Course on “Statistical Physics of Fields In and Out of Equilibrium”
School of Physics, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran
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Subjects covered in this course:

Statistical Physics of Fields In Equilibrium

Part I — Preliminaries

- Statistical physics and thermodynamics of many-particle systems
  - A conceptual review in four lectures based on the classic text by Landau & Lifshitz (Vol. 5)
- Statistical fields: A scenery (slide presentation)
  - Spin & particle models: From classical to modern examples
  - Modern applications of statistical field theory in soft & condensed matter physics
- Statistical field theory: Basic methods
  - Construction of continuum field actions
    - Coarse-graining of microscopic Hamiltonians
    - Symmetries & phenomenological actions (or effective ‘Hamiltonians’)
  - Classical field theory
    - Calculus of variations
    - The least-action principle & Euler-Lagrange equations
    - Continuous symmetries & Noether’s theorem
    - Energy-momentum (or stress-energy) tensor
  - Fluctuating fields & functional-integral methods
    - Path integrals: An overview
    - Functional integrals & the partition function
    - Generating functionals & connected correlation (Green) functions
    - Response functions & the fluctuation-dissipation theorem
    - Gaussian integrals & Wick’s theorem (‘free field theory’)
    - Formal perturbative expansion of field interaction
    - Feynman diagrams
  - Other routes to constructing field theories
    - A modern (re)formulation of the Hubbard-Stratonovich transformation
    - Construction of exact field theories: From O(n) model to Coulomb fluids
    - Edwards-Gupta method: Effective field theory for Ising & O(n) models
- Saddle-point method
  - Mean field theory as a steepest descent (saddle-point) approximation
  - Loop expansion & Gaussian-fluctuation (one-loop) corrections
  - Saddle-point approximation in the presence of zero modes (included in Homework #15)
- Other useful techniques (applicable to Part II)
  - Schwinger-Dyson equations
  - Ward identities (included in Homework #7)

Further information about this course, including problem sets and term-essay topics, are available at: http://physics.ipm.ac.ir/~naji/fields_SS2016.html
• Functional determinants (applicable to Part III)
  - Van Vleck-Pauli-Morette (VVPM) formula
  - Spectral $\zeta$-functions & heat kernel: An overview
  - $\zeta$-function regularization
  - Gel’fand-Yaglom (GY) formula: Contour integration approach
  - Coleman’s proof of GY formula (included in Homework #9)
  - Equivalence of VVPM & GY formulas (included in Homework #9)
• Zero modes (applicable to Part IV) (included in Homework #15)
  - Continuous symmetry & Goldstone modes
  - Gaussian-fluctuation (one-loop) corrections in the presence of zero modes
  - Perturbative expansion in the presence of zero modes

✦ Part II — Field theory for Coulomb fluids (or “Coulomb gases”)
  ‣ Coulomb interactions & Coulomb fluids in soft matter and biology (slide presentation)
  ‣ Coulomb fluids: General formalism
    • Exact field action via Hubbard-Stratonovich transformation
  ‣ Mean-field theory
    • Multi-component Coulomb fluids (“ionic mixtures” or “electrolytes”)
      - Saddle-point approximation: The nonlinear Poisson-Boltzmann equation
      - Electrical double layers: The Gouy-Chapman theory
      - Linearization approximation: The Debye-Hückel theory
      - Generalized electrostatic stress tensor (not covered in this Course)
    • Confined one-component (“counterion-only”) Coulomb fluids
      - Counterions at a single charged wall: Mean-field density profile
      - Counterions between two charged walls: Effective counterion-mediated interactions
  ‣ Beyond mean-field theory (I): Weak-coupling regime
    • Bulk ionic mixtures (“Yukawa” plasmas)
      - Quadratic Debye-Hückel field action
      - Break down of the virial expansion & the origin of Debye screening
      - Singular correlation corrections to the equation of state (bulk limiting laws)
    • Confined one-component (“counterion-only”) Coulomb fluids: One-loop corrections
      - Fluctuation corrections to counterion density: One & two walls (included in Homework #8)
      - Gaussian-fluctuation corrections to counterion-mediated interactions between two walls
  ‣ Beyond mean-field theory (II): Strong-coupling regime (slide presentation)
    • Weak vs strong couplings: A unified field-theoretic approach
    • Strong-coupling theory: Virial expansion restored for confined Coulomb fluids
    • Recent advances in the theory & simulations of confined Coulomb fluids

✦ Part III — Fluctuation-induced phenomena (Casimir & pseudo-Casimir effects)
  ‣ A plethora of fluctuation-induced phenomena (slide presentation)
    • Examples from quantum vacuum fluctuations to superfluid films to liquid crystals to Derjaguin-Landau-Verwey-Overbeek theory of colloidal stability
  ‣ Quantum vacuum & electromagnetic field fluctuations
    • Non-retarded Casimir forces: Perfect “mirrors” at zero temperature
    • Thermal (classical) Casimir forces: Massless fields at high temperature
    • Lifshitz theory for dielectrics at finite temperature: Retardation effects
  ‣ Fluctuation-induced effects in correlated liquids
    • Pseudo-Casimir forces in superfluid helium films
    • Pseudo-Casimir forces in nematic liquid-crystalline films (included in Homework #10)
Fluctuation-induced effects in Coulomb fluids
- Massive field fluctuations & Pseudo-Casimir effect in “Yukawa” plasmas
- Fluctuations on a non-uniform background: Confined Coulomb fluids revisited

