Strong magnetic background field and modified noncommutativity

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Based on

- *A. Jafari Salim and N.S.; Phys. Rev. D 73, 065023 (2006)*

Also on

- *Phys. Rev. D72, 065014 (2005)*
- *Phys. Lett. B611, 207 (2005)*
- *Phys. Rev. D70, 105007 (2004)*

by V.A. Miransky et al.

I. Noncommutative (NC) field theory

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- Noncommutativity of space-time ▸ $[x_i, x_j] = i\Theta_{ij}$

  ▸ Noncommutative Moyal $\star$-product

$$f(x) \star g(x) \equiv f(x+\xi)\, \exp\left(i\Theta^{\mu\nu}\partial_\mu^\xi \partial_\nu^\zeta\right) g(x+\zeta)\Big|_{\xi=\zeta=0}$$

- Action of NC-U(1)

$$\mathcal{S}[A_\mu, \bar{\psi}, \psi] = -\frac{1}{4}\int d^4x\; F_{\mu\nu} \star F^{\mu\nu} + \int d^4x\; \bar{\psi}(x) \star (i\not{D} - m)\,\psi(x)$$

  ▸ Field strength tensor

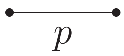
$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right]_\star$$

  ▸ Covariant derivative

$$D_\mu \psi(x) \equiv \partial_\mu \psi(x) + igA_\mu(x) \star \psi(x)$$

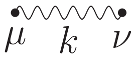
Perturbative aspects

- Fermion propagator (not modified)



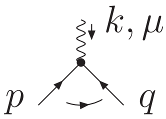
$$S(p) = \frac{i}{\not p - m}$$

- Photon propagator ($\xi = 1$) (not modified)



$$D_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2}$$

- Vertex (modified by a momentum dependent phase factor)



$$V_\mu = (2\pi)^4 \delta^4 (p + q + k)\, ig\gamma_\mu \, \mathrm{exp}\left(i\theta_{\eta\sigma} p^\eta q^\sigma\right)$$

  - Consequence: UV / IR mixing

UV / IR mixing

Conventional UV terms in a commutative theory are distributed into two parts:

- UV divergent part
- UV finite part ► But these UV finite terms become singular in the IR limit

Consequence

- Noncommutative gauge theories are non-renormalizable

II. Modified noncommutative field theory

$\hookrightarrow$ *V.A. Miransky et al.; Phys. Rev. D70, 105007 (2004)*

II. Modified noncommutative field theory

- They study the connection between the dynamics of NJL model in a strong magnetic field and that in NCFT
  - Strong magnetic field $\triangleq$ LLL dominant regime
- The NJL consists of two composites π and σ
- The low energy effective action of π and σ in the LLL dominant regime has noncommutative properties
- No UV / IR mixing occurs here
  - This is mainly due to certain field redefinition (fields are smeared with certain Gaussian factor which provides us from the notorious UV/IR mixing)
- Same phenomena for QED

III. Dynamics of $O(N)$-model in a strong magnetic field

$\hookrightarrow$ *A. Jafari Salim and N.S.; Phys. Rev. D 73, 065023 (2006)*

- In the presence of a magnetic background field this model consists of a condensate
- Look for:
  - Effective action of this condensate in the LLL approximation
  - Effective action of the composite field from the world-line formalism (new method)
- Comparison with the ordinary NCFT

$O(N)$-Model

- Lagrangian density

$$\mathcal{L} = -|D_\mu \Phi|^2 - m^2 \Phi^* \Phi - \frac{1}{2}\lambda(\Phi^*\Phi)^2,$$

with $\Phi = (\varphi_1, \cdots, \varphi_N)$ and $D_\mu \Phi = \partial_\mu \Phi + igA_\mu \Phi$

- Introducing
  - Composite field $\sigma \equiv \lambda \Phi^* \Phi$
  - 't Hooft Coupling $g \equiv \lambda N$
- Lagrangian density in terms of σ

$$\mathcal{L} = -|D_\mu \Phi|^2 - m^2 \Phi^* \Phi - \sigma \Phi^* \Phi + \frac{N}{2g}\sigma^2$$

