Influence of trapped electrons on the non-stationary evolution of the ion-acoustic solitons train

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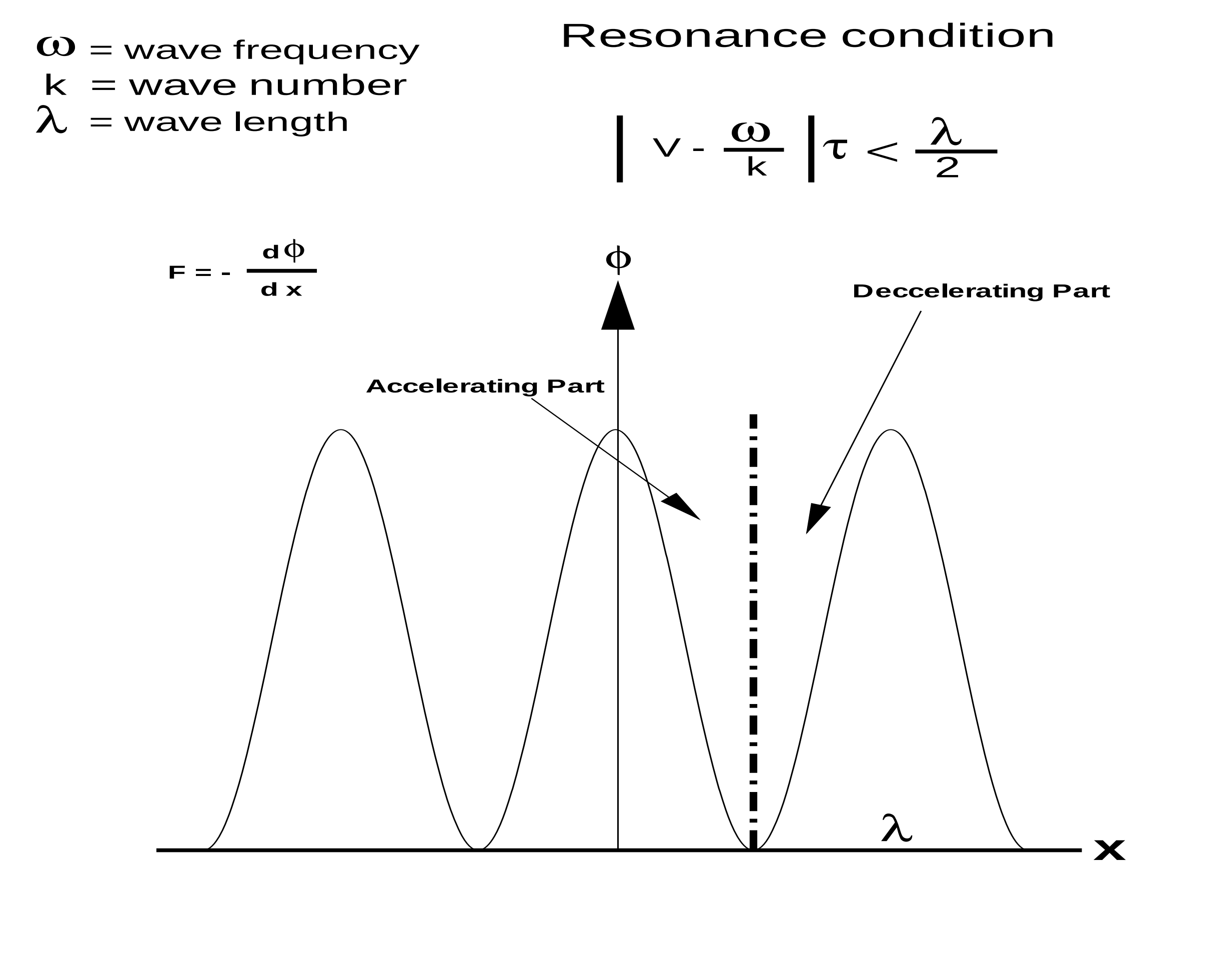
Trapping Mechanism

A nonlinear
Wave-particle
interaction!



There are two types of particles:

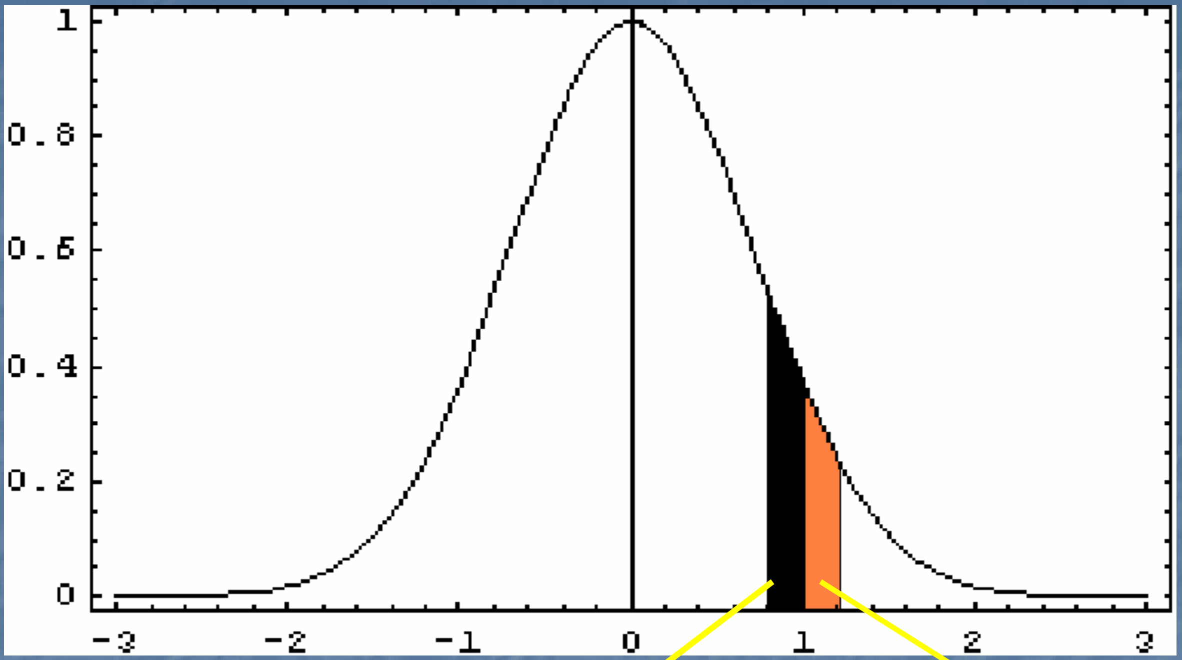
Type I:
Particle velocity < phase velocity

