Q^2)\right] N(p),
\end{aligned}$$

where $Q^2 = -q^2$, is the negative of the square of the virtual photon momentum, $q = p - p'$ and F_1 and F_2 are the Dirac and Pauli form factors, respectively.

- Another set of nucleon form factors is the so called Sa form factors, which are defined in terms of the $F_1(Q^2)$ $F_2(Q^2)$ as follows:

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2),$$

At the static limit, values at $Q^2 = 0$ are $G_E^P(0) = 1$, $G_E^n(0) = 0$, $G_M^P(0) = \mu_P = 2.792847337(29)$ and $G_M^n(0) = \mu_n = -1.91304272(45)$, where μ_P and μ_n ar anomalous magnetic moments of the proton and neut in units of the Bohr magneton.

- $$J^N(x) = 2\varepsilon^{abc}\sum_{\ell=1}^{2}(u^{Ta}(x)CA_1^\ell d^b(x))A_2^\ell u^c(x),$$

  where $A_1^1 = I$, $A_1^2 = A_2^1 = \gamma_5$, $A_2^2 = \beta$, and C is the cha conjugation operator, and a, b, c are the color indices. electromagnetic current is:

  $$J_\lambda^{el}(x) = e_u\bar{u}\gamma_\lambda u + e_d\bar{d}\gamma_\lambda d\,,$$

  and the choice $\beta = -1$ corresponds to the Ioffe curren

- Analysis of the experimental results lead that the mag form factors of the nucleon are very well described by dipole formula

$$G_M^{n,p}(Q^2) = \frac{\mu_{n,p}}{\left(1 + \frac{Q^2}{(0.71\ GeV)^2}\right)^2} = \mu_{n,p} G_D.$$

$$\Pi_\lambda(p,q) = i\int d^4x e^{iqx}\langle 0 \mid T\{J^N(0)J_\lambda^{el}(x)\} \mid N(p)\rangle,$$

Physical or phenomenological side

$$\Pi_\lambda(p,q) = \sum_s \frac{\langle 0 \mid J^N(0) \mid N(p',s)\rangle\langle N(p',s) \mid J_\lambda^{el}(0) \mid N(p)\rangle}{m_N^2 - p'^2}$$

$$\langle 0 \mid J^N(0) \mid N(p',s)\rangle = \lambda_N N(p',s)$$

$$\sum_s N(p',s)\overline{N}(p',s) = \not{p}' + m_N,$$

$$\Pi_\lambda(p,q) = \frac{\lambda_N}{m_N^2 - p'^2}(\not{p}' + m_N)\left[\gamma_\lambda F_1(Q^2) - \frac{i}{2m_N}\sigma_{\lambda\nu}q^\nu F_2(Q^2)\right] N(p) + \cdots$$

QCD or theoretical side

$$
\begin{aligned}
(\Pi_\lambda)_\rho &= \frac{i}{2}\int d^4x e^{iqx}\sum_{\ell=1}^{2}\Big\{ \\
& e_u(CA_1^\ell)_{\alpha\gamma}\Big[A_2^\ell S_u(-x)\gamma_\lambda\Big]_{\rho\phi}4\epsilon^{abc}\langle 0|u_\alpha^a(0)u_\phi^b(x)d_\gamma^c(0)|N(p)\rangle \\
+ & e_u(A_2^\ell)_{\rho\alpha}\Big[(CA_1^\ell)^T S_u(-x)\gamma_\lambda\Big]_{\gamma\phi}4\epsilon^{abc}\langle 0|u_\alpha^a(0)u_\phi^b(x)d_\gamma^c(0)|N(p)\rangle \\
+ & e_d(A_2^\ell)_{\rho\phi}\Big[CA_1^\ell S_d(-x)\gamma_\lambda\Big]_{\alpha\gamma}4\epsilon^{abc}\langle 0|u_\alpha^a(0)u_\phi^b(0)d_\gamma^c(x)|N(p)\rangle\Big\},
\end{aligned}
$$

-
$$
\begin{aligned}
& 4\langle 0|\epsilon^{abc}u_\alpha^a(a_1x)u_\beta^b(a_2x)d_\gamma^c(a_3x)|P\rangle \\
= \;& \mathcal{S}_1 m_N C_{\alpha\beta}(\gamma_5 N)_\gamma + \mathcal{S}_2 m_N^2 C_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma \\
+ \;& \mathcal{P}_1 m_N(\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{P}_2 m_N^2(\gamma_5 C)_{\alpha\beta}(\not{x}N)_\gamma \\
+ \;& (\mathcal{V}_1 + \frac{x^2 m_N^2}{4}\mathcal{V}_1^M)(\not{p}C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
+ \;& \mathcal{V}_2 m_N(\not{p}C)_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma + \mathcal{V}_3 m_N(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu\gamma_5 N \\
+ \;& \mathcal{V}_4 m_N^2(\not{x}C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
+ \;& \ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

- where, the calligraphic functions are defined in terms o nucleon distribution amplitudes:

$$\begin{aligned}
\mathcal{S}_1 &= S_1, \\
2px\mathcal{S}_2 &= S_1 - S_2, \\
&\cdots\cdots \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots &\quad \cdots\cdots\cdots\cdots\cdots\cdots, \\
4(px)^2\mathcal{V}_6 &= -V_1 + V_2 + V_3 + V_4 + V_5 - V_6 \\
\cdots\cdots &
\end{aligned}$$

- $$\begin{aligned}
V_1(x_i,\mu) &= 120x_1x_2x_3[\phi_3^0(\mu)+\phi_3^+(\mu)(1-3x_3)],\\
V_2(x_i,\mu) &= 24x_1x_2[\phi_4^0(\mu)+\phi_3^+(\mu)(1-5x_3)],\\
V_3(x_i,\mu) &= 12x_3\{\psi_4^0(\mu)(1-x_3)+\psi_4^-(\mu)[x_1^2+x_2^2-\\
&\quad +\psi_4^+(\mu)(1-x_3-10x_1x_2)\},\\
V_4(x_i,\mu) &= 3\{\psi_5^0(\mu)(1-x_3)+\psi_5^-(\mu)[2x_1x_2-x_3(1-\\
&\quad +\psi_5^+(\mu)[1-x_3-2(x_1^2+x_2^2)]\},\\
V_5(x_i,\mu) &= 6x_3[\phi_5^0(\mu)+\phi_5^+(\mu)(1-2x_3)],\\
V_6(x_i,\mu) &= 2[\phi_6^0(\mu)+\phi_6^+(\mu)(1-3x_3)],\\
A_1(x_i,\mu) &= 120x_1x_2x_3\phi_3^-(\mu)(x_2-x_1),\\
A_2(x_i,\mu) &= 24x_1x_2\phi_4^-(\mu)(x_2-x_1),
\end{aligned}$$

- They contain the following functions parameterized in terms of 8 independent parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_d^2 and f_u^1 as

$$
\begin{aligned}
\phi_3^0 &= \phi_6^0 = f_N \\
\phi_4^0 &= \phi_5^0 = \frac{1}{2}(\lambda_1 + f_N) \\
\xi_4^0 &= \xi_5^0 = \frac{1}{6}\lambda_2 \\
\psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_N - \lambda_1) \\
\phi_3^- &= \frac{21}{2}A_1^u,
\end{aligned}
$$

- The numerical values are obtained using:

$$
\begin{aligned}
f_N &= (5.0 \pm 0.5) \times 10^{-3}\ GeV^2, \\
\lambda_1 &= -(2.7 \pm 0.9) \times 10^{-2}\ GeV^2, \\
\lambda_2 &= (5.4 \pm 1.9) \times 10^{-2}\ GeV^2.
\end{aligned}
$$

For other five independent parameters, we have used
three sets as:

$$
\begin{aligned}
\text{Set 1 } : A_1^u &= 0.38 \pm 0.15, \quad V_1^d = 0.23 \pm 0.03, \\
f_2^d &= 0.22 \pm 0.05, \quad f_1^u = 0.07 \pm 0.05, \\
f_1^d &= 0.40 \pm 0.05,
\end{aligned}
$$

- $$\begin{aligned} \text{Set 2 }: A_1^u &= \frac{1}{14}, \;\; V_1^d = \frac{13}{42}, \;\; f_1^d = 0.40 \pm 0.05 \\ f_2^d &= 0.22 \pm 0.05, \;\; f_1^u = 0.07 \pm 0.05, \end{aligned}$$

$$\begin{aligned} \text{Set 3 (asymptotic)}: A_1^u &= 0, \;\; V_1^d = \frac{1}{3}, \;\; f_1^d = \frac{3}{10}, \;\; f_2 \\ f_1^u &= \frac{1}{10} \end{aligned}$$

$$
\begin{aligned}
F_1(Q^2) = \frac{-1}{2\lambda_N} e^{m_N^2/M_B^2} \Big\{ & e_u m_N \int_{t_0}^{1} dx_2 \int_0^{1-x_2} dx_1 e^{-s(x_2,Q^2)/M_B^2} \Big[2\mathcal{H}_{5,-7}(x_i)(1-\beta) \\
& +4(\mathcal{H}_{17}(x_i) - 2\mathcal{H}_{19}(x_i))(1+\beta) \Big] + e_u m_N \int_{t_0}^{1} dx_2 \int_0^{1-x_2} dx_1 \int_{t_0}^{x_2} \frac{dt_1}{t_1} e^{-s(t_1,Q^2)/M_B^2} \Big(\\
& -2 \Big[\mathcal{H}_{20,-18}(x_i)(1+\beta) - \mathcal{H}_6(x_i)(-1+\beta) \Big] \\
& -\frac{1}{M_B^2} \Big[\Big\{ 2\mathcal{H}_{20,18}(x_i)(1+\beta)(Q^2 + s(t_1,Q^2) + m_N^2(-1+t_1)) \Big\} \\
& +m_N^2 \left\{ \mathcal{H}_{15,-14}(x_i) t_1 (1-\beta) - 4\mathcal{H}_{21,24}(x_i) t_1 (1+\beta) \right. \\
& \left. +2\mathcal{H}_{10}(x_i)(-1+\beta)(t_1 - x_2) + 2(\mathcal{H}_{16}(x_i)(-1+\beta) + 2\mathcal{H}_{24}(x_i)(1+\beta))x_2 \right\} \Big] \Big) \\
- \quad & e_u m_N \int_{t_0}^{1} dx_2 \int_0^{1-x_2} dx_1 e^{-s_0/M_B^2} \frac{t_0}{Q^2 + m_N^2 t_0^2} \Big(2\mathcal{H}_{20,18}(x_i)(1+\beta)(Q^2 + s_0 + m_N^2(-1+t_0)) \\
& +m_N^2 \Big[\left\{ \mathcal{H}_{-8,9}(x_i)(1-\beta) - (3\mathcal{H}_{21,24}(x_i) + 8\mathcal{H}_{23}(x_i))(1+\beta) \right\} t_0 \\
& +2\mathcal{H}_{10}(x_i)(-1+\beta)(t_0 - x_2) + 2(\mathcal{H}_{16}(x_i)(-1+\beta) + \mathcal{H}_{24}(x_i)(1+\beta))x_2 \Big] \Big)
\end{aligned}
$$

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....

