Hydrodynamics Series for Critical Systems

30’th IPM Physics Spring Conference

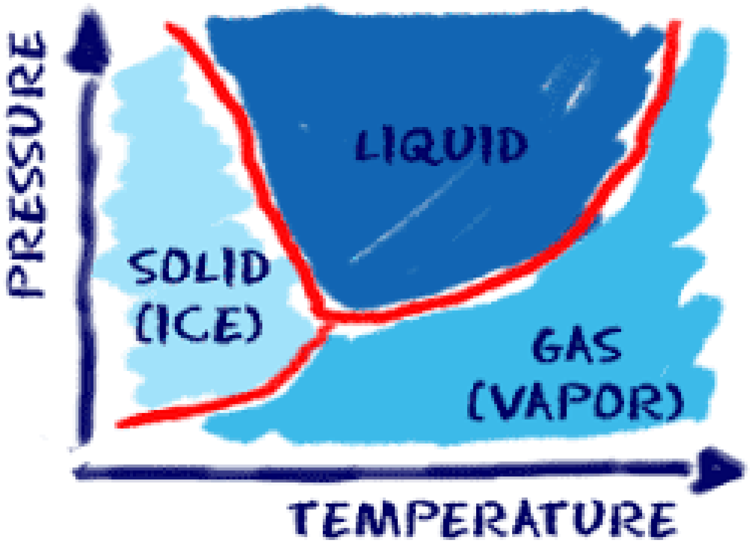
Presenter: Farid Taghinavaz

In collaboration with H. Soltanpanahi, Jagellonian univerity

Institute for Research in Fundamental Sciences

❖ Phase transition

Every substance can take various forms under extreme conditions.



Forms → Dynamical macroscopic fields,
Low energy action principle,
Underlying physics appears in some coefficients (transports).

❖ Ginzburg-Landau description:

$$Z = \int \mathcal{D}\phi \, e^{-\beta S_{eff}(\phi)}, \qquad \beta = 1/T$$

$$\mathcal{S}_{eff}(\phi) = \int d^d x \left(a(T, \mu, \cdots)\phi^2(x) + b(T, \mu, \cdots)\phi^4(x) + c(T, \mu, \cdots)\phi^6(x) + \cdots \right)$$

$\phi(x)$ Slowly varying fields

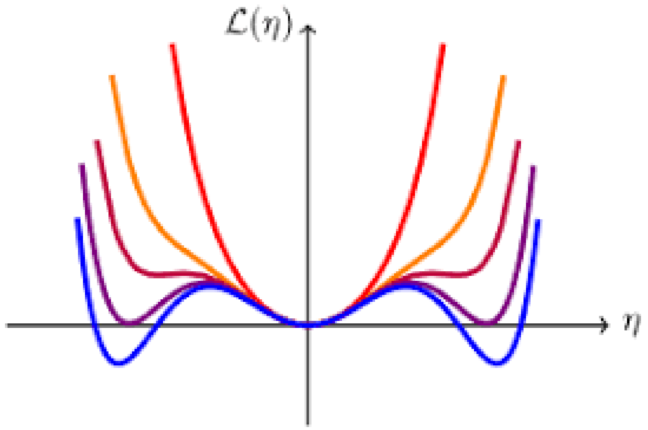
$\left.\frac{\delta S_{eff}}{\delta \phi}\right|_{\phi=\phi_0} = 0$ Equations of motion

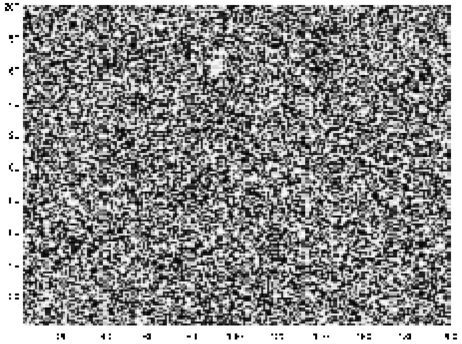
✓ **Saddle-point approximation**

$$Z \simeq e^{-\beta S_{eff}(\phi_0)} \left(\frac{2\pi}{\beta S''(\phi_0)} \right)^{1/2}$$