Wave gives energy to particle

Type II:
Particle velocity > phase velocity

Particle gives energy to wave

Maxwellian distribution function



$\omega/k = v$

Type I $>$ Type two

Landau Damping

Nonlinear Case:

The force is large and consequently each particle in the Characteristic time of dampingexperiences both acceleration and deceleration parts.

Theoretical prediction:

O' Neil (T. M. O'neil, Phys. Fluids 8, 2255 (1965))

1. Damping can altered because of nonlinear exchange of energy
2. Particle bouncing causes a type of ocillation on the wave amplitude
3. Asymptotic amplitude is non-zero

A mathematical proof:

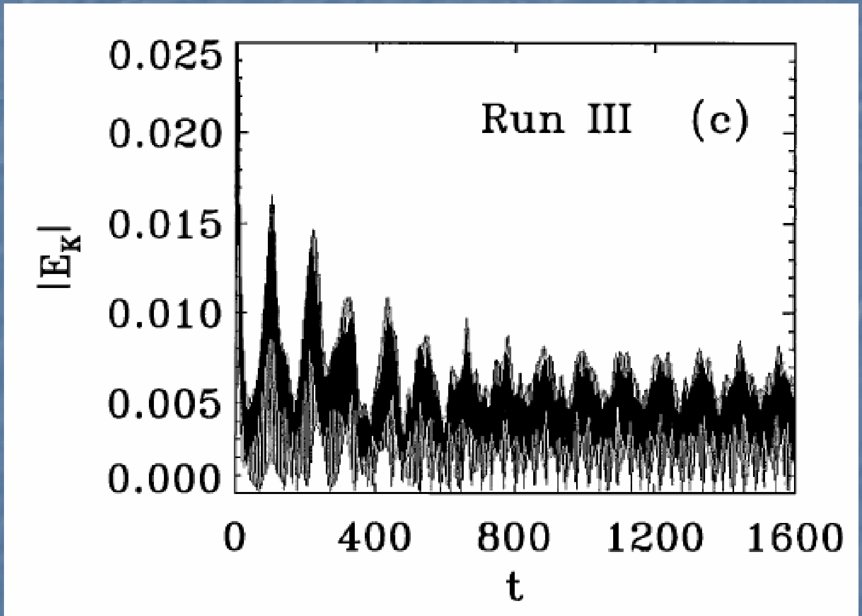
Lancellotti, et. Al. Phys. Rev. Lett. 81, 5137 (1998)

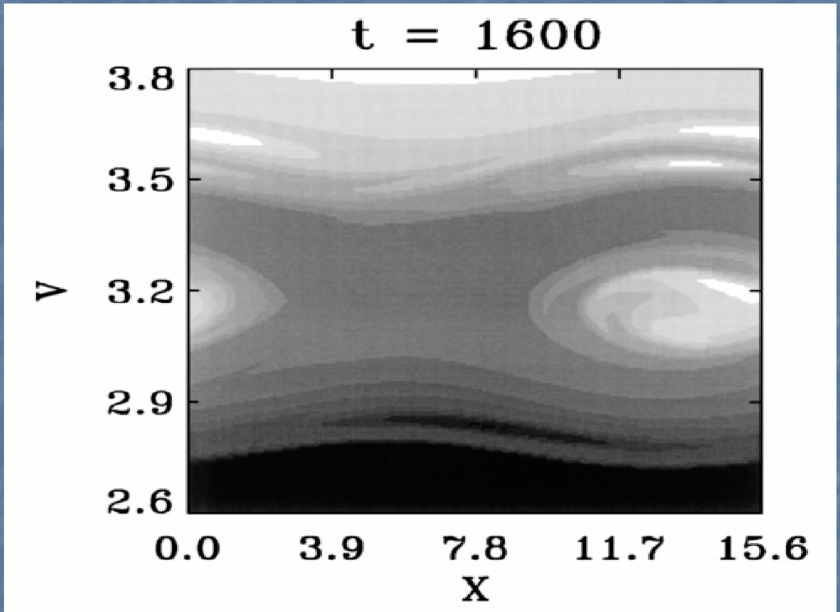
The existence of a critical initial states which leads to a non-zero Asymptotic state.

Simulation result:

G. Manfredi, Phys. Rev. Lett. 79, 2815 (1997)

Simulating the Vlasov-Poisson set of equatios and demonstrating the non-zero level of the field amplitude





The breakup of plasma wave through the trapped-particle instability.

