بسمه تعالی

Primordial black hole: constraint from accretion and merging rate

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Outline

- A Brief Review
- Primordial Black Hole formation
- PBH mass fraction rate
- Primordial black holes accretion
- Primordial Black merger rate

A Brief Review

- In the early universe, high densities could have led sufficiently dense regions to undergo gravitational collapse, forming black holes.

- Zel'dovich and Novikov in 1966 first proposed the existence of such black holes.
- The mechanism was elaborated and developed later by Hawking in 1971.

- PBHs could thus span an enormous mass range: those formed at the Planck time (10–43 s) would have the Planck mass (10^–5 g), whereas those formed at 1 s would be as large as 10^5 Msun.
- PBHs 10^17-10^23 g could be 100% DM.

PBH as a dark matter

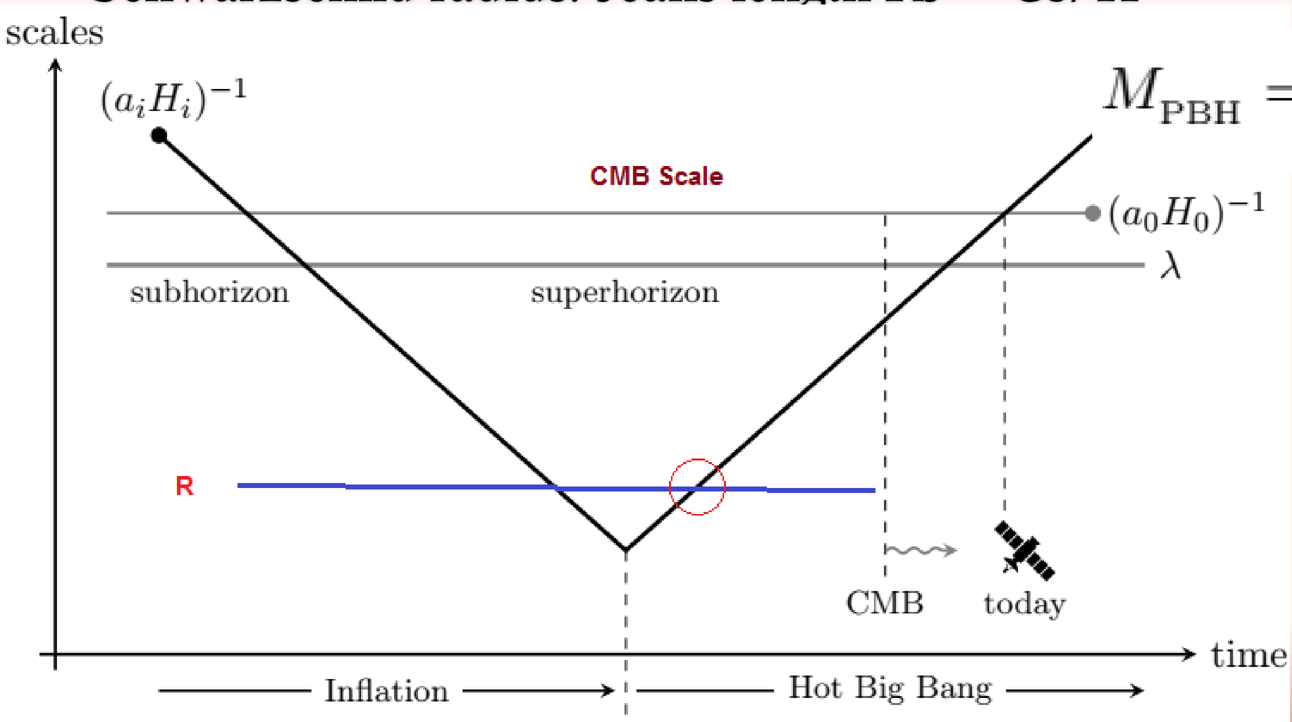
- Since Primordial black holes (PBHs) are non-baryonic they interact with their environment by gravitational force, they represent a natural candidate for one of the components of the dark matter (DM) in the Universe.

- Indeed, this idea that goes back to the earliest days of PBH research, with Chapline suggesting this in 1975.

- Such primordial black holes could account for all or part of dark matter, be responsible for some of the observed gravitational waves signals, and seed supermassive black holes found in the center of our Galaxy and other galaxies.

Pbh formation

- When in the radiation-dominated era, a highly over-dense region re-enters the cosmological horizon, it may overcome the pressure and collapse to find itself as a PBH. Mathematically, these over-dense regions are described by sufficiently large cosmological perturbations.
- Because only perturbations on scales larger than Jeans length are able to collapse to form PBHs, we need Cs k ≈ aH
- The density required for a region of mass M to fall within its Schwarzschild radius. Jeans length RJ = Cs/H



$$M_{\rm PBH} = \gamma_* M_H\big|_{t=t_f} = \frac{\gamma_*}{2GH(t_f)}$$

Pbh scale dependency

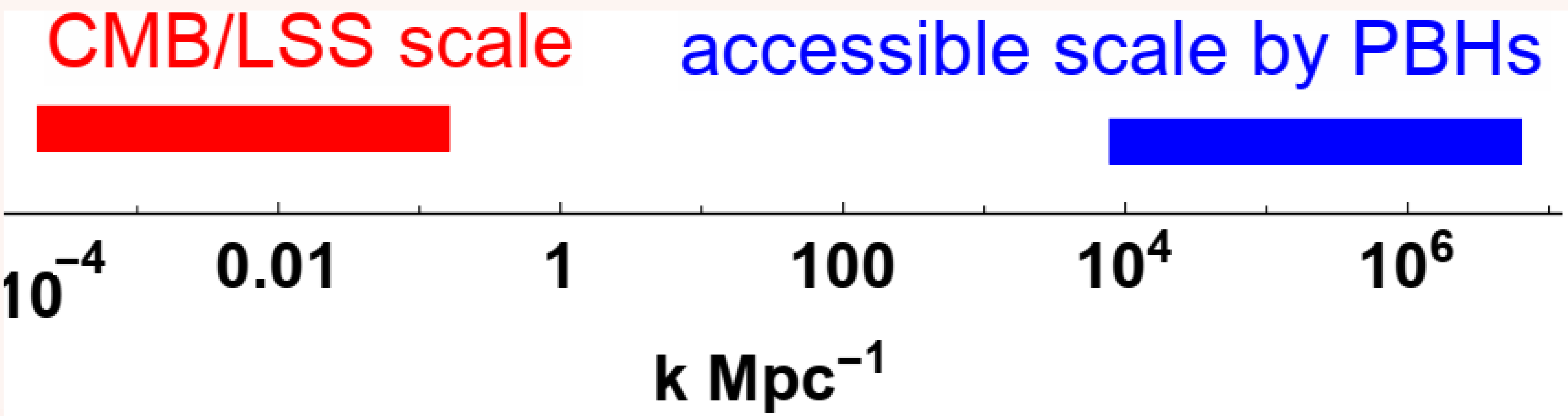
- Mass in term of wavenumber

$$M_{\rm PBH}(k) \approx 30\ M_\odot \left(\frac{\gamma_*}{0.2}\right)\left(\frac{g_{*,\rm form}}{10.75}\right)^{-1/6}\left(\frac{k}{2.9\times 10^5 {\rm Mpc}^{-1}}\right)^{-2}$$