- In a strong magnetic field a condensate is built even in $N = 1$

$$\langle \varphi^*(x)\varphi(x)\rangle \sim |eB|$$

- Look for the effective action for the composite

Effective action for the composite field σ

- Take the Lagrangian density in terms of the composite field σ

$$\mathcal{L} = -|D_\mu \Phi|^2 - m^2 \Phi^* \Phi - \sigma \Phi^* \Phi + \frac{N}{2g}\sigma^2$$

- Integrating over Φ^* and Φ

$$\begin{aligned} e^{\widetilde{\Gamma}[\sigma]} &= \int \mathcal{D}\Phi^* \, \mathcal{D}\Phi \ \exp\left(- \int d^4x \left\{ \Phi^* [-D_\mu D^\mu + m^2 + \sigma] \Phi + \frac{N}{2g} \, \sigma^2 \right\} \right) \\ &= \exp\left(-\text{tr} \ln[-D_\mu D^\mu + m^2 + \sigma] + \frac{N}{2g} \int d^4x \ \sigma^2 \right), \end{aligned}$$

- The full effective action

$$\widetilde{\Gamma}[\sigma] = \Gamma[\sigma] + \frac{N}{2g} \int d^4x \ \sigma^2$$

with

$$\Gamma[\sigma] = -\text{tr} \ln \left(-D_\mu D^\mu + m^2 + \sigma \right)$$

III.A. Effective action for n composites σ in the LLL approximation

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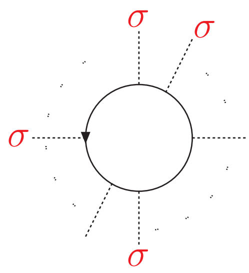
- Effective action for n composite fields σ

$$\Gamma_n[\sigma] = \int d^4x_1 \cdots d^4x_n \left[\sigma(x_1)G(x_1,x_2)\sigma(x_2)G(x_2,x_3)\cdots\sigma(x_n)G(x_n,x_1)\right]$$

- Full effective action for σ

$$\Gamma[\sigma] \equiv \sum_{n=0}^{\infty} \frac{1}{n!}\Gamma_n[\sigma];$$



$= \Gamma_n[\sigma]$

Method

- ▸ Use Schwinger proper time formalism ▸ Full propagator $G(x_i, x_j)$
- ▸ Determine $G(x_i, x_j)$ in the LLL approximation

▸▸ Effective action for n-composites in the LLL approximation

Full propagator from Schwinger proper time formalism

- Schwinger propagator

$$G(x', x'') = P(x', x'')D(x' - x'')$$

  - Schwinger line integral

$$P(x', x'') \equiv \exp\left(ie\int_{x''}^{x'} d\xi^\mu A_\mu(\xi)\right)$$

  - Translational invariant part

$$D(x'-x'') \equiv \frac{1}{(4\pi)^2}\int_0^\infty \frac{ds}{s^2}e^{-ism^2}\exp\left(\frac{-1}{2}\mathrm{tr}\ln\left[\frac{\sinh eFs}{eFs}\right]\right)\exp\left(-\frac{i}{4}(x'-x'')eF\coth(eFs)(x'-x'')\right)$$

- Fixing the gauge (symmetric gauge for $\vec{B} = B\hat{e}_3$)

$$A_\mu = \frac{B}{2}(0, x_2, -x_1, 0)$$

►►►

- Schwinger line integral

$$P(x',x'') = e^{\frac{ieB}{2}\epsilon^{ab}x'_a x''_b}, \qquad\qquad a,b=1,2$$

- Translational invariant part in momentum space

$$\tilde{D}(k) = -\frac{1}{eB}\int_0^\infty \frac{ds}{\cosh s}\, e^{-s\rho-\alpha\tanh(s)} \qquad \left\{\begin{array}{rcl} \mathbf{k}_\parallel^2 & \equiv & k_0^2 - k_3^2 \\ \mathbf{k}_\perp^2 & \equiv & k_1^2 + k_2^2 \\ \alpha & \equiv & \mathbf{k}_\perp^2/|eB| \\ \rho & \equiv & \left(m^2 - \mathbf{k}_\parallel^2\right)/|eB| \end{array}\right.$$