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T. M. Aliev, K. Azizi, A. Ozpineci, M. Savcı
Phys. Rev. D 77 (2008)114014.

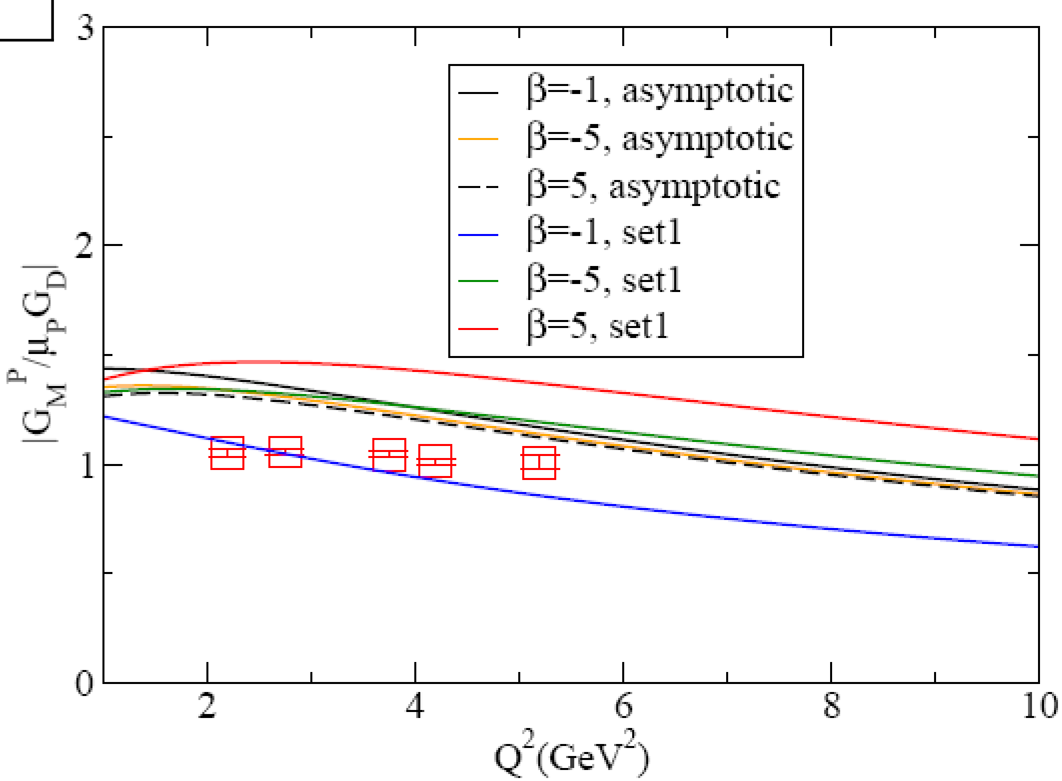
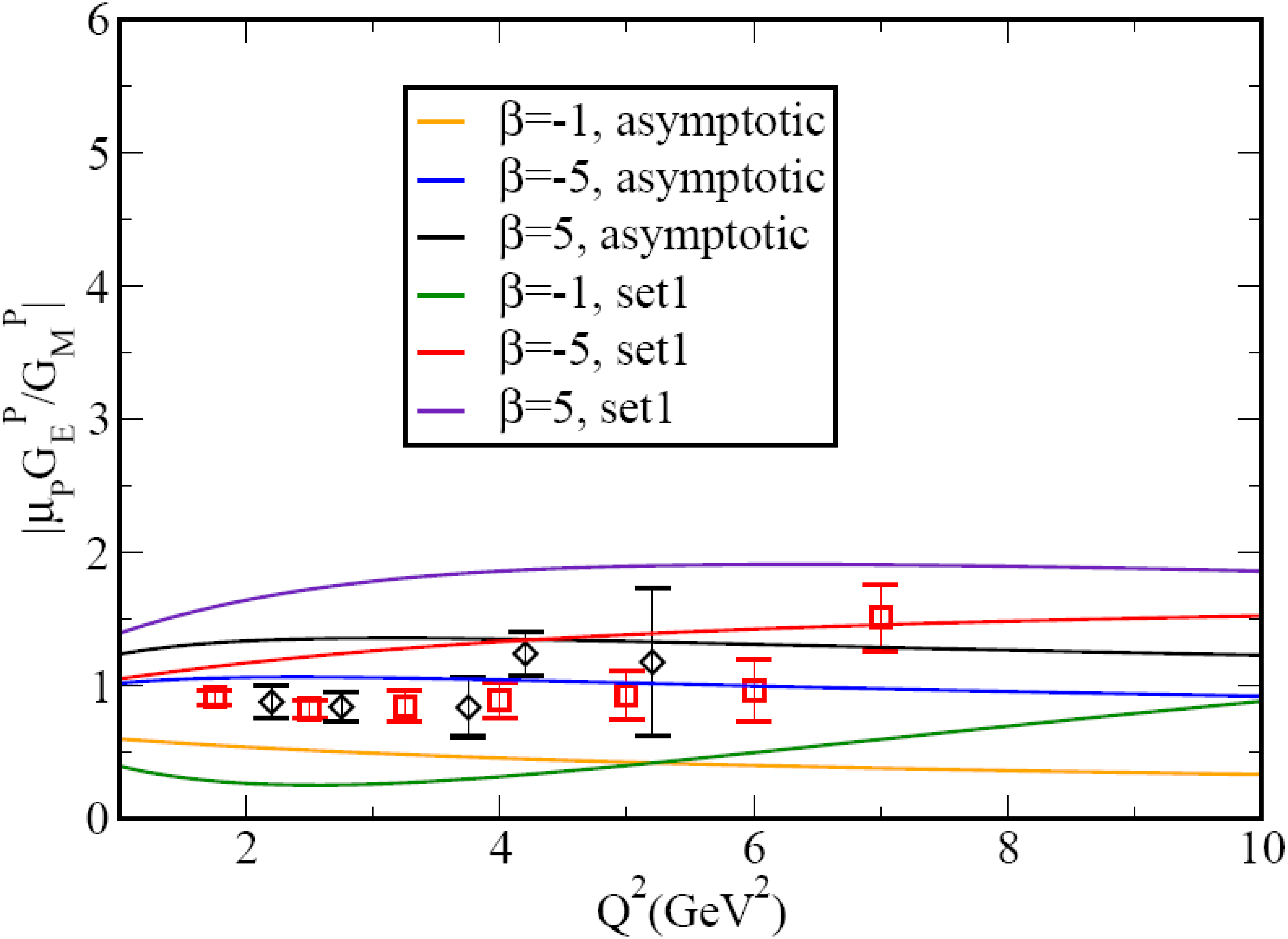
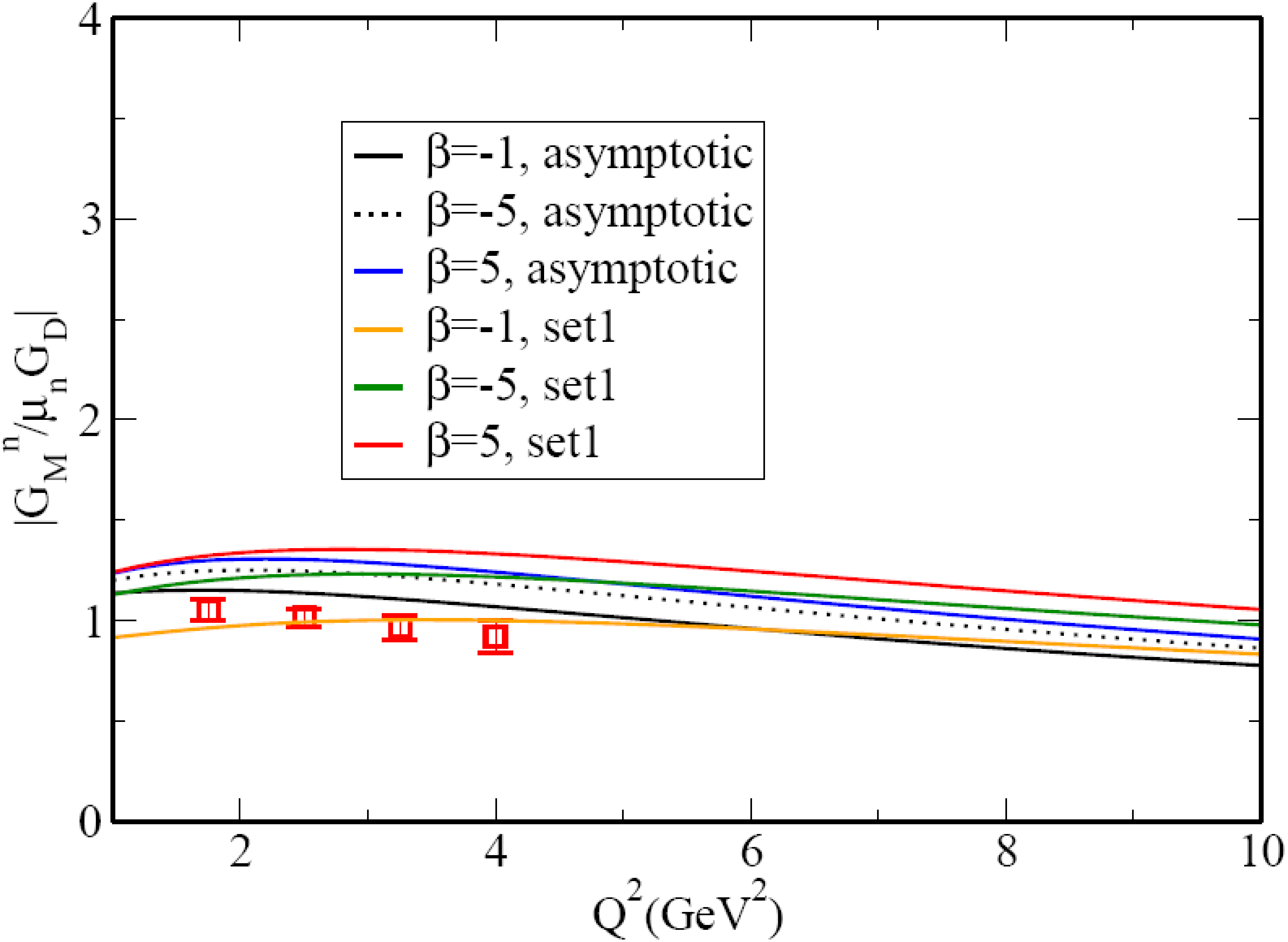
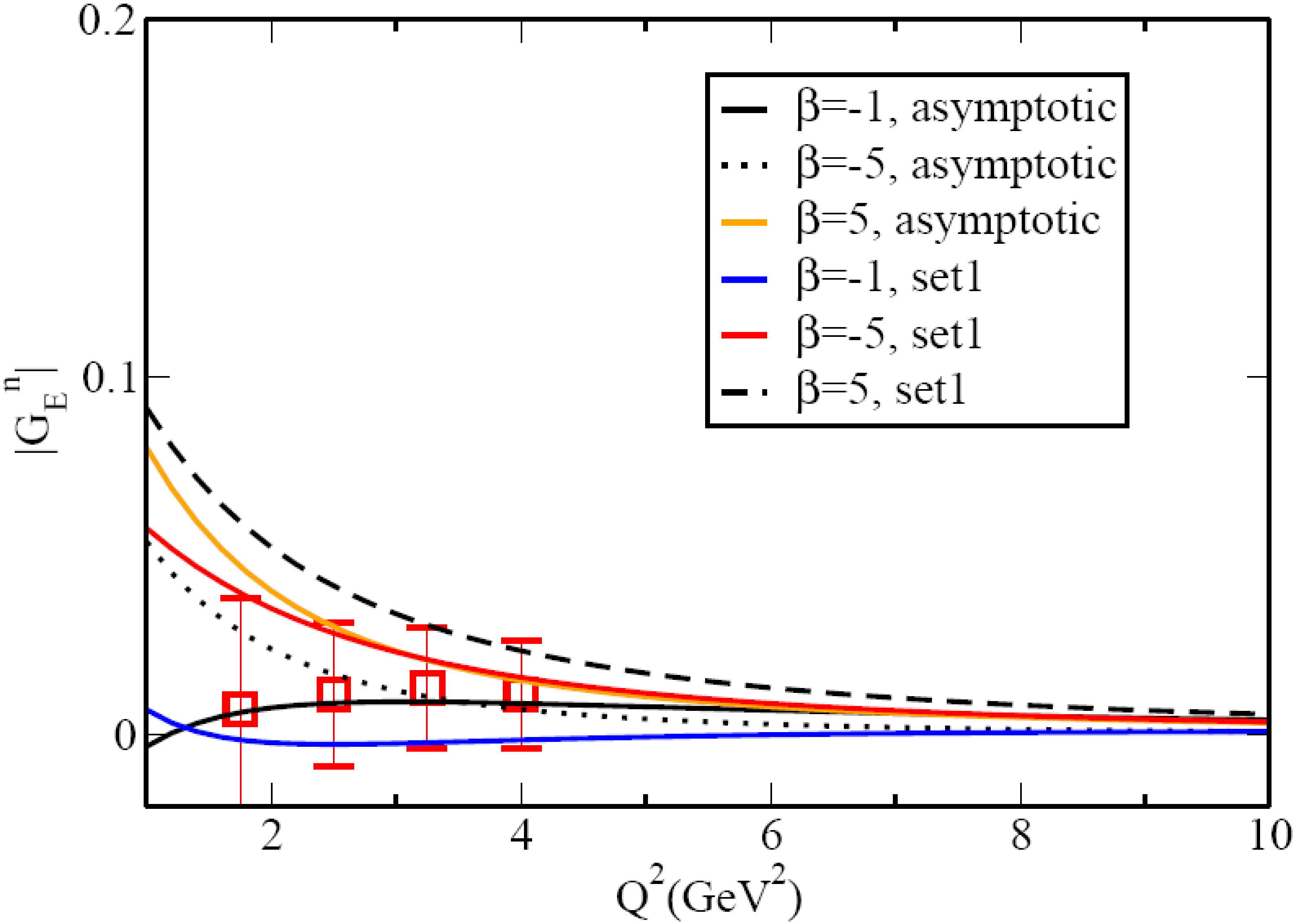


