

# Global Analysis of Fragmentation Functions and their Applications in Top quark Decay

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# Outline

## 1- Annihilation

- ✓ Electron Positron Annihilation
- ✓ Hadronization
- ✓ Single Inclusive Electron Positron Annihilation

## 2- QCD Analysis

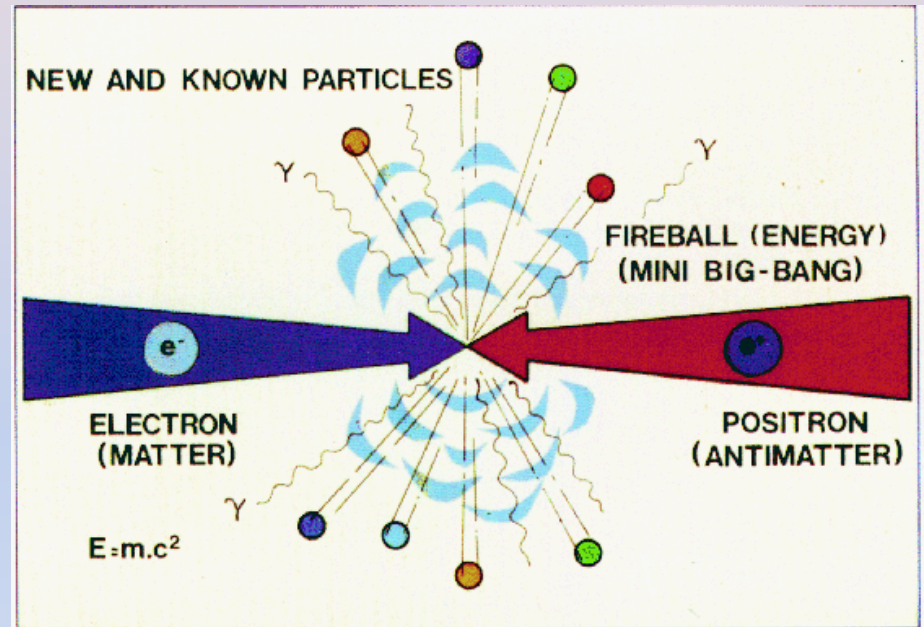
- ✓ Fragmentation Functions
- ✓ Fragmentation Densities
- ✓ Parametrization Method
- ✓ Results for Pion FFs

## 3- Top Quark Decay

- ✓ The Decay Width of Process
- ✓ Result of The Decay Width

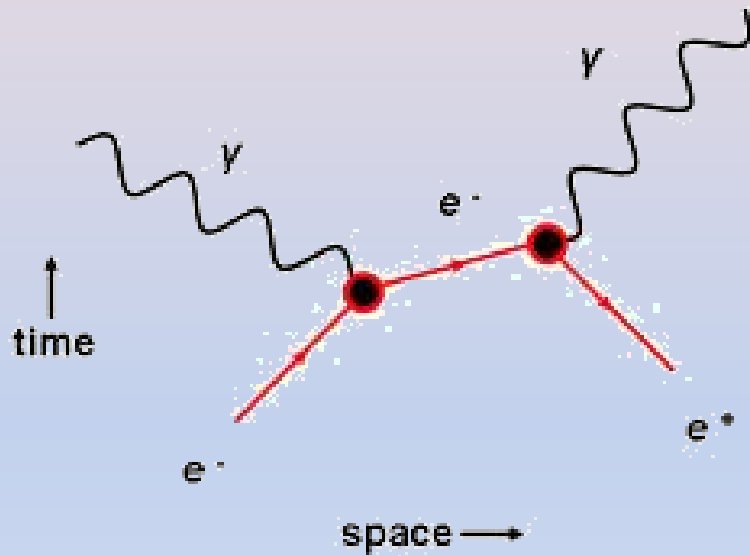
# Annihilation

Annihilations are of course not decays, but they too occur via virtual particles. In an annihilation a matter and an antimatter particle completely annihilate into energy.



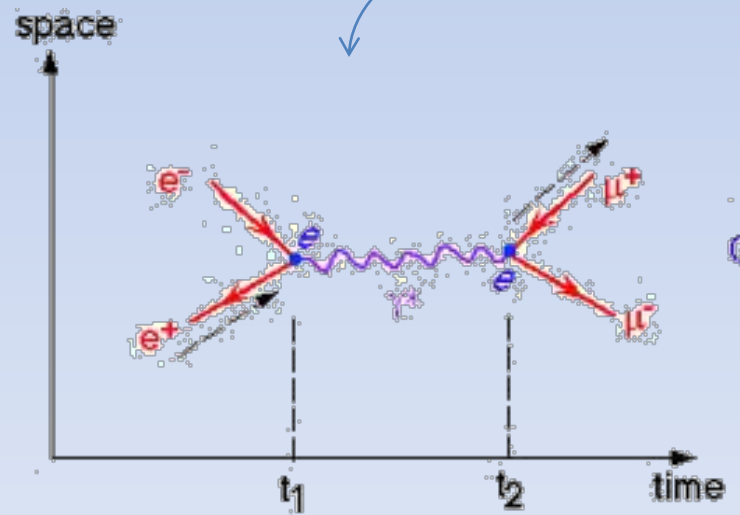
That is, they interact with each other, converting the energy of their previous existence into a very energetic force carrier particle (a gluon, W/Z, or photon). These force carriers, in turn, are transformed into other particles. Quite often, physicists will annihilate two particles at tremendous energies in order to create new, massive particles.