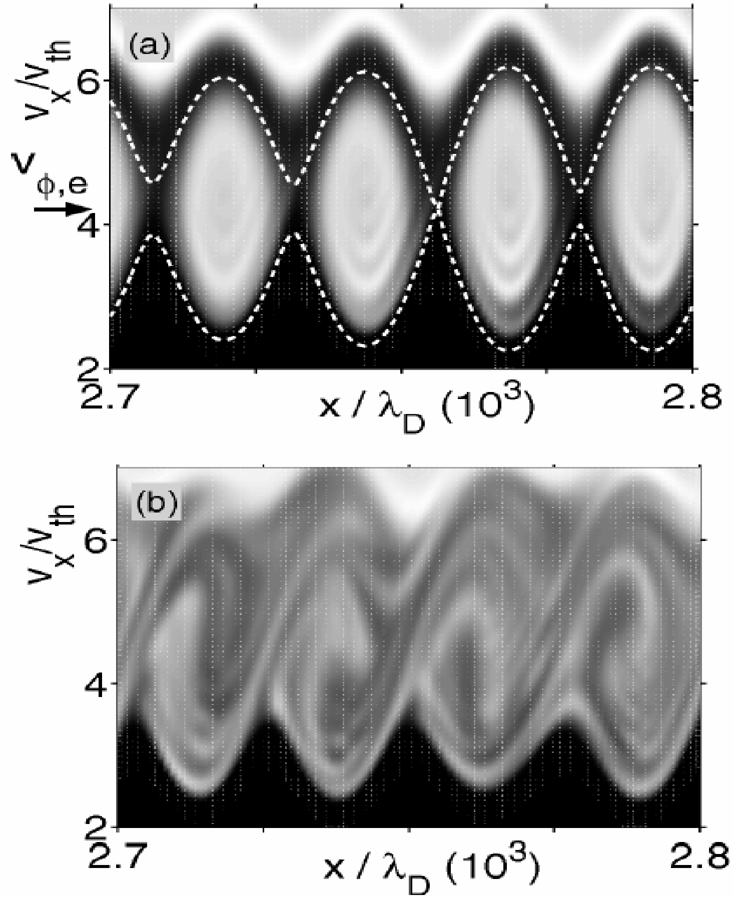
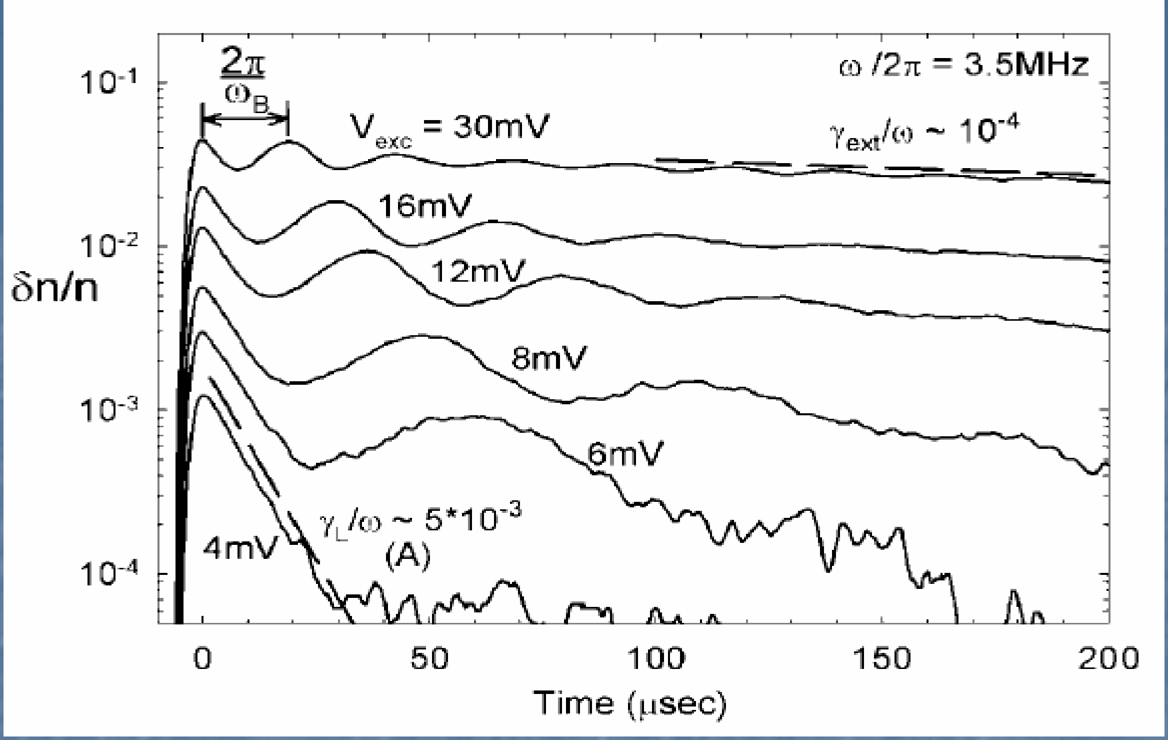


FIG. 3. Electron distribution at time (a) $t\omega_p = 4.8 \times 10^3$ and (b) $t\omega_p = 6.2 \times 10^3$ [times pointed out in Fig. 1(a)], in a limited region of phase space (x, v_x), corresponding to approximately four wavelengths of the main plasma mode. The separatrix of the trapping region is represented by dashed lines in (a).

S. Brunner and E. J. Valeo, *Phys. Rev. Lett.* **93**, 145003 (2004)



• In the lowest amplitude mode it seems that the wave decay exponentially with a linear damping rate.

• For higher amplitude modes an oscillating state create.

