UNIVERSITY OF WATERLOO
FACULTY OF SCIENCE
Department of Physics & Astronomy

IQC Institute *for* Quantum Computing

Micro-Macro Entanglement

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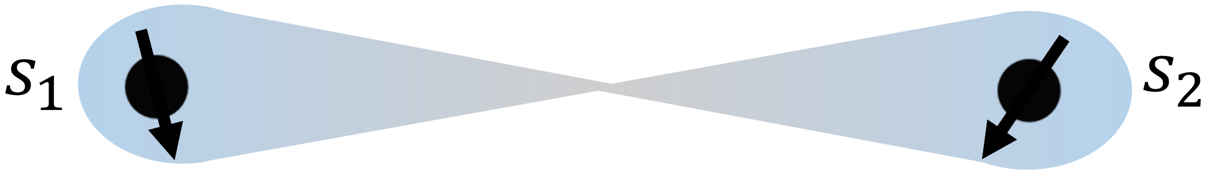
IPM Spring Conference

May 17th 2023

SHARIF
UNIVERSITY OF
TECHNOLOGY

What is Entanglement?

Pure quantum correlation between two or more quantum systems



$$|\phi\rangle_{s_1 s_2} \neq |\phi\rangle_{s_1} \otimes |\phi\rangle_{s_2}$$

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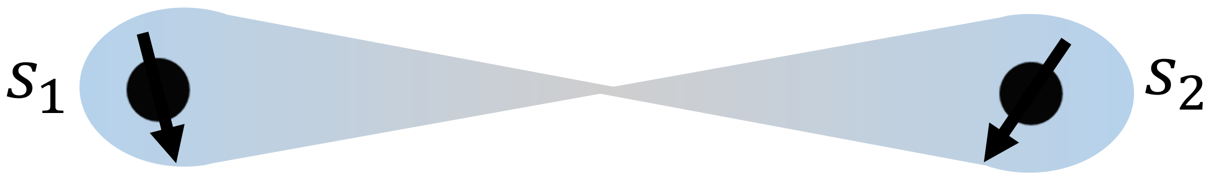
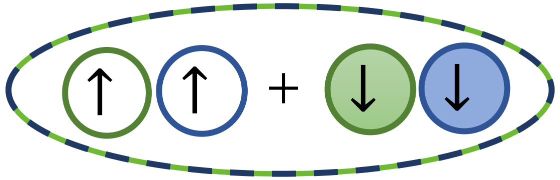


$$|\phi\rangle_{s_1 s_2} \neq |\phi\rangle_{s_1} \otimes |\phi\rangle_{s_2}$$

Maximally Bipartite Entangled State

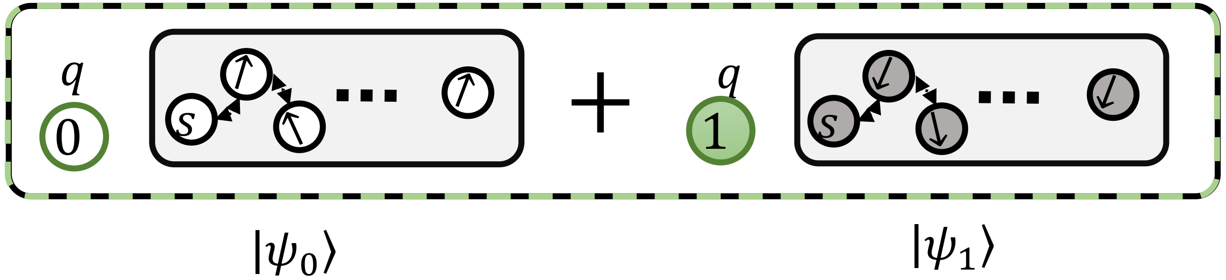


$$|\phi\rangle_{s_1 s_2} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{s_1}|\uparrow\rangle_{s_2} + |\downarrow\rangle_{s_1}|\downarrow\rangle_{s_2}\right)$$

What is Micro-Macro Entanglement?

micro-macro entangled states

Qubit: $\{|0\rangle, |1\rangle\}$
MS: $\{|\uparrow\rangle, |\downarrow\rangle\}$



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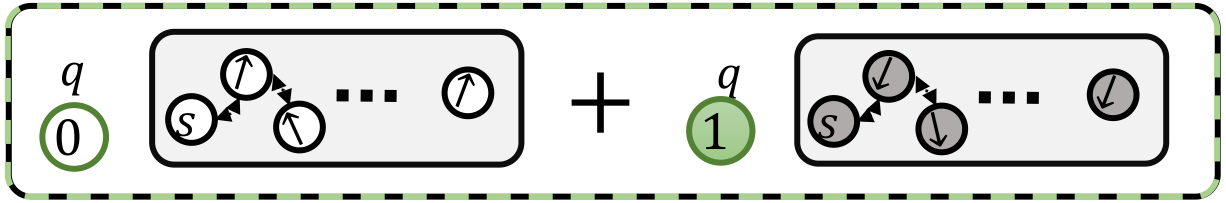

$|\psi_0\rangle$ $|\psi_1\rangle$

Ideal Pure Maximally Entangled State

$$|\phi\rangle_{q,\mathrm{MS}} = \frac{1}{\sqrt{2}}(|0\rangle|\psi_0\rangle + |1\rangle|\psi_1\rangle)$$

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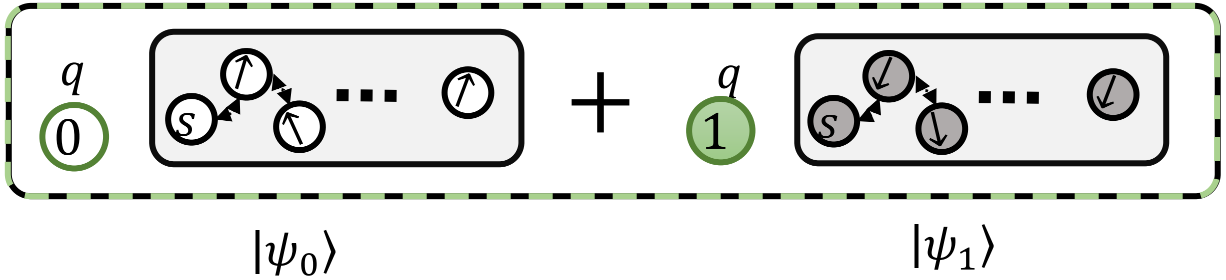

Ideal Pure Maximally Entangled State $$|\phi\rangle_{q,\mathrm{MS}} = \frac{1}{\sqrt{2}}(|0\rangle|\psi_0\rangle + |1\rangle|\psi_1\rangle)$$

1. **Bipartite Entanglement,** $|\psi_0\rangle$ orthogonal to $|\psi_1\rangle$
$$\langle\psi_0|\psi_1\rangle = 0$$

What is Micro-Macro Entanglement?