Figure: The dependence of $G_M^P/\mu_P G_D$ on Q^2 at $s_0 = 2.25\ GeV^2$, $M_B^2 = 1.2\ GeV^2$ for $\beta = -1,\ -5$ and 5. Th boxes correspond to experimental data

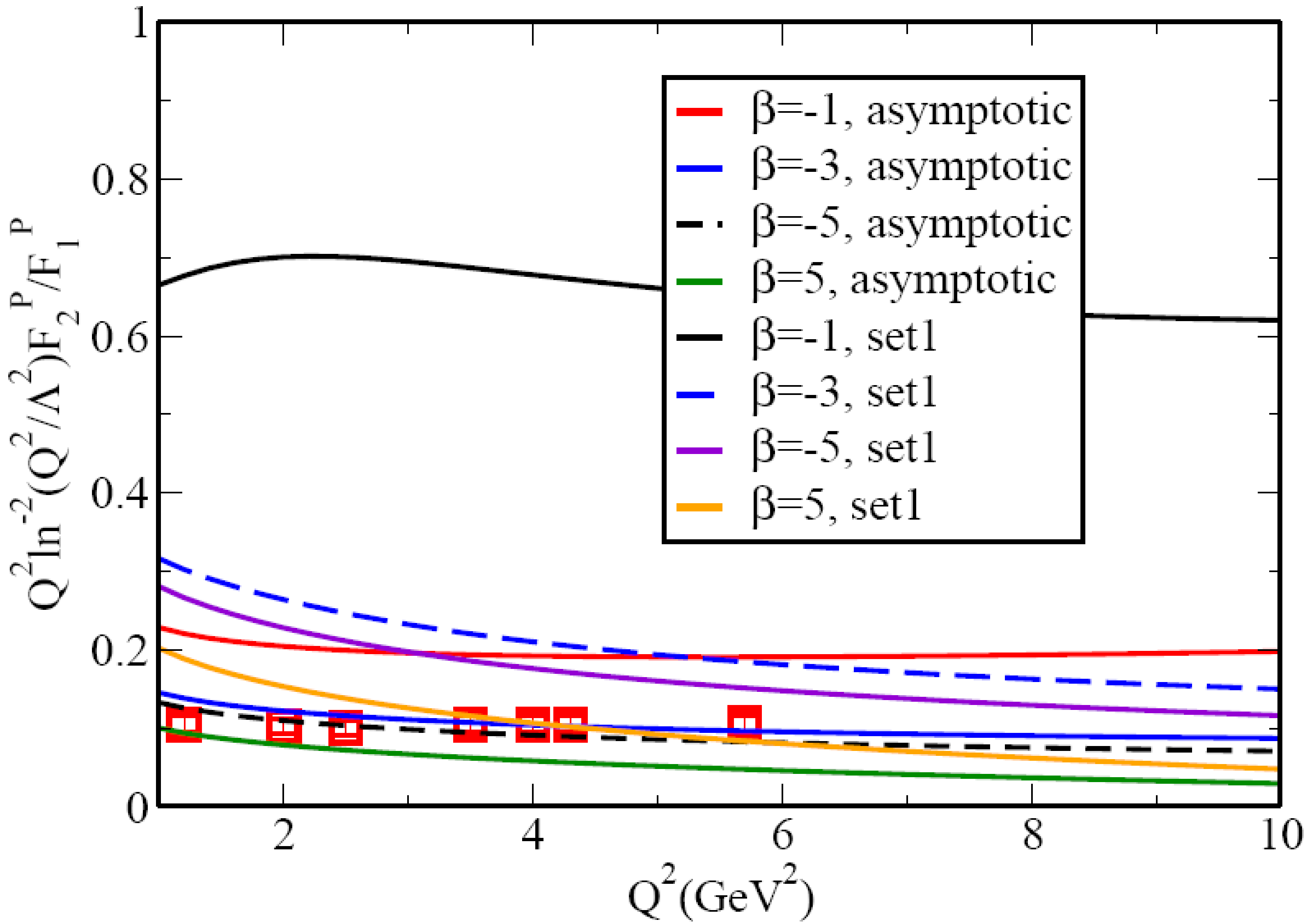


β=-1, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-5, set1
β=5, set1
$|\mu_P G_E^P/G_M^P|$
$Q^2(GeV^2)$
0
1
2
3
4
5
6
2
4
6
8
10