1) J. R. Danielson, F. Anderegg and C. F. Driscoll, *Phys.Rev. Lett.***92**, 245003 (2004)

The best analytical work (BGK Mode):

I. B. Bernstein, J. M. Greens, M. D. Kruskal, Phys. Rev. 108, 546 (1957)

How to find the distribution function of trapped particles?

1. Find the experimental profile of the electric potential
2. Solve the associated Poisson equation for density
3. Density is the first moment of the distribution function
4. Solving the abel integral equation

Problem: sigular or negative distribution functions!!

Follow the recipe up side down

Consequently,
we have functional form of density respect to electric potential

Basic equation:

$$\partial_t n_i + \partial_x (n_i v_i) = 0,$$

$$\partial_t v_i + v_i \partial_x v_i = -\partial_x \phi,$$

$$\partial_{xx} \phi = n_e - n_i,$$

$$n_e(\phi) = I(\phi) + |\beta|^{-1/2} \begin{cases} \exp(\beta\phi)\mathrm{erf}(\sqrt{\beta\phi}), & \beta \geq 0 \\ 2\pi^{-1/2} W(\sqrt{-\beta\phi}), & \beta < 0 \end{cases}$$

$$I(\phi) = \exp(\phi)[1 - \mathrm{erf}(\sqrt{\phi})]$$

$$W(y) = \exp(-y^2) \int_0^y dt \exp(t^2)$$

Weak nonlinear case $\phi < 1$

Electron density

$$n_e = 1 + \phi - \frac{4}{3} b \phi^{3/2} + \frac{1}{2}\phi^2 \ldots,$$

Stretched coordinates

$$\xi = \epsilon^{1/4}(x - v_0 t) \quad \text{and} \quad \tau = \epsilon^{3/4} t,$$

Reductive perturbation scheme

$$n_i = 1 + \epsilon n^{(1)} + \epsilon^{3/2} n^{(2)} + \ldots,$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^{3/2} \phi^{(2)} + \ldots,$$
$$v_i = \epsilon v^{(1)} + \epsilon^{3/2} v^{(2)} + \ldots,$$

Modified Korteweg-de Vries

$$\partial_\tau \Phi + b\sqrt{\Phi}\partial_\xi \Phi + \frac{1}{2}\partial_{\xi\xi\xi}\Phi = 0.$$

Stationary solution

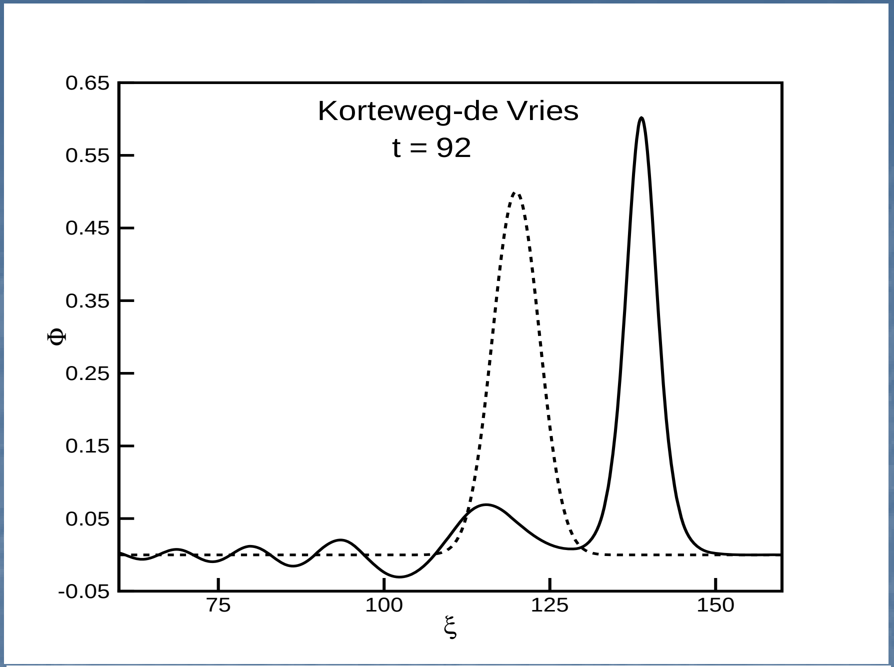
$$\Phi = \Phi_m \mathrm{Sech}^4 \left[\sqrt{\frac{b\sqrt{\Phi_m}}{15}} (\xi - M\tau) \right]$$