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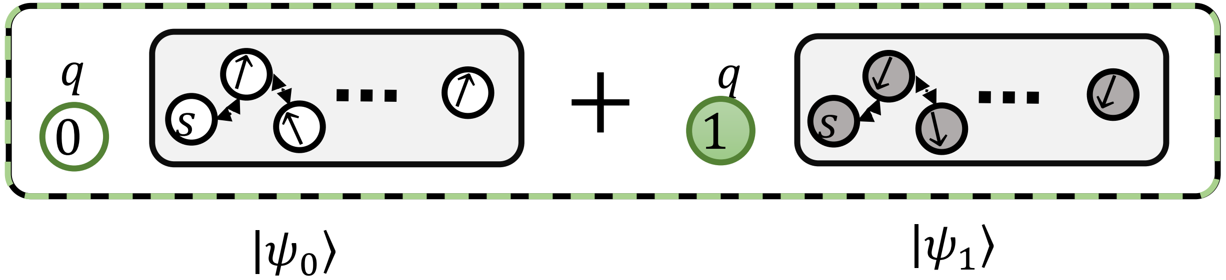
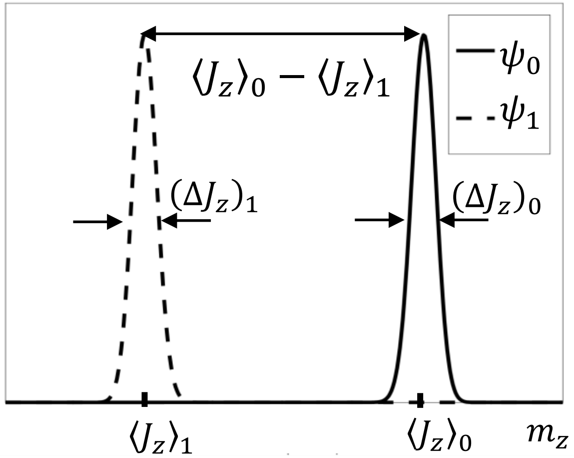


Collective physical observable: J_z



$$|\phi\rangle_{q,\mathrm{MS}} = \frac{1}{\sqrt{2}}(|0\rangle|\psi_0\rangle + |1\rangle|\psi_1\rangle)$$

2. Macroscopic Distinctness

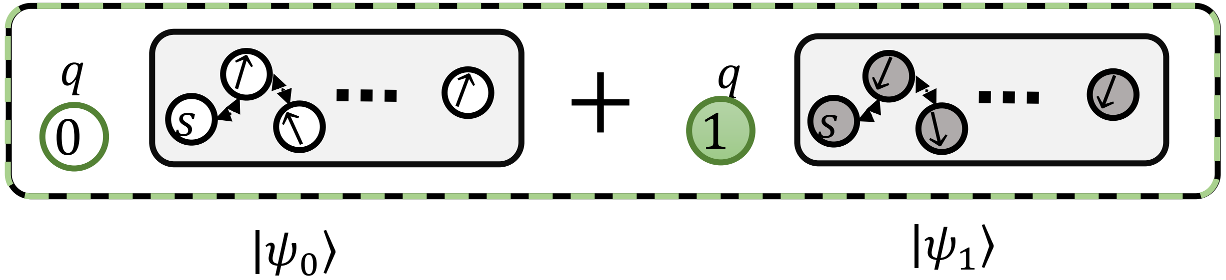
- **Macroscopic difference** between $|\psi_0\rangle$ and $|\psi_1\rangle$ when measured by a **physical observable (e.g. collective magnetization, J_z)**

$$\textbf{distinctness} = \frac{\langle J_z\rangle_0 - \langle J_z\rangle_1}{(\Delta J_z)_0 + (\Delta J_z)_1} \gg 1$$

A. J. Leggett, J. Phys. Condens. Matter **14**, R415 (2002).

What is Micro-Macro Entanglement?

micro-macro entangled states



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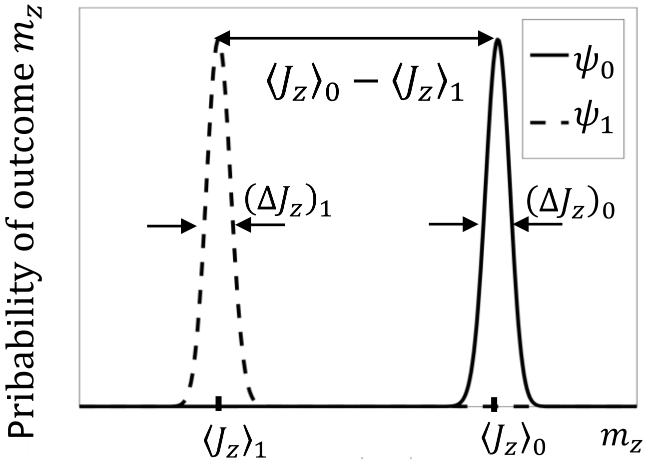