- Comparing with CMB scale

$$k_{\rm CMB} \sim 0.002{\rm Mpc}^{-1}$$

$$k_{30M_\odot} \sim e^{20} k_{\rm CMB}$$



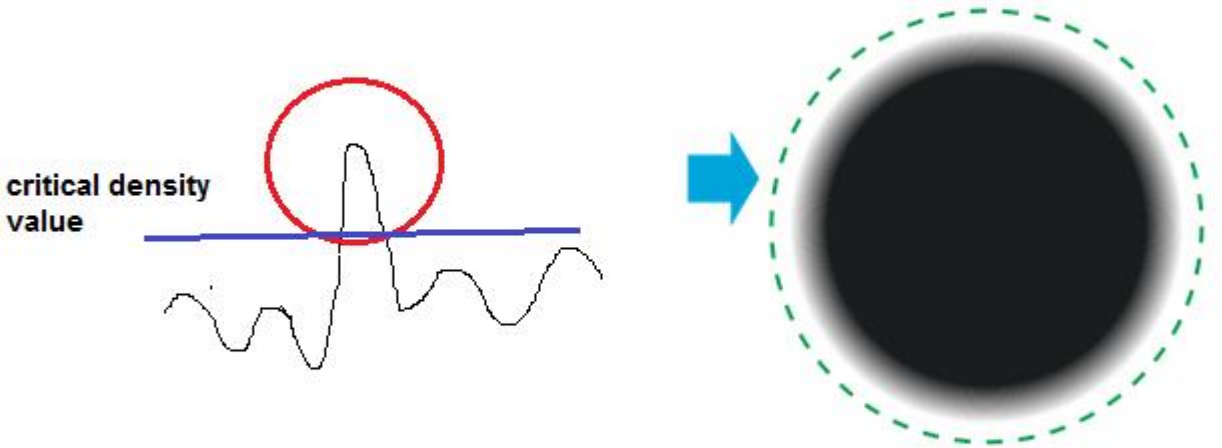
A Ashoorioon, A Rostami, JT Firouzjaee JHEP 2021 (7), 1-23

Mass fraction

$$\beta' = \frac{\rho_{\rm PBH}}{\rho_{cr}}|_{formation\ time}$$

- To state the condition for a collapse to a PBH is usually stated in terms of the smoothed density contrast at horizon crossing (HC), $\delta_{\rm HC}(R)$. If $\delta_{\rm HC}(R) > \delta_{th} \sim O(1)$ a fluctuation on a scale $R(M_{\rm PBH})$ will collapse to form a PBH, with mass $M_{\rm PBH}$ around the horizon mass

$$\frac{R}{1\ \rm Mpc} = 5.54 \times 10^{-24} \frac{1}{\gamma_*} \left(\frac{M_{\rm PBH}}{1\rm g}\right)^{1/2} \left(\frac{g_*}{3.36}\right)^{1/6}$$



Allahyari, A, JT Firouzjaee, and A. A. Abolhasani. *JCAP* (2017): 041.

A Brief Review

the variance of the density fluctuations on the mass scale M_{PBH}

$$\sigma^2(R) = \int_0^\infty \tilde{W}^2(k,R)\mathcal{P}_S(k,t)\frac{\mathrm{d}k}{k} \, ,$$

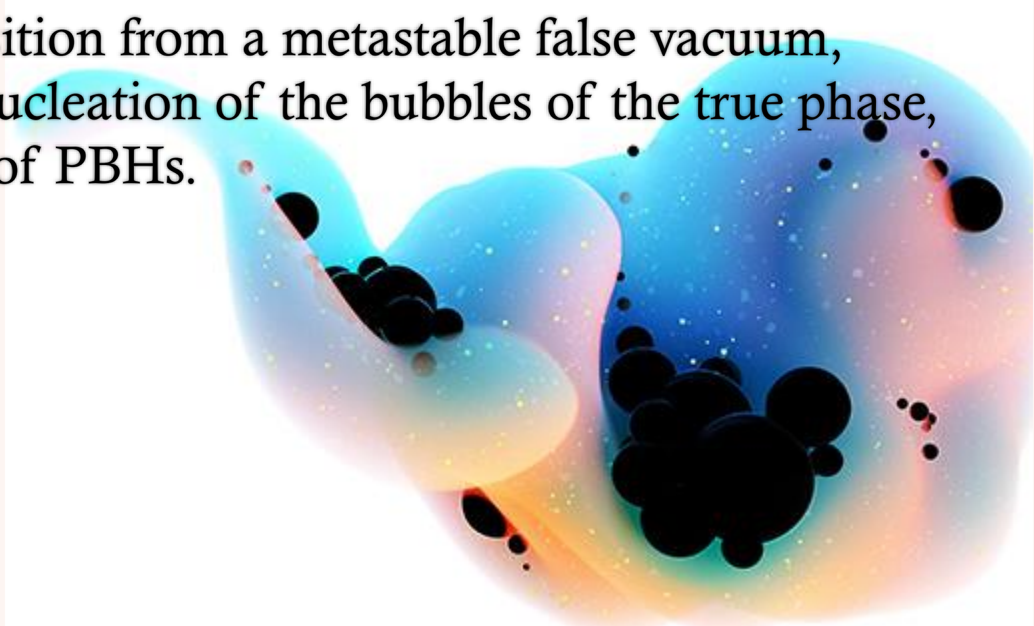
- The mass fraction of PBHs for a Gaussian distribution

$$\beta'(M_{\text{PBH}}) = \int_{\delta_{th}}^\infty P(\delta_{\text{HC}}(R))\,\mathrm{d}\delta_{\text{HC}}(R) = \frac{1}{\sqrt{2\pi}\sigma_{\text{HC}}(R)}\int_{\delta_{th}}^\infty \exp\left(-\frac{\delta^2_{\text{HC}}(R)}{2\sigma^2_{\text{HC}}(R)}\right)\mathrm{d}\delta_{\text{HC}}(R)$$

- One can proposed a mechanism to enhance the power spectrum at the relevant scales to get enough the PBH abundance

Formation mechanism

- The idea of Effective Field Theories (EFTs) is that the effect of the higher energy scales within a theory that a mechanism for enhancing the power spectrum during inflation that does not use the flattening of the potential or reduction of the sound speed of scalar perturbations.
- The first-order phase transition from a metastable false vacuum, which proceeds through nucleation of the bubbles of the true phase, can lead to the formation of PBHs.