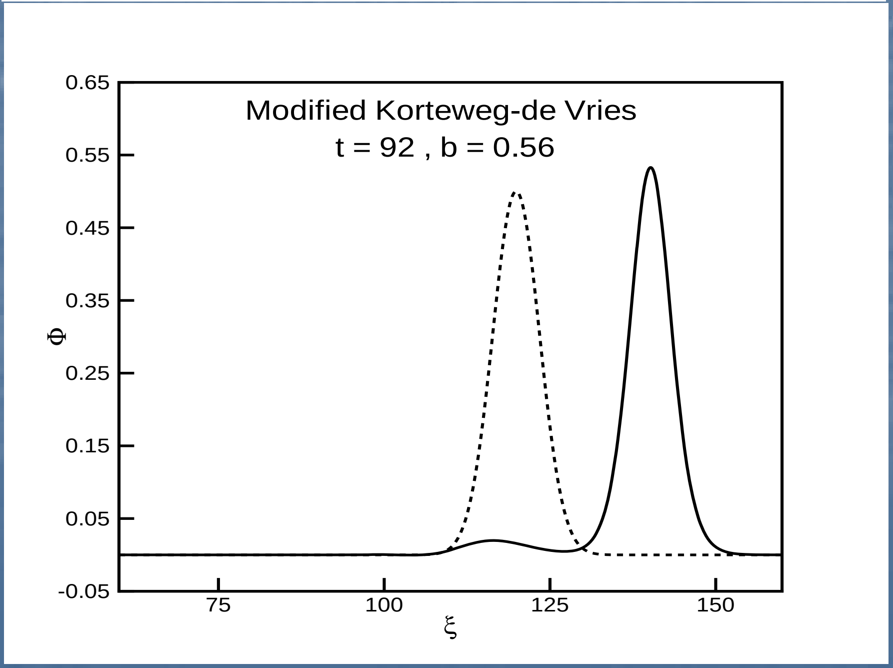
$$M = (8/15) b \sqrt{\Phi_m}$$

$$\Delta = \sqrt{\frac{15}{b\sqrt{\Phi_m}}} \mathrm{Sech}^{-1} \left(\frac{1}{2} \right)^{1/4}$$

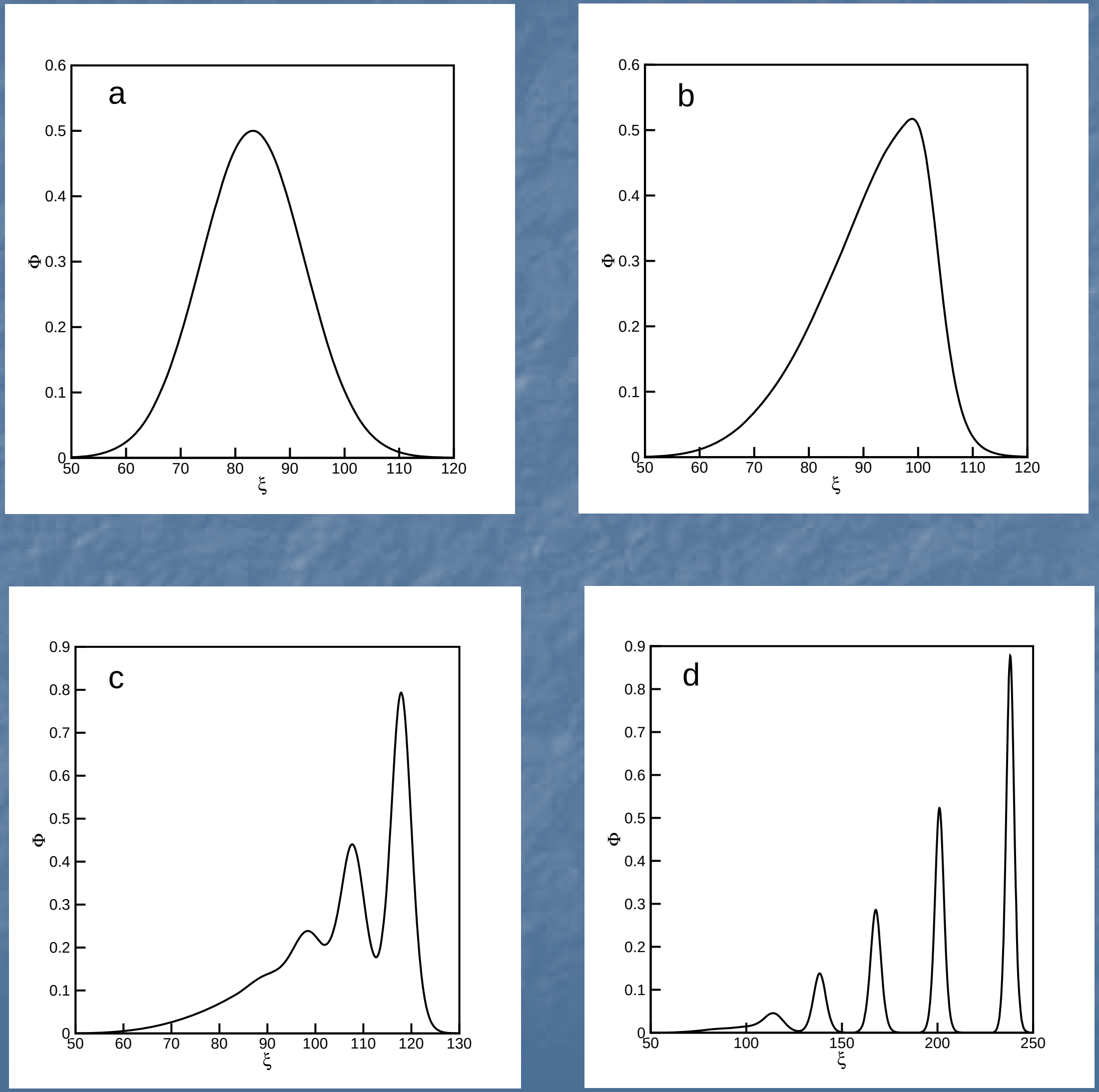
Non-Stationary solutions:

