Effect of the Non-axial Hexadecpole Deformations on Total Energy

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Abstract: Potential energy surface of the nucleus ^{168}Hf is analyzed in a 5-dimensional deformation space at different spins using the configuration-constrained Cranked Nilsson-Strutinsky (CNS) model. The space includes two components of the quadrupole deformation (ε_2, γ) and three components of the hexadecapole deformation $(\varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$. Main attention is given to the influence of non-axial hexadecapole shapes on the total energy of the analyzed nucleus. It is found that this influence is very small (a few keV) for axially symmetric shapes. Whereas it leads to a decrease of the total energy about 500 keV at large triaxial deformation.

1 Introduction

The high-spin structure of deformed nuclei shows a variety of interesting phenomena caused by the interplay between collective and single-particle excitations. Potential energy surface (PES) calculations, predict that these nuclei constitute a new region of exotic shapes [1, 2] coexisting with normal prolate deformation ($\varepsilon_2 \sim 0.23$). At high spins these nuclei may assume stable triaxial superdeformed (TSD) shapes. Recently a high-spin band has been observed in ^{168}Hf [3] which appears to correspond to triaxial shape with a deformation which is considerably larger than that of the TSD bands in $^{161-167}$Lu. Partly because of the large deformation of the TSD band in ^{168}Hf, some approximations of the Cranked Nilsson-Strutinsky (CNS) approach become somewhat questionable. Most important, however, is that, for the first time to our knowledge, a complete minimization in the three hexadecapole degrees of freedom has been carried out at a large triaxial deformation.

2 The standard CNS formalism

In the configuration-dependent cranked Nilsson-Strutinsky (CNS) model [4, 5], the nucleons are moving independently of each other in a deformed and rotating mean-field generated by the nucleons themselves. The rotation or the effect of the rotation is treated as an external potential. The mean-field Hamiltonian used to describe a nucleon in the rotating nucleus is the cranked modified oscillator Hamiltonian [4]

$$H = h_{HO}(\varepsilon_2, \gamma) - \kappa\hbar\omega_0[2\ell_t \cdot s + \mu(\ell_t^2 - \langle \ell_t^2 \rangle_N)] + V_4(\varepsilon_4, \gamma) - \omega j_x. \tag{1}$$

In this Hamiltonian, the cranking term ωj_x is introduced to make the deformed potential rotate uniformly around a principal axis with the angular velocity ω. Also, $h_{HO}(\varepsilon_2, \gamma)$ is an anisotropic harmonic-oscillator Hamiltonian. Equation (1) represents the rotating modified oscillator Hamiltonian in terms of the quadrupole ε_2, non-axial γ, and the hexadecapole ε_4, deformation parameters. The dependence of the Hamiltonian on the hexadecapole deformation parameters is written as [4]

$$\varepsilon_{40} = \varepsilon_4 \frac{1}{6}(5\cos^2\gamma + 1), \ \varepsilon_{42} = -\varepsilon_4 \frac{1}{12}\sqrt{30}\sin 2\gamma, \ \varepsilon_{44} = \varepsilon_4 \frac{1}{12}\sqrt{70}\sin^2\gamma, \tag{2}$$

Because the ε_{4i} parameters depend on one parameter ε_4, there is only one hexadecapole degree of freedom. In a standard calculation, the total energy is minimized by varying three parameters: ε_2, γ and one hexadecapole parameter, ε_4 [4].

