The renormalization group (RG), one of the conceptual pillars of statistical mechanics and quantum field theory, explores how the behavior of an extended system depends on the scale of observation. Important notions such as universality, criticality or stability of phases are explained in terms of the existence of fixed-points of the RG flow. On the other hand, studies of entanglement in the context of quantum information have resulted in the development of tensor networks, a framework to efficiently describe many-body wave-functions. In this Colloquium I will review a decade-long research program based on applying tensor networks to the renormalization group. This program has produced novel non-perturbative, real-space RG approaches of current interest in a wide range of research areas, from statistical mechanics to condensed matter and quantum field theory, from quantum gravity (holography), to classical and quantum information theory (error correction).