❖ Kinds of phase transition

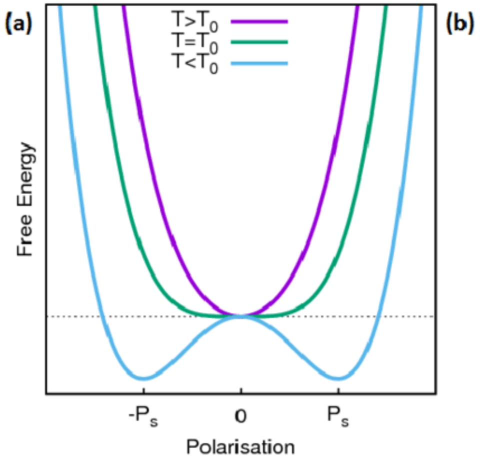
✓ *First-order phase transition*

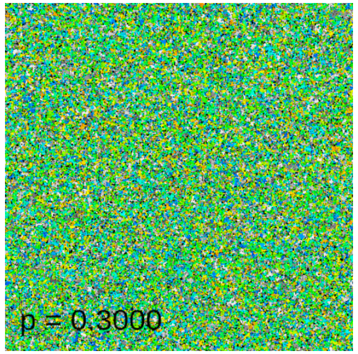




https://en.wikipedia.org/wiki/Spinodal_decomposition

✓ *Second-order phase transitions*



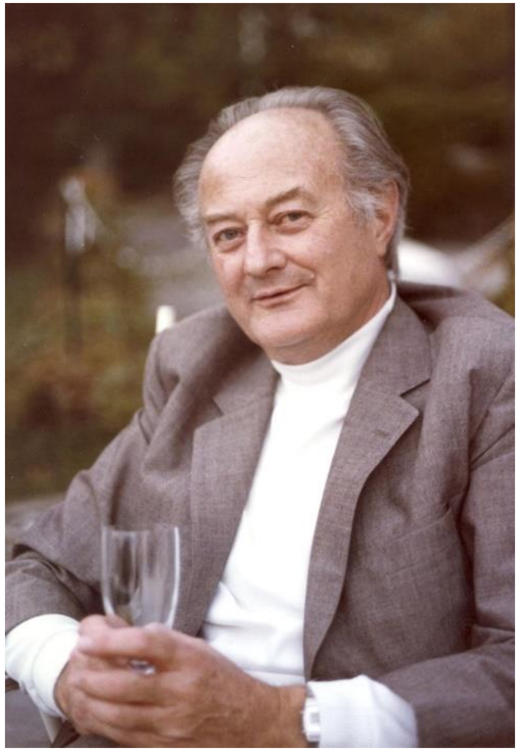


https://en.wikipedia.org/wiki/Percolation_theory

❖ Brief history of many-body QCD

✓ **No clear understanding of high-temperature matter**

✓ **Self consistency requirements**
Mass spectrum of fireballs = # of thermodynamic states in a specific energy



$$Z(T) \to \frac{1}{T_0 - T}.$$

T_0 is the highest accessible temperature, T_0 ~160MeV.

I can't believe that all this is a purely accidental coincidence, ...
Strong interactions are indeed governed by some extremal principles which is not yet understood.

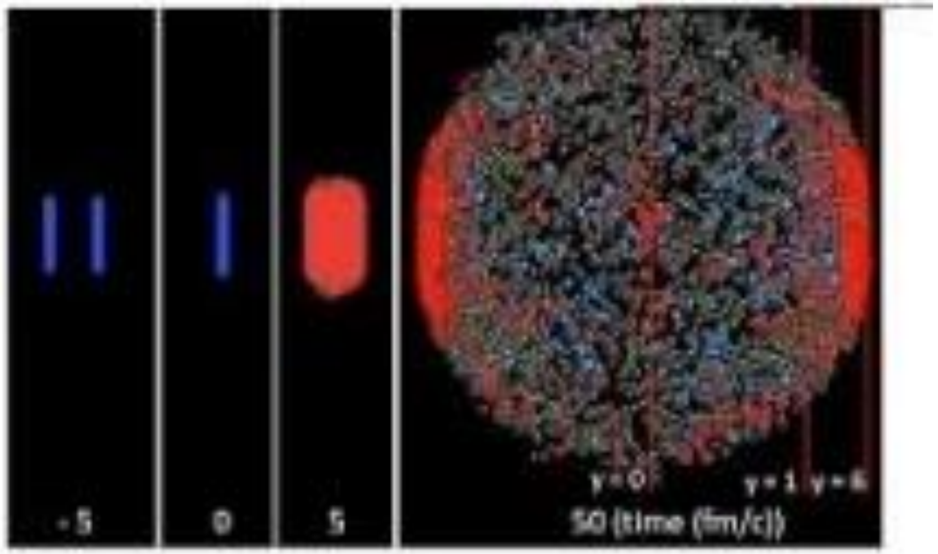
R. Hagedorn, Nuovo Cim. Suppl. **3**, 147-186 (1965).

❖ Collision of two heavy ions : Pb+Pb

Diameter: 10 fm

Thickness: $\frac{10}{\gamma}$fm, $\gamma \sim 1000$

Quantum fluctuations are “almost real” due to the time dilation.