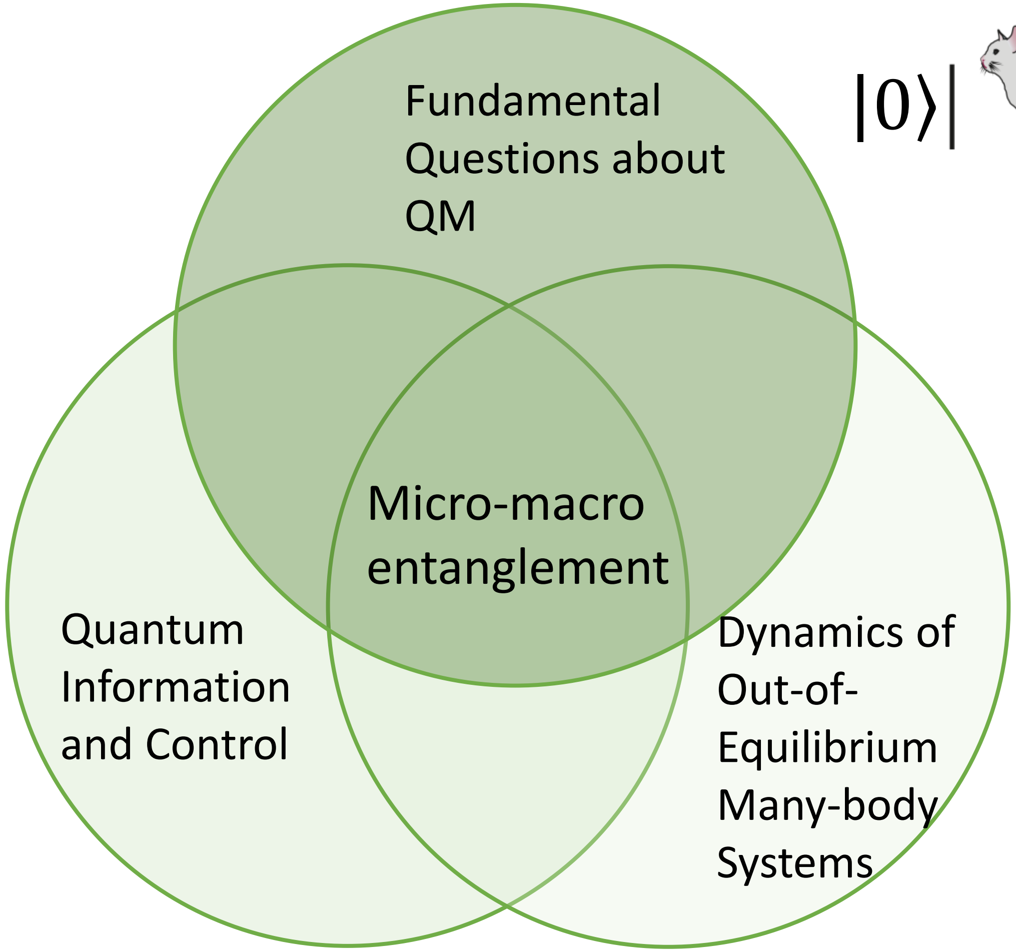
J_z : Collective observable measuring total magnetization along z

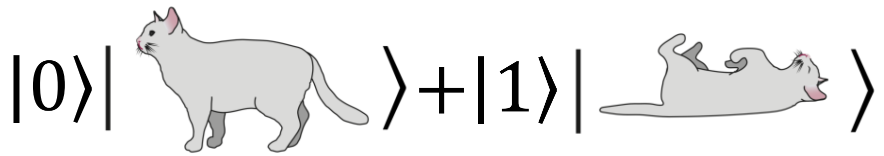
3 Spins: $|\uparrow\uparrow\uparrow\rangle: m_z = \frac{3}{2}, |\uparrow\downarrow\downarrow\rangle: m_z = -\frac{1}{2}, \quad \frac{|\uparrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\rangle}{\sqrt{2}}, \ m_z = \begin{cases} \frac{3}{2} \\ -\frac{1}{2} \end{cases}$

Eigenvalues of J_z for N spin $-\frac{1}{2}$ particles:

$$m_z: \ \hbar\left\{-\frac{N}{2}, -\frac{N}{2}+1, \dots, \frac{N}{2}-1, \frac{N}{2}\right\}$$

Why Micro-Macro Entanglement?





- Quantum Measurement and collapse of the wavefunction (Schrodinger’s thought cat-in-the-box experiment)
- Quantum to Classical Transition

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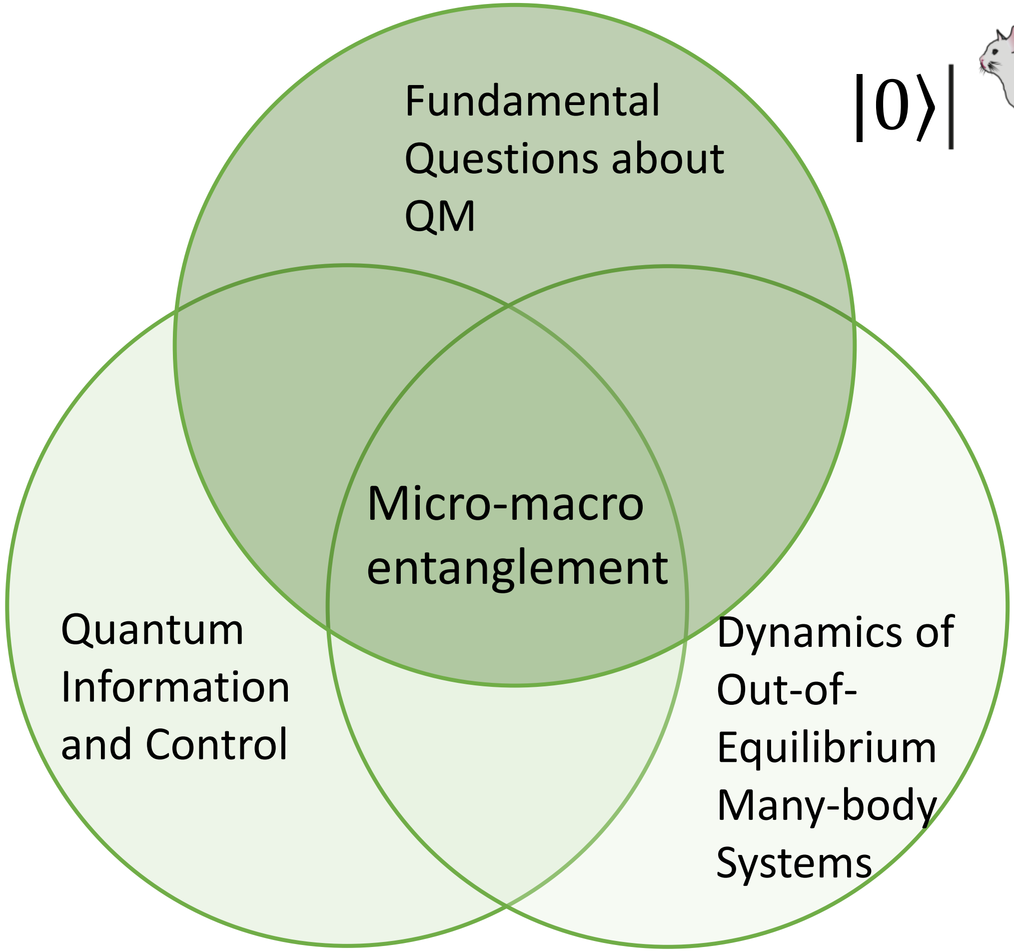