# Electron-Positron Annihilation



*Low energy case*

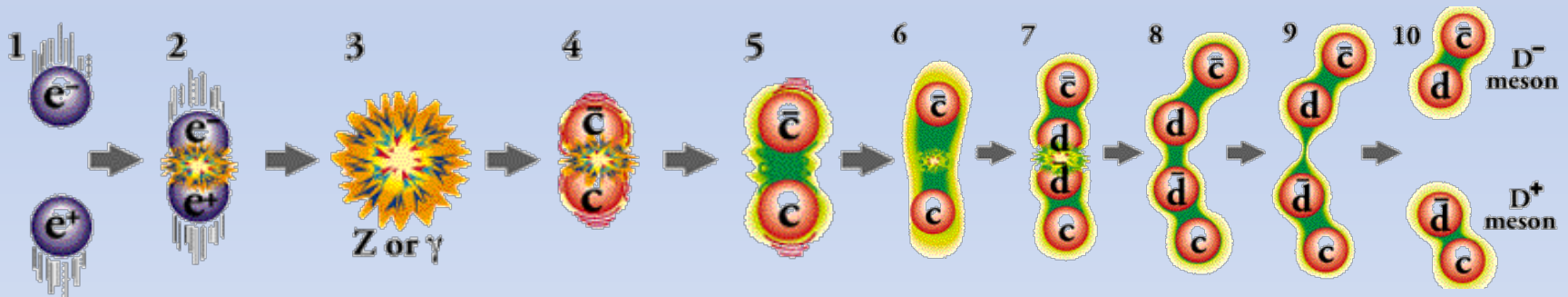
*High energy case*



# Annihilation and Hadronization

When an electron and positron (anti electron) collide at high energy, they can annihilate to produce different pairs quarks.

A charm quark and a charm anti quark emerge from the virtual force carrier particle.

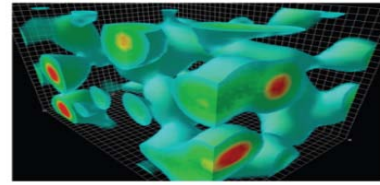


The energy in the force field increases with the separation between the quarks..

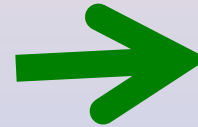
Then the energy is converted into a quark and an anti-quark.

# Hadronization

Quarks and gluons produced in a short distance process from themselves into hadrons.



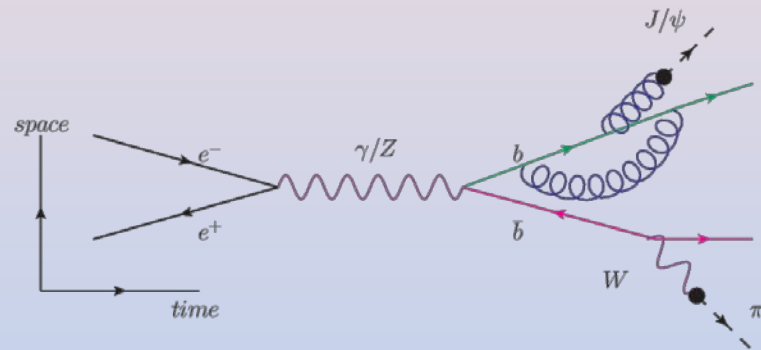
**Quark**



**Hadrons**

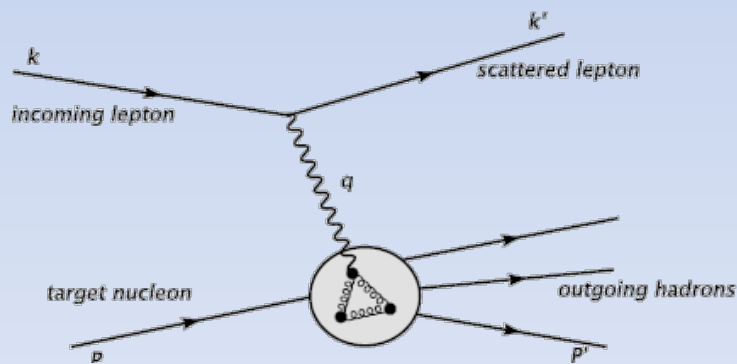
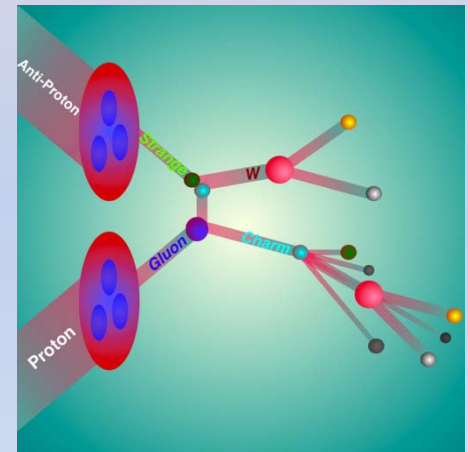
In the hard scattering reactions, partons turn into the colorless and non-perturbative hadronic bound state and hadrons are detected in the final state.