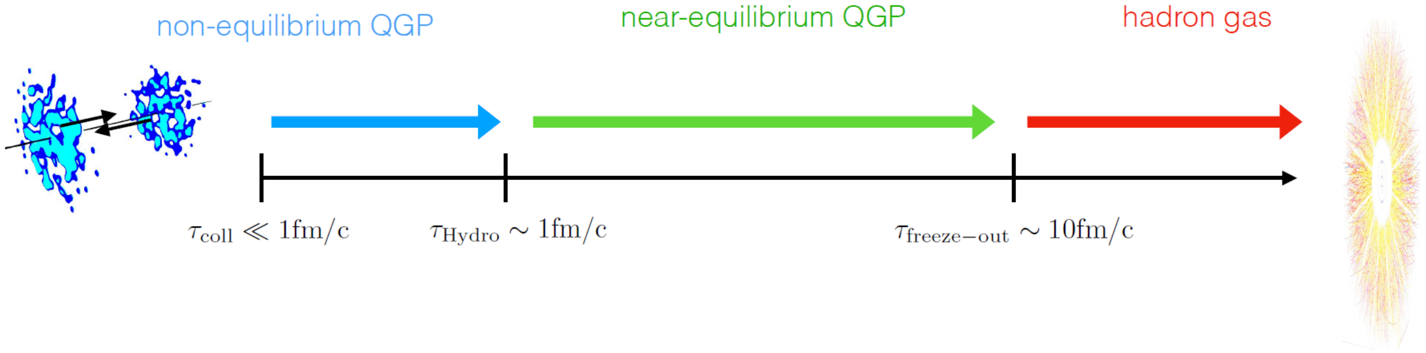


W. Busza, K. Rajagopal and W. van der Schee, Ann. Rev. Nucl. Part. Sci. **68**, 339 (2018).

Large energy density $\sim 10\ \frac{GeV}{fm^3}$

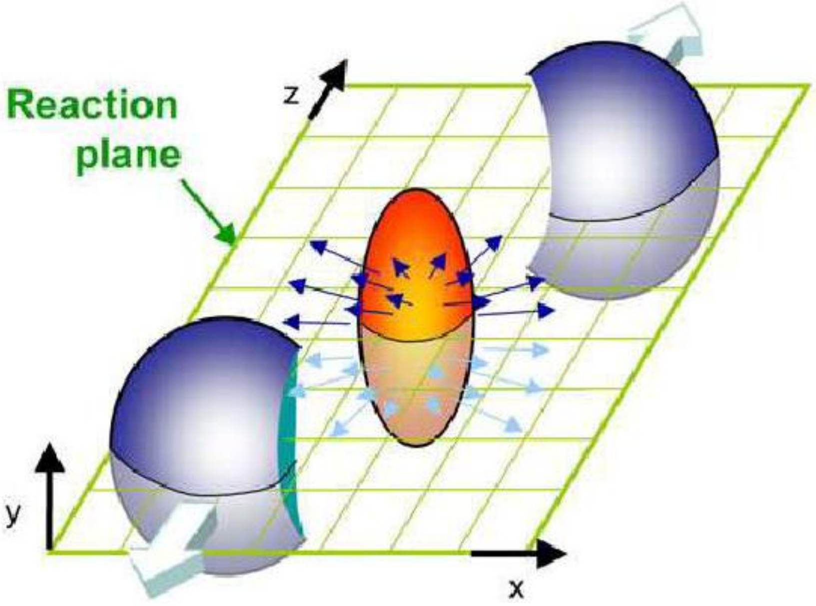
Large entropy production~10000 particles

