Chiral Nonet Mixing in $\eta' \to \eta\pi\pi$ Decay

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Underlying mixing of scalar mesons is studied in $\eta' \to \eta\pi\pi$ decay within a generalized linear sigma model of low-energy QCD which contains two nonets of scalar mesons and two nonets of pseudoscalar mesons (a quark-antiquark nonet and a four quark nonet). The model has been previously employed in various investigations of the underlying mixings among scalar mesons below and above 1 GeV (as well as those of their pseudoscalar chiral partners) and has provided a coherent global picture for the physical properties and quark substructure of these states. At the leading order, which corresponds to neglecting terms in the potential with higher than eight quark and antiquark lines, the free parameters of the model have been previously fixed in detailed global fits to scalar and pseudoscalar experimental mass spectra below and above 1 GeV together with several low-energy parameters. In the present work, the same order of potential with fixed parameters is used to further explore the underlying mixings among scalar mesons in the $\eta' \to \eta\pi\pi$ decay. It is found that the prediction of the linear sigma model with only a single lowest-lying nonet is not accurate in predicting the decay width, but inclusion of the mixing of this nonet with the next-to-lowest lying nonet significantly improves this prediction and agrees with experiment up to about 1%. It is also shown that while the prediction of the leading order of the generalized model for the Dalitz parameters is not close to the experiment, the model is able to give a reasonable prediction of the energy dependencies of the normalized decay amplitude squared and that this is expected to improve with further refinement of the complicated underlying mixings. Overall this investigation provides further support for the global picture of scalar mesons: those below 1 GeV are predominantly four-quark stats and significantly mix with those above 1 GeV which are closer to the conventional p-wave quark-antiquark states.

I. INTRODUCTION

The scalar mesons continue to attract attention of many investigators to their important roles in low-energy QCD. Although not all their properties have been fully uncovered, nevertheless a great deal of progress has been made over the past couple of decades [1]-[5]. In short, the scalars below 1 GeV appear to be close to four quark states with some distortions and those above 1 GeV appear to be close to quark-antiquark states with some distortions. The natural question would be whether such distortions on the quark substructure of both of these sets of states is due to a mixing among these states. The idea of mixing is intuitively understandable since some of the scalars below and above 1 GeV are very broad (such as, for example, $f_0(500)$ and $f_0(1370)$, or $K_0^*(800)$) and there is no reason that they should not refrain from mixing with members with the same quantum numbers in a nearby nonet. The idea of such mixings and their effects on the properties of isovectors and isodoublets was studied in [6] within a nonlinear chiral Lagrangian model. In ref. [7] such mixing patterns were further studied in a generalized linear sigma model.

The tree-level Feynman diagrams representing the $\eta' \to \eta\pi\pi$ decay are shown in Fig. 1. These include a four-point interaction diagram (contact diagram) together with diagrams representing the contributions of isovector and isosinglet scalar mesons. This is a suitable decay channel for studying the role of scalar mesons and their underlying mixing patterns. We take up this task in this paper. To probe the effect of such underlying mixings, we use both a single-nonet SU(3) linear sigma model, as well as a generalized version that contains two nonets of scalar mesons (a two-quark nonet and a four-quark nonet).

In either case, the computation of the partial decay width, and that of the energy dependencies of

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the normalized decay amplitude, are the points of contact with experiment (Table I).

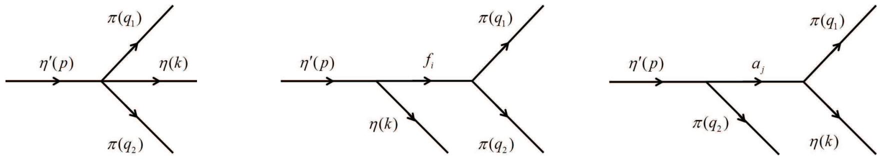

FIG. 1: Feynman diagrams representing the decay $\eta' \to \eta\pi\pi$: Contact term (left), contribution of isosinglet scalars (middle) and contribution of isovectors (right).

