Physics Colloquium
IPM, Tehran

A mathematician's approach to general relativistic quantum mechanics

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Some Recent Results in GRQM

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- Fundamental objects that are highly singular in both gravity and electromagnetism: point mass and point charge.

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- *“Writers have occasionally noted the possibility that material particles might be considered as singularities of the field. This point of view, however, we cannot accept at all...Every field theory must adhere to the fundamental principle that singularities of the field are to be excluded."*

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- Einstein himself! and Herman Weyl

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- PS: If you dislike a Weyl theory of matter, just wait five minutes!

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  2. Electrovacuum spacetimes are highly singular

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- Are the predictions of such a theory in agreement with physical experiments?

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- Negative Continuum: Dirac's Sea, Hole Theory, Positron

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- Pathologies of well-known solutions to Einstein's equations
- Likely culprit: Infinite self-energy of point charges (linear electromagnetics) causes the spacetime to be infinitely curved close to the charge.

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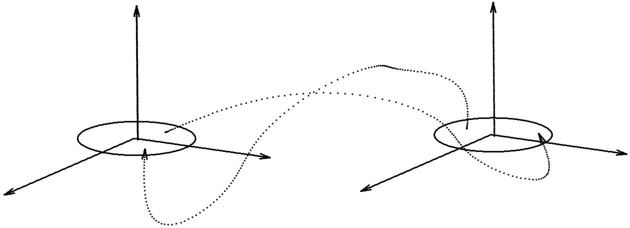

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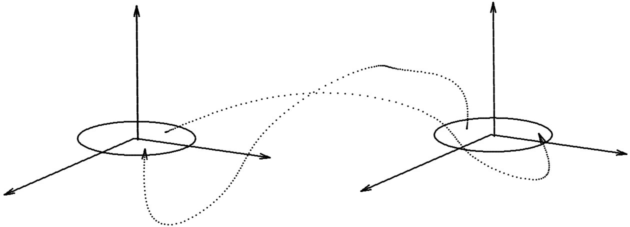

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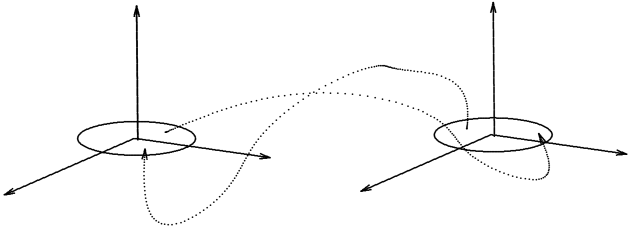

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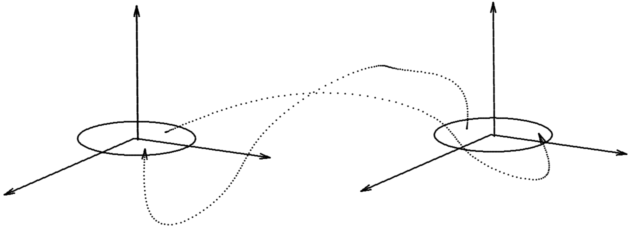

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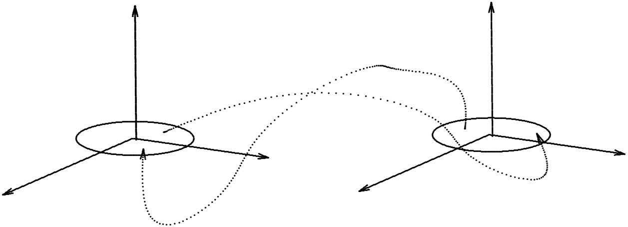

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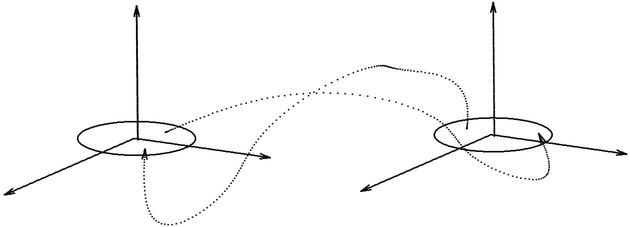

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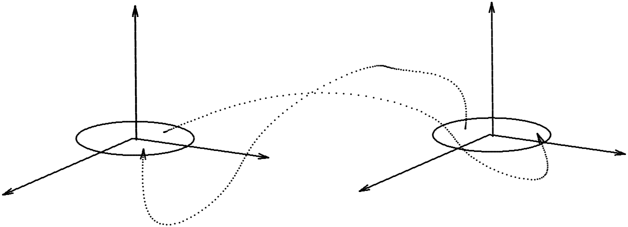

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- zGKN = This spacetime + EM fields on it (P. Appell, 1888)

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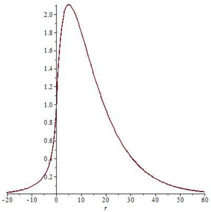

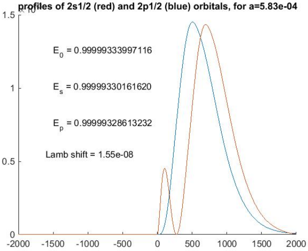

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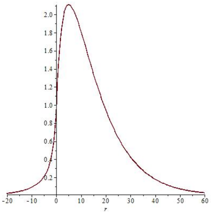

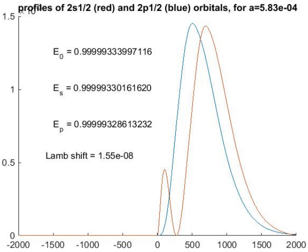

- Excited states. Numerical approximation. Hyperfine splitting and Lamb shift without QED!