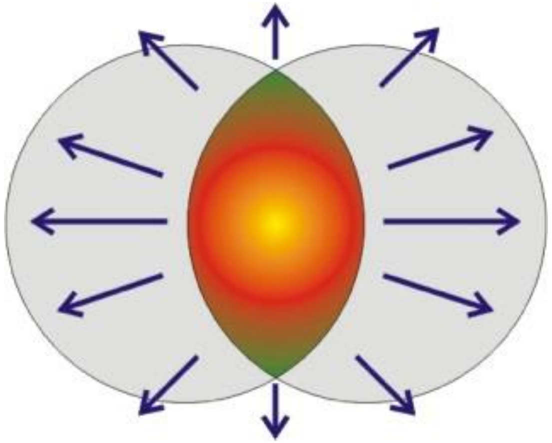
Standard model of HIC



❖ QGP is a (almost perfect) fluid

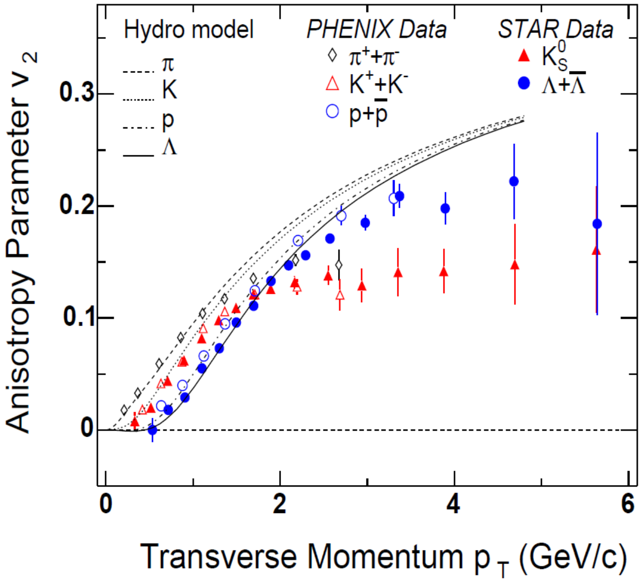
Collective flow





Final particle flow

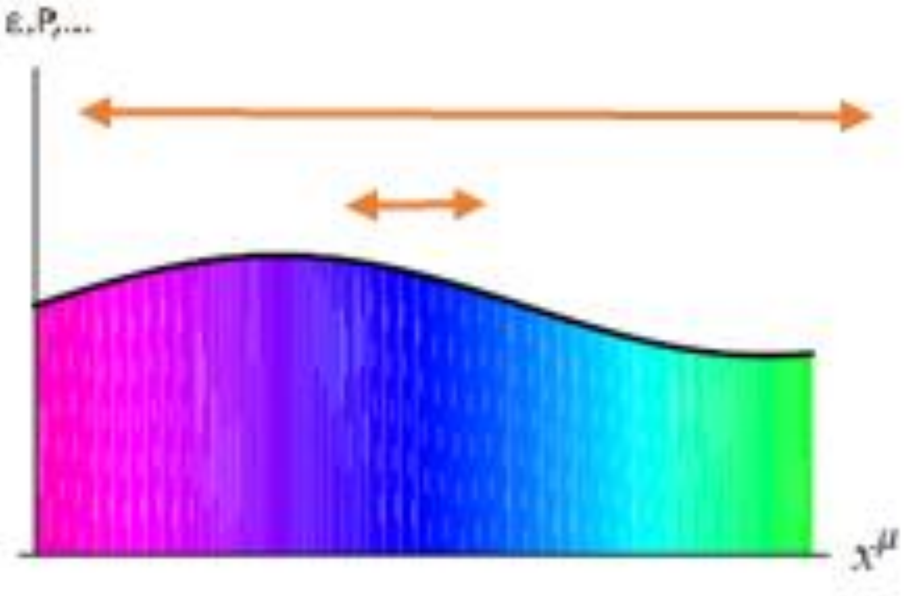
Elliptic flow (v2)



M. Gyulassy and L. McLerran, Nucl. Phys. A **750**, 30-63 (2005).

❖ Relativistic Hydrodynamics(RH)

RH is an effective description of any high-energy systems at long distances.