Linear wave packet part of Korteweg-de Vries equation

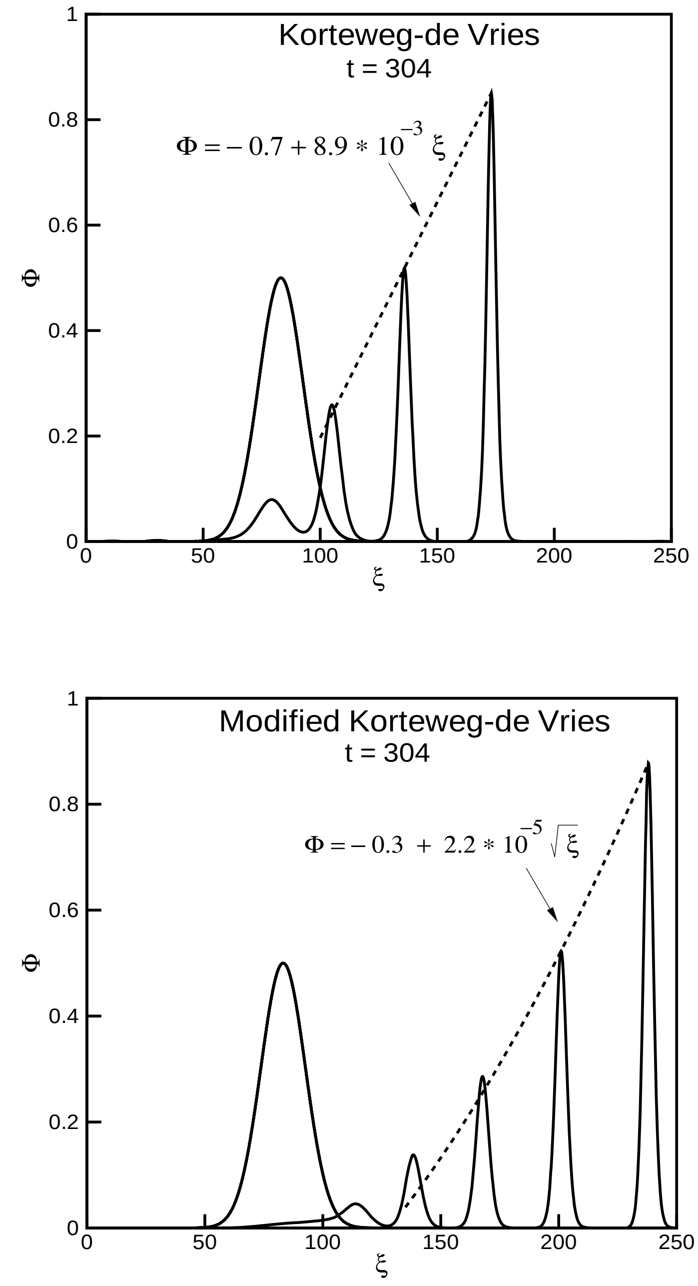




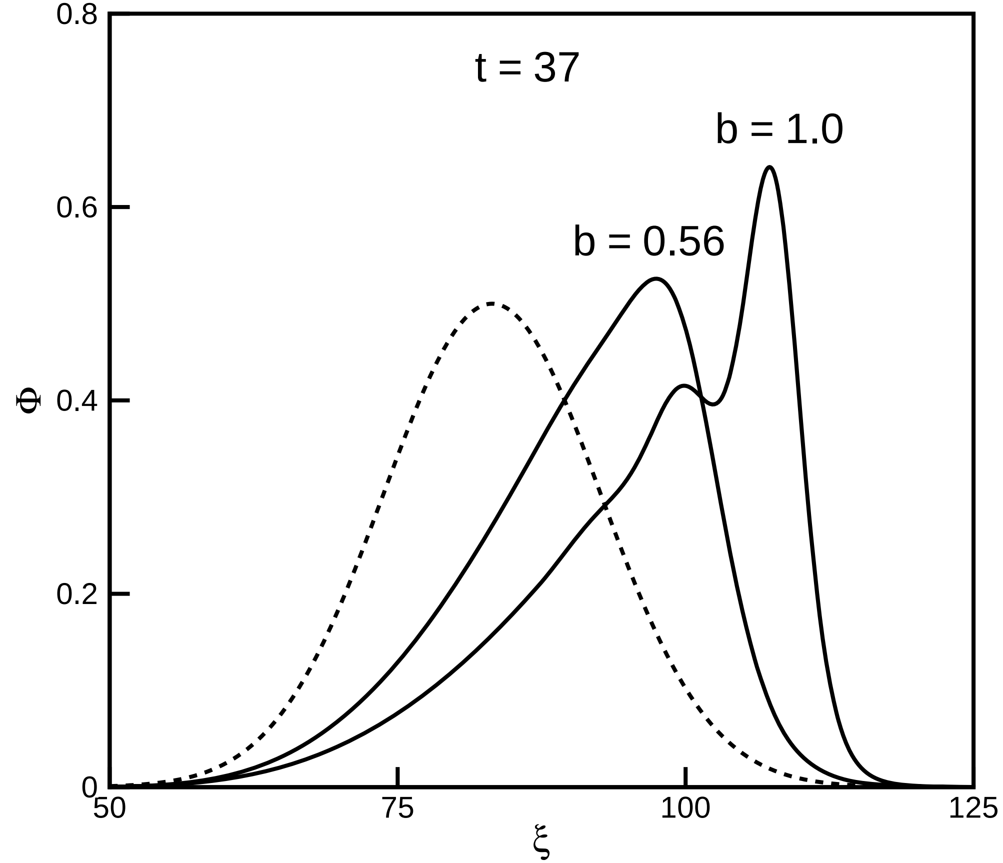
Three different stages of the non-stationary nonlinear evolution



A test that proves the localized structures resulting from the initial condition disintegration are solitons.