# Hadronization



$$e^+e^- \rightarrow (\gamma, Z) \rightarrow H,$$

$$P\bar{P} \rightarrow \text{Hadron}$$



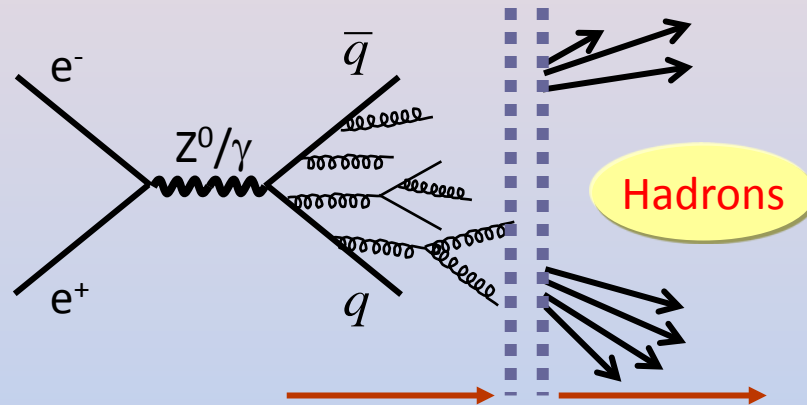
$$eN \rightarrow e'HX,$$

SIA

**SINGLE  
INCLUSIVE  
ELECTRON-  
POSITRON  
ANNIHILATION**



# Perturbative and Non-Perturbative Phase



Perturbative phase  
 $\alpha_s < 1$  (Parton Level)

Non-perturbative phase  
 $\alpha_s \geq 1$

We are going to be able to describe the early time (short distance) part of phenomena where the QCD coupling is small.

Perturbative QCD can tell us little about the later time, large distance, behavior in terms of a simple Parton fragmentation model.

# Perturbative and Non-Perturbative Phase

The reaction  $e^+e^- \rightarrow (Z^0/\gamma)^* \rightarrow \text{hadrons}$  is described by a two step processes:

The first part is to create a quark-antiquark pair by the reaction

$$(e^- e^+ \rightarrow \gamma / Z^0 \rightarrow q\bar{q})$$

*Perturbation QCD*

The second part is to create a hadron  $h$  from quark and antiquark or gluon and This process is called fragmentation.

$$(q\bar{q} \rightarrow \text{Hadron} + \text{Jets})$$

*Fragmentation*

# Cross Section in SIA

The cross section in semi-inclusive electron-positron annihilation is given in terms of fragmentation functions

$$\frac{1}{\sigma_0} \frac{d^2\sigma^h}{dx d\cos\theta} = \frac{3}{8}(1 + \cos^2\theta)F_T^h + \frac{3}{4}\sin^2\theta F_L^h$$

The transverse and longitudinal fragmentation functions

$$x = 2E_h/\sqrt{s} \leq 1$$

M. Soleymaninia, Ali N. Khorramian, M. Moosavinejad, JPCS 347, 012017 (2012).

# Fragmentation Densities

Integration of before equation over  $\theta$  yields the total fragmentation function

$$F^h = F_T^h + F_L^h$$

short distance cross section for producing a parton  $i$  and they are calculated in perturbative QCD.

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{dx} = F^h(x, s) = \sum_i \int_x^1 \frac{dz}{z} C_i(z, \alpha_s(\mu), \frac{s}{\mu^2}) D_i^h(\frac{x}{z}, \mu^2)$$

the parton fragmentation functions (or fragmentation densities)

The function  $D_i^h(z, \mu_f)$  *encodes the probability that the parton  $i$  fragments into a hadron  $h$  carrying a fraction  $z$  of the parton's momentum.*