$$\frac{L_{mic}}{L_{mac}} \ll 1$$

$$\nabla_\mu T^{\mu\nu}(x) = 0,$$

$$\nabla_\mu J^{\mu}(x) = 0.$$

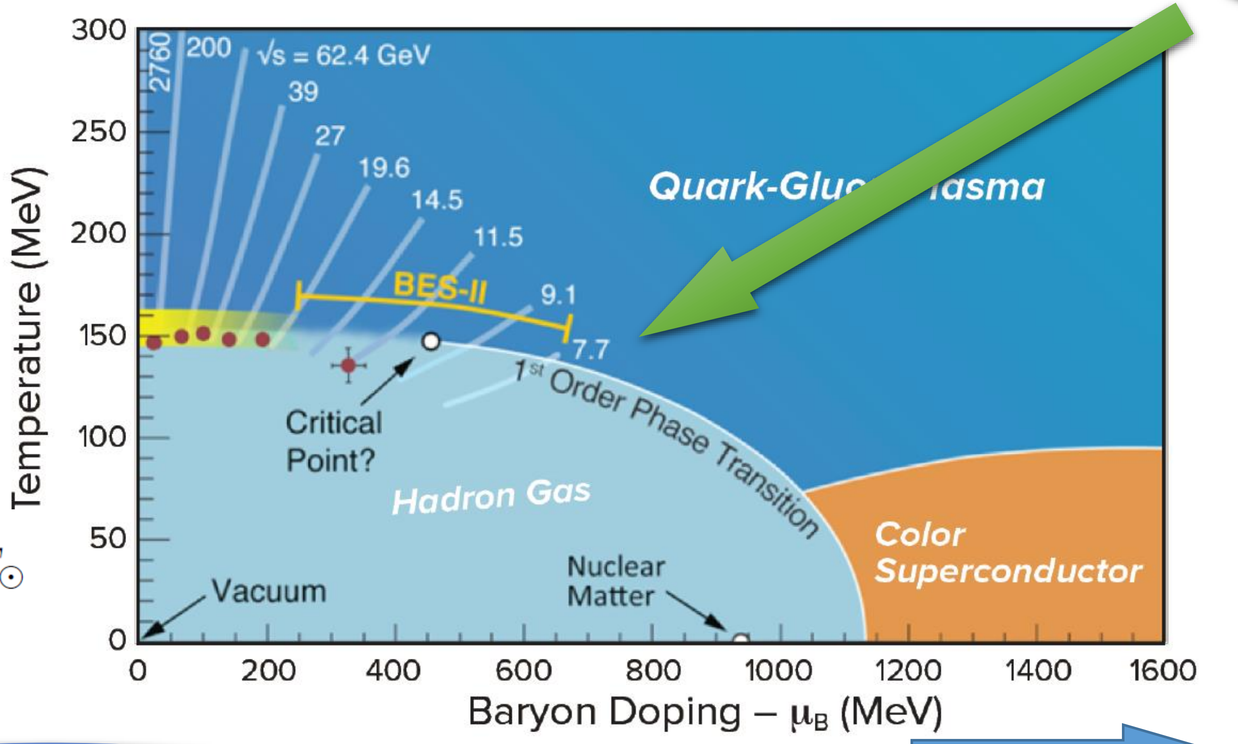
Consequence of Diff invariance(External symmetries)

Consequence of internal symmetries

❖ Quark-matter phase diagram

FAIR, NICA

Nowadays colliders



$$T_c \sim 150\,\mathrm{MeV} \sim 10^{12}\,\mathrm{K} \sim 10^5\,T_\odot$$

The QM phase diagram is just a schematic. Observations have not confirmed it so far.

Neutron star mergers

W. Busza, K. Rajagopal and W. van der Schee, Ann. Rev. Nucl. Part. Sci. **68**, 339 (2018).

❖ Gradient expansion is less valid near the critical points

✓ Invalidity of separation of scales near the critical points, due to the growing of correlation length

$$\tau_{loc} \sim \xi^z \sim \tau_{glob}, \qquad z \simeq 3$$

✓ Breakdown of gradient expansion

$$T_{\mu\nu}^{(0)} \simeq T_{\mu\nu}^{(1)}(\partial), \qquad \zeta \nabla . u \sim \frac{\xi^z}{\ell_{mac}} \sim 1$$

M. Stephanov and Y.Yin, Phys. Rev. D **98**, no.3, 036006 (2018).

➢ Dynamic critical phenomena:

Time-dependent critical behavior, includes transport, relaxation times and so on. They are necessary for understanding the heavy-ion collisions near the critical point.

$$\tau \sim \xi^z, \qquad \zeta \sim t^{-z\nu+\alpha}, \qquad \lambda \sim t^{x_\lambda}, \qquad \eta \sim t^{x_\eta}$$

P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435-479 (1977).

❖ Where is the radius of convergence?

Consider spectral curve $\mathcal{F}(\omega_\star, q_\star^2) = 0,$ where we are to find $\omega_\star = \omega(q_\star^2)$. ω and q^2 are in the complex plane.

- Regular points: $\mathcal{F}(\omega_\star, q_\star^2) = 0, \quad \left.\frac{\partial \mathcal{F}}{\partial \omega}\right|_{\omega=\omega_\star} \neq 0, \quad \Rightarrow \quad \omega(q^2) = \sum_{n=0}^{\infty} a_n (q^2 - q_\star^2)^n$
- Singular points: $\mathcal{F}(\omega_\star, q_\star^2) = 0, \quad \left.\frac{\partial \mathcal{F}}{\partial \omega}\right|_{\omega=\omega_\star} = 0, \quad \ldots \left.\frac{\partial^p \mathcal{F}}{\partial \omega^p}\right|_{\omega=\omega_\star} = 0, \quad \Rightarrow \omega_j(q^2) = \sum_{n=0}^{\infty} a_n^{(j)} (q^2 - q_\star^2)^{\frac{n}{j}}, \;\; j = 1, \cdots p$

❑ Imaginarize ω and q, continue analytically.

The radius of convergence is distance between the origin and the smallest critical point.