- Full propagator

$$G(x',x'') = e^{\frac{ieB}{2}\epsilon^{ab}x'_a x''_b} \int \frac{d^4k}{(2\pi)^4}\widetilde{D}(k)\; e^{-ik(x'-x'')}$$

- To determine Schwinger propagator in LLL approximation ►►►

Schwinger propagator in LLL approximation

- $\tilde{D}(k) = \frac{e^{-\alpha}}{2|eB|} \sum_{n'=0,m'=1}^{\infty} (-1)^{n'} L_{n'}^{(-1)}(2\alpha) \left(\frac{\rho + 2n' - 3}{m'(4m' + \rho + 2n' - 3)} - \frac{\rho + 2n' - 1}{m'(4m' + \rho + 2n' - 1)} \right)$

  $L_{n'}^{\beta}$ generalized Laguerre polynomial, m' is related to the expansion of a digamma function $\psi(z)$

- To determine (m', n') related to LLL form of $\widetilde{D}(k)$:
  - E_0 from the energy spectrum of a KG particle in an external B field

    $\equiv$

    Poles of the propagator
  - Energy spectrum

    $$E_{\ell'} = \sqrt{m^2 + |eB|(2\ell' + 1) + k_3^2}, \qquad \ell' = 0, 1, 2, \cdots, \infty$$
  - LLL energy $\ell' = 0$ ► $m' = 1$ and $n' = 0$ in $\widetilde{D}(k)$

- LLL propagator

- LLL propagator

$$G(x', x'') = G_{\perp}(\mathbf{x}'_{\perp}, \mathbf{x}''_{\perp}) G_{\parallel}(\mathbf{x}'_{\parallel} - \mathbf{x}''_{\parallel}),$$

  - The transverse part ▸ $\mathbf{x}_{\perp} = (x_1, x_2)$

$$G_{\perp}(\mathbf{x}'_{\perp}, \mathbf{x}''_{\perp}) = \frac{|eB|}{2\pi} e^{\frac{ieB}{2}\epsilon^{ab} x'_a x''_b} \exp\left(-\frac{|eB|}{4}(\mathbf{x}'_{\perp} - \mathbf{x}''_{\perp})^2\right)$$

  - The longitudinal part ▸ $\mathbf{x}_{\parallel} = (x_0, x_3)$

$$G_{\parallel}(\mathbf{x}'_{\parallel} - \mathbf{x}''_{\parallel}) = \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \frac{1}{\mathbf{k}_{\parallel}^2 - (m^2 + |eB|)} e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{x}' - \mathbf{x}'')_{\parallel}}$$

- n-point vertex function for n-composites σ in LLL approximation

Result

- n-point contribution to the effective action in the LLL approximation

$$
\begin{aligned}
\Gamma_n[\sigma] = 2\pi N|eB| \int d^2\mathbf{x}_{1\|}\cdots d^2\mathbf{x}_{n\|}\,\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_n}{(2\pi)^4} \\
\times\delta^2\left(\sum_{i=1}^{n}\mathbf{p}_{i\perp}\right)\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right) \\
\times\exp\left(i\sum_{i=1}^{n}\mathbf{p}_{i\|}\cdot\mathbf{x}_{i\|}\right)\left[\sigma(p_1)G_{\|}(\mathbf{x}_{1\|},\mathbf{x}_{2\|})\sigma(p_2)G_{\|}(\mathbf{x}_{2\|},\mathbf{x}_{3\|})\cdots\sigma(p_n)G_{\|}(\mathbf{x}_{n\|},\mathbf{x}_{1\|})\right]
\end{aligned}
$$

Here $\mathbf{p}_i\times\mathbf{p}_j \equiv \epsilon_{ab}p_i^a p_j^b$ and $a,b=1,2$

- Full effective action in the LLL approximation

$$
\Gamma[\sigma] \equiv \sum_{n=0}^{\infty}\frac{1}{n!}\Gamma_n[\sigma]
$$

III.B. Effective action for σ from worldline formalism

$\hookrightarrow$ *Phys. Rev. D73, 065023 (2006) for more details*

Comparing two results

- 1PI n-point function for n composites σ from worldline formalism

$$\Gamma_{1\mathrm{PI}}[p_1,\cdots,p_n] = N|eB|(-1)^n(2\pi^2)\,\delta\left(\sum_{i=1}^{n} p_i\right)$$
$$\times\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right)$$
$$\times\int_0^\infty\frac{dT}{T^{2-n}}e^{-(m^2+|eB|)T}\prod_{i=1}^{n}\int_0^1 du_i\exp\left(\sum_{i<j=1}^{n}[G_{Bij}\,g_{\|}]\,p_i\cdot p_j\right)$$