TABLE I: Experimental decay width and experimental Dalitz slope parameters (a, b and d) for $\eta' \to \eta\pi^+\pi^-$ (column 2), $\eta' \to \eta\pi^0\pi^0$ (column 3) and $\eta' \to \eta\pi\pi$ in the isospin invariant limit (column 4).

	Exp. $[\eta' \to \eta\pi^+\pi^-]$	Exp. $[\eta' \to \eta\pi^0\pi^0]$	Exp. (averaged)
Γ (MeV)	0.086 ± 0.004	0.0430 ± 0.0022	0.086 ± 0.003
a	$-0.066 \pm 0.016 \pm 0.003$	$-0.127 \pm 0.016 \pm 0.008$	-0.094 ± 0.012
b	$-0.063 \pm 0.028 \pm 0.004$	$-0.106 \pm 0.028 \pm 0.014$	-0.082 ± 0.021
d	$-0.067 \pm 0.020 \pm 0.003$	$-0.082 \pm 0.017 \pm 0.008$	-0.075 ± 0.014

II. SINGLE NONET APPROACH

The Lagrangian has the general structure [8]

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(\partial_\mu M \partial_\mu M^\dagger\right) - V_0\left(M\right) - V_{SB}, \tag{1}$$

where the chiral field M is constructed out of scalar nonet S and pseudoscalar nonet ϕ: $M = S + i\phi$ and transforms linearly under chiral transofrmation: $M \to U_L M U_R^\dagger$ and V_0 is an arbitrary function of the independent $\mathrm{SU(3)_L \times SU(3)_R \times U(1)_V}$ invariants

$$I_1 = \mathrm{Tr}\left(MM^\dagger\right), \quad I_2 = \mathrm{Tr}\left(MM^\dagger MM^\dagger\right), \quad I_3 = \mathrm{Tr}\left[\left(MM^\dagger\right)^3\right] \quad I_4 = 6\left(\det M + \det M^\dagger\right) \tag{2}$$

The symmetry breaker V_{SB} has the minimal form $V_{SB} = -2\mathrm{Tr}(AS)$, where $A = \mathrm{diag}\,(A_1, A_2, A_3)$ are proportional to the three "current" type quark masses. The vacuum values satisfy $\langle S_a^b \rangle = \alpha_a \delta_a^b$. We find:

$$\Gamma^{\mathrm{Single\ \ nonet}}[\eta' \to \eta\pi\pi] = 0.61 \pm 0.01\,\mathrm{MeV}. \tag{3}$$

Clearly, despite the success of the nonrenormalizable single nonet SU(3) linear sigma model in describing the low-energy scatterings such as $\pi\pi$, πK and $\pi\eta$, it estimates the partial decay width about seven times higher than the experimental value displayed in Table I.

The Dalitz parameters that characterize the energy expansion of this amplitude squared are given in Table II. Comparing with the averaged experimental values of Table I, we see that there is a qualitative order of magnitude agreement, at best. This lack of accuracy of the single nonet approach raises the natural question of whether the underlying mixing among scalar mesons (which are clearly important players in this decay) has a noticeable effect on these estimates. Moreover, the eta systems (both the two below 1 GeV as well as those above 1 GeV) can mix and have a nontrivial effect on this decay estimate. The single nonet approach does not take these mixing effects among the scalars and among the pseudoscalars into account and these mixings can have important consequences for this partial decay width. This motivates us further to study this decay within the generalized linear sigma model (that contains two scalar nonets and two pseudoscalar nonets).

TABLE II: The predicted Dalitz parameters in single nonet linear sigma model of ref. [8].