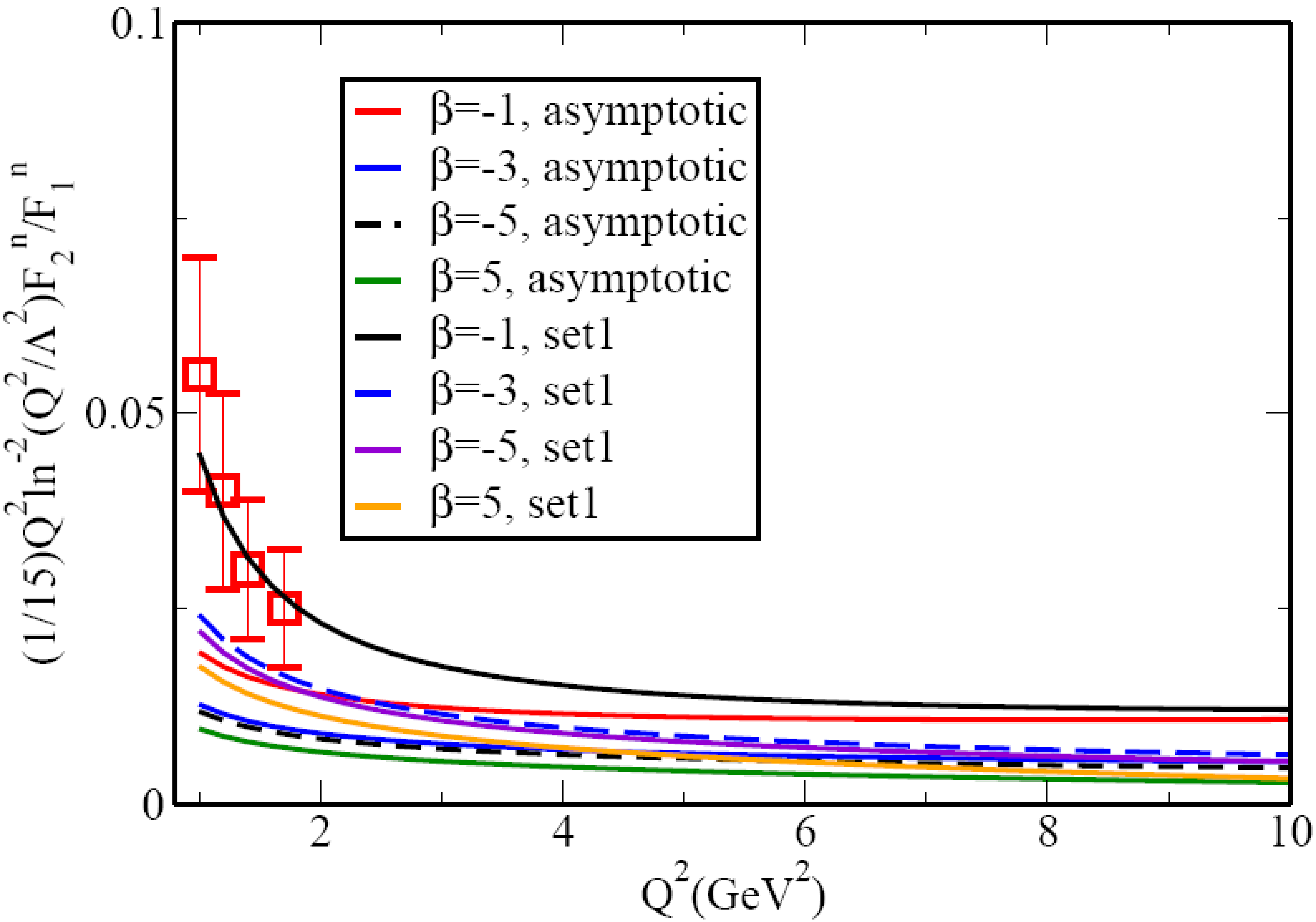

4
3
2
1
0
2
4
6
8
10
$|G_M{}^n/\mu_n G_D|$
$Q^2(GeV^2)$
β=-1, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-5, set1
β=5, set1



0.2
0.1
0
$|G_E^n|$
2
4
6
8
10
$Q^2(\mathrm{GeV}^2)$
β=-1, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-5, set1
β=5, set1



$Q^2 \ln^{-2}(Q^2/\Lambda^2)F_2^P/F_1^P$
$Q^2(GeV^2)$
0
0.2
0.4
0.6
0.8
1
2
4
6
8
10
β=-1, asymptotic
β=-3, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-3, set1
β=-5, set1
β=5, set1



T. M. Aliev, K. Azizi, A. Ozpineci, M. Savci, Phys.Rev.D77:114014,2008

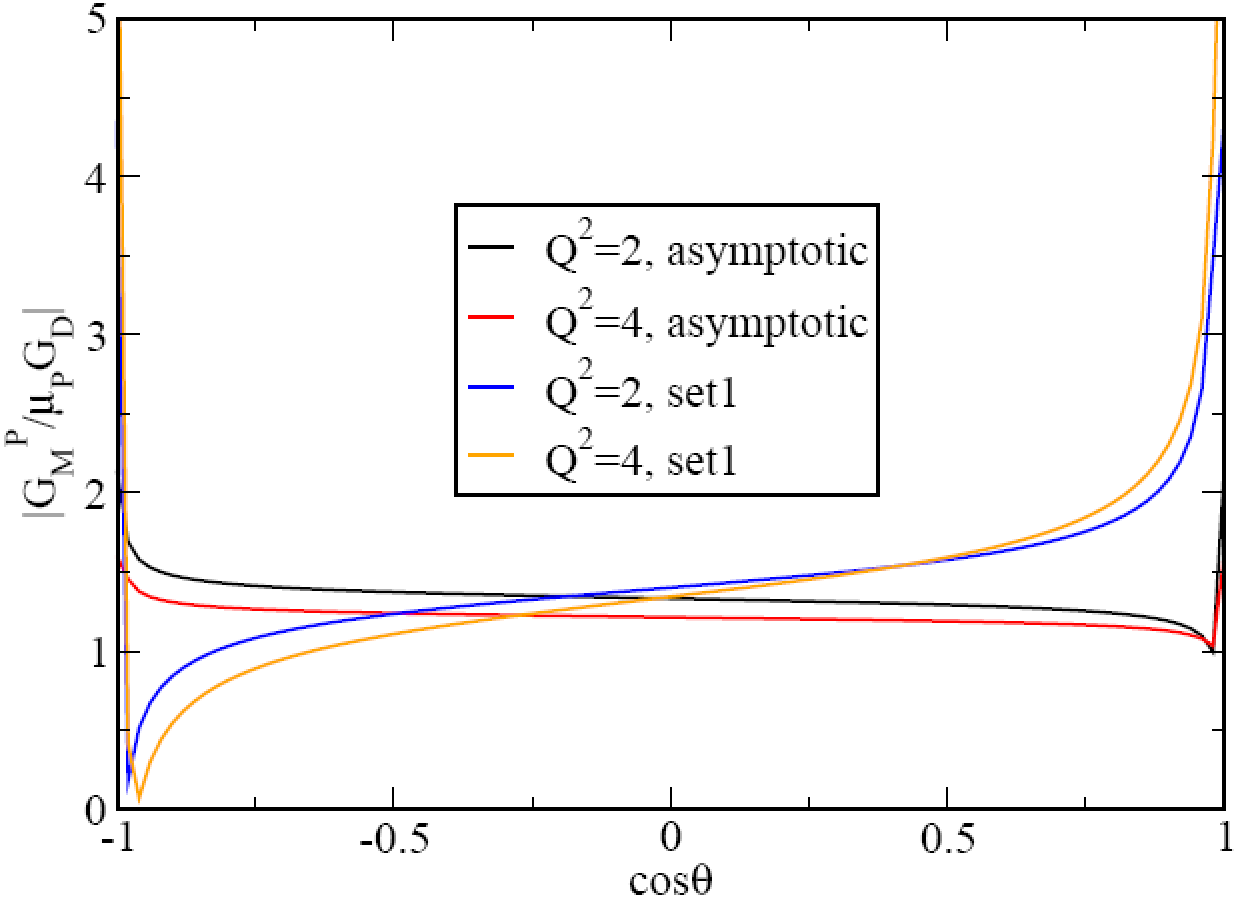


Figure: The dependence of $G_M^P/\mu_P G_D$ on $cos\theta$ at $s_0 = 2.25\ GeV^2, M_B^2 = 1.2\ GeV^2$ for two different values of Q^2, i.e. $Q^2 = 2\ GeV^2$ and $Q^2 = 4\ GeV^2$.

QCD sum rules

relativistic three-quark model

constituent quark model

HQET

	$\lvert G_M \rvert$	$\lvert G_E \rvert$
$\Sigma_b^{*0} \to \Sigma_b^0 \gamma$	1.0 ± 0.4	0.005
$\Sigma_b^{*-} \to \Sigma_b^- \gamma$	2.1 ± 0.7	0.016
$\Sigma_b^{*+} \to \Sigma_b^+ \gamma$	4.2 ± 1.4	0.026
$\Sigma_c^{*+} \to \Sigma_c^+ \gamma$	1.2 ± 0.2	0.014
$\Sigma_c^{*0} \to \Sigma_c^0 \gamma$	0.5 ± 0.1	0.003
$\Sigma_c^{*++} \to \Sigma_c^{++} \gamma$	2.8 ± 0.8	0.030
$\Xi_b^{*0} \to \Xi_b^0 \gamma$	8.5 ± 3.0	0.085
$\Xi_b^{*-} \to \Xi_b^- \gamma$	0.9 ± 0.3	0.011
$\Xi_c^{*+} \to \Xi_c^+ \gamma$	4.0 ± 1.5	0.075
$\Xi_c^{*0} \to \Xi_c^0 \gamma$	0.45 ± 0.15	0.007
$\Sigma_b^{*0} \to \Lambda_b^0 \gamma$	7.3 ± 2.8	0.075
$\Sigma_c^{*+} \to \Lambda_c^+ \gamma$	3.8 ± 1.0	0.060

	Γ (present work)	Γ [12]	Γ [13]	Γ [14]
$\Sigma_b^{*0} \to \Sigma_b^0 \gamma$	0.028 ± 0.016	0.15	-	0.08
$\Sigma_b^{*-} \to \Sigma_b^- \gamma$	0.11 ± 0.06	-	-	0.32
$\Sigma_b^{*+} \to \Sigma_b^+ \gamma$	0.46 ± 0.22	-	-	1.26
$\Sigma_c^{*+} \to \Sigma_c^+ \gamma$	0.40 ± 0.16	0.22	0.14 ± 0.004	-
$\Sigma_c^{*0} \to \Sigma_c^0 \gamma$	0.08 ± 0.03	-	-	-
$\Sigma_c^{*++} \to \Sigma_c^{++} \gamma$	2.65 ± 1.20	-	-	-
$\Xi_b^{*0} \to \Xi_b^0 \gamma$	135 ± 65	-	-	-
$\Xi_b^{*-} \to \Xi_b^- \gamma$	1.50 ± 0.75	-	-	-
$\Xi_c^{*+} \to \Xi_c^+ \gamma$	52 ± 25	-	54 ± 3	-
$\Xi_c^{*0} \to \Xi_c^0 \gamma$	0.66 ± 0.32	-	0.68 ± 0.04	-
$\Sigma_b^{*0} \to \Lambda_b^0 \gamma$	114 ± 45	251	-	344
$\Sigma_c^{*+} \to \Lambda_c^+ \gamma$	130 ± 45	233	151 ± 4	-

Table 4: The results for the decay rates of the corresponding radiative transitions in KeV.