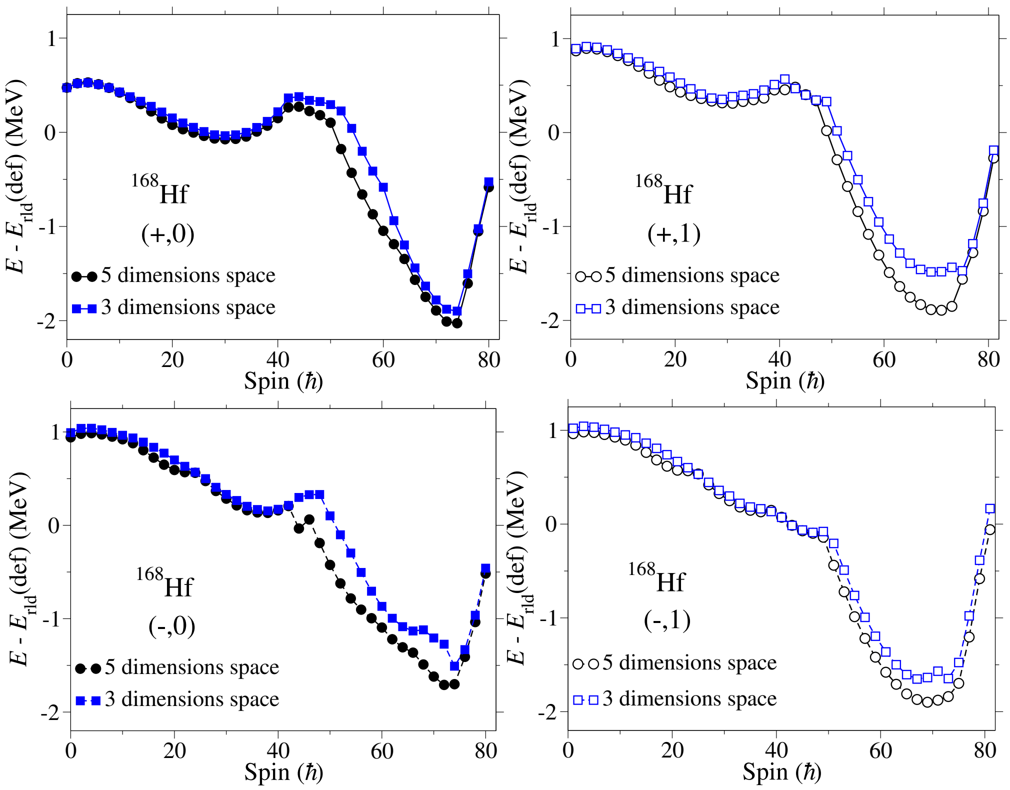

Figure 1: The ^{168}Hf yrast energies relative to a rotating liquid drop energy E_{rld} as a function of spin I for the four combinations of parity and signature, $(+,0)$, $(+,1)$, $(-,0)$ and $(-,1)$. The circles show the minimum energy in $(\varepsilon_2, \gamma, \varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$ space of deformation and the squares in $(\varepsilon_2, \gamma, \varepsilon_4)$. Solid lines correspond to positive-parity configurations and broken lines correspond to negative parity. Similarly, solid symbols correspond to signature $\alpha = 0$ and open symbols correspond to signature $\alpha = 1$.

3 Minimization in five dimensions

In general, for axial symmetric shapes it is only the ε_{40} (with quantization around the symmetry axis) shape degree of freedom which is expected to be of major importance because the energy is even (independent of the sign) in ε_{42} and ε_{44}. This is only valid at no rotation around the perpendicular axis, but if the rotational frequency is not extremely high, it is still expected that only the ε_{40} degree of freedom will be of major importance. Furthermore, shapes corresponding to small quadrupole deformations are never far away from a symmetry axis in the (ε_2, γ) plane, so it should be sufficient to minimize the energy in only one ε_4 degree of freedom also in this case [6]. For a triaxial shape and large quadrupole deformation on the other hand, the full minimization in the ε_{4i} parameter space might be more important. In order to make a full minimization in the five-dimensional deformation space, we have changed the standard CNS code and then the total energy of ^{168}Hf calculated at the following grid points:

$$x = 0.18[0.02]0.44, \;\; y = 0.08[0.02]0.42, \tag{3}$$

$$\varepsilon_{40} = 0.005[0.01]0.045, \;\; \varepsilon_{42} = -0.02[0.01]0.02, \;\; \varepsilon_{44} = -0.01[0.01]0.03, \tag{4}$$

where (x, y) are Cartesian coordinates in the (ε_2, γ) plane.