S. Grozdanov, P. K. Kovtun, A. O. Starinets and P. Tadic, Phys. Rev. Lett. **122**, no.25, 251601 (2019).

❖ Holographic model

- ❑ Einstein-Klein-Gordon action

$$S = \frac{1}{16\pi G_5}\int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + \frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-\gamma}\,K + I_{\mathrm{ct}}$$

- ❑ Globally Superpotential form

$$V(\phi) = -\frac{1}{3}W(\phi)^2 + \frac{1}{2}W'(\phi)^2$$

- ❑ Explore wealthy physics by choosing

$$W(\phi) = -6 - \frac{1}{2}\phi^2 + B_4\,\phi^4$$

H. Soltanpanahi, F. Taghinavaz, ``Hydrodynamics and phase transition'', in preparation.

❖ Equations of motion

$$\mathrm{d}s^2 = e^{2A(u)}\left(-H(u)\,\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) - e^{A(u)+B(u)}\,\mathrm{d}u\,\mathrm{d}t$$

$$\begin{aligned}
H\left(B' - 4A'\right) - H' + e^{2B}V' &= 0\\
6\left(A'B' - A''\right) - 1 &= 0\\
H'' + (4A' - B')H' &= 0\\
6A'H' + H\left(24A'^2 - 1\right) + 2e^{2B}V &= 0
\end{aligned}$$

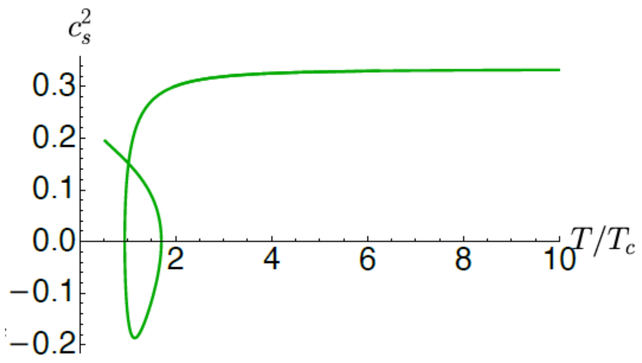
- Gubser gauge: $\phi(u) = u.$

H. Soltanpanahi, F. Taghinavaz, ``Hydrodynamics and phase transition'', in preparation.

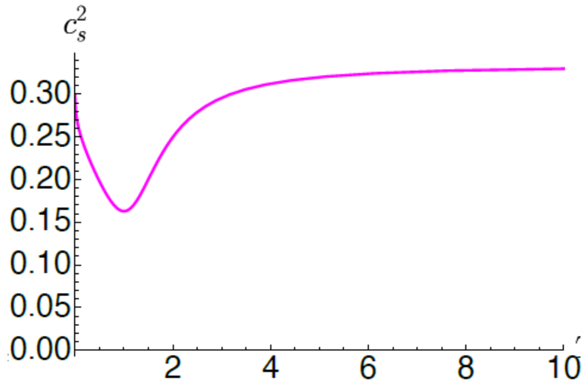
❖ Thermodynamics

$$s = \frac{1}{4G_5} e^{3A(\phi_h)} \qquad T = \frac{1}{4\pi} e^{A(\phi_h)+B(\phi_h)} |V'(\phi_h)| \qquad c_s^2 = \frac{\mathrm{d}\ \ln T}{\mathrm{d}\ \ln s}$$

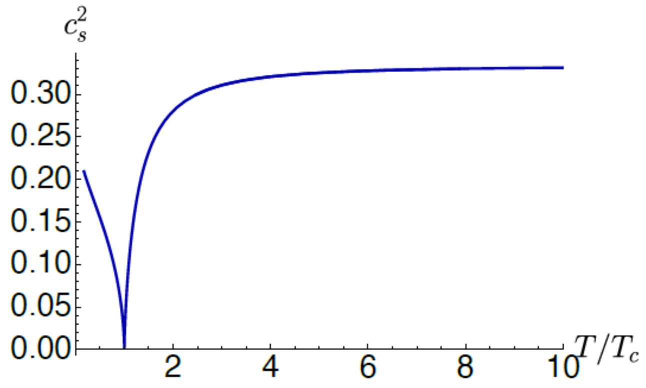
B4 = - 0.02



B4 =0



B4 = - 0.0098



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❖ Quasi-Normal Modes (QNM):

$$g_{\mu\nu}(t,u,z) = g^0_{\mu\nu}(t,u,z) + e^{-i\omega t+ikz}h_{\mu\nu}(u), \qquad \phi(t,u,z) = u + e^{-i\omega t+ikz}\Phi(u)$$