M. Hirai and S. Kumano, *Nucl. Phys. B* **813** (2009).

D. de Florian, R. Sassot, and M. Stratmann, *Phys. Rev. D* **75** (2007).

# Scaling Violation

The simplest parton-model approach would predict scale-independent  $x$ -distributions ('scaling') for both the fragmentation function *and the parton fragmentation functions*  $D_i^h(x, \mu_f)$ . *Perturbative QCD corrections lead, after factorization of the final-state collinear singularities for light partons, to logarithmic scaling violations via the evolution equations*

$$\frac{\partial}{\partial \ln \mu^2} D_i(x, \mu^2) = \sum_j \int_x^1 \frac{dz}{z} P_{ji}(z, \alpha_s(\mu^2)) D_j\left(\frac{x}{z}, \mu^2\right)$$

the singlet splitting-function matrix

# Scaling Violation

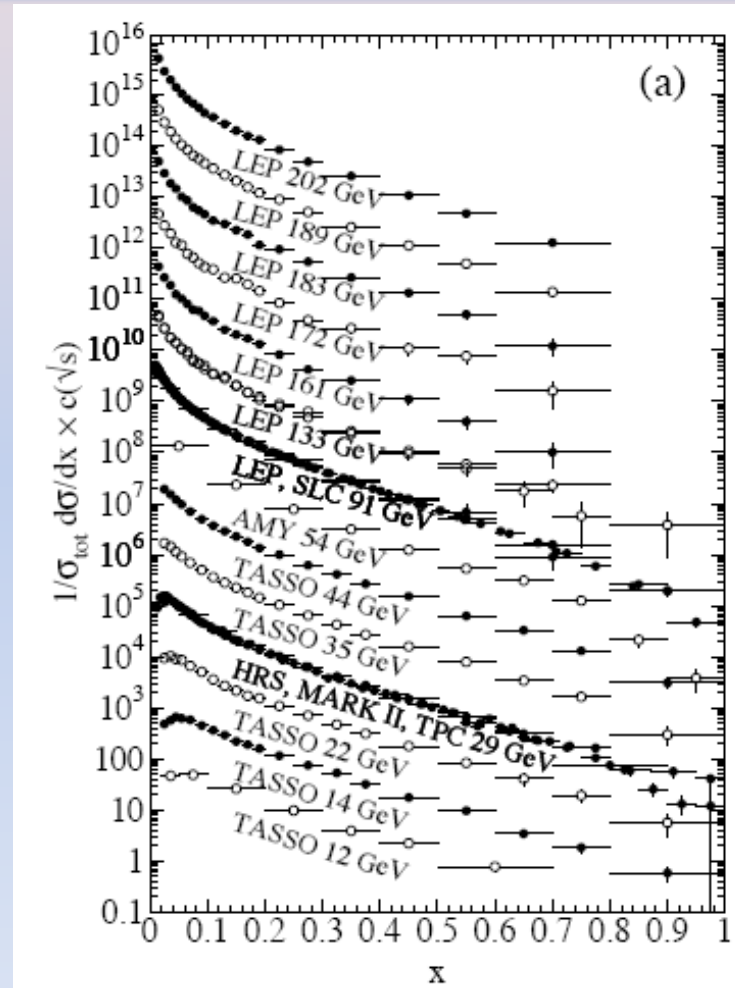


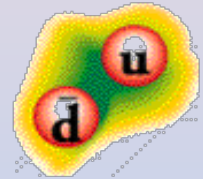
FIG. 1 The  $e^+e^-$  fragmentation function for all charged particles is shown.