T. M. Aliev, K. Azizi, A. Ozpineci, Phys. Rev. D 79, 056005 (2009)

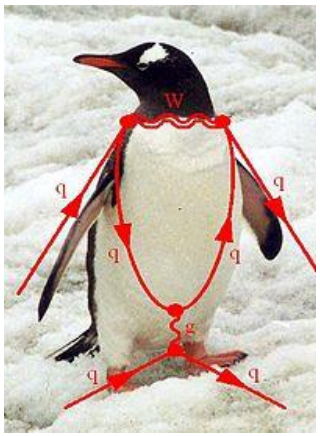
We have no experimental data yet.

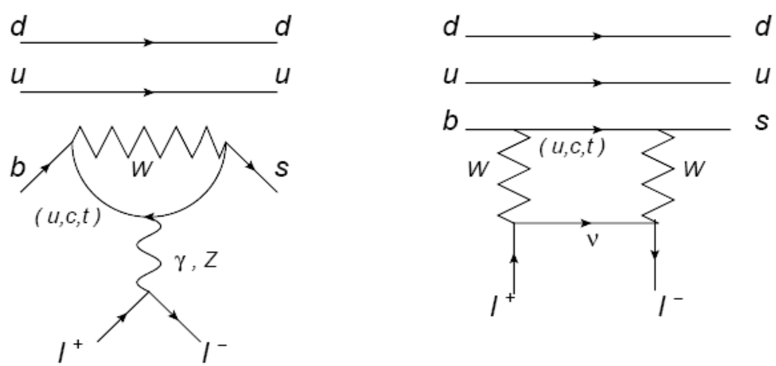
✓ Weak intractions of hadrons/ their semileptonic decays (SM)

✓heavy spin 1/2 → heavy ½ (neutral current) : ½→ ½ Z (γ)→ ½ l^+l^-

[FCNC transitions: $\boldsymbol{\Lambda_b}$→$\boldsymbol{\Lambda}$ l^+l^-] → This decay is in agenda of different experiments nowadays.

Figure: Penguin and box diagram responsible for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ transition





u	c	t
d	s	b

upper line←→lower line: tree level

transitions inside a line: loop

$$\mathcal{H}_{eff} = \frac{G_F \ \alpha_{em} V_{tb} \ V_{ts}^*}{2\sqrt{2}\ \pi}\Big\{ C_9^{eff} \ \bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu l + C_{10} \ \bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu\gamma_5 l$$
$$- 2m_b \ C_7 \frac{1}{q^2} \ \bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\bar{l}\gamma^\mu l \Big\}$$

$$\langle \Lambda_b(p+q)|\mathcal{H}_{eff}|\Lambda(p)\rangle$$

Transition matrix elements

$$\langle \Lambda(p) \mid \bar{s}\gamma_\mu(1-\gamma_5)b \mid \Lambda_b(p+q)\rangle = \bar{u}_\Lambda(p)\Big[\gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu}q^\nu f_2(Q^2) + q^\mu f_3(Q^2$$
$$- \gamma_\mu\gamma_5 g_1(Q^2) - i\sigma_{\mu\nu}\gamma_5 q^\nu g_2(Q^2) - q^\mu\gamma_5 g_3(Q^2)\Big]u_{\Lambda_b}(p+q)\ ,$$

$$\langle \Lambda(p) \mid \bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \mid \Lambda_b(p+q)\rangle = \bar{u}_\Lambda(p)\Big[\gamma_\mu f_1^T(Q^2) + i\sigma_{\mu\nu}q^\nu f_2^T(Q^2) + q^\mu f_3^T(Q^2)$$
$$+ \gamma_\mu\gamma_5 g_1^T(Q^2) + i\sigma_{\mu\nu}\gamma_5 q^\nu g_2^T(Q^2) + q^\mu\gamma_5 g_3^T(Q^2)\Big]u_{\Lambda_b}(p+q)\ ,$$

Correlation functions

$$\Pi_\mu^I(p,q) = i\int d^4x e^{-iqx}\langle 0 \mid T\{J^{\Lambda_b}(0), \bar{b}(x)\gamma_\mu(1-\gamma_5)s(x))\} \mid \Lambda(p)\rangle\ ,$$
$$\Pi_\mu^{II}(p,q) = i\int d^4x e^{-iqx}\langle 0 \mid T\{J^{\Lambda_b}(0), \bar{b}(x)i\sigma_{\mu\nu}q^\nu(1+\gamma_5)s(x)\} \mid \Lambda(p)\rangle\ ,$$

$$J^{\Lambda_b}(x) = \frac{1}{\sqrt{6}}\epsilon_{abc}\Big\{2\Big[(q_1^{aT}(x)Cq_2^b(x))\gamma_5 b^c(x) + \beta(q_1^{aT}(x)C\gamma_5 q_2^b(x))b^c(x)\Big]$$
$$+ (q_1^{aT}(x)Cb^b(x))\gamma_5 q_2^c(x) + \beta(q_1^{aT}(x)C\gamma_5 b^b(x))q_2^c(x)$$
$$+ (b^{aT}(x)Cq_2^b(x))\gamma_5 q_1^c(x) + \beta(b^{aT}(x)C\gamma_5 q_2^b(x))q_1^c(x)\Big\}\ ,$$

cuurent

$$\frac{d\Gamma}{ds} = \frac{G^2\alpha_{em}^2 m_{\Lambda_b}}{8192\pi^5}|V_{tb}V_{ts}^*|^2 v\sqrt{\lambda}\left[\Theta(s) + \frac{1}{3}\Delta(s)\right]$$

Functions of form factors

T.M. Aliev, K. Azizi, M. Savcı, ***Phys. Rev. D 81 (2010) 056006***.

	Present work	HQET
$Br(\Lambda_b \to \Lambda e^+ e^-)$	$(4.6 \pm 1.6) \times 10^{-6}$	$(2.23 \div 3.34) \times 10^{-6}$
$Br(\Lambda_b \to \Lambda \mu^+ \mu^-)$	$(4.0 \pm 1.2) \times 10^{-6}$	$(2.08 \div 3.19) \times 10^{-6}$
$Br(\Lambda_b \to \Lambda \tau^+ \tau^-)$	$(0.8 \pm 0.3) \times 10^{-6}$	$(0.179 \div 0.276) \times 10^{-6}$

Our comment (2010): Order of Br shows that this transition can be detected at LHC

$$\mathcal{B}(\Lambda_b^0 \to \Lambda\mu^+\mu^-) = [1.73 \pm 0.42(\mathrm{stat}) \pm 0.55(\mathrm{syst})] \times 10^{-6} \qquad (\mathrm{CDF\ \ Collaboration}^\dagger)$$

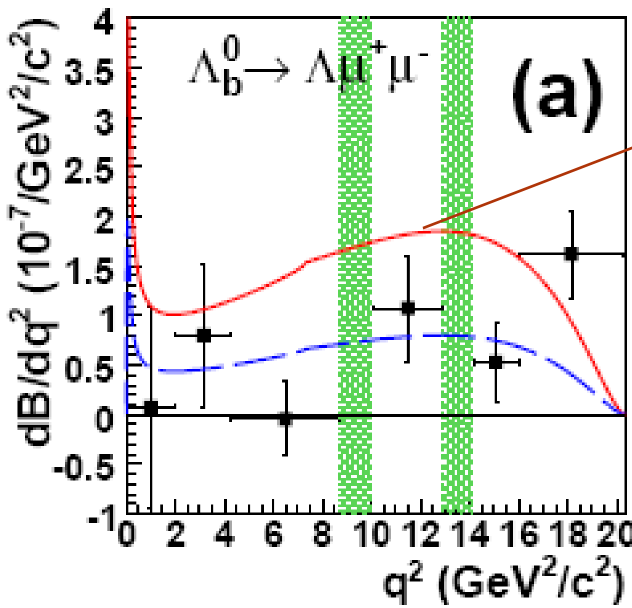
10 November 2011

CDF Collaboration at Fermilab: First observation on baryonic FCNC

Phys. Rev. Lett. 107, 201802 (2011)

Observation of the Baryonic Flavor-Changing Neutral Current Decay $\Lambda_b^0 \to \Lambda\mu^+\mu^-$