A Ashoorioon, A Rostami, JTF Physical Review D 103 (12), 123512

Evaporation dynamics

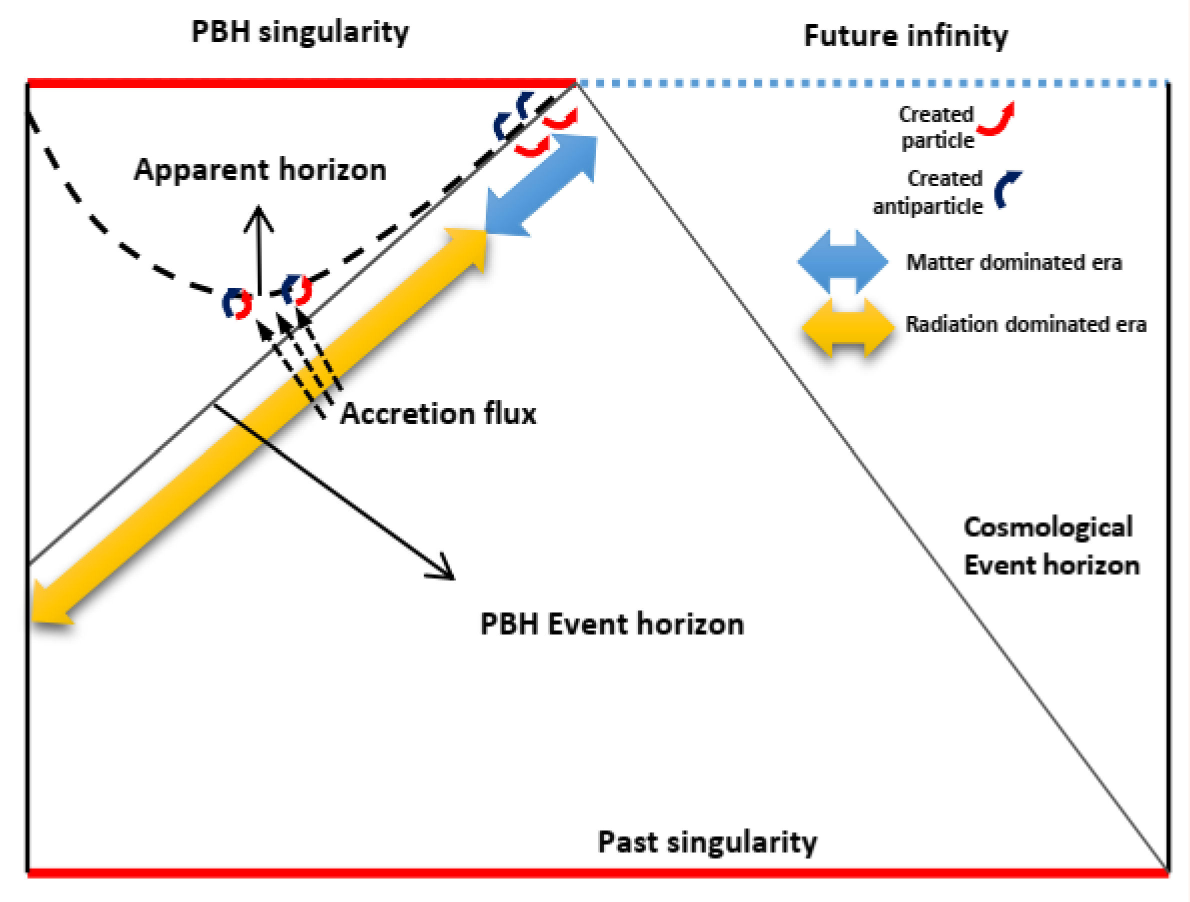
- Due to this process called Hawking evaporation for ideal Schwartzchild BH, PBHs lose mass at the rate given by

$$\frac{dM_{PBH}}{dt} = -\frac{2\pi^3}{15}\frac{f_{eva}g_*(T_{PBH})M_{PBH}^2(k_BT_{PBH})^4}{c^5\hbar c M_{Pl}^4},$$

- PBH lifetime

$$\tau(M) \simeq (10^{-26}s)(\frac{M}{1g})^3.$$

BH in cosmological background



J. T. Firouzjaee and G. F. R. Ellis, Phys. Rev. D 91, no.10, 103002 (2015)

Essential conditions

- In the case of a fully dynamical black hole, we can not apply Hawking's quantum field theory approach to black hole radiation, which applies to late-time stationary black holes and is not a suitable method for calculating the Hawking radiation thermal aspect. In such cases, new approaches were developed to calculate Hawking radiation in a dynamical background. In these approaches which are based on the semi-classical approach based on the adiabatic vacuum in quantum field theory in curved space-time, the radiation is plausibly emitted from the vicinity of apparent horizons rather than near the event horizon. In this approach consider a null curve that comes from past null infinity parameterized by u and reflects off the center at r = 0 and goes to the future null infinity which is parameterized by U. Adiabatic condition

$$U = U^* + C^* \int exp(-\int \kappa(\tilde{u})\ d\tilde{u})\ d\bar{u},$$

$$\frac{|\dot{\kappa_*}|}{\kappa_*^2} \ll \epsilon \ll 1.$$

PBH growth

$M_i(g)$	$R_i(m)$	$Model$	$Final Raduis$		
			$f_{acc} = 0.05$	$f_{acc} = 0.1$	$f_{acc} = 0.15$
10^{16}	1.48×10^{-11}	I	1.60×10^{-11}	1.74×10^{-11}	1.91×10^{-11}
		II	2.01×10^{-11}	3.14×10^{-11}	7.14×10^{-11}
10^{17}	1.48×10^{-10}	I	1.60×10^{-10}	1.74×10^{-10}	1.91×10^{-10}
		II	2.01×10^{-10}	3.14×10^{-10}	7.14×10^{-10}
10^{20}	1.48×10^{-17}	I	1.60×10^{-7}	1.74×10^{-7}	1.91×10^{-7}
		II	2.01×10^{-7}	3.14×10^{-7}	7.14×10^{-7}
10^{24}	1.48×10^{-3}	I	1.60×10^{-3}	1.74×10^{-3}	1.91×10^{-3}
		II	2.01×10^{-3}	3.14×10^{-3}	7.14×10^{-3}
10^{33}	1.48×10^{6}	I	1.60×10^{6}	1.74×10^{6}	1.91×10^{6}
		II	2.01×10^{6}	3.14×10^{6}	7.14×10^{6}
10^{36}	1.48×10^{9}	I	1.60×10^{9}	1.47×10^{9}	1.91×10^{9}
		II	2.01×10^{9}	3.14×10^{9}	7.14×10^{9}

S S Tabasi, JTF, EPJC 2023

PBH temperature

- As shown in this table, in all mass windows of PBHs to provide the dark matter, the temperature of the universe is much higher than the temperature of PBHs, therefore only the accretion process can be effective.

$M_i(g)$	10^{16}	10^{17}	10^{20}	10^{24}	10^{33}	10^{36}
$t_i(s)$	2.47×10^{-23}	2.47×10^{-22}	2.47×10^{-19}	2.47×10^{-15}	2.47×10^{-6}	2.47×10^{-3}
$T_{PBH}(K)$	1.23×10^{10}	1.23×10^{9}	1.23×10^{6}	1.23×10^{2}	1.23×10^{-7}	1.23×10^{-10}
$T_U(K)$	3.06×10^{21}	9.67×10^{20}	3.06×10^{19}	3.06×10^{17}	9.67×10^{12}	9.67×10^{11}

- We have shown that the lower mass limit for PBHs that have not yet evaporated should approximately be 10^14g rather than 10^15g. Finally, we study the effects of Hawking radiation quiescence in cosmology and reject models based on the evaporation of PBHs in the radiation-dominated era.