# PION FRAGMENTATION FUNCTIONS

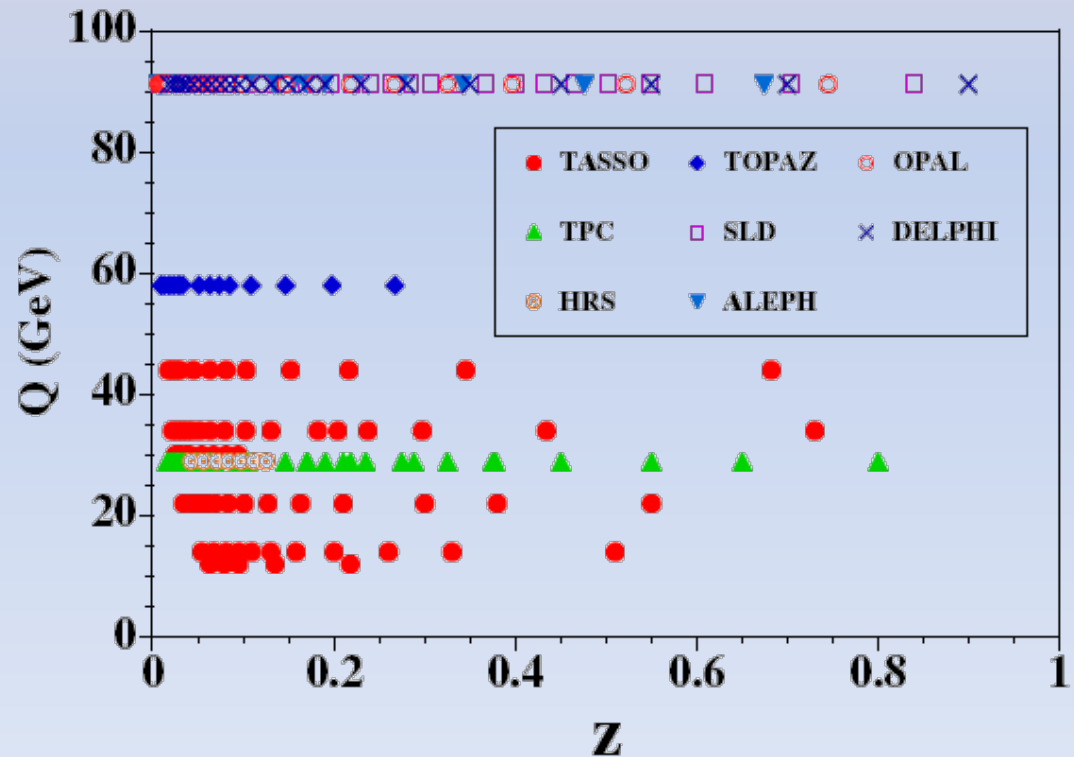
# Pion Fragmentation Functions

Mesons are bound states of quark-antiquark  $\pi^+ \Rightarrow u\bar{d}$

We will determine pion fragmentation functions and our global analysis are based on single inclusive annihilation SIA data.



$$e^+e^- \rightarrow (Z^0/\gamma)^* \rightarrow \pi^+$$





# Method of Parametrization

We choose a functional form for FFs at initial scale

$$D_i^H(z, Q_0) = N_i z^{\alpha_i} (1-z)^{\beta_i} [1 - e^{-\gamma_i z}]$$

The parameters are determined by fitting  $\chi^2$  of SIA data. The initial scale  $Q_0$  is different for quarks. The initial scale for  $Q^2$  evolution for light quarks  $u, d, s, u, d, s$  and  $g$  is taken to be  $Q_0^2 = 1\text{GeV}^2$  and it is taken at  $Q_0^2 = m_c^2$  and  $Q_0^2 = m_b^2$  for charm and bottom quarks. We Choose  $m_c = 1.29\text{GeV}$  and  $m_b = 4.19\text{GeV}$  in our analysis.

# Method of Parametrization

In our analysis we take the same fragmentation functions for valence quarks. Since pion production Possibility from valence quarks is more than sea quarks we determine separate fragmentation functions for light sea quarks and gluon. Also different functions are specified for heavy quarks fragmentation because of mass difference. According to the pion structure  $\pi^+ = u\bar{d}$  ,functional forms are

$$\begin{aligned} D_{u}^{\pi^+} &= D_{\bar{d}}^{\pi^+} , \\ D_{\bar{u}}^{\pi^+} &= D_{d}^{\pi^+} = D_{s}^{\pi^+} = D_{\bar{s}}^{\pi^+} \\ D_{g}^{\pi^+} & \\ D_{c}^{\pi^+} &= D_{\bar{c}}^{\pi^+} , \\ D_{b}^{\pi^+} &= D_{\bar{b}}^{\pi^+} . \end{aligned}$$