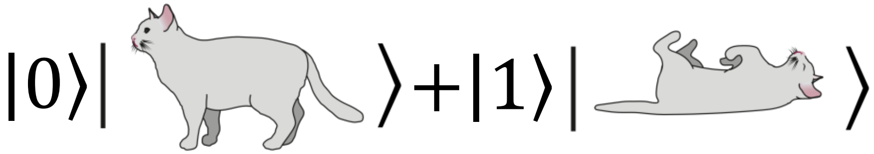

- Quantum Measurement and collapse of the wavefunction
  (Schrodinger's thought cat-in-the-box experiment)
- Quantum to Classical Transition

➢ Experiments by Serge Haroche (Nobel 2012)
  Rydberg atoms with ~10 photons in superconducting cavity

M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali,C.Wunderlich, J. M. Raimond, and S. Haroche, Phys. Rev. Lett.**77**, 4887 (1996).
S. Haroche, Phys. Scr. **1998**, 159 (1998).

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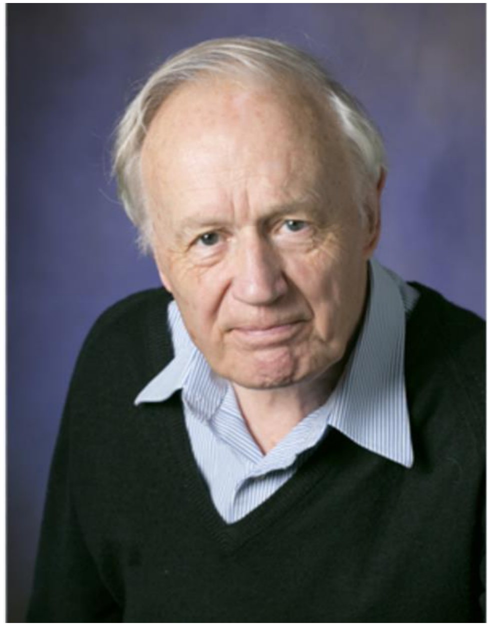