S Sajad Tabasi, JTF, EPJC 2023

Bondi accretion

- We focus on accretion equations, and all calculations are performed by considering spherical symmetric condition. The Bondi-Hoyle accretion model is used for this goal

$$\frac{dM_{PBH}}{dt} = 4\pi R_{PBH}^2 \rho v.$$

In accretion of radiation $v = c/\sqrt{3}$ and $R_{PBH} = R_s = 2GM_{PBH}/c^2$.

Barynonic matter accretion

- To obtain the rate of mass increases by baryonic matter, we use following equation

$$\frac{dM_b}{dt} = \lambda 4\pi m_H n_{gas} v_{eff} r_B^2.$$

Here, m_H and n_{gas} are the mass of hydrogen and its number density, $r_B = GM_{PBH} v_{eff}^{-2}$ is the Bondi-Hoyle radius and $v_{eff} = (v_{rel}^2 + c_s^2)^{\frac{1}{2}}$ is the effective velocity of PBH, expressed in terms of the PBH relative velocity v_{rel} with regard to the gas with sound speed c_s

Accretion in RD

- It was possible to enhance the power spectrum to the threshold needed for PBH formation

$$M_{R-RD}(t) = (\frac{1}{M_i} + 1.3 \times 10^{-35} f_{acc}(\frac{1}{t} - \frac{1}{t_i}))^{-1},$$
$$for \quad t_i < t < t_1$$

$$M_{R-MD}(t) = (\frac{1}{M_{i-MD}} + 3.5 \times 10^{-27} f_{acc}(\frac{1}{t^{\frac{5}{3}}} - \frac{1}{t_1^{\frac{5}{3}}}))^{-1},$$
$$for \quad t_1 < t < t_2$$

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Accretion in RD

- It was possible to enhance the power spectrum to the threshold needed for PBH formation

$$M_{R-RD}(t) = (\frac{1}{M_i} + 1.3 \times 10^{-35} f_{acc}(\frac{1}{t} - \frac{1}{t_i}))^{-1}\,;$$

$$for \quad t_i < t < t_1$$

$$M_{R-MD}(t) = (\frac{1}{M_{i-MD}} + 3.5 \times 10^{-27} f_{acc}(\frac{1}{t^{\frac{5}{3}}} - \frac{1}{t_1^{\frac{5}{3}}}))^{-1}\,,$$

$$for \quad t_1 < t < t_2$$

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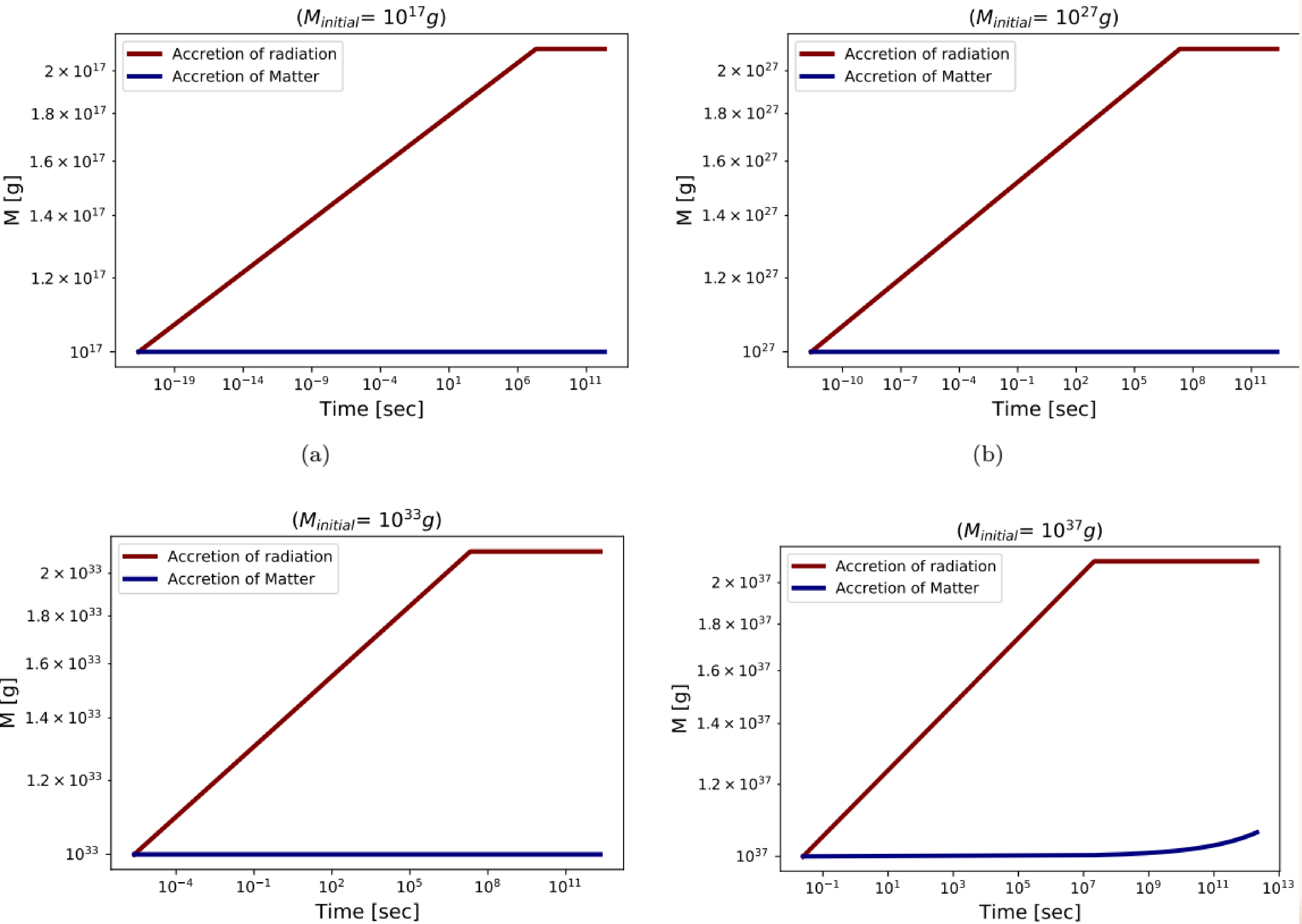


($M_{initial}$= $10^{17}g$)
Accretion of radiation
Accretion of Matter
M [g]
Time [sec]
(a)
($M_{initial}$= $10^{27}g$)
Accretion of radiation
Accretion of Matter
M [g]
Time [sec]
(b)
($M_{initial}$= $10^{33}g$)
Accretion of radiation
Accretion of Matter
M [g]
Time [sec]
($M_{initial}$= $10^{37}g$)
Accretion of radiation
Accretion of Matter
M [g]
Time [sec]