Part IV — Phase transitions and critical phenomena

General aspects and examples (blackboard & slide presentation)
- Bulk phases and phase transitions in simple fluids & ferromagnets
- Classifications of (bulk) phase transitions: Thermodynamic non-analyticities
- Discontinuous (first-order) transitions: Phase separation & coexistence region
- Continuous (second-order) transitions: Critical points
- Multicritical points & other typical features of phase diagrams
- Modern perspective on phase transitions
  - Criticality, correlations, scaling & universality
  - Critical exponents: Experiments, simulations & theory
  - Critical exponents: Thermodynamic inequalities
  - Examples from liquid crystals to lipids to superfluids to superconductors
  - Order parameter & broken symmetry

Ising model
- Ising model in one dimension
  - Exact transfer matrix solution, spin correlations & correlation length
  - Kinks & the absence of finite-$T$ spontaneous magnetization
  - Nature of the singularity & phase transition at $T = 0$
- Ising model in two dimensions
  - Domain walls & the existence of finite-$T$ phase transition
  - Critical exponents (Onsager’s exact results)
- Mean-field theory in arbitrary dimension
  - Weiss molecular-field theory
  - Bragg-Williams (variational) approximation
  - Infinite-range (or infinite-dimensional) Ising model
  - Mean-field phase diagrams: First- & second-order transitions
- Ising critical exponents: Mean field $vs$ exact $vs$ simulation results
- Yang-Lee theory of phase transitions (not covered in this Course)

Landau mean-field theory
- Landau-Ginzburg phenomenology
- $\phi^4$ theory
  - Spontaneous symmetry breaking
  - Thermodynamic limit & the ergodicity breaking
  - Critical exponents: Mean-field Ising universality class
- $\phi^3$ theory: Continuous $vs$ discontinuous transition
- $\phi^6$ theory: Tricritical point (included in Homework #11)
- Liquid-vapor transition
  - Virial expansion & the van der Waals equation of state
  - Maxwell construction, phase coexistence & the critical point
  - Comparison with $\phi^4$ theory of ferromagnets
- Ginzburg-Landau theory of superconductivity (included in Homework #11)

Gaussian-fluctuation (one-loop) corrections
- Landau-Ginzburg-Wilson Hamiltonian
  - Field fluctuations & stiffness
  - Correlation functions & susceptibility
  - Modified singularities: Free energy & heat capacity
- Liquid-vapor system: Critical opalescence
  • Upper critical dimension & the Ginzburg criterion
  • Dimensional analysis & anomalous dimensions

› Widom scaling
  • Homogeneous functions
  • Widom scaling hypothesis
  • Critical exponents: Scaling laws
  • Spatial scale-invariance & hyperscaling laws
  • Hyperscaling above dimension four: A paradox?

› Real-space renormalization group (RG) transformation
  • Kadanoff block spins & derivation of Widom scaling
  • Wilson block spins, fixed points & calculation of critical exponents
  • General properties of RG flows & some characteristic fixed points
  • Ising model on a triangular lattice: Real-space RG in two dimensions

› Momentum-space RG transformation
  • RG scheme revisited: Coarse grain, rescale & renormalize
  • Gaussian model: Exact solution & RG analysis
  • Dangerous irrelevant variables: Hyperscaling revisited

› \( \phi^4 \) theory
  • Perturbative RG & Feynman diagrams
  • \( \epsilon \)-expansion & critical exponents to order \( O(\epsilon) \)
  • Supplementary remarks
    - Finite anomalous dimension to order \( O(\epsilon^2) \)
    - Asymptotic behavior of \( \epsilon \)-expansion
    - Results from resummation techniques
    - Ising universality class: Comparison with experiments & simulations

› Continuous symmetry
  • Discrete symmetry breaking: Domain walls revisited
  • Continuous symmetry breaking
    - Goldstone modes: Examples from crystalline solids to (Heisenberg) ferromagnets to liquid crystals to superfluid Helium
    - Goldstone’s theorem revisited
    - Massless fluctuations & the lower critical dimension
    - Mermin-Wagner (-Hohenberg-Coleman) theorem
  • Coupling to gauge fields: Anderson-Higgs mechanism (included in Homework #12)

› Other subjects (not covered in this Course / potential subjects for term essays)
  • Nonlinear \( \sigma \) model: Critical behavior near dimension two
  • XY model, topological defects & the Kosterlitz-Thouless transition
  • RG for the two-dimensional Coulomb gas
  • Two-dimensional solids & melting

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**Statistical Physics of Fields Out of Equilibrium**

	• Part V — Dissipative dynamics of fields near and far from equilibrium

› Dissipative (Brownian) dynamics of particles
  • Elements of the theory of stochastic processes
  • Langevin equation: From phenomenological to formal derivation
  • (Smoluchowski-) Fokker-Planck equation
  • Path-integral formulation of Brownian motion
Dissipative dynamics of fields: A scenery (slide presentation)
- Models of dissipative field dynamics: Examples from nucleation & spinodal decomposition to elastic membranes & growing surfaces to active fluids & bacterial suspensions

Critical dynamics & dynamic scale-invariance near equilibrium
- Non-conserved dynamics (Model A: Stochastic time-dependent Landau-Ginzburg equation)
- Conserved dynamics (Model B: Stochastic Cahn–Hilliard equation)
- An overview of dynamic perturbation theory

Critical dynamics & dynamic scale-invariance far from equilibrium (slide presentation)
- Kardar-Parisi-Zhang equation
- Dynamic RG at a glance: The KPZ universality class

Field theory for stochastic partial differential equations
- Langevin-type partial differential equations
- Generalized (Smoluchowski-) Fokker-Planck equation (included in Homework #16)
- Basics of functional-integral representation & the Martin-Siggia-Rose action