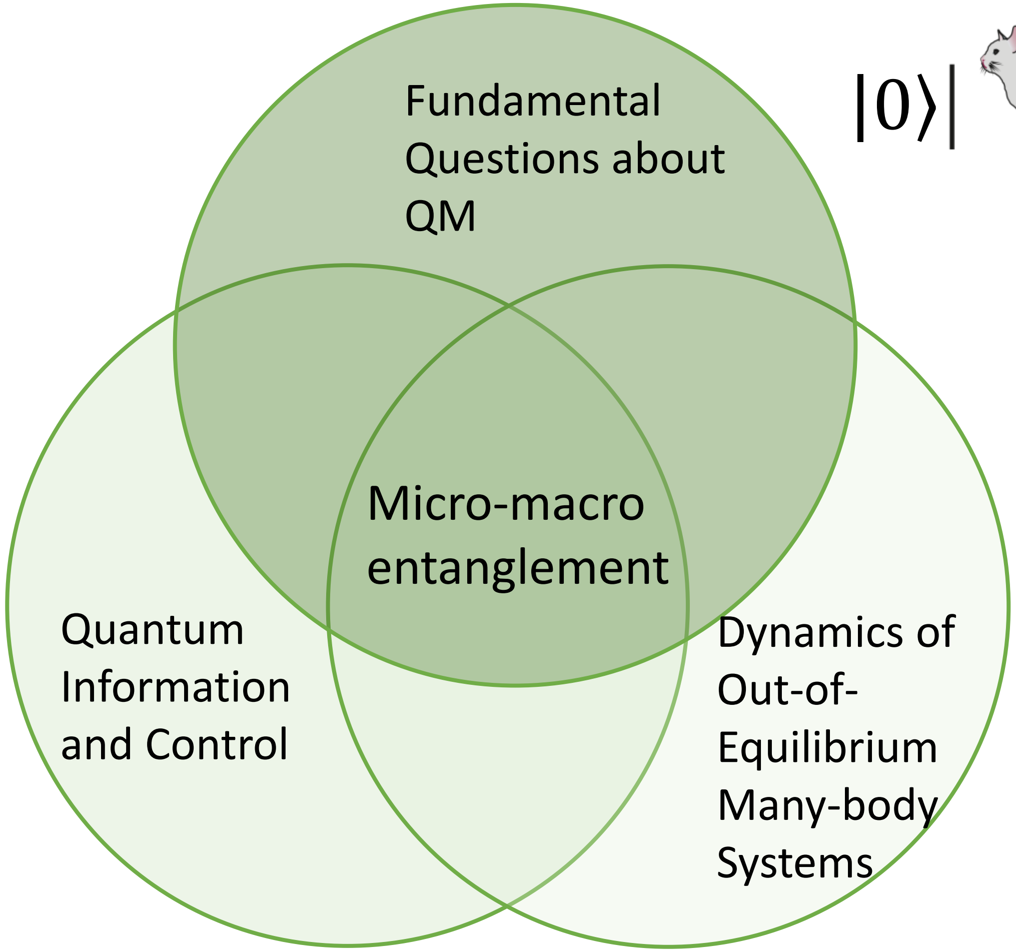

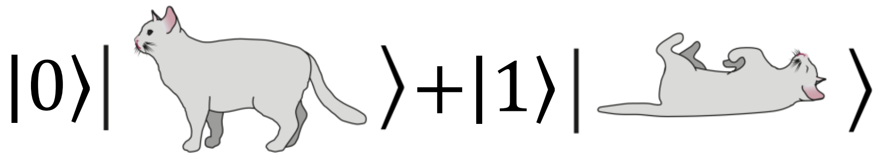

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➢ Formulations by Anthony Leggett (Nobel 2003)

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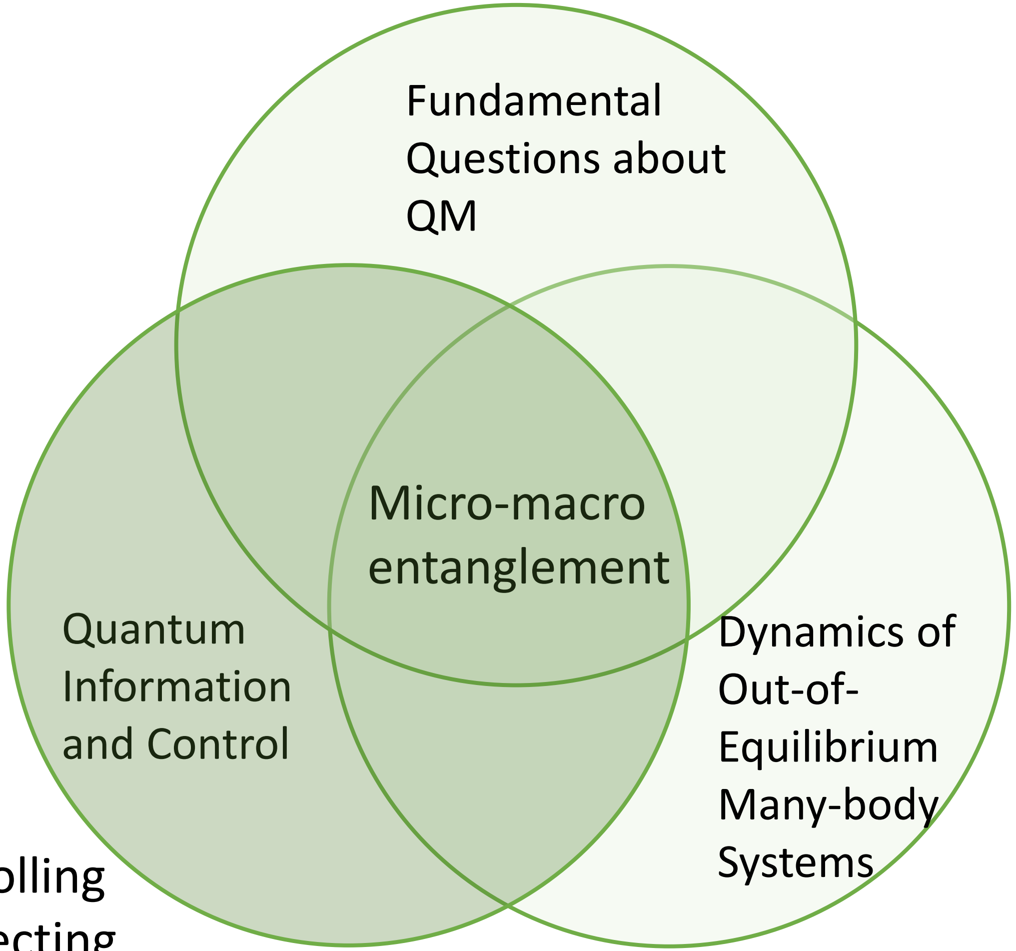
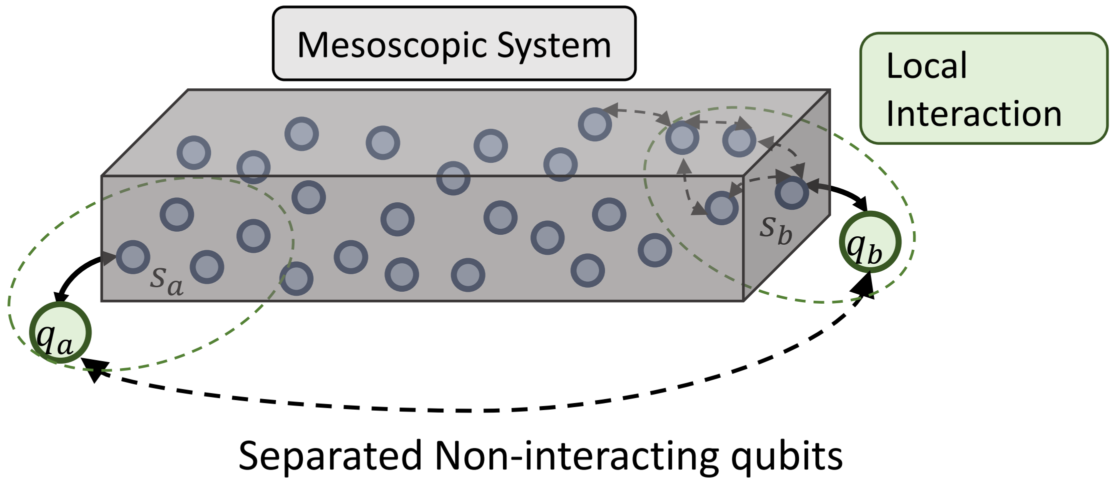


- New approaches to controlling qubits, in particular connecting non-interacting separated spin qubits

Why Micro-Macro Entanglement?