Accretion in MD

the mass evolution of PBHs in the radiation-dominated era

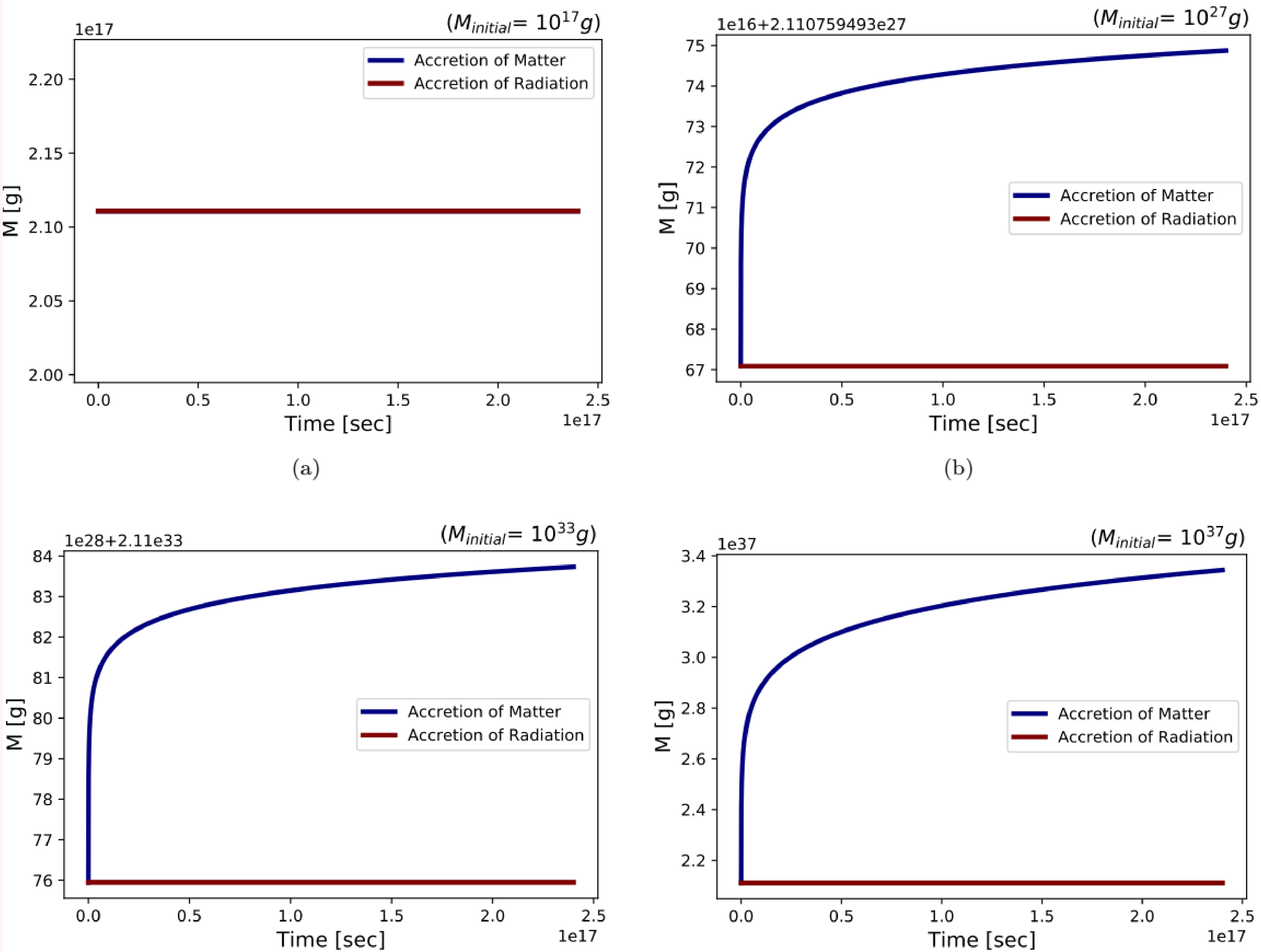
$$M_{M-RD}(t) = M_i(1 + \frac{123}{25 \times 10^{39}} M_i(\sqrt[4]{t_i} - \sqrt[4]{t}))^{-1} .$$

$$for \quad t_i < t < t_1$$

Besides, the mass evolution equation in terms of time in the matter-dominated era by using $1 + z = (a_0/a) = e^{H_0(t_0-t_2)}(t_2/t)^{2/3}$ is

$$M_{M-MD}(t) = M_{i-MD}(1 + 1.5 \times 10^{-36} M_{i-MD} \ln \frac{t_1}{t})^{-1} .$$

$$for \quad t_1 < t < t_2$$



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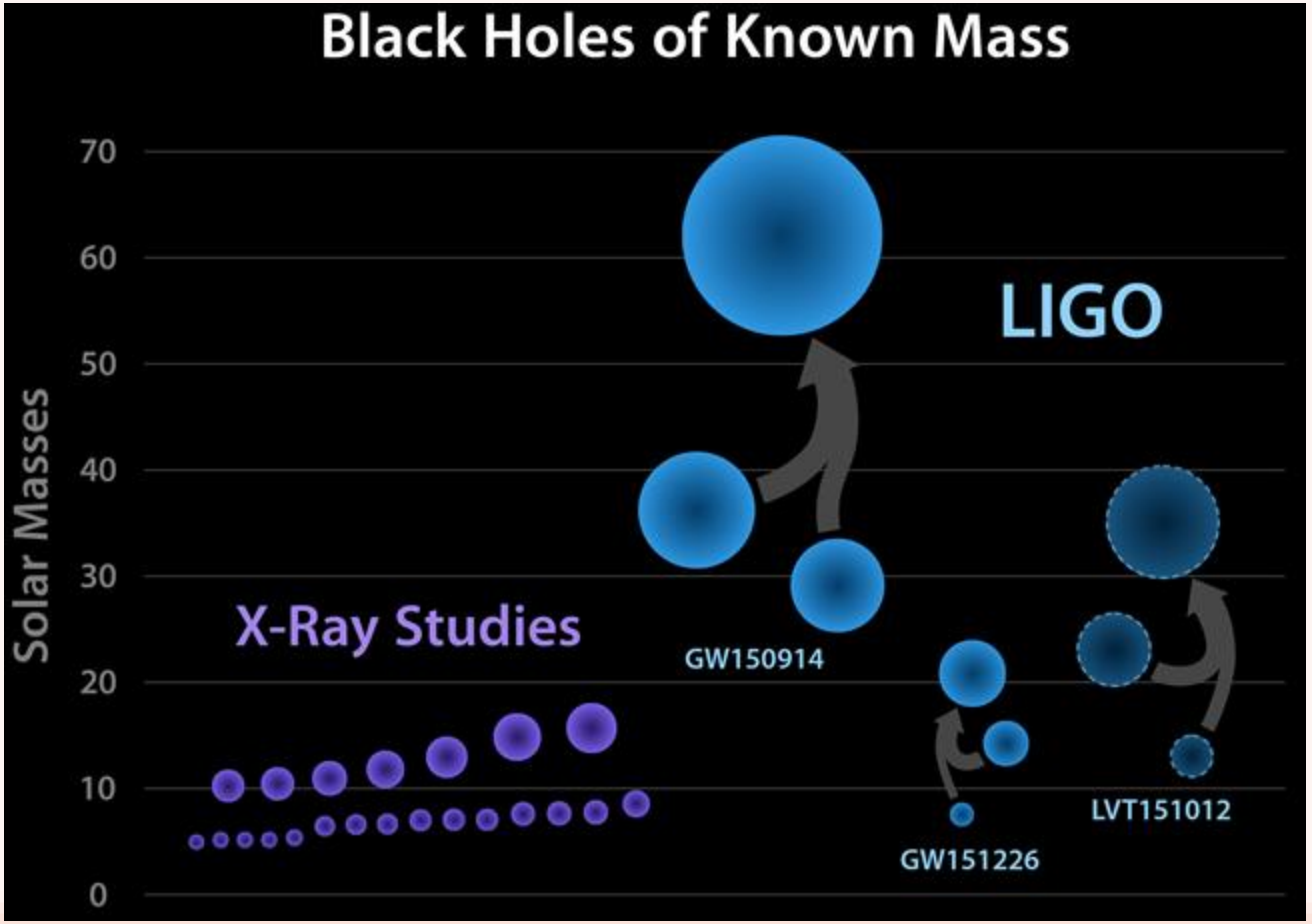
A Brief Review

- In March 2016, one month after the announcement of the detection by Advanced LIGO/VIRGO of gravitational waves emitted by the merging of two 30 $M_{\odot}$ black holes (about 6×10^{31} kg), three groups of researchers proposed independently that the detected black holes had a primordial origin
- Is all the dark matter made of primordial black holes or primordial black holes could only contribute to a part of the total dark matter?