Parameter	single nonet model
a	-0.114 ± 0.001
b	-0.001 ± 0.001
d	-0.063 ± 0.001

III. THE GENERALIZED LINEAR SIGMA MODEL APPROACH

The model employs the 3×3 matrix chiral nonet fields [7]:

$$M = S + i\phi, \qquad M' = S' + i\phi'. \tag{4}$$

The matrices M and M' transform in the same way under chiral SU(3) transformations but may be distinguished by their different $U(1)_A$ transformation properties. M describes the "bare" quark antiquark scalar and pseudoscalar nonet fields while M' describes "bare" scalar and pseudoscalar fields containing two quarks and two antiquarks. At the symmetry level in which we are working, it is unnecessary to further specify the four quark field configuration. The four quark field may, most generally, be imagined as some linear combination of a diquark-antidiquark and a "molecule" made of two quark-antiquark "atoms". The general Lagrangian density which defines our model is

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(\partial_\mu M \partial_\mu M^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M' \partial_\mu M'^\dagger\right) - V_0\left(M, M'\right) - V_{SB}, \tag{5}$$

where $V_0(M, M')$ stands for a function made from $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$ (but not necessarily $\mathrm{U(1)_A}$) invariants formed out of M and M'.

As we previously discussed [4], the leading choice of terms corresponding to eight or fewer underlying quark plus antiquark lines at each effective vertex reads:

$$\begin{aligned} V_0 = &- c_2\,\mathrm{Tr}(MM^\dagger) + c_4^a\,\mathrm{Tr}(MM^\dagger MM^\dagger) \\ &+ d_2\,\mathrm{Tr}(M'M'^\dagger) + e_3^a(\epsilon_{abc}\epsilon^{def} M_d^a M_e^b M'^c_f + h.c.) \\ &+ c_3\left[\gamma_1 \ln(\frac{\det M}{\det M^\dagger}) + (1-\gamma_1)\ln\frac{\mathrm{Tr}(MM'^\dagger)}{\mathrm{Tr}(M'M^\dagger)}\right]^2. \end{aligned} \tag{6}$$

All the terms except the last two (which mock up the axial anomaly) have been chosen to also possess the $\mathrm{U(1)_A}$ invariance. A possible term $\left[\mathrm{Tr}(MM^\dagger)\right]^2$ is neglected for simplicity because it violates the OZI rule. The symmetry breaking term which models the QCD mass term takes the form as in single nonet. The model allows for two-quark condensates, $\alpha_a = \langle S_a^a \rangle$ as well as four-quark condensates $\beta_a = \langle {S'}_a^a \rangle$. Here we assume isotopic spin symmetry so $A_1 = A_2$ and:

$$\alpha_1 = \alpha_2 \neq \alpha_3, \qquad \beta_1 = \beta_2 \neq \beta_3 \tag{7}$$

We also need the "minimum" conditions,

$$\left\langle \frac{\partial V_0}{\partial S} \right\rangle + \left\langle \frac{\partial V_{SB}}{\partial S} \right\rangle = 0, \qquad \left\langle \frac{\partial V_0}{\partial S'} \right\rangle = 0. \tag{8}$$

There are twelve parameters describing the Lagrangian and the vacuum: Six coupling constants given in Eq.(6), the two quark mass parameters, $(A_1 = A_2, A_3)$ and the four vacuum parameters $(\alpha_1 = \alpha_2, \alpha_3, \beta_1 = \beta_2, \beta_3)$. Ten of these parameters (c_2, c_4^a, d_2, e_3^a, α_1, α_3, β_1, β_3, A_1, A_3) are determined using the four minimum equations together with the following six experimental inputs for the masses, pion decay constant and the ratio of strange to non-strange quark masses. $m[\pi(1300)]$ and A_3/A_1 have large uncertainties which in turn dominate the uncertainty of predictions.

The remaining two parameters (c_3 and γ_1) only affect the isosinglet pseudoscalars (whose properties also depend on the ten parameters discussed above).