Quantum Information and Control



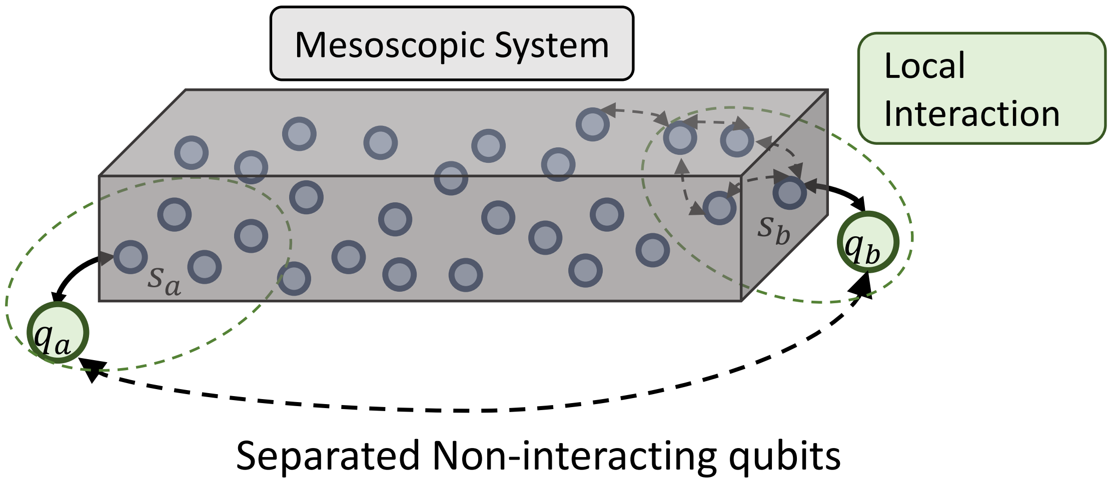
What are Mesoscopic Systems?

Intermediate size many-body systems that

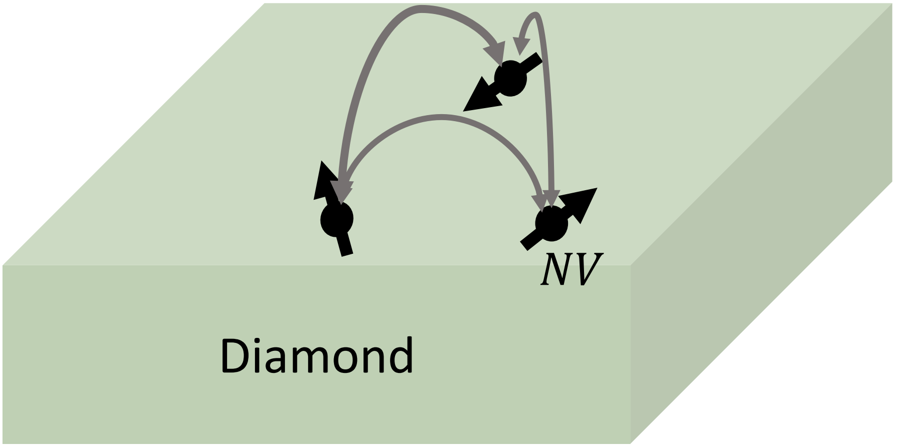
- Contain $10^3 - 10^{10}$ two-level systems e.g. spin-1/2 particles
- Exhibit collective quantum characteristics.
- Can be controlled and measured collectively.
- Lies on the boundary of quantum and classical

Why Micro-Macro Entanglement?

Quantum Information and Control



It is a challenging task of keeping the qubits isolated yet connected as desired.