(arXiv: 1603.00464), (arXiv: 1603.05234) and (arXiv: 1603.08338)

- LIGO has definitively detected two sets of black hole mergers (bright blue). The black holes shown with a dotted border represent a LIGO candidate event that was too weak to be conclusively claimed as a detection. Black hole masses determined via x-ray measurements are shown in purple. (Image Credit: LIGO.)
- Contrary to the astrophysical processes (i.e. collapse of stars) for which only BHs heavier than a particular mass (around 3 solar mass) are possible to form in PBH formation.



Bird etal., PRL 116, 201301; Sasaki etal., PRL 117, 061101;

Primordial Black Hole Merger Rate (Late-Time)

- The merger rate inside a halo with mass M_h is given by

$$\mathcal{R}_h(M_h) = \int_0^{R_{\rm vir}} dr\ 4\pi r^2 \frac{1}{2}\left(\frac{\rho_{\rm PBH}(r)}{M_{\rm PBH}}\right)^2 \langle \sigma v_{\rm PBH} \rangle,$$

- where ρ_{pbh} is density profile of PBHs inside the halo, and the angle bracket denotes the average over relative velocity distribution.
- The total merger rate per unit volume and unit time is given by

$$\mathcal{R} = \int_{M_{\rm min}} dM_h\ \frac{dn}{dM_h} \mathcal{R}_h(M_h),$$

- where $\frac{dn}{dM_h}$ is the halo mass function and M_{min} is the minimum mass of halos that have not yet evaporated by the present time.

PBHs Merger & DM Halo Models

- S. Bird, et al. calculated the merger rate of the PBHs for the spherical collapse halo models.
- But what if the model is a non-spherical (Let's say ellipsoidal) collapse model??? (What we were looking for)
- Two crucial functions should be defined based on the model of halo collapse.

(a) Halo Mass Function $\frac{dn}{dM_h} \propto g(\sigma)$

(b) Halo Mass-Concentration Relation $C(M_h)$

S Fakhry, JT Firouzjaee, M Farhoudi - Physical Review D, 2021
S Fakhry, M Naseri, JT Firouzjaee, M Farhoudi - Physical Review D, 2022
S Fakhry, Z Salehnia, A Shirmohammadi, JT Firouzjaee, The Astrophysical Journal 941 (1), 36 2022

Primordial Black Hole Merger Rate (Late-Time)

- Let us consider a situation where a PBH traveling in space accidentally has a near-miss with another PBH.
- These PBHs may be concentrated in local region like inside larger dark matter halo or simply moving freely in space.
- Near the periastron, relative acceleration of the PBHs becomes the largest and dominant emission of gravitational radiation occurs.
- The time-averaged energy loss rate of the binary in the Keplerian orbit due to gravitational radiation is given by

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32}{5} \frac{G^4 m_1^2 m_2^2 (m_1 + m_2)}{a^5 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

- m_1 and m_2 are masses of the PBHs and e is orbital eccentricity.

Primordial Black Hole Merger Rate (Late-Time)

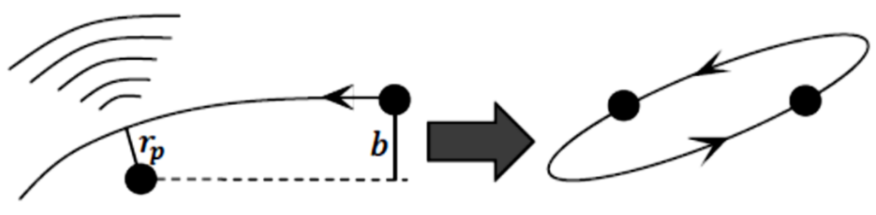
- Then, the energy loss by the close-encounter during one orbital period T is obtained by plugging e = 1

$$\Delta E = \frac{85\pi\sqrt{G(m_1+m_2)}G^3 m_1^2 m_2^2}{12\sqrt{2}r_p^{7/2}}.$$

- r_p is periastron.
- If this energy is greater than the kinetic energy then the PBHs form a binary. This imposes a condition on r_p as

$$r_p < r_{p,\mathrm{max}} = \left[\frac{85\pi}{6\sqrt{2}}\frac{G^{7/2}(m_1+m_2)^{3/2}m_1 m_2}{v^2}\right]$$

Primordial Black Hole Merger Rate (Late-Time)



- In the Newtonian approximation, relation between impact parameter b and r_p is given by (if $m_1 = m_2 = M$)

$$b^2(r_p) = r_p^2 + \frac{2GM\,r_p}{v^2}.$$

- The encounter with the impact parameter less than $b(r_{p,max})$ yields a binary.
- The cross section for forming a binary becomes (strong gravitational limit)

$$\sigma = \pi b^2(r_{p,\mathrm{max}}) \simeq \left(\frac{85\pi}{3}\right)^{2/7} \frac{\pi(2GM_{\mathrm{PBH}})^2}{v^{18/7}}.$$

Primordial Black Hole Merger Rate (Late-Time)

- The merger rate inside a halo with mass M_h is given by

$$\mathcal{R}_h(M_h) = \int_0^{R_{\rm vir}} dr\ 4\pi r^2 \frac{1}{2}\left(\frac{\rho_{\rm PBH}(r)}{M_{\rm PBH}}\right)^2 \langle \sigma v_{\rm PBH} \rangle,$$

- where ρ_{pbh} is density profile of PBHs inside the halo, and the angle bracket denotes the average over relative velocity distribution.
- The total merger rate per unit volume and unit time is given by

$$\mathcal{R} = \int_{M_{\rm min}} dM_h\ \frac{dn}{dM_h} \mathcal{R}_h(M_h),$$

- where $\frac{dn}{dM_h}$ is the halo mass function and M_{min} is the minimum mass of halos that have not yet evaporated by the present time.