T. Aaltonen,[22] B. Álvarez González[v,10] S. Amerio,[42] D. Amidei,[33] A. Anastassov,[37] A. Annovi,[18] J. Antos,[13]
G. Apollinari,[16] J.A. Appel,[16] A. Apresyan,[47] T. Arisawa,[56] A. Artikov,[14] J. Asaadi,[52] W. Ashmanskas,[16]
B. Auerbach,[59] A. Aurisano,[52] F. Azfar,[41] W. Badgett,[16] A. Barbaro-Galtieri,[27] V.E. Barnes,[47] B.A. Barnett,[24]
P. Barria[ee,45] P. Bartos,[13] M. Bauce[cc,42] G. Bauer,[31] F. Bedeschi,[45] D. Beecher,[29] S. Behari,[24] G. Bellettini[dd,45]
J. Bellinger,[58] D. Benjami … [cc,42] I. Bizjak[ii,29] K.R. Bland,[5]
C. Blocker,[7] B. Blumenfe … au,[46] A. Boveia,[12] B. Brau[a,16]
L. Brigliadori[bb,6] A. Bris … [45] J. Budagov,[14] H.S. Budd,[48]
S. Budd,[23] K. Burkett,[16] … C. Calancha,[30] S. Camarda,[4]
M. Campanelli,[34] M. … Carlsmith,[58] R. Carosi,[45]
S. Carrillo[k,17] S. Carron … D. Cauz,[53] V. Cavaliere[ee,45]
M. Cavalli-Sforza,[4] A. … hiarelli,[45] G. Chlachidze,[16]
F. Chlebana,[16] K. … Chung,[48] C.I. Ciobanu,[43]
M.A. Ciocci[ee,45] A. Clark,[1] … way,[8] M.Corbo,[43] M. Cordelli,[18]
C.A. Cox,[8] D.J. Cox,[8] F … ulbertson,[16] D. Dagenhart,[16]
N. d'Ascenzo[t,43] M. Da … M. Dell'Orso[dd,45] C. Deluca,[4]
L. Demortier,[49] J. De … Canto[dd,45] B. Di Ruzza,[45]
J.R. Dittmann,[5] M. D' … a,[56] A. Elagin,[52] A. Eppig,[33]
R. Erbacher,[8] D. Errede,[2] … [7] S. Farrington,[41] M. Feindt,[25]
J.P. Fernandez,[30] C. … J. Frank,[5] M. Franklin,[21]
J.C. Freeman,[16] I. Fu … Garfinkel,[47] P. Garosi[ee,45]
H. Gerberich,[23] E. Gerchte … K. Gibson,[46] C.M. Ginsburg,[16]
N. Giokaris,[3] P. Giromini … ski,[16] M. Gold,[36] D. Goldin,[52]
N. Goldschmidt,[17] A. … oncharov,[31] O. González,[30]
I. Gorelov,[36] A.T. Goshaw,[15] K. Goulianos,[49] A. Gresele,[42] S. Grinstein,[4] C. Grosso-Pilcher,[12] R.C. Group,[16]
J. Guimaraes da Costa,[21] Z. Gunay-Unalan,[34] C. Haber,[27] S.R. Hahn,[16] E. Halkiadakis,[51] A. Hamaguchi,[40]
J.Y. Han,[48] F. Happacher,[18] K. Hara,[54] D. Hare,[51] M. Hare,[55] R.F. Harr,[57] K. Hatakeyama,[5] C. Hays,[41] M. Heck,[25]
J. Heinrich,[44] M. Herndon,[58] S. Hewamanage,[5] D. Hidas,[51] A. Hocker,[16] W. Hopkins[g,16] D. Horn,[25] S. Hou,[1]
R.E. Hughes,[38] M. Hurwitz,[12] U. Husemann,[59] N. Hussain,[32] M. Hussein,[34] J. Huston,[34] G. Introzzi,[45] M. Iori[gg,50]
A. Ivanov[o,8] E. James,[16] D. Jang,[11] B. Jayatilaka,[15] E.J. Jeon,[26] M.K. Jha,[6] S. Jindariani,[16] W. Johnson,[8]
M. Jones,[47] K.K. Joo,[26] S.Y. Jun,[11] T.R. Junk,[16] T. Kamon,[52] P.E. Karchin,[57] Y. Kato[n,40] W. Ketchum,[12]
J. Keung,[44] V. Khotilovich,[52] B. Kilminster,[16] D.H. Kim,[26] H.S. Kim,[26] H.W. Kim,[26] J.E. Kim,[26] M.J. Kim,[18]
S.B. Kim,[26] S.H. Kim,[54] Y.K. Kim,[12] N. Kimura,[56] S. Klimenko,[17] K. Kondo,[56] D.J. Kong,[26] J. Konigsberg,[17]



Fit of our work

LHCb at CERN has also started to study this decay mode

Theories beyond the SM

The Standard Model (SM) of particle physics describes all known particles and their interactions except than gravity. The SM is the only minimal model which is in perfect consistency with all confirmed collider data despite it needs a missing ingredient, the Higgs boson or something else to give masses to the elementary particles.

But

There are some problems such as origin of the matter in the universe, gauge and fermion mass hierarchy, number of generations, matter-antimatter asymmetry, neutrino oscillations, unification, quantum gravity and so on, which can not addressed by the SM.

The SM can not be the ultimate theory of nature and it can be considered as a low energy manifestation of some fundamental theories

Some new physics scenarious:

various extensions of the standard model through supersymmetry, such as the Minimal Supersymmetric Standard Model (MSSM) and Next-to-Minimal Supersymmetric Standard Model (NMSSM), SM4 or entirely novel explanations, such as string theory, M-theory and extra dimensions.

FCNC transitions → loop level → New physics such as extra dimensions, Suppersymmetric particles, fourth family quarks, light dark matter etc. can contribute

Consider: UED which is a kind of ED

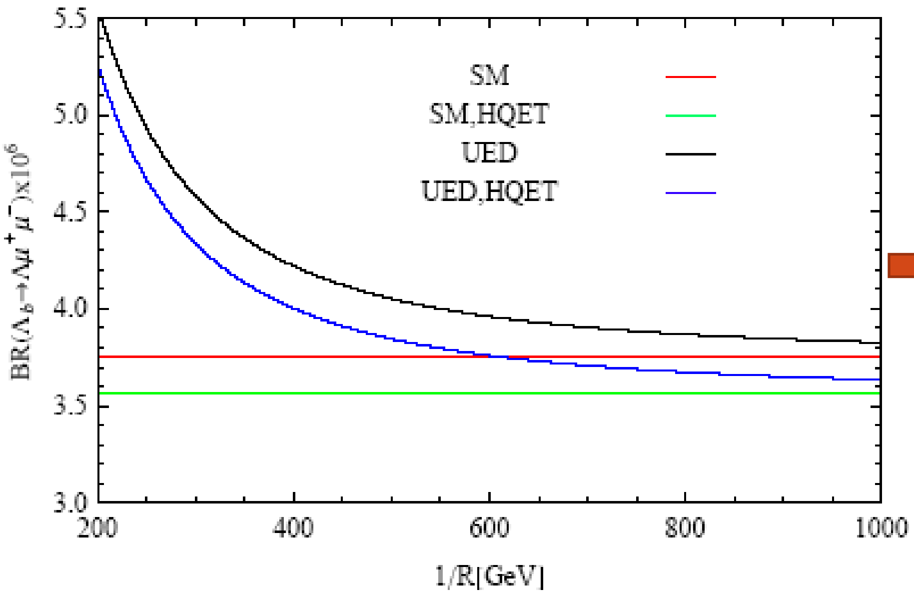
$$\left.\begin{array}{l} C_7^{eff} \\ C_9^{eff} \\ C_{10}, \end{array}\right\} \rightarrow \text{are modified}$$

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n), \quad x_n = \frac{m_n^2}{m_W^2}$$

$$m_n = \frac{n}{R}$$

Compactification radius

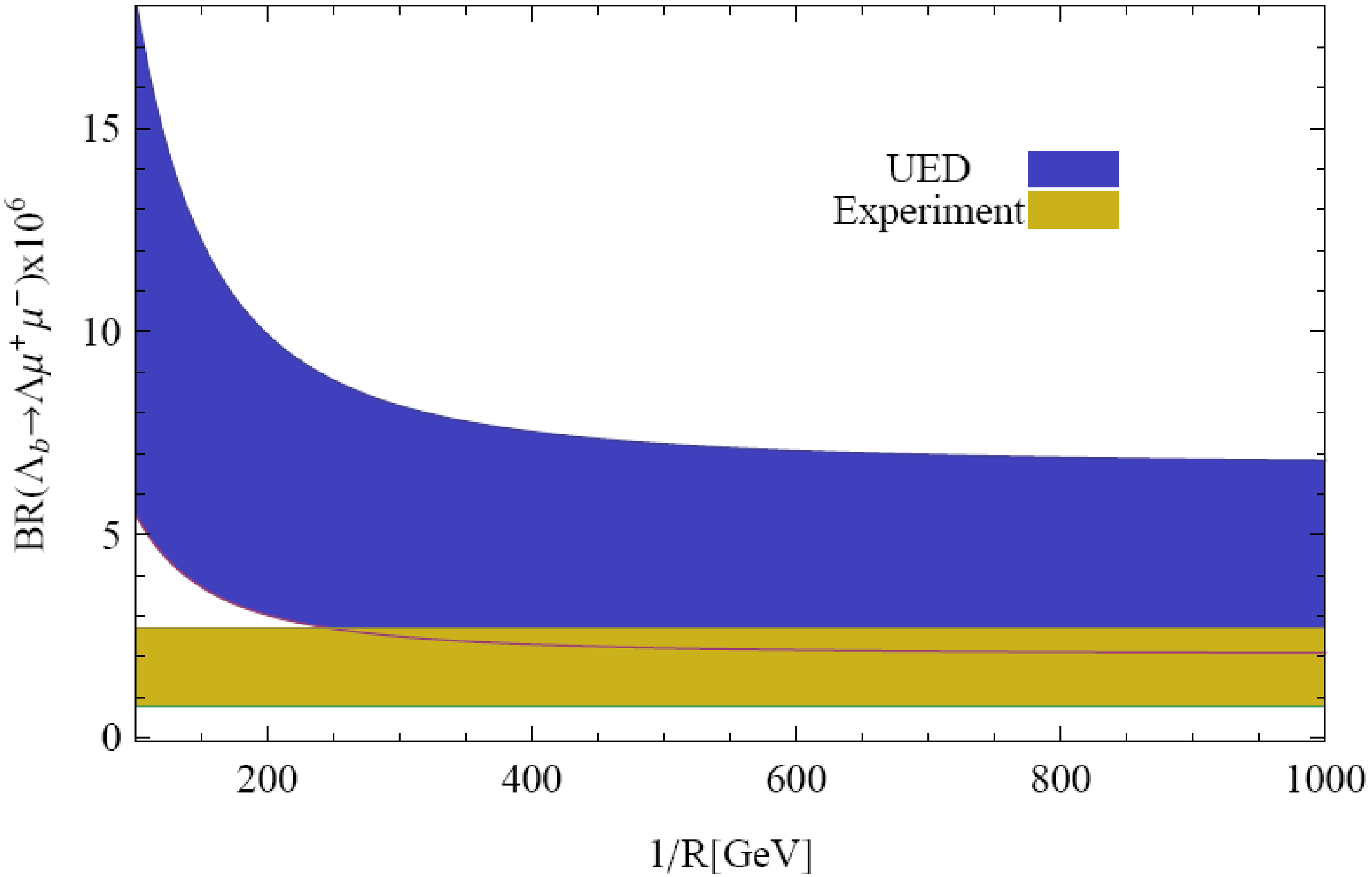
Form factors remain the same



→ Such discrepancy at low values of $1/R$ can be considered as a signal for the existence of extra dimensions

K. Azizi, N. Katirci, JHEP 1101 (2011) 087.

Constraint on the Compactification Factor via $\Lambda_b \to \Lambda\mu^+\mu^-$ Decay Channel



Lower limit on the compactification scale1/R: 250 GeV

K. Azizi, S. Kartal, N. Katirci, A. T. Olgun, Z. Tavukoğlu, JHEP 1205 (2012) 024.

✓ Strong interactions of hadrons/ their hadronic decays

Light baryons:

✓ $D-D-PS$
✓ $D-D-V$
✓ $D-O-PS$

Heavy Baryons:

✓ $3/2-3/2-V$
✓ $3/2-3/2-PS$
✓ $3/2-1/2-V$
✓ $3/2-1/2-PS$
✓ $1/2-1/2-V$
✓ $1/2-1/2-PS$

We calculated the strong coupling constant characterizing these strong hadronic transitions. In each group, we have about 80 coupling constant. Using different symmetry arguments, we could explain all 80 coupling constant in terms on only one invariant universal function. This is the first in the literature. In the case of light baryons, our method includes the SU(3)_f symmetry breaking, automatically. This help us predict the order of SU(3)_f breaking. In our calculations, we used the generalized interpolating currents and saw that the Ioffe Current remains out of the reliable regions in all cases. The progress in heavy baryon spectroscopy reported by BaBar, BELLE, CDF and D0 Collaborations and also progress at LHC, show a possibility to study such channels in the near future .