In Fig. 1, the ^{168}Hf yrast energies are compared with the corresponding energies from the minimization in the $(\varepsilon_2, \gamma, \varepsilon_4)$ parameter space. In our calculations, the reference energy E_{rld} is minimized in a deformation space $(\varepsilon_2, \gamma, \varepsilon_4)$ for each spin value. As one can see, at spins $10 \lesssim I \lesssim 45$, the yrast states in the deformation space $(\varepsilon_2, \gamma, \varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$ are only a few keV lower in energy than that in the $(\varepsilon_2, \gamma, \varepsilon_4)$ deformation space. On the other hand, the gain in energy in the high-spin region, $I \gtrsim 45$, is important and amounts to 0.5 MeV at some spin values. These findings are consistent with the

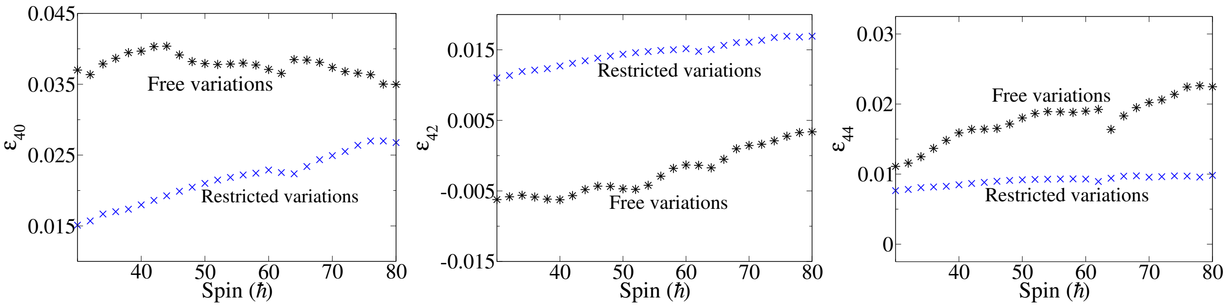

Figure 2: The ε_{4i} parameters as a function of spin I for the TSD configuration [8(22),(22)6(11)]. In the calculations, the triaxiality parameter of Eq. (2) is $(\gamma + 120°)$. The $\times$ symbols are for the minimization process in the space $(\varepsilon_2, \gamma, \varepsilon_4)$ while the $*$ symbols are used for the minimization process in the space $(\varepsilon_2, \gamma, \varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$.

general expectations discussed above. Thus, according to the potential-energy surfaces in the CNS calculations for ^{168}Hf [7], the yrast states are built from configurations which have prolate shape with $\varepsilon_2 \sim (0.23 - 0.26)$ for spin values below $I \sim 45$ but at non axial shape with $(\varepsilon_2, \gamma) \sim (0.44, 20°)$ (TSD shapes) for spins $I \gtrsim 45$. Therefore, in the following, we do the minimization process in the deformation space $(\varepsilon_2, \gamma, \varepsilon_4)$ to study the bands close to axial shape and in the deformation space $(\varepsilon_2, \gamma, \varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$ to study the TSD bands in ^{168}Hf. In order to illustrate the variation of the ε_{4i} parameters in the two cases, they are drawn in Fig. 2 as functions of spin I for the TSD configuration, [8(22),(22)6(11)]. In the complete minimization, the ε_{4i} parameters get different values relative to Eq. (2) in the full spin range, $I = 30 - 80$. The value of the ε_{40} parameter becomes considerably larger, $\varepsilon_{40} \sim 0.035$ compared with $\varepsilon_{40} \sim 0.020$ in the restricted variation. The ε_{42} parameter changes sign over most of the spin range while the ε_{44} parameter varies faster and gets larger values.

4 Conclusions

In this study, we have made a modifications when solving the Hamiltonian in the configuration-dependent cranked Nilsson-Strutinsky formalism in order to test the accuracy of some approximations. The nucleus ^{168}Hf is used as a test case and the observed highest-spin bands in this nucleus are analyzed. The total energy was minimized in the full hexadecapole space indicating that, for axially symmetric shape, it is generally sufficient to include only the standard ε_{40} degree of freedom, which does not break the axial symmetry. On the other hand, at large triaxial deformation, it appears necessary to minimize the energy in all three hexadecapole deformation parameters $(\varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$. The study of the deformation space shows that, in ^{168}Hf, the energy of the axially symmetric bands with normal deformation could as well be minimized in the restricted deformation space $(\varepsilon_2, \gamma, \varepsilon_4)$, while the energy of the TSD bands should be minimized in the deformation space $(\varepsilon_2, \gamma, \varepsilon_{40}, \varepsilon_{42}, \varepsilon_{44})$.

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