S. Kretzer, Phys. Rev. D 62, 054001 (2000).

B. A. Kniehl and G. Kramer, Phys. Rev. D 71, 094013 (2005).

# Fragmentation Function Results

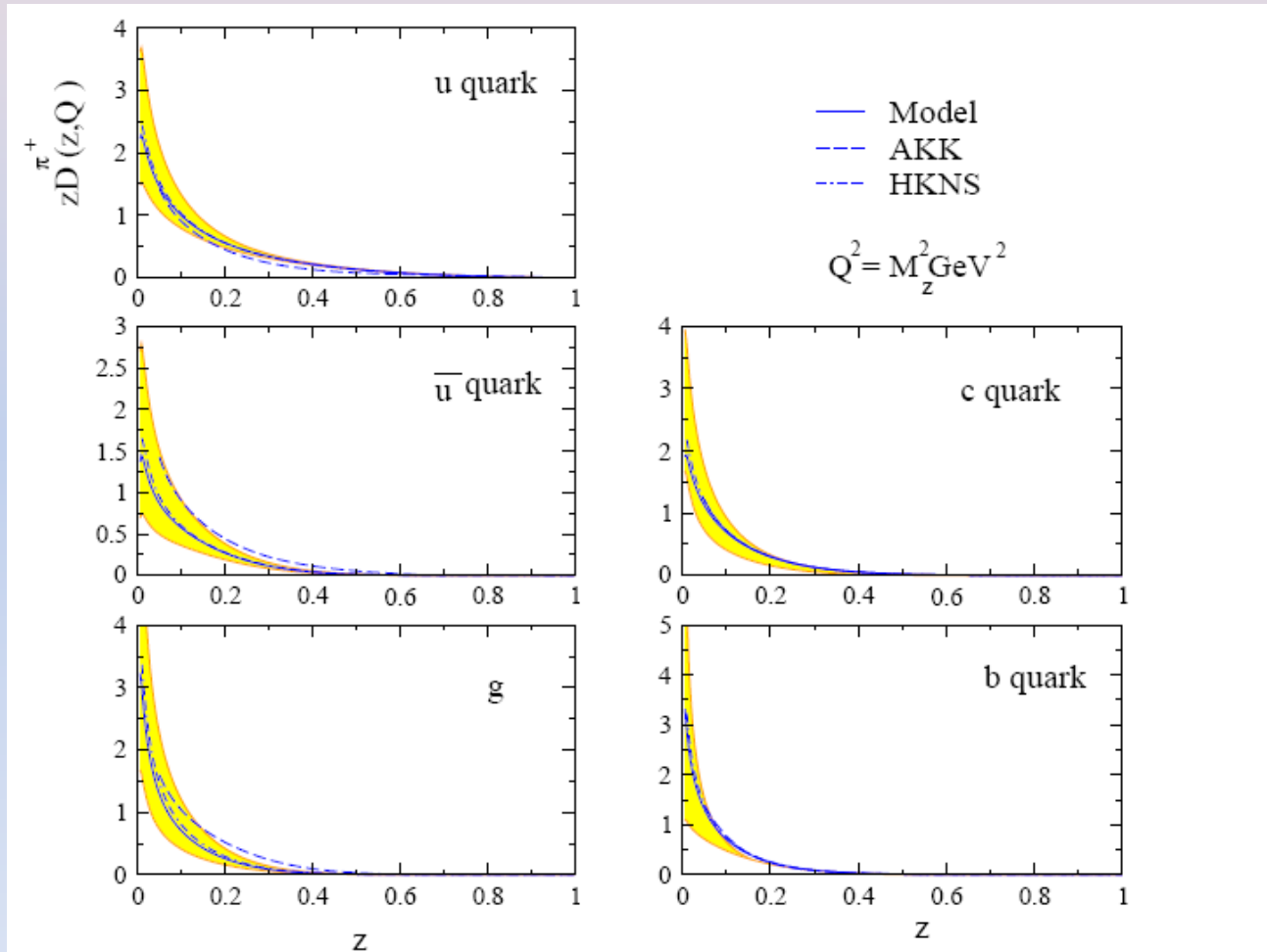


FIG. 2 Pion fragmentation functions in  $Q = M_z \text{GeV}$  in NLO.