- Classify according to the gauge+diff transformations

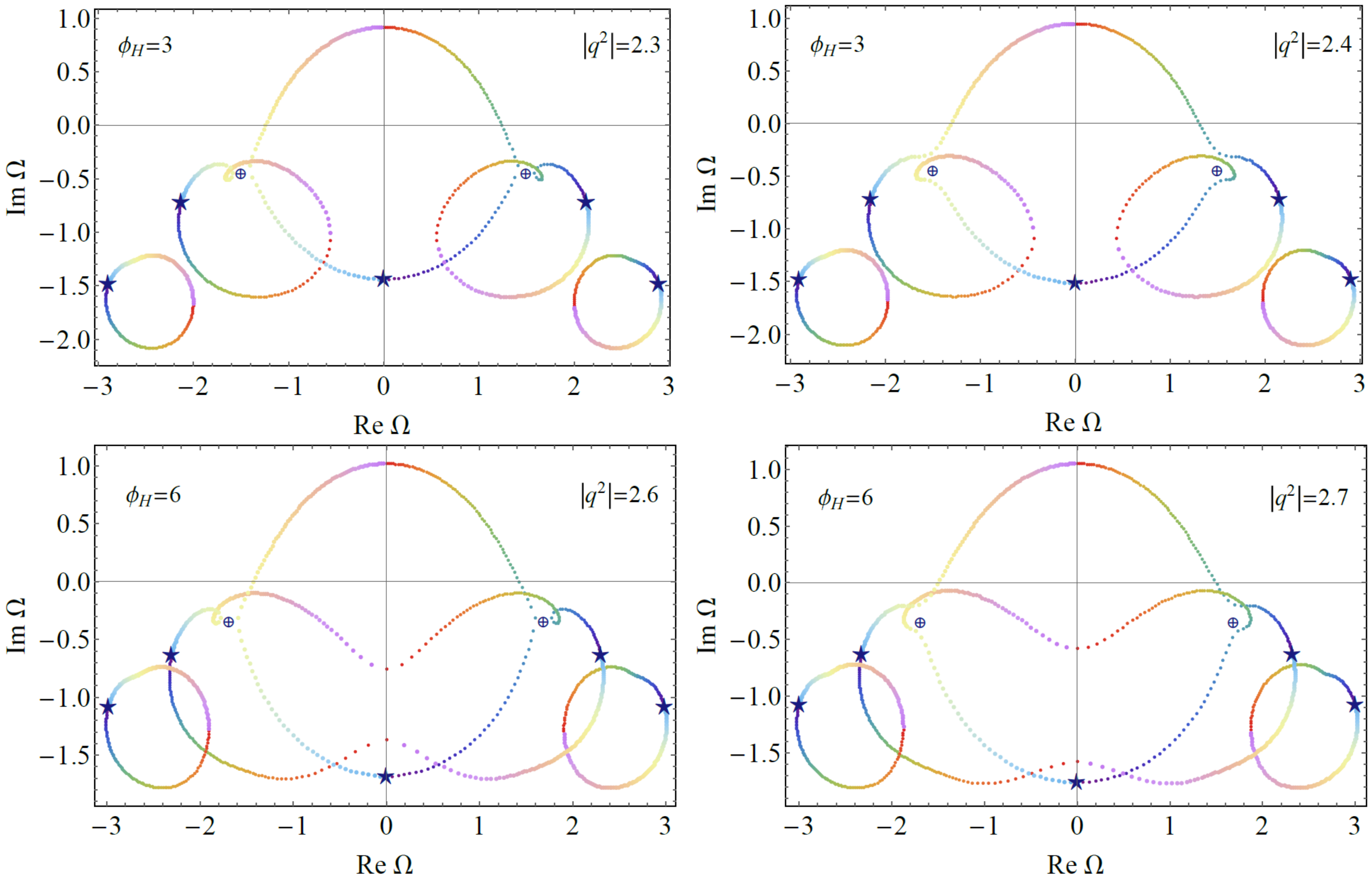
$$h_{\mu\nu} \to h_{\mu\nu} - \nabla_\mu\xi_\nu - \nabla_\nu\xi_\mu, \qquad \Phi(u) \to \Phi - \xi^\nu\nabla_\nu\phi_0.$$

- ✓ Spin-2 sector: tensor under the little SO(2) group,
- ✓ Spin-1 sector: vector under the little SO(2) group,
- ✓ Spin-0 sector: scalar under the little SO(2) group .