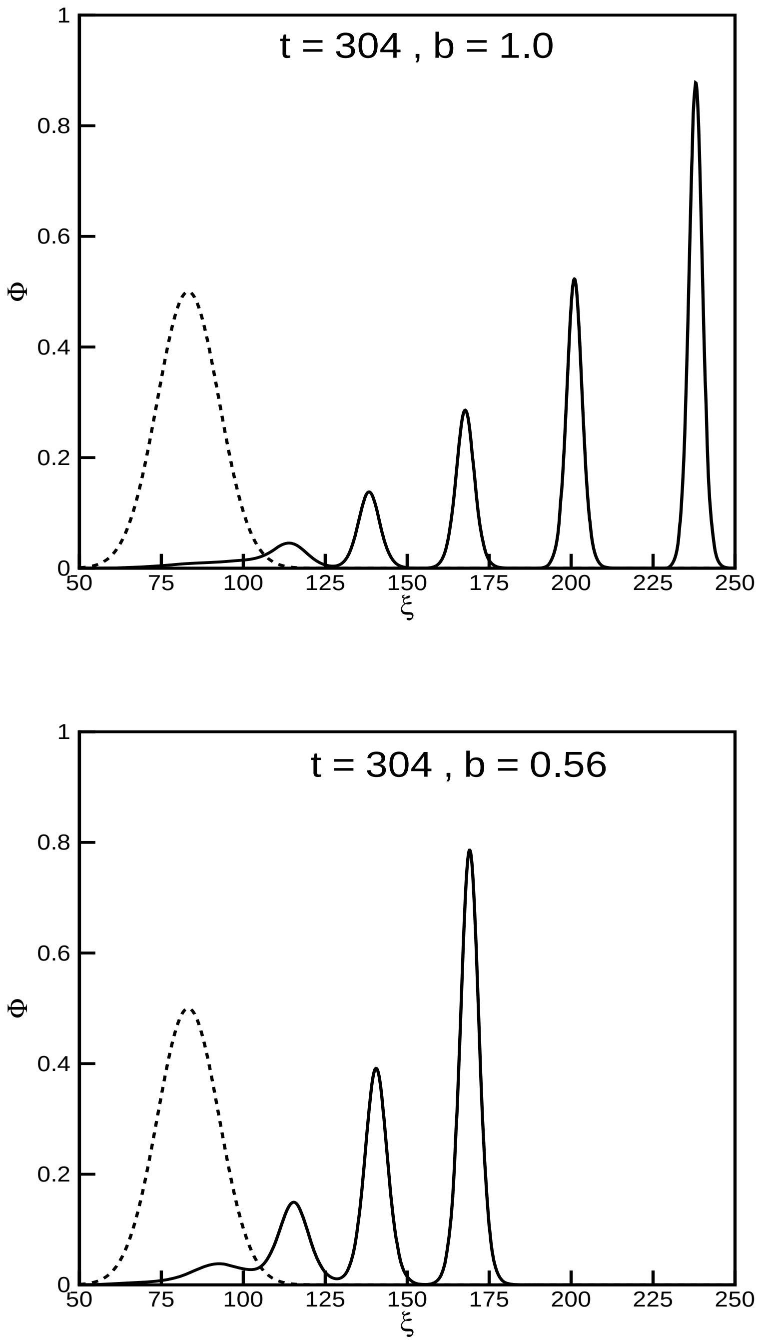
Trapped electrons makes the steepening process faster