PBH-Neutron Star Merger Rate

- Accordingly, the binary formation rate in a single galactic halo is given by the following relation

$$\Gamma = 4\pi \int_0^{r_{\rm vir}} \left(\frac{f_{\rm PBH}\rho_{\rm halo}}{m_1} \right) \left(\frac{\rho_{\rm NS}}{m_2} \right) \langle \xi v_{\rm rel} \rangle \, r^2 \, dr,$$

PBH-Neutron Star Merger Rate

- Also, ρNS is the NS density profile that we characterize by the following spherically-symmetric form

$$\rho_{\rm NS}(r) = \rho^*_{\rm NS} \exp\left(-\frac{r}{r^*_{\rm NS}}\right)$$

- For the normalizing the distribution of NS,we use the time-independent form of the initial Salpeter stellar mass-function, which is described as

$$\chi(m_*) \sim m_*^{-2.35}$$

PBH-Neutron Star Merger Rate

- Thus, the number of NSs in a galaxy with stellar mass M∗ is specified by the following definition

$$n_{\rm NS} = M_* \int_{m_*^{\rm min}}^{m_*^{\rm max}} \chi(m_*) dm_*,$$

PBH-Neutron Star Merger Rate

- GW detectors is an accumulation of the merger rates. In this case, by convolving the halo mass function, dn(M)/dM, with the rate of binary formation within each halo, Γ(M), and integrating over a minimum mass of halos, one can calculate the total merger rate of PBH-NS binaries as follows

$$\mathcal{R} = \int_{M_{\rm min}} \frac{dn}{dM_{\rm vir}} \Gamma(M_{\rm vir}) dM_{\rm vir}.$$

PBH-Neutron Star Merger Rate

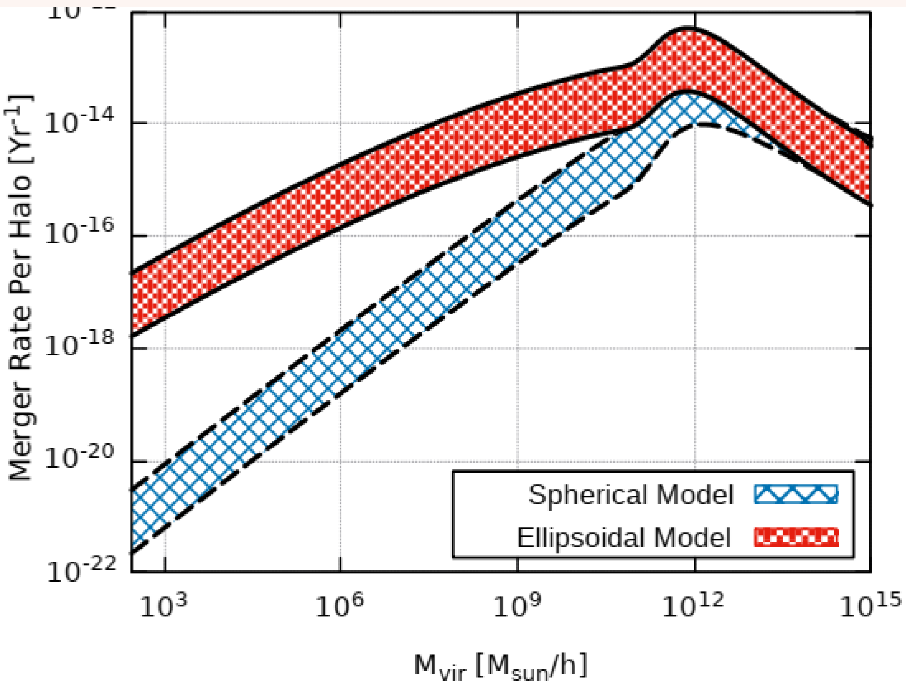


Figure 1. Merger rate of PBH-NS binaries within each halo for ellipsoidal and spherical-collapse halo models as a function of halo virial mass in the present-time Universe. The shaded red band shows this relation for ellipsoidal-collapse halo models, while the shaded blue band indicates it for spherical-collapse halo models. The NFW density profile has been utilized.

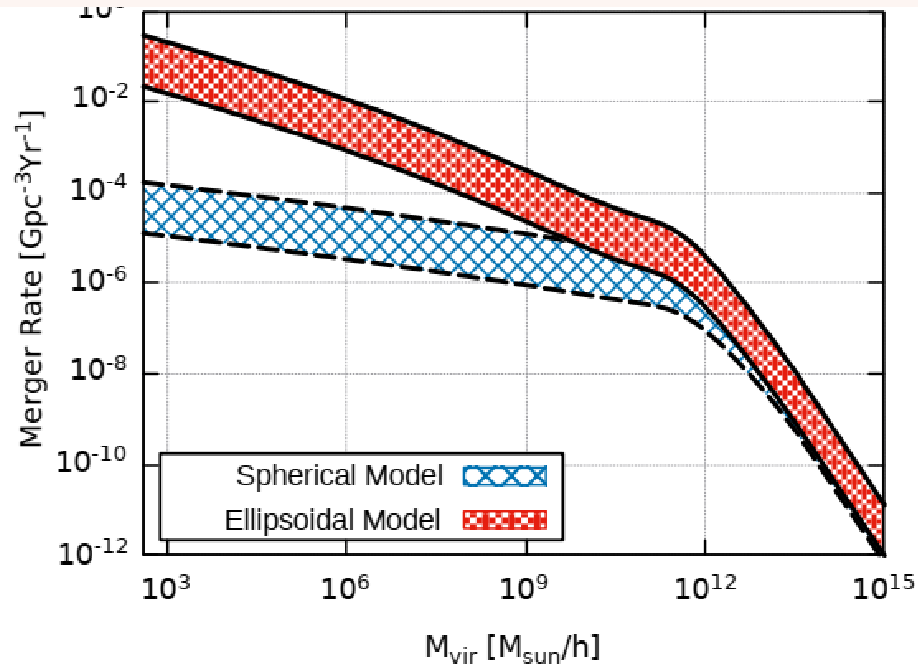


Figure 2. Merger rate of PBH-NS binaries per unit time and per unit volume for ellipsoidal and spherical-collapse halo models as a function of halo virial mass in the present-time Universe. The shaded red band shows this relation for ellipsoidal-collapse halo models, while the shaded blue band represents it for spherical-collapse halo models. The NFW density profile has been utilized.

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PBH-Neutron Star Merger Rate

Table 1. Total merger event rate of PBH-NS binaries per unit time and per unit volume for a range of PBH masses, i.e., (5-50) $M_\odot$ while considering ellipsoidal and spherical-collapse dark matter halo models. The mass of NS is considered to be $1.4\, M_\odot$. The results are related to the present-time Universe and both NFW and Einasto density profiles have been considered separately.