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- Our radically new idea: Electron and Positron are not distinct particles but in fact "two different sides of the same coin"
- This resolves the paradox that Dirac's equation "for the electron" also seems to describe "a positron" in many situations, while it is a true one-particle equation.

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- Model other particles (protons, etc) as singularities of Riemann spaces branched over knots.

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- Eigenfunctions with positive energy are 99% supported in one sheet, and those with negative energy are 99% supported in the other sheet

Proof of existence of discrete spectrum

- $\Psi(t,r,\theta,\varphi) = R(r)S(\theta)e^{-i(Et-\kappa\varphi)} \begin{pmatrix} \cos(\Theta(\theta)/2)e^{+i\Omega(r)/2} \\ \sin(\Theta(\theta)/2)e^{-i\Omega(r)/2} \\ \cos(\Theta(\theta)/2)e^{-i\Omega(r)/2} \\ \sin(\Theta(\theta)/2)e^{+i\Omega(r)/2} \end{pmatrix}$

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- Two equilibrium points on each boundary: $f(x_-) = f(x_+) = 0$ and $g_\mu(x_\pm, y) = 0 \implies y \in \{s_\pm, n_\pm\}$

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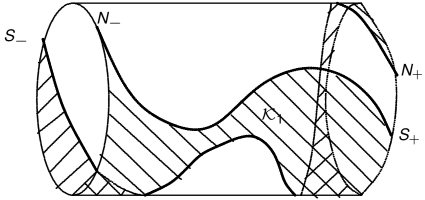

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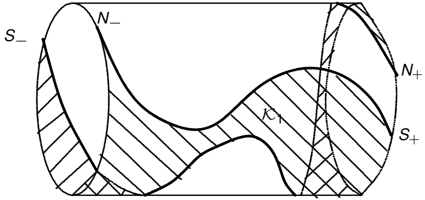

- Saddle-Saddle connector exists iff the corridor collapses.

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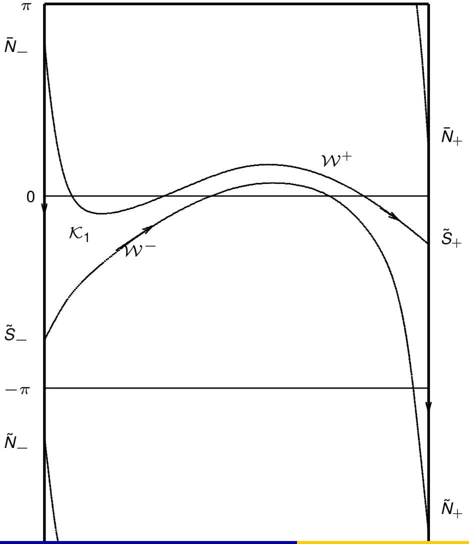
- Construction of barriers to prove existence of corridors with given winding number.

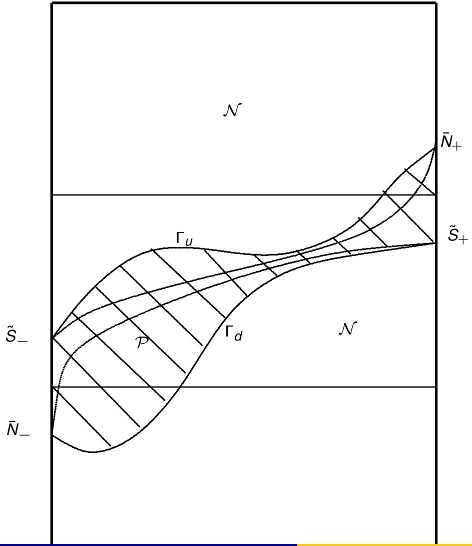
Area and Winding Number for Corridors

Working in the universal cover of the cylinder:

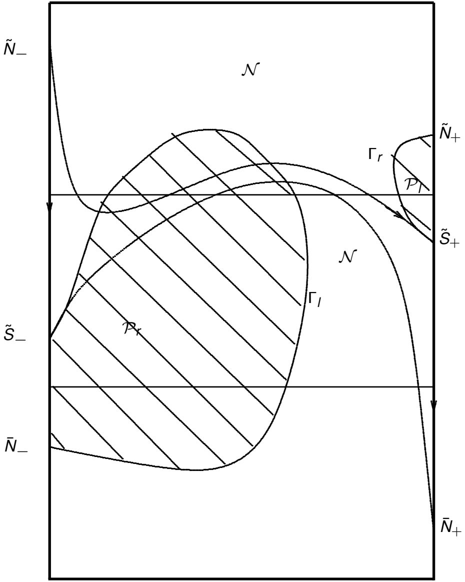


Topology of Nullclines

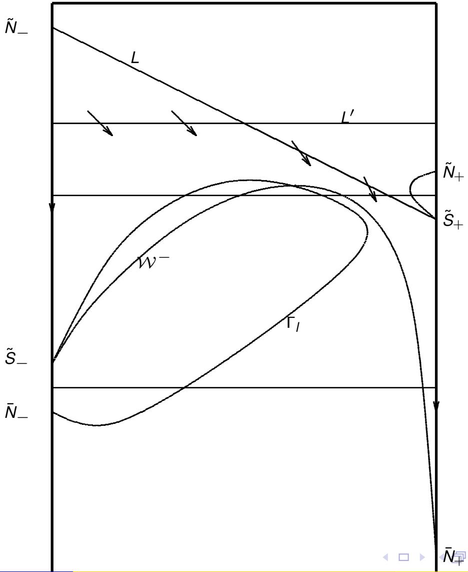
Orbits must increase while in the shaded region



Change in Nullcline Topology and Corridor Winding



Barrier construction



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- Novel proposal: Dirac’s equation describes a single “particle / anti-particle” structure: two “topo-spin” states

Fin!

THANK YOU FOR LISTENING!