- Effective action for n composites σ in the LLL approximation

$$\Gamma_n[\sigma] = 2\pi N|eB|\int d^2\mathbf{x}_{1\|}\cdots d^2\mathbf{x}_{n\|}\,\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_n}{(2\pi)^4}\delta^2\left(\sum_{i=1}^{n}\mathbf{p}_{i\perp}\right)$$
$$\times\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right)$$
$$\times\exp\left(i\sum_{i=1}^{n}\mathbf{p}_{i\|}\cdot\mathbf{x}_{i\|}\right)\left[\sigma(p_1)G_{\|}(\mathbf{x}_{1\|},\mathbf{x}_{2\|})\sigma(p_2)G_{\|}(\mathbf{x}_{2\|},\mathbf{x}_{3\|})\cdots\sigma(p_n)G_{\|}(\mathbf{x}_{n\|},\mathbf{x}_{1\|})\right]$$

IV. Modified noncommutative $O(N)$-model

Ordinary noncommutative field theory

- General structure of an n-point vertex

$$\int d^Dx\, \overbrace{\phi(x)\star\cdots\star\phi(x)}^{n\text{-times}} =$$
$$= \int \frac{d^Dp_1}{(2\pi)^D}\cdots\frac{d^Dp_n}{(2\pi)^D}\,\delta^D\left(\sum_{i=1}^n p_i\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^n \mathbf{p}_i\times\mathbf{p}_j\right)\phi(p_1)\cdots\phi(p_n)$$

Modified noncommutative field theory

- The new vertex (as we have seen)

$$\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^n \mathbf{p}_{i\perp}^2 - \frac{i}{2}\sum_{i<j=1}^n \mathbf{p}_i\times\mathbf{p}_j\right)\equiv\exp\left(-\frac{\mathbf{i}}{\mathbf{2}}\sum_{i<j=1}^n \mathbf{p}_i\widehat{\times}\mathbf{p}_j\right),$$

Ordinary noncommutative field theory

- The ordinary vertex

$$\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i \times \mathbf{p}_j\right)$$

arises from

$$[x_i, x_j] = i\theta_{ij} \qquad \text{with} \qquad (\theta) = \theta \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

Modified noncommutative field theory

- The new vertex

$$\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2 - \frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i \times \mathbf{p}_j\right) \equiv \exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i \widehat{\times} \mathbf{p}_j\right)$$

arises from

$$[x_i, x_j] = i\widehat{\theta}_{ij} \qquad \text{with} \qquad (\widehat{\theta}) = \frac{1}{|eB|}\begin{pmatrix} i & \mathrm{sign}(eB) \\ -\mathrm{sign}(eB) & i \end{pmatrix}$$

Modified noncommutative $O(N)$ model

- The effective action of n composites

$$\Gamma_{n\sigma} \sim \int d^2x_\parallel \, d^2x_\perp \, \underbrace{\sigma(\mathbf{x}_\perp) \,\widehat{\star} \cdots \widehat{\star}\, \sigma(\mathbf{x}_\perp)}_{n\text{-times}},$$

- Alternatively

  Use the ordinary noncommutativity

$$[x_i, x_j] = i\theta_{ij} \qquad \text{with} \qquad \theta_{ij} = \theta\epsilon_{ij}$$

  BUT modify the fields with a Gaussian damping factor

$$\Sigma(x) \equiv e^{\frac{\vec{\nabla}_\perp^2}{4|eB|}} \sigma(x)$$

  to write

$$\Gamma_{n\Sigma} \sim \int d^2x_\parallel \, d^2x_\perp \, \underbrace{\Sigma(\mathbf{x}_\perp) \star \cdots \star \Sigma(\mathbf{x}_\perp)}_{n\text{-times}},$$