$M_{\rm PBH}(M_\odot)$	Density Profile	Total Merger Rate ($\mathrm{Gpc^{-3}yr^{-1}}$) Spherical Model	Total Merger Rate ($\mathrm{Gpc^{-3}yr^{-1}}$) Ellipsoidal Model
5	NFW	$1.01 \times 10^{-4} - 1.35 \times 10^{-3}$	$0.35 - 4.70$
5	Einasto	$1.39 \times 10^{-4} - 1.86 \times 10^{-3}$	$0.56 - 7.51$
10	NFW	$9.39 \times 10^{-5} - 1.25 \times 10^{-3}$	$0.27 - 3.63$
10	Einasto	$1.28 \times 10^{-4} - 1.71 \times 10^{-3}$	$0.41 - 5.59$
20	NFW	$8.77 \times 10^{-5} - 1.17 \times 10^{-3}$	$0.21 - 2.89$
20	Einasto	$1.19 \times 10^{-4} - 1.59 \times 10^{-3}$	$0.28 - 4.50$
30	NFW	$8.27 \times 10^{-5} - 1.10 \times 10^{-3}$	$0.18 - 2.37$
30	Einasto	$1.13 \times 10^{-4} - 1.51 \times 10^{-3}$	$0.25 - 3.64$
50	NFW	$7.94 \times 10^{-5} - 1.06 \times 10^{-3}$	$0.15 - 2.07$
50	Einasto	$1.09 \times 10^{-4} - 1.47 \times 10^{-3}$	$0.23 - 3.16$

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PBH-Neutron Star Merger Rate

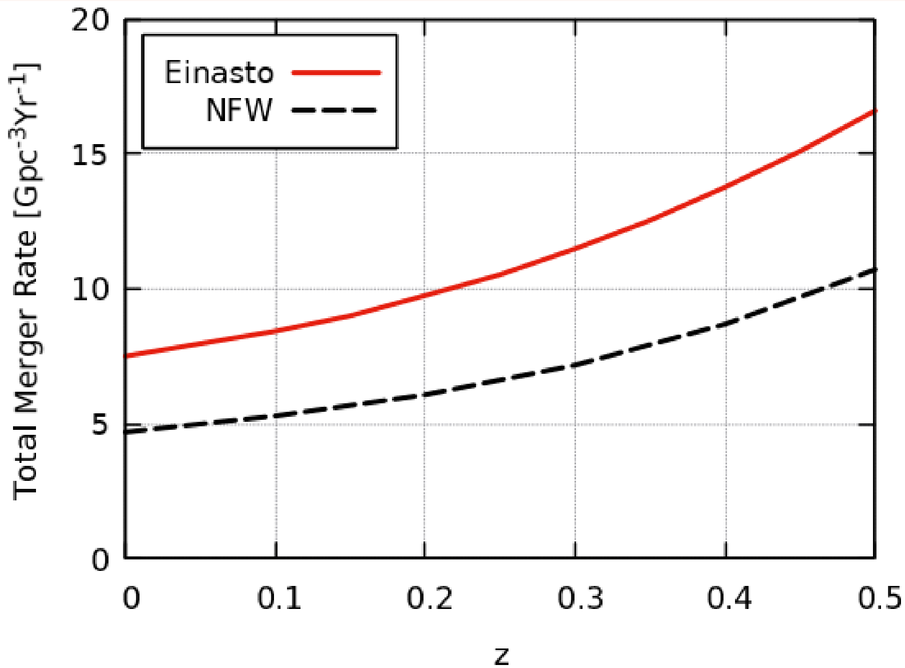

Figure 3. Upper limit of the merger event rate of PBH-NS binaries for ellipsoidal-collapse dark matter halo models as a function of redshift. The solid (red) line represents this relation for the Einasto density profile, while the dashed (black) line shows it for the NFW density profile.

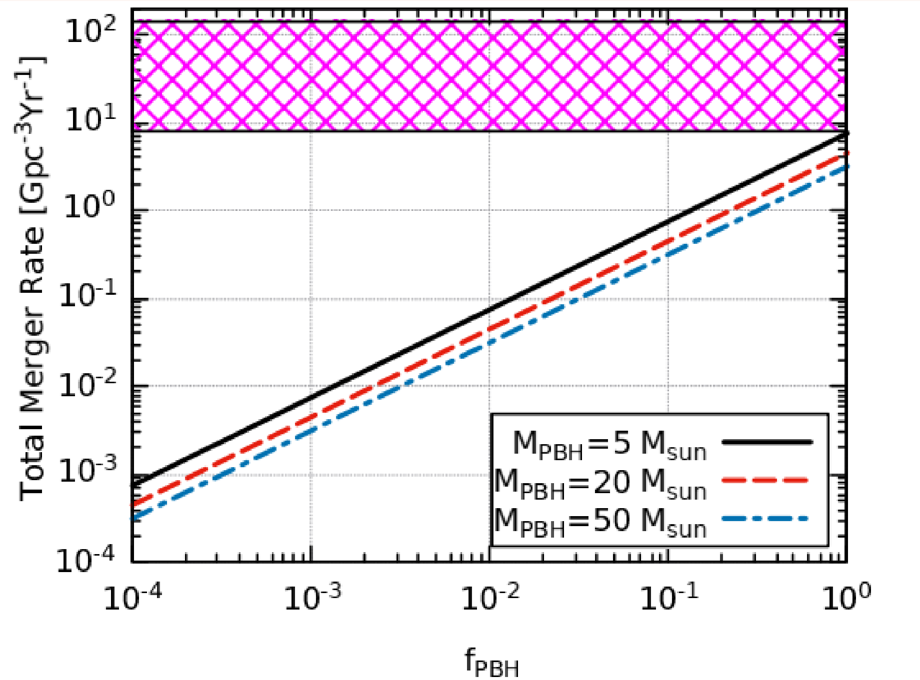

Figure 4. Total merger event rate of PBH-NS binaries for ellipsoidal-collapse dark matter halo models as a function of the upper bounds on PBH fraction and their masses. The solid (black), dashed (red), and dot-dashed (blue) lines indicate this relation while considering PBH mass to be $M_{\rm PBH} = 5, 20$, and $50 M_{\odot}$, respectively. The shaded band indicates the total merger rate of BH-NS binaries recorded by the LIGO-Virgo detectors, i.e., (7.8-140) $\rm Gpc^{-3} yr^{-1}$.

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Conclusions

- Considering the dynamical nature of the PBHs is unavoidable for astrophysical study.
- we study the effects of Hawking radiation quiescence in cosmology and reject models based on the evaporation of PBHs in the radiation-dominated era.
- We have shown that the lower mass limit for PBHs that have not yet evaporated should approximately be 10^14g rather than 10^15g.
- These calculations show well the mass evolution of primordial black holes from the time of formation to the end of the matterdominated era, taking into account both the main processes governing black holes, evaporation and accretion.

Conclusions

- we have calculated the merger rate of PBH-NS binaries in the ellipsoidal-collapse of dark matter halo models and compared it with the corresponding results obtained from spherical-collapse dark matter halo models.
- The results indicate that the merger rate of PBH-NS binaries in the context of ellipsoidalcollapse dark matter halo models is far higher than that extracted from spherical-collapse dark matter halo models.
- Moreover, it has been confirmed that for halos with smaller masses, the merger rate of PBH-NS binaries in ellipsoidal-collapse dark matter halo models has more deviations from that obtained for spherical-collapse dark matter halo models.
- the merger rate of PBH-NS binaries for the Einasto density profile is slightly higher than that derived from the NFW density profile.
- Smaller PBHs are more likely to involve in PBH-NS binary formations than larger ones.
- Th PBHs at higher redshifts have been more likely to encounter NSs than in the present-time Universe.
- It has been inferred that ellipsoidal-collapse dark matter halo models will be able to justify the merger rate of PBH-NS binaries with masses of ($M_{PBH} \leq 5\ M_{\odot}$, $M_{NS} \simeq 1.4\ M_{\odot}$) in the framework of LIGO-Virgo sensitivity if $f_{PBH} \simeq 1$.

با تشکر