# Differential cross Section Result

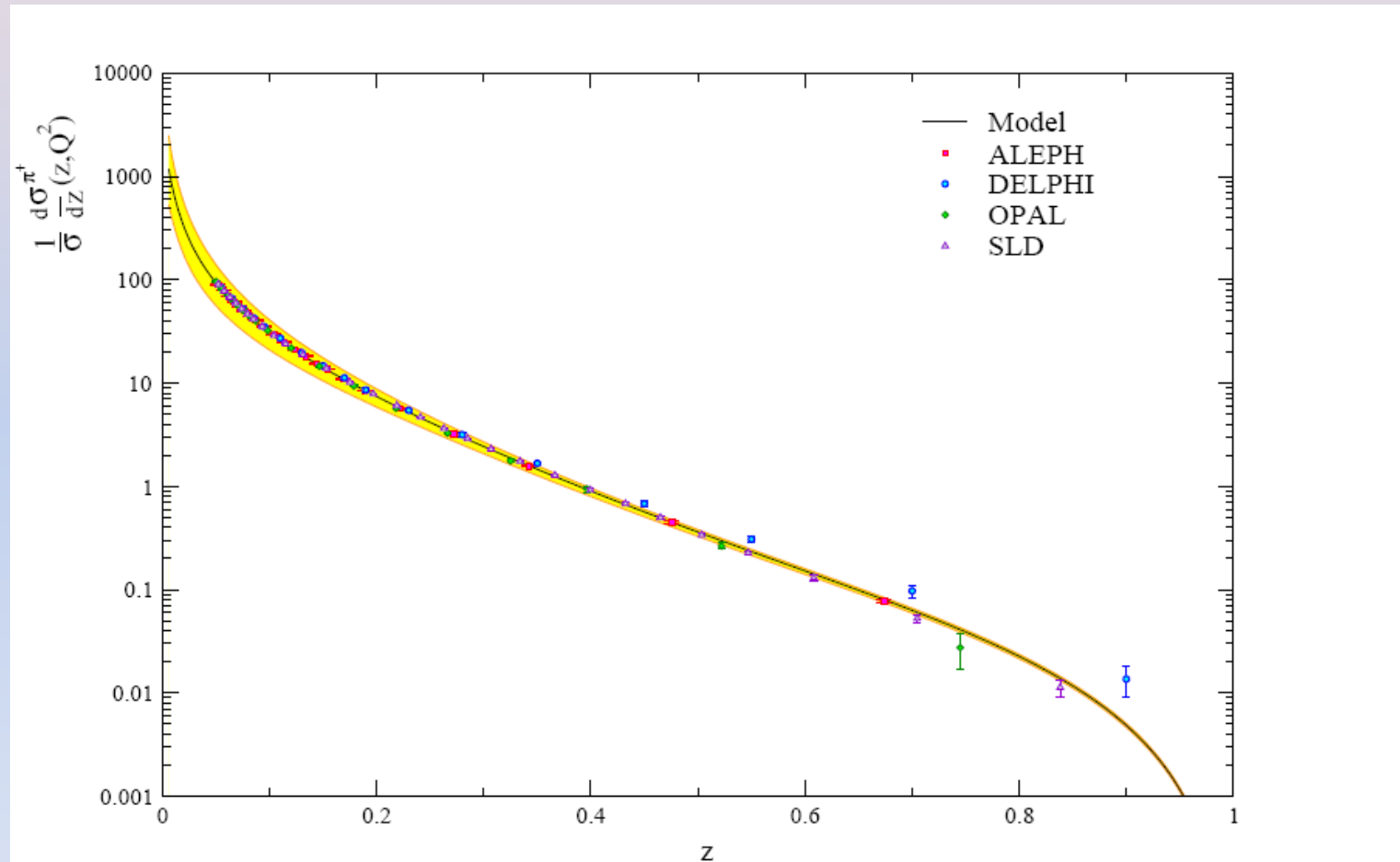


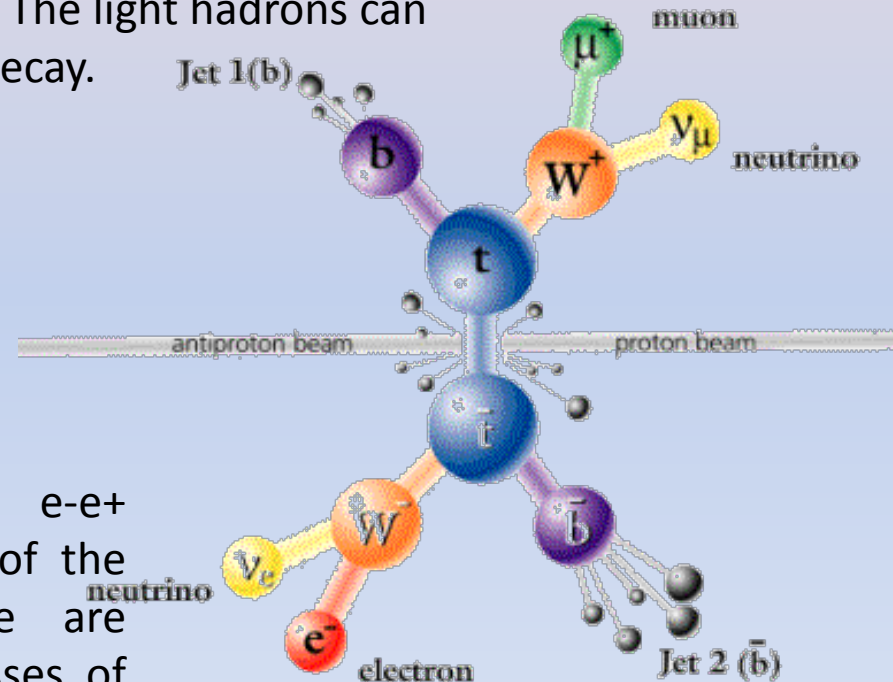
FIG. 3 Compression of our model differential cross section for pion with inclusive pion production in  $e^+e^-$  in  $Q = M_z GeV$  in NLO.

# Top Quark Decay

✓ At NLO, both b-quark can directly fragment into the b-flavored hadrons and the gluon, emitted from the b-quark, can produce a pair  $qq$  and this pair can create a light hadron. The light hadrons can also obtain from B-meson and D-meson decay.

$$(t \rightarrow b + W^+ (g) \rightarrow \pi^\pm + X)$$

✓ Since much precise data from e-e+ annihilation exists for the production of the three lightest charged hadrons we are interested in the fragmentation processes of the b-quark into  $\pi$  hadron.



## the partial decay width of process

We wish to calculate the partial decay width of process

$$(t \rightarrow b + W^+ (g) \rightarrow \pi^\pm + X)$$

differential in  $\frac{d\Gamma}{dx_h}$  at NLO in the ZM-VFN scheme.

It is convenient to introduce the scaled energy fractions  $x_i = E_i / E_b^{\max}$  ( $i = b, g, h$ ) where  $h$  denotes the light hadrons.

The gluon in above equation contributes to the real radiation at NLO and both the  $b$  quark and the gluon may hadronize to the light mesons.