H. Soltanpanahi, F. Taghinavaz, ``Hydrodynamics and phase transition'', in preparation.

❖ Mode collision: collision between hydro and non-hydro modes

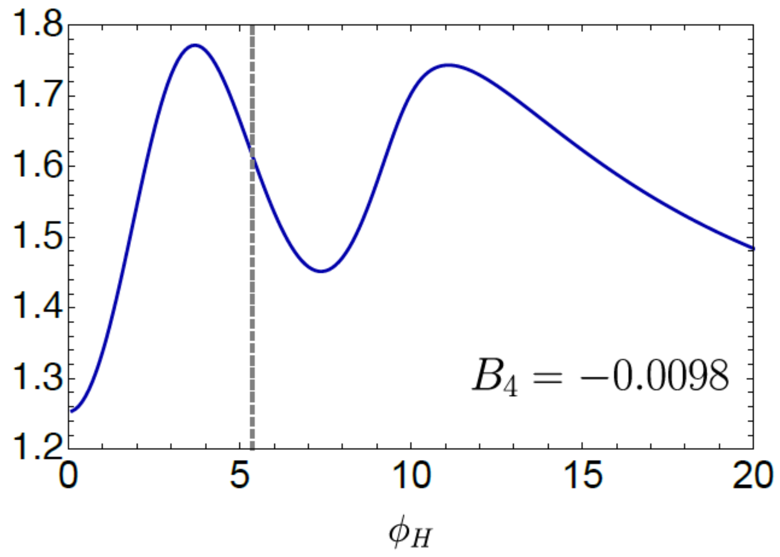
- B4=0, Spin-1 sector

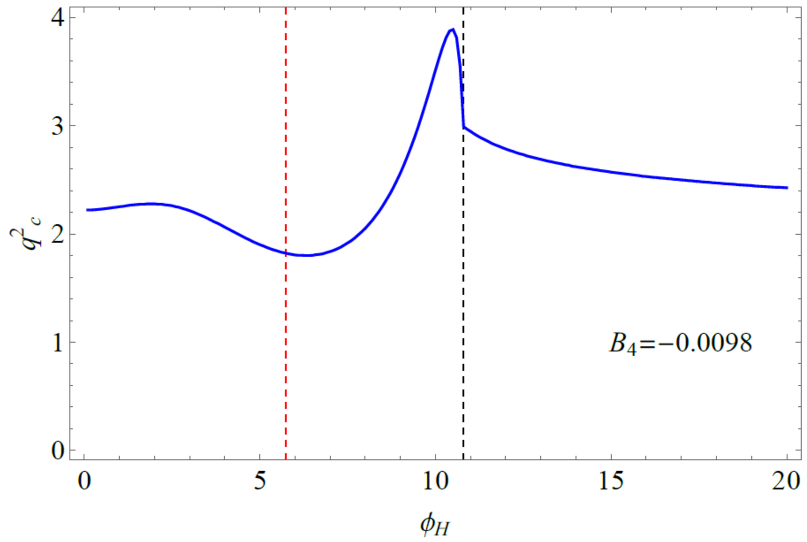


H. Soltanpanahi, F. Taghinavaz, ``Hydrodynamics and phase transition'', in preparation.

❖ Second-order phase transition: B4=-0.0098

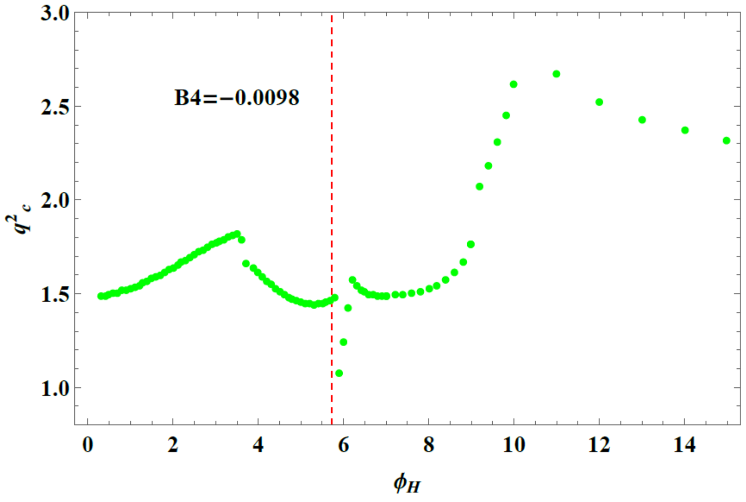
Spin-2





Spin-1

Spin-0



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Conclusion

- ✓ Validity of hydrodynamics series as an EFT, Bounds on transport
- ✓ We study within a holographic model having different kinds of phase transition the radius of convergence of hydrodynamics series by using the collision between imaginary QNM spectra
- ✓ Decrease near the transition points for first and second-order phase transitions, while increase in crossover.
- ✓ Pattern of collision changes according to the spin sections and φh values.

Open questions:

- ✓ Study more real systems, including chemical potential to mimic the QCD phase diagram