Comparing our results with any experimental data can give valuable information about the internal structures of the considered baryons as well as the nature of strong interactions inside them.

My Collaborators:
Valeri Zamiralov (Moscow state university)
T. M. Aliev, A. Özpineci, M. Savci (ODTÜ)

Phys.Rev. D 83 (2011) 096007;

Phys. Lett. B 696 (2011) 220 ;

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Strong coupling constants of light pseudoscalar mesons with heavy baryons in QCD

T. M. Aliev, K. Azizi, M. Savci, Phys.Lett.B696:220-226,2011

$$\Pi^{(ij)} = i \int d^4x e^{ipx} \left\langle \mathcal{P}(q) \left| \mathcal{T} \left\{ \eta^{(i)}(x) \bar{\eta}^{(j)}(0) \right\} \right| 0 \right\rangle \ , \qquad (1)$$

where $\mathcal{P}(q)$ is the pseudoscalar–meson with momentum q, η is the interpolating current for the heavy baryons and $\mathcal{T}$ is the time ordering operator. Here, $i=1$, $j=1$ describes the sextet–sextet, $i=1$, $j=2$ corresponds to sextet–triplet, and $i=2$, $j=2$ describes triplet–triplet transitions. For convenience we shall denote $\Pi^{(11)} = \Pi^{(1)}$, $\Pi^{(12)} = \Pi^{(2)}$ and $\Pi^{(22)} = \Pi^{(3)}$. The sum rules for the coupling constants of pseudoscalar mesons with heavy baryons can be obtained by calculating the correlation function (1) in two different ways, namely, in terms of the hadrons and in terms of quark gluon degrees of freedom, and then matching these two representations.

Hadronic side

$$\Pi^{(ij)} = \frac{\left\langle 0 \left| \eta^{(i)}(0) \right| B_2(p) \right\rangle \left\langle B_2(p) \mathcal{P}(q) \mid B_1(p+q) \right\rangle \left\langle B_1(p+q) \left| \bar{\eta}^{(j)}(0) \right| 0 \right\rangle}{(p^2 - m_2^2)\left[(p+q)^2 - m_1^2\right]} + \cdots \ , \qquad (2)$$

$$\begin{aligned} \left\langle 0 \left| \eta^{(i)} \right| B(p) \right\rangle &= \lambda_i u(p) \ , \\ \left\langle B(p+q) \left| \eta^{(j)} \right| 0 \right\rangle &= \lambda_j \bar{u}(p+q) \ , \\ \left\langle B(p) \mathcal{P}(q) \mid B(p+q) \right\rangle &= g \bar{u}(p) i \gamma_5 u(p+q) \ , \end{aligned}$$

$$\Pi^{(ij)} = i\frac{\lambda_i\lambda_j g}{(p^2-m_2^2)\left[(p+q)^2-m_1^2\right]}\left\{\not{q}\not{p}\gamma_5 + \text{other structures}\right\},$$

OPE side

We start our discussion by considering the sextet–sextet transition, concretely. Consider he $\Sigma_b^0 \to \Sigma_b^0\pi^0$ transition. The invariant function for this transformation can be written in he following form

$$\Pi^{\Sigma_b^0\to\Sigma_b^0\pi^0} = g_{\pi\bar{u}u}\Pi_1^{(1)}(u,d,b) + g_{\pi\bar{d}d}\Pi_1^{\prime(1)}(u,d,b) + g_{\pi\bar{b}b}\Pi_2^{(1)}(u,d,b)\ , \tag{6}$$

vhere the interpolating current of π^0 meson is written as

$$J_{\pi^0} = \sum_{u,d} g_{\pi\bar{q}q}\bar{q}\gamma_5 q\ .$$

$$g_{\pi\bar{u}u} = -g_{\pi\bar{d}d} = \frac{1}{\sqrt{2}},\ g_{\pi\bar{b}b} = 0$$

invariant functions Π_1, Π_1' and Π_2 describe the radiation of π^0 meson from u, d and b quarks of Σ_b^0 baryon, respectively, and they can formally be defined as:

$$\begin{aligned}
\Pi_1^{(1)}(u,d,b) &= \left\langle \bar{u}u \left| \Sigma_b^0 \bar{\Sigma}_b^0 \right| 0 \right\rangle \ , \\
\Pi_1'^{(1)}(u,d,b) &= \left\langle \bar{d}d \left| \Sigma_b^0 \bar{\Sigma}_b^0 \right| 0 \right\rangle \ , \\
\Pi_2^{(1)}(u,d,b) &= \left\langle \bar{b}b \left| \Sigma_b^0 \bar{\Sigma}_b^0 \right| 0 \right\rangle \ .
\end{aligned} \tag{7}$$

It follows from the definition of the interpolating current of Σ_b baryon that it is symmetric under the exchange $u \leftrightarrow d$, hence $\Pi_1'^{(1)}(u,d,b) = \Pi_1^{(1)}(d,u,b)$. Using this relation, we immediately get from Eq. (6) that,

$$\Pi^{\Sigma_b^0 \to \Sigma_b^0 \pi^0} = \frac{1}{\sqrt{2}} \left[\Pi_1^{(1)}(u,d,b) - \Pi_1^{(1)}(d,u,b) \right] , \tag{8}$$

in the $SU_2(2)_f$ limit, $\Pi^{\Sigma_b^0 \to \Sigma_b^0 \pi^0} = 0$.

The invariant function responsible for the $\Sigma_b^+ \to \Sigma_b^+ \pi^0$ transition can be obtained from the $\Sigma_b^0 \to \Sigma_b^0 \pi^0$ case by making the replacement $d \to u$, and using $\Sigma_b^0 = -\sqrt{2}\Sigma_b^+$, from which we get,

$$4\Pi_1^{(1)}(u,d,b) = -2\left\langle \bar{u}u \left| \Sigma_b^+ \bar{\Sigma}_b^+ \right| 0 \right\rangle \ . \tag{9}$$

Appearance of the factor 4 on the left hand side is due to the fact that each Σ_b^+ contains two u quark, hence there are 4 possible ways for radiating π^0 from the u quark. Making use of Eq.(8), we get

$$\Pi^{\Sigma_b^+ \to \Sigma_b^+ \pi^0} = \sqrt{2}\Pi_1^{(1)}(u,u,b). \tag{10}$$

The invariant function describing $\Sigma_b^- \to \Sigma_b^- \pi^0$ can easily be obtained from the $\Sigma_b^0 \to \Sigma_b^0 \pi^0$ transition by making the replacement $u \to d$ and taking into account $\Sigma_b^0(u \to d) = \sqrt{2}\Sigma_b^-$. Performing calculation similar to the previous case, we get

$$\Pi^{\Sigma_b^- \to \Sigma_b^- \pi^0} = \sqrt{2}\Pi_1^{(1)}(d,d,b). \tag{11}$$

Now, let us proceed to obtain the results for the invariant function involving $\Xi_b^{\prime -(0)} \to \Xi_b^{\prime -(0)} \pi^0$ transition. The invariant function for this transition can be obtained from the $\Sigma_b^0 \to \Sigma_b^0 \pi^0$ case using the fact that $\Xi_b^{\prime 0} = \Sigma_b^0 (d \to s)$ and $\Xi_b^{\prime -} = \Sigma_b^0 (u \to s)$. As a result, we obtain

$$\begin{aligned}
\Pi^{\Xi_b^{\prime 0} \to \Xi_b^{\prime 0} \pi^0} &= \frac{1}{\sqrt{2}} \Pi_1^{(1)}(u,s,b) \ , \\
\Pi^{\Xi_b^{\prime -} \to \Xi_b^{\prime -} \pi^0} &= -\frac{1}{\sqrt{2}} \Pi_1^{(1)}(d,s,b) \ .
\end{aligned} \tag{12}$$

Using similar arguments one can find the symmetry relations for other transitions.

$$g^{(i)} = \frac{1}{\lambda_1^{(i)} \lambda_2^{(i)}} e^{\frac{m_1^{(i)2}}{M_1^2} + \frac{m_2^{(i)2}}{M_2^2}} \, \Pi_1^{(i)} \ ,$$

$$
\begin{aligned}
& e^{m_Q^2/M^2 - m_{\mathcal{P}}^2/M^2}\Pi_1^{(1)}(u,d,b) = \\
& \frac{(1-\beta)^2}{32\sqrt{2}\pi^2} M^4 m_Q^3 f_{\mathcal{P}} \Big[I_2 - m_Q^2 I_3 \Big] \phi_\eta(u_0) + \frac{(1-\beta^2)}{64\sqrt{2}\pi^2} M^4 m_Q^2 \mu_{\mathcal{P}} \Big\{ \Big(i_3(\mathcal{T},1) - 2 i_3(\mathcal{T},v) \Big) I_2 \\
& - 2 m_Q^2 \Big[i_3(\mathcal{T},1) - 2 i_3(\mathcal{T},v) + (1 - \widetilde{\mu}_{\mathcal{P}}^2) \phi_\sigma(u_0) \Big] I_3 \Big\} \\
& - \frac{(1-\beta)^2}{128\sqrt{2}\pi^2} M^2 m_{\mathcal{P}}^2 m_Q f_{\mathcal{P}} \Big\{ m_Q^2 \mathbb{A}(u_0) I_2 - 2 \Big(i_2(\mathcal{V}_\parallel,1) - 2 i_2(\mathcal{V}_\perp,1) \Big) I_1 \\
& - 2 m_Q^2 \Big[i_2(\mathcal{A}_\parallel,1) - 2 \Big(i_2(\mathcal{V}_\parallel,1) - i_2(\mathcal{V}_\perp,1) + i_2(\mathcal{A}_\parallel,v) \Big) \Big] I_2 \Big\}
\end{aligned}
$$