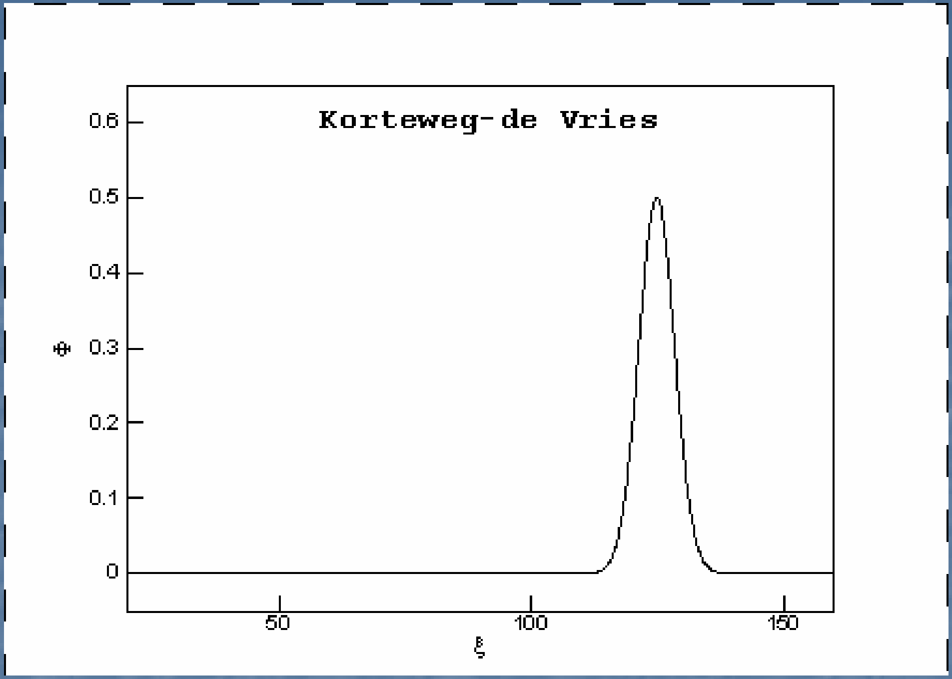


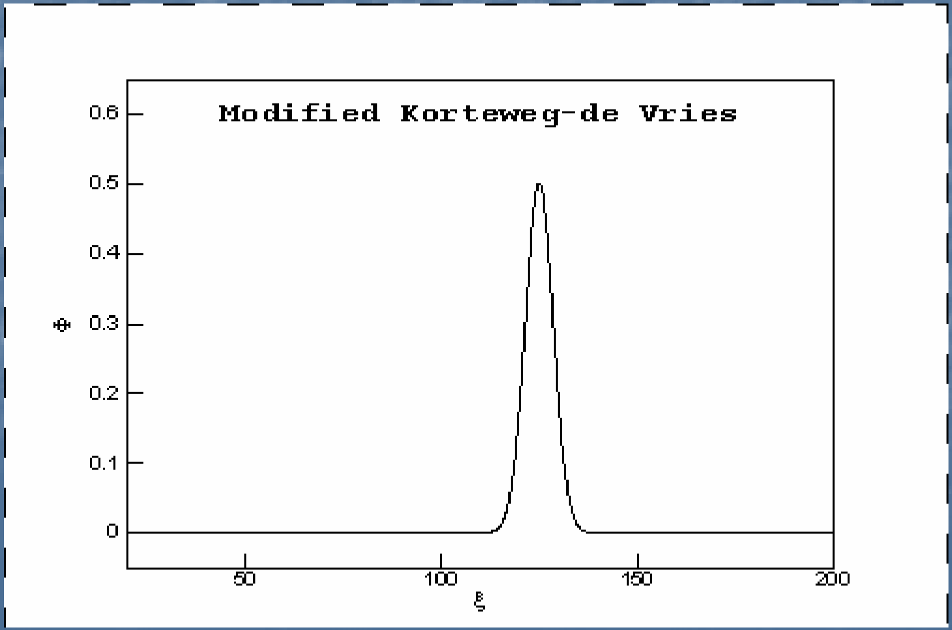
Trapped electrons cause the train of solitons to move faster.

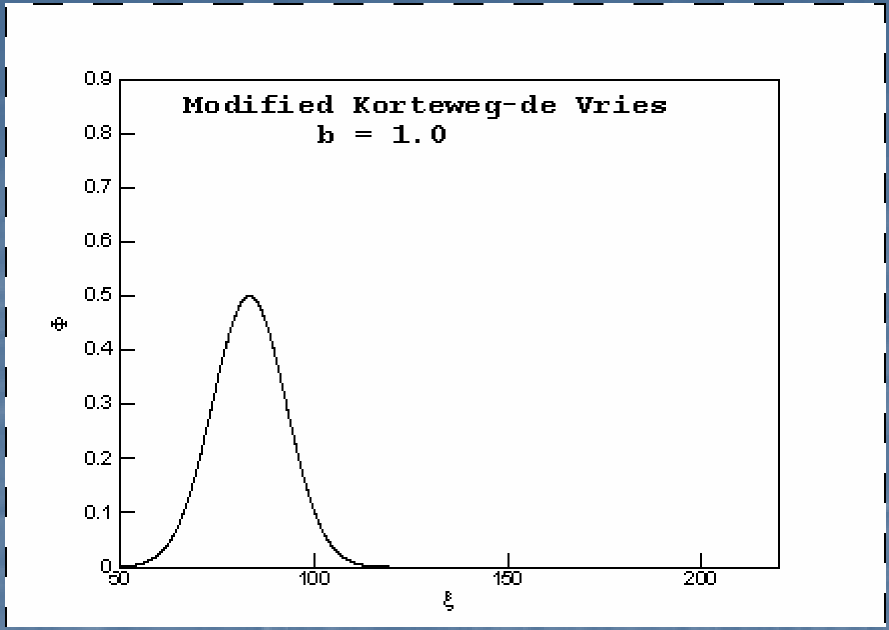
Number of solitons has are proportional to trapping parameter.

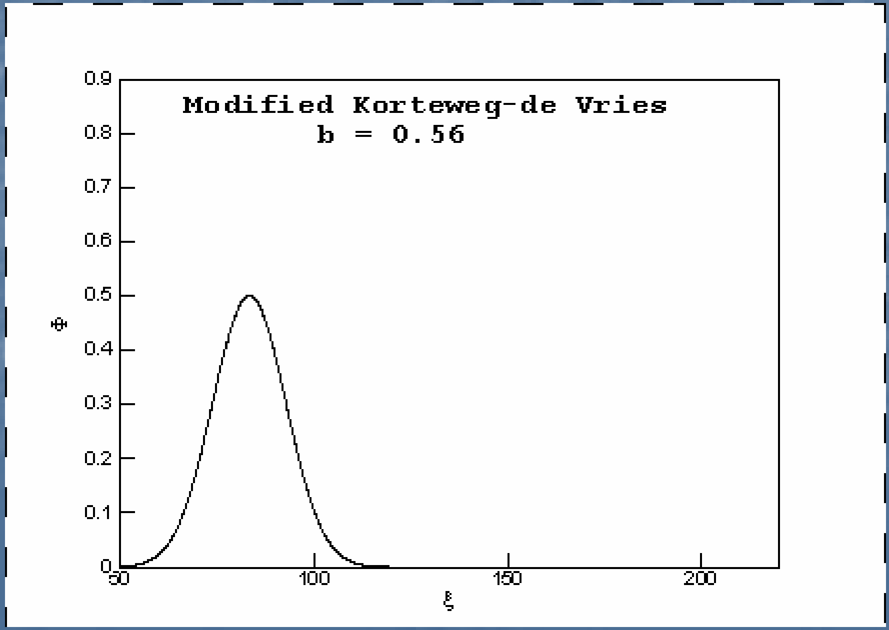
Solitons associated with larger trapping Parameter have larger amplitude and are slimmer.




Korteweg-de Vries
0.6
0.5
0.4
0.3
0.2
0.1
0
Φ
50
100
150
ξ


Modified Korteweg-de Vries
0.6
0.5
0.4
0.3
0.2
0.1
0
Φ
50
100
150
200
ξ


Modified Korteweg-de Vries
b = 1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Φ
50
100
150
200
ξ


Modified Korteweg-de Vries
b = 0.56
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Φ
50
100
150
200
ξ