## the partial decay width of process

According to the factorization theorem of the QCD-improved parton model, the energy distribution of a hadron  $H$  can be expressed as the convolution of the parton-level spectrum with the nonperturbative fragmentation function,  $D_i^h(z, \mu_f)$  describing the hadronization  $i \rightarrow h$ ,

*the parton-level differential width of the process  $t \rightarrow i + W^+$  ( $i = b, g$ )*

$$\frac{d\Gamma}{dx_H} = \sum_{i=b,g} \int_{x_i^{\min}}^{x_i^{\max}} \frac{dx_i}{x_i} \frac{d\hat{\Gamma}_i}{dx_i}(\mu_R, \mu_F) D_i^H\left(\frac{x_H}{x_i}, \mu_F\right)$$

the nonperturbative fragmentation function

S. M. Moosavi Nejad, *Phys.Rev.D* **85**:054010, 2012

# Differential Decay Rate Results

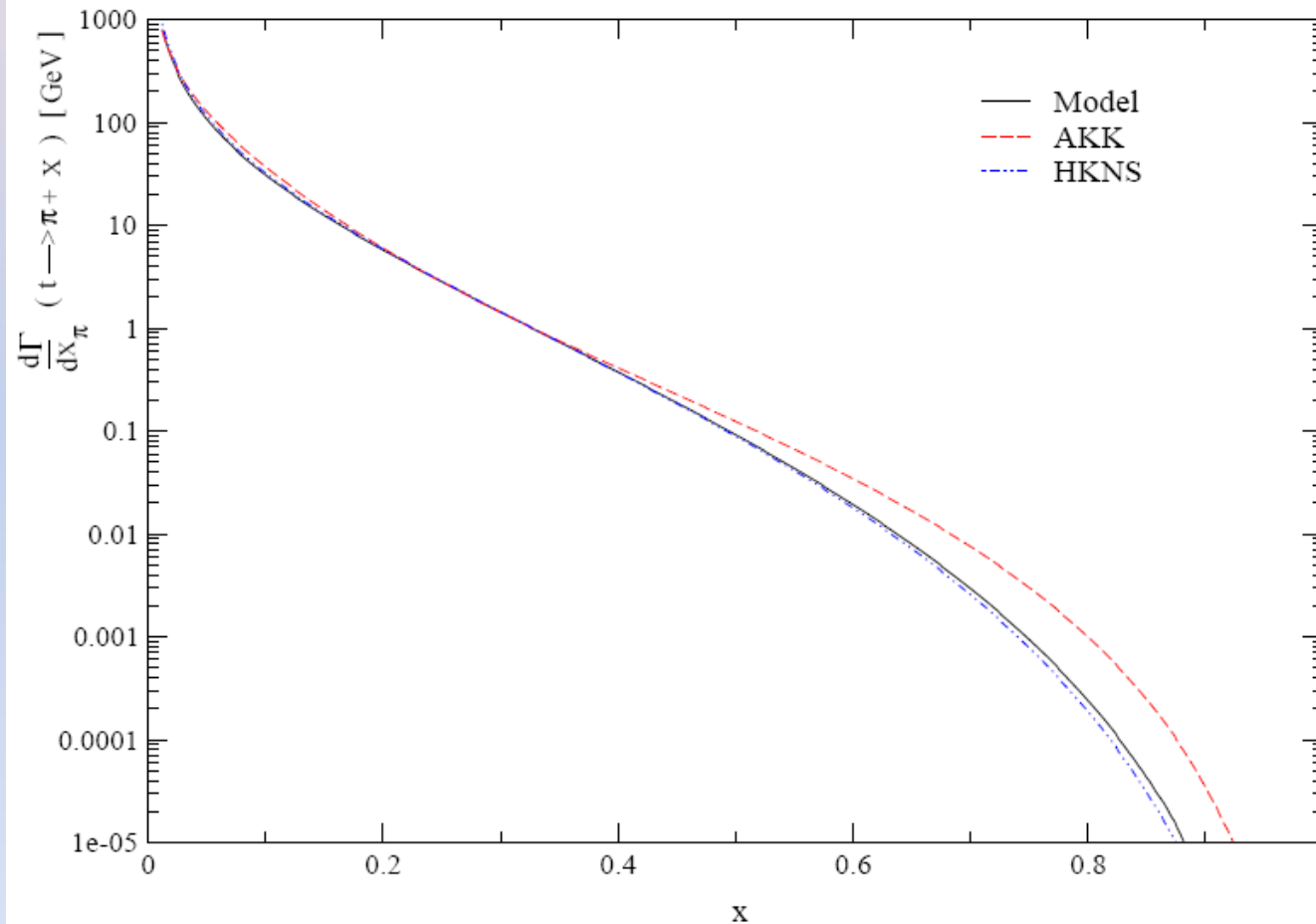


FIG. 4The differential decay rate of inclusive  $\pi$  meson production in top decay.



# Summery

- We defined Single inclusive electron-positron annihilation (SIA).
- Based on QCD analysis, fragmentation functions have role in SIA cross section.
- We used new functional form for pion parton FFs in initial scale.
- Pion FFs results are good agreement with other models and experimental data.
- We use of Pion FFs to calculate decay width of the top quark decay into Pion .