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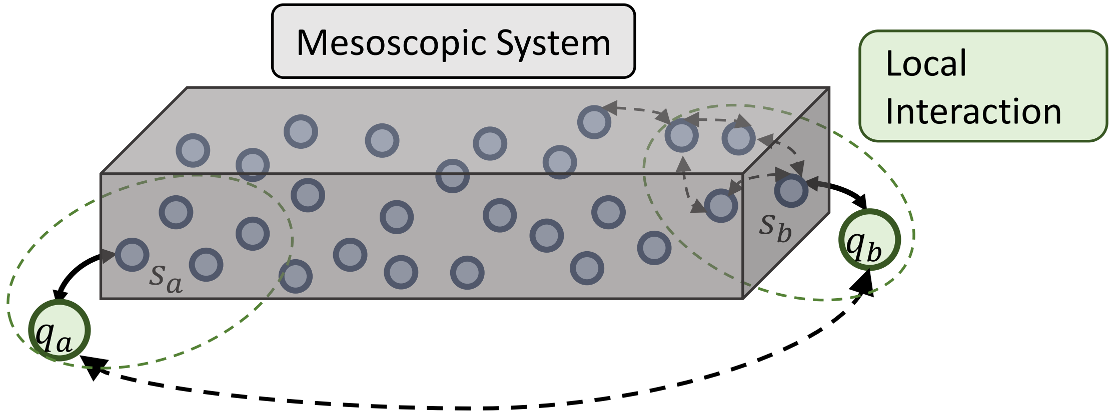
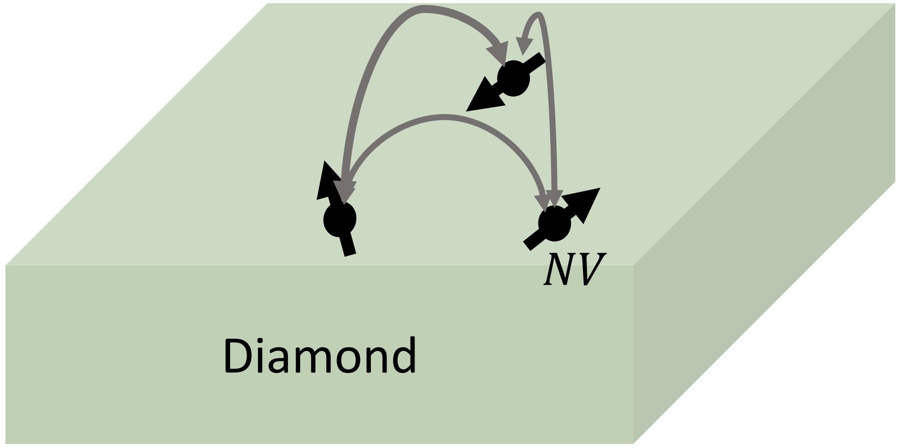
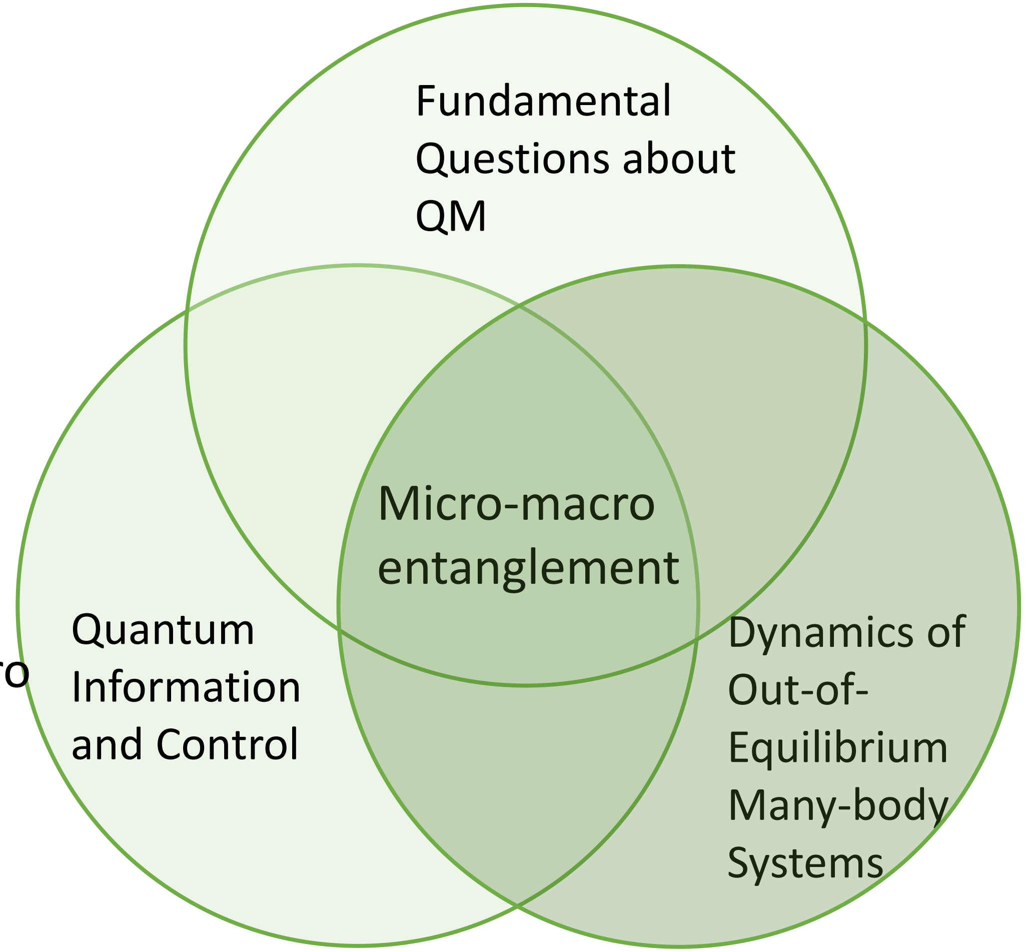




- Indirect Measurement
- Indirect CNOT Gate

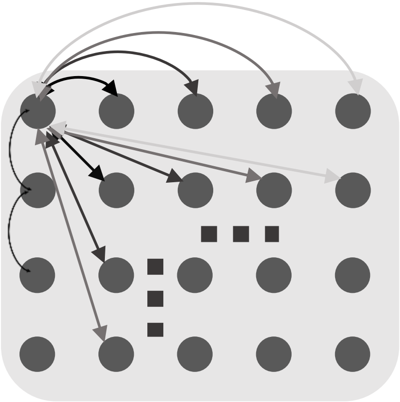
M. S. Mirkamali and D. G. Cory, US Patent 9792558 (2017)
M. S. Mirkamali and D. G. Cory, J. Emerson, doi.org/10.1103/PhysRevA.98.042327 (2018)
M. S. Mirkamali and D. G. Cory, doi.org/10.1103/PhysRevA.101.032320 (2020)

Why Micro-Macro Entanglement?



