Neutron star is one of the most fascinating objects in the universe created by the collapsed core of a giant star. Neutron stars are incredibly dense and compact objects representing an ideal laboratory to test alternative theories of gravity. Moreover, studying these mysterious compact objects will expand our limited knowledge of the properties and the physics of nuclear matter under extreme conditions. We establish the effect of scalarization on static and slowly rotating neutron stars in scalar-tensor theories of gravity, implementing various realistic Equations Of State (EOSs). Beside a polytropic EOS and some EOSs for pure nuclear matter and pure quark matter, for the first time in this context, we include several EOSs describing nuclear matter with hyperons and hybrid matter. We explore the onset of scalarization for these different EOSs, presenting a universal (independent of the EOS) relation for the critical coupling parameter versus the compactness. Then, we identify that the most significant universal feature of the onset and the magnitude of the scalarization is the correlation with the value of the gravitational potential at the center of the star. We also analyze the moment-of-inertia--compactness relations and confirm universality for the nuclear matter, hyperon and hybrid equations of state.