We investigate the osmotic (electrostatic) pressure acting on the proteinaceous shell of a generic model of virus-like particles (VLPs), comprising a charged outer shell and a metallic nanoparticle core, coated by a charged layer and bathed in an aqueous electrolyte (salt) solution. Motivated by the recent studies accentuating the role of multivalent ions for the stability of VLPs, we focus on the effects of multivalent cations and anions in an otherwise monovalent ionic bathing solution. We perform extensive Monte-Carlo simulations based on appropriate Coulombic interactions that consistently take into account the effects of salt screening, the dielectric polarization of the metallic core, and the strong-coupling electrostatics due to the presence of multivalent ions. We specifically study the intricate roles these factors play in the electrostatic stability of the model VLPs. It is shown that while the insertion of a metallic nanoparticle by itself can produce negative, inward-directed, pressure on the outer shell, addition of only a small amount of multivalent counterions can robustly engender negative pressures, enhancing the VLP stability across a wide range of values for the system parameters.