Generation of micro-macro entangled states with mesoscopic ***spin*** systems

Local interaction
+
Internal dipolar coupling

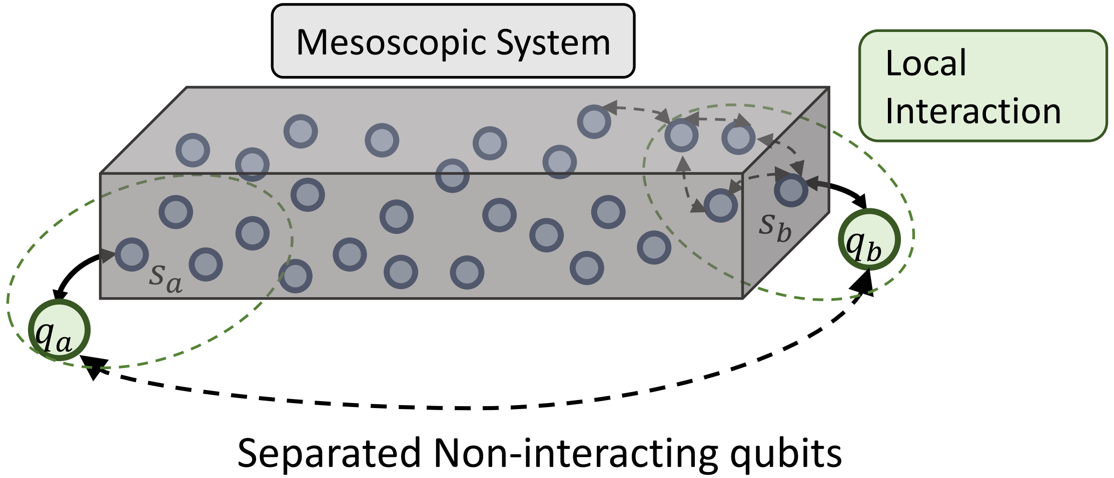


Short range coupling:
Lieb-Robinson bound, Light cone

E. H. Lieb and D. W. Robinson, Commun. Math. Phys. **28**, 251 (1972).

M. S. Mirkamali and D. G. Cory, doi.org/10.1103/PhysRevA.101.032320 (2020)

Useful Micro-Macro Entangled States

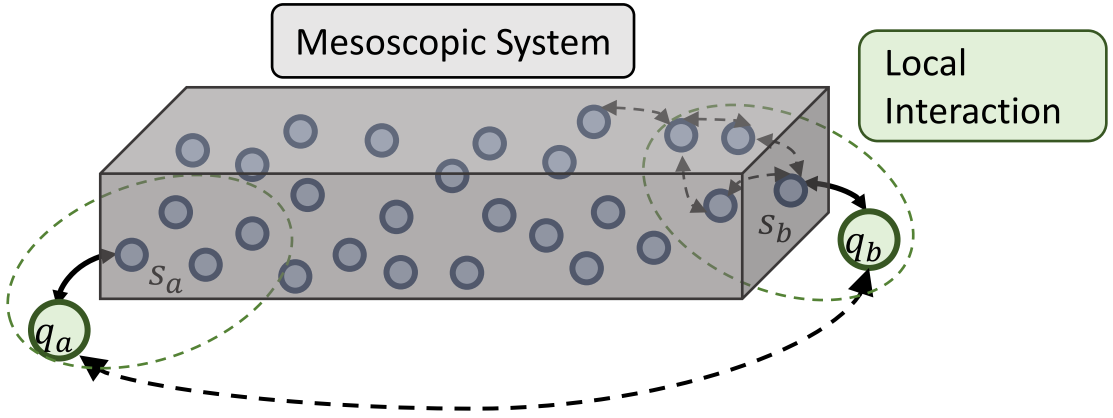


Task: Creating an entangled pair of target qubits

Method:

- Indirect Measurement
- Indirect CNOT Gate

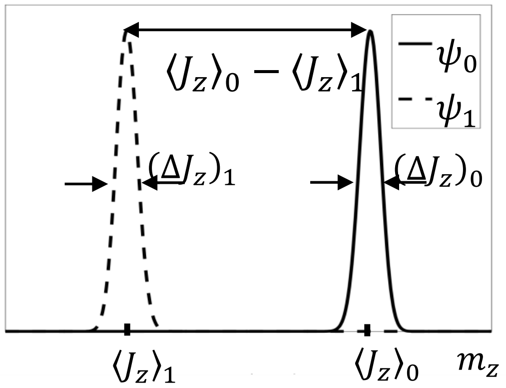
Useful Micro-Macro Entangled States



Task: Creating an entangled pair of target qubits

Method:
- Indirect Measurement
- Indirect CNOT Gate

The bigger the distinctness is, the closer the target qubits are to a maximally entangled state

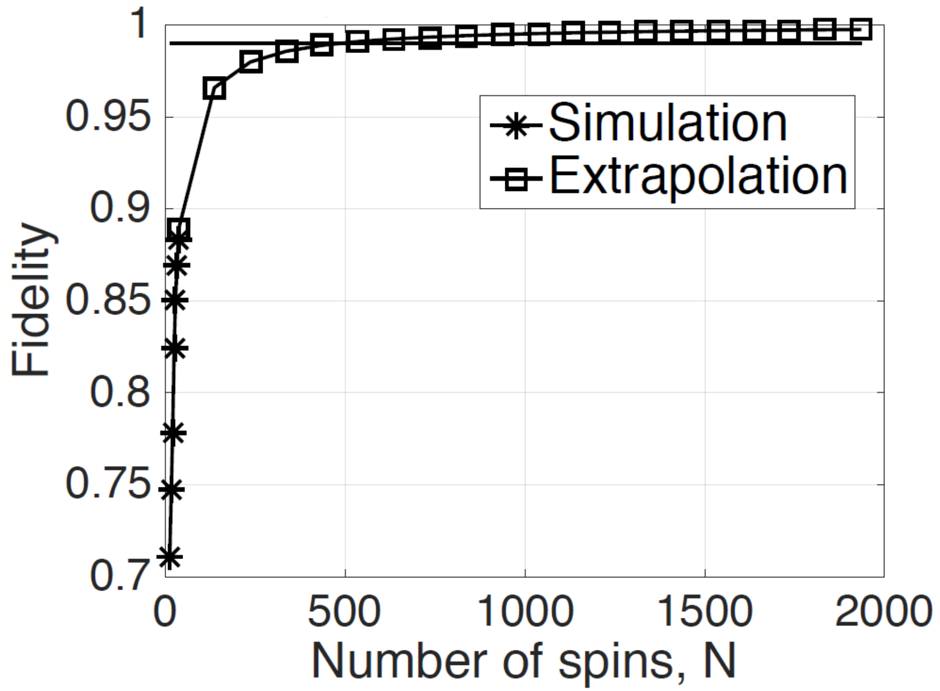


$$\textbf{distinctness} = \frac{\langle J_z\rangle_0 - \langle J_z\rangle_1}{(\Delta J_z)_0 + (\Delta J_z)_1}$$

More Distinct, More Useful

Useful Micro-Macro Entangled States

Based on Indirect Joint Measurement