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$$
I_n = \int_{m_Q^2}^{\infty} ds \frac{e^{m_Q^2/M^2 - s/M^2}}{s^n} ,
$$

✓ 1/2-1/2−PS

$$\langle B(p)\mathcal{P}(q) \mid B(p+q)\rangle = g\bar{u}(p)i\gamma_5 u(p+q)$$

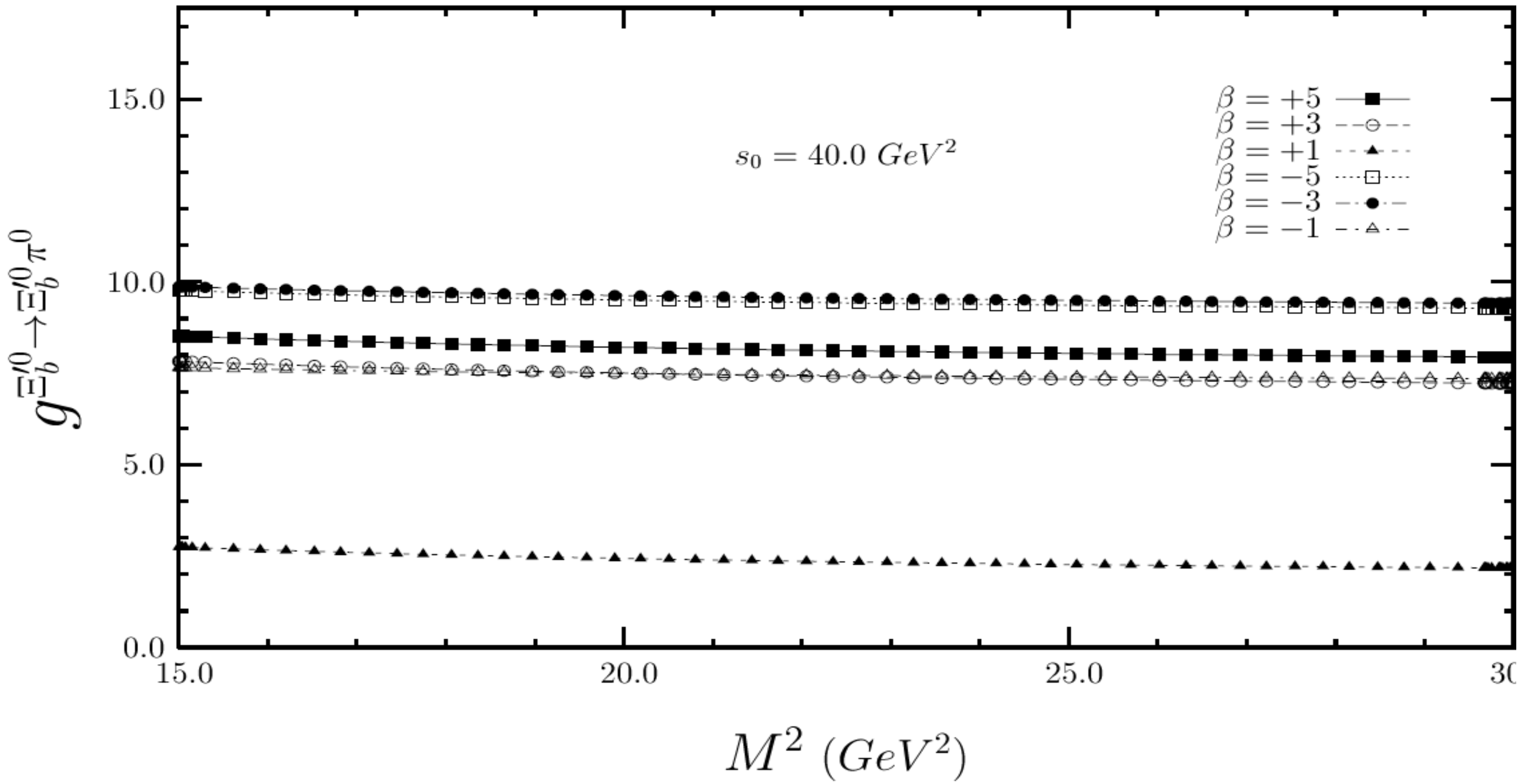

Figure 2: The dependence of the strong coupling constant for the $\Xi_b^{\prime 0} \to \Xi_b^{\prime 0}\pi^0$ transi at several different fixed values of β, and at $s_0 = 40.0\ GeV^2$.

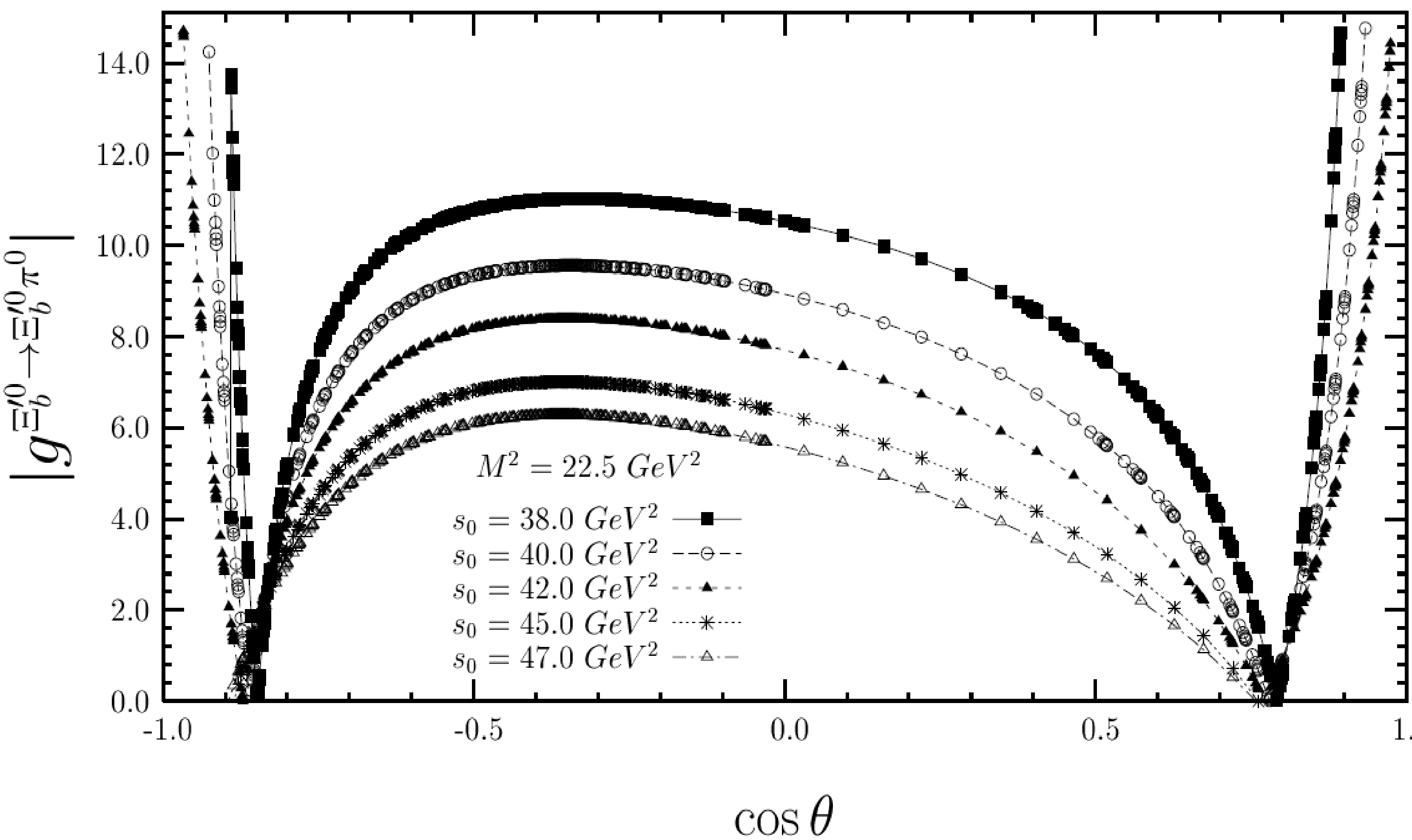

Figure 3: The dependence of the strong coupling constant for the $\Xi_b'^0 \to \Xi_b'^0 \pi^0$ transition on $\cos\theta$ at several different fixed values of s_0, and at $M^2 = 22.5\ GeV^2$.

g channel	Bottom Baryons		g channel	Charmed Baryons	
	General current	Ioffe current		General current	Ioffe current
$g^{\Xi_b^{\prime 0}\to\Xi_b^0\pi^0}$	7.5±2.6	6.1±2.2	$g^{\Xi_c^{\prime +}\to\Xi_c^+\pi^0}$	3.1±1.1	2.0±0.7
$g^{\Sigma_b^-\to\Lambda_b^0\pi^-}$	15.0±4.9	11.5±3.9	$g^{\Sigma_c^0\to\Lambda_c^+\pi^-}$	6.5±2.4	5.6±1.8
$g^{\Sigma_b^0\to\Xi_b^0\bar{K}^0}$	11.5±3.9	8.9±3.1	$g^{\Sigma_c^+\to\Xi_c^+\bar{K}^0}$	5.0±1.7	3.7±1.3
$g^{\Omega_b^-\to\Xi_b^-\bar{K}^0}$	17.0±4.5	13.5±4.8	$g^{\Omega_c^0\to\Xi_c^0\bar{K}^0}$	6.5±2.3	3.0±1.1
$g^{\Xi_b^{\prime 0}\to\Xi_b^-K^+}$	12.0±4.3	9.8±3.5	$g^{\Xi_c^{\prime +}\to\Xi_c^0K^+}$	4.5±1.6	2.1±0.8
$g^{\Xi_b^{\prime 0}\to\Xi_b^0\eta_1}$	16.0±5.6	12.0±4.3	$g^{\Xi_c^{\prime +}\to\Xi_c^+\eta_1}$	6.7±2.4	4.3±1.5

$$g^{\Sigma_c\to\Lambda_c\pi} = \begin{cases} 8.88, & \text{(relativistic three–quark model)}\ , \\ 6.82, & \text{(light–front quark model)}, \\ 6.5\pm 2.4, & \text{(our result) (LCSR)} \end{cases}$$

Our results can be checked by future experiments

Thank You