First we present the "bare" prediction of the model (i.e. without unitarity corrections) for decay width and the energy dependencies of the normalized decay amplitude. Next we include the effect of unitarity corrections due to the final state interactions.

The best predicted decay widths from χ and χ^2-fit are found with $m[\pi(1300)] = 1.22 \pm 0.0025$ and $A_3/A_1 = 30 \pm 0.25$:

$$\Gamma^{\mathrm{MM}'}[\eta' \to \eta\pi\pi] = 0.15 \pm 0.01\,\mathrm{MeV} \tag{9}$$

where

$$\chi(m[\pi(1300)], A_3/A_1) = \sum_i \frac{|\Gamma^{\mathrm{theo}}(m[\pi(1300)], A_3/A_1) - \Gamma^{\mathrm{exp.}}|}{\Gamma^{\mathrm{exp.}}} \tag{10}$$

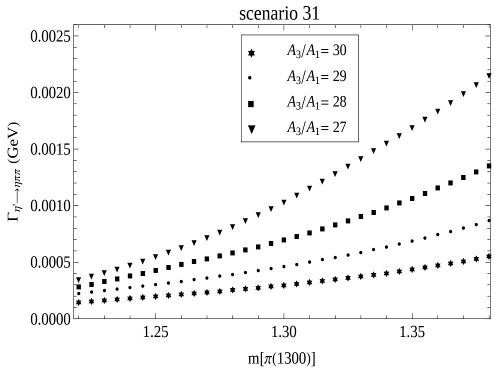

FIG. 2: "Bare" prediction (without unitarity corrections) of the generalized linear sigma model for partial decay width of $\eta' \to \eta\pi\pi$.

The "bare" prediction for the energy dependency of the normalized decay amplitude is shown in Fig. 3 and is compared with experiment. The best fits to the Dalitz parameters result in best values of $m[\pi(1300)] = 1.38$ GeV and $A_3/A_1 = 28.75$ are within the parameter space of the model however do not coincide with the best values of these parameters found in the partial decay width analysis. This shows that although inclusion of mixing among scalar and pseudoscalars clearly improves the model predictions, nevertheless, it is necessary to account for the effect of final state interactions.

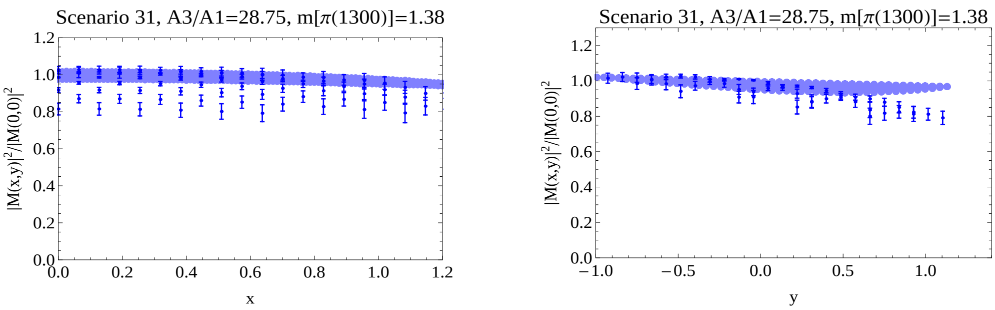

FIG. 3: The "bare" prediction of the generalized linear sigma model for the partial decay width of $\eta' \to \eta\pi\pi$.

In principle there are corrections due to the final-state interactions of $\pi\pi$ and $\pi\eta$. In the $\pi\pi$ analysis it is found that the effect of the final-state interactions is large on sigma meson and this manifests itself in the substantial difference between the "bare" sigma mass (Lagrangian mass) and the sigma mass found from the pole of the K-matrix unitarized $I = J = 0$, $\pi\pi$ scattering amplitude which for sigma significantly shifts.

TABLE III: The best predicted Dalitz parameters in bare generalized linear sigma model from χ-fit (for $m[\pi(1300)] = 1.38 \pm 0.015$ and $A_3/A_1 = 28.75^{+1.25}_{-1.75}$) and χ^2-fit (for $m[\pi(1300)] = 1.38 \pm 0.0025$ and $A_3/A_1 = 27.25^{+1.5}_{-0.25}$).

Parameter	χ-fit	χ^2-fit
a	$-0.024^{+0.025}_{-0.017}$	$-0.039^{+0.015}_{-0.003}$
b	$0.0001^{+0.0110}_{-0.0034}$	$0.008^{+0.002}_{-0.008}$
d	$-0.029^{+0.012}_{-0.001}$	$-0.020^{+0.003}_{-0.009}$

As sigma is very broad and decays dominantly into two pions, we account for the effect of the final-state interactions of pions by adjusting the "bare" mass of sigma (m_σ) and its "bare" coupling constant to two pions ($\gamma_{\sigma\pi\pi}$) such that they coincide with their pole properties found in the K-matrix unitarized $I = J = 0$, $\pi\pi$ scattering amplitude, i.e.

$$\begin{aligned} m_\sigma &\rightarrow \tilde{m}_\sigma \\ \gamma_{\sigma\pi\pi} &\rightarrow \tilde{\gamma}_{\sigma\pi\pi} \end{aligned} \tag{11}$$

where the physical sigma mass ($\tilde{m}_\sigma$) and physical sigma to two pion coupling ($\tilde{\gamma}_{\sigma\pi\pi}$) are those found from the lowest pole (z_1) of the scattering amplitude

$$z_1 = \tilde{m}_\sigma^2 - i\tilde{m}_\sigma\Gamma_\sigma \tag{12}$$

and since $\Gamma_\sigma \approx \Gamma[\sigma \rightarrow \pi\pi]$,

$$\tilde{\gamma}_{\sigma\pi\pi} = \sqrt{\frac{16\pi\tilde{m}_\sigma^2\Gamma_\sigma}{3\sqrt{\tilde{m}_\sigma^2 - 4m_\pi^2}}} \tag{13}$$

Recalculating the partial decay of $\eta' \rightarrow \eta\pi\pi$ with the new substitutions (11) we find the results displayed in Fig. 4, showing that the model predictions cross into the experimental range. The same effect can be taken into account for the $f_0(980)$, but that has a negligible effect on the results presented. Within the model parameter space, the best value for decay width that minimize χ [defined in Eq.(10)], as well as the conventional χ^2, with $m[\pi(1300)] = 1.29$ GeV and $A_3/A_1 = 29.8$, ($m[\pi(1300)] = 1.2925 \pm 0.025$ and $A_3/A_1 = 29.75 \pm 0.75$) is

$$\Gamma^{\mathrm{MM}'}_{\mathrm{Unitarized}}[\eta' \rightarrow \eta\pi\pi] = 0.085^{+0.003}_{-0.002}\,\mathrm{MeV} \tag{14}$$

The energy dependencies of the normalized decay amplitude squared are plotted in Fig. 5, and fits to the Dalitz parameters are given in Table IV. It is found that $m[\pi(1300)] = 1.38$ GeV and $A_3/A_1 = 29.7$ give the best agreement with the experiment. Although the values of these parameters are both within the parameter space of the model, they do not coincide, showing the need for further improvement of this complicated decay.

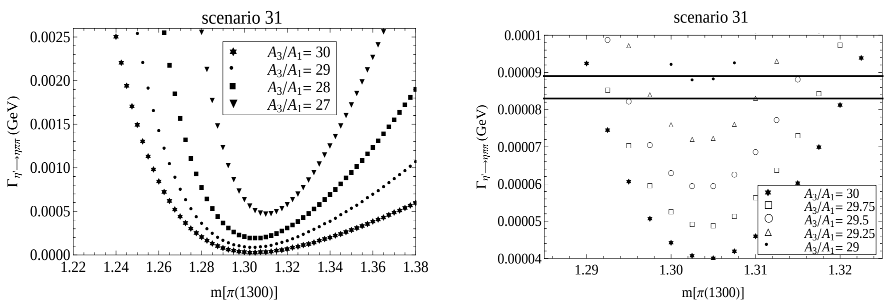

FIG. 4: Prediction of the generalized linear sigma model for the partial decay width of $\eta' \rightarrow \eta\pi\pi$. The final-state interactions of pions are taken into account by shifting the mass and coupling constant of sigma meson according to Eq. (11).

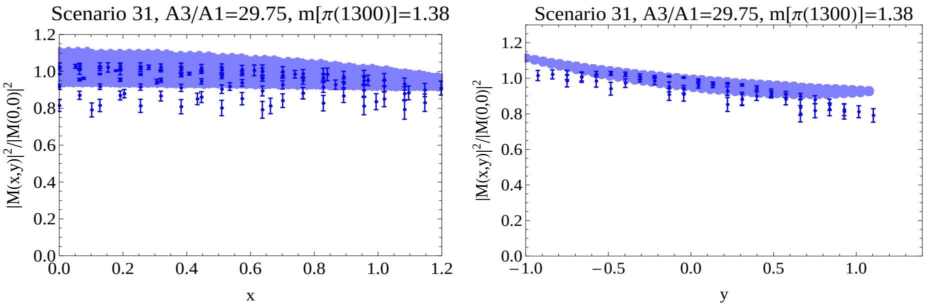

FIG. 5: Projects of the normalized decay amplitude squared onto planes containing x and y parameters (shaded regions) are compared with the experimental data (error bars). The final-state interactions of pions are taken into account by shifting the mass and coupling constant of sigma meson according to Eq. (11).

TABLE IV: The best predicted Dalitz parameters in unitarized generalized linear sigma model from χ-fit (for $m[\pi(1300)] = 1.38 \pm 0.0025$ and $A_3/A_1 = 29.75 \pm 0.25$), and χ^2-fit (for $m[\pi(1300)] = 1.38 \pm 0.0025$ and $A_3/A_1 = 29.75 \pm 0.25$).

Parameter	χ-fit	χ^2-fit
a	$-0.079^{+0.019}_{-0.021}$	-0.079 ± 0.019
b	$0.024^{+0.010}_{-0.009}$	0.024 ± 0.009
d	-0.028 ± 0.001	-0.028 ± 0.001

IV. CONCLUDING DISCUSSION

In this work, we examined the $\eta' \to \eta\pi\pi$ decay as a probe of scalar mesons substructure and mixing patterns within a generalized linear sigma model of low-energy QCD that is formulated in terms of two scalar meson nonets and two pseudoscalar meson nonets (a two- and a four-quark nonet for each spin). We first showed that the single nonet model of, despite its considerable success in describing $\pi\pi$, πK and $\pi\eta$ low-energy scatterings, gives inaccurate predictions for the partial decay width of $\eta' \to \eta\pi\pi$ as well as the energy dependencies of its normalized decay amplitude squared. Since this decay involves η and η'as well as intermediate scalar mesons and these states are known to have nontrivial mixings with states with the same quantum numbers above 1 GeV, and since such mixings have been previously given important insights into the physical properties of both scalar as well as pseudoscalar mesons, in this work we explored the effect of these mixings on this decay. We investigated whether the inclusion of mixing can have a tangible effect and whether such effects improve the predictions of the single nonet linear sigma model for this decay. We showed that inclusion of the underlying mixings (even without unitarity corrections) considerably improves the partial decay width prediction as well as the energy dependencies of the normalized decay amplitude squared. We then showed that inclusion of final state interaction of pions further improves the predictions and brings the partial decay width to within 1.2% of its experimental value, and considerably improves the predictions for the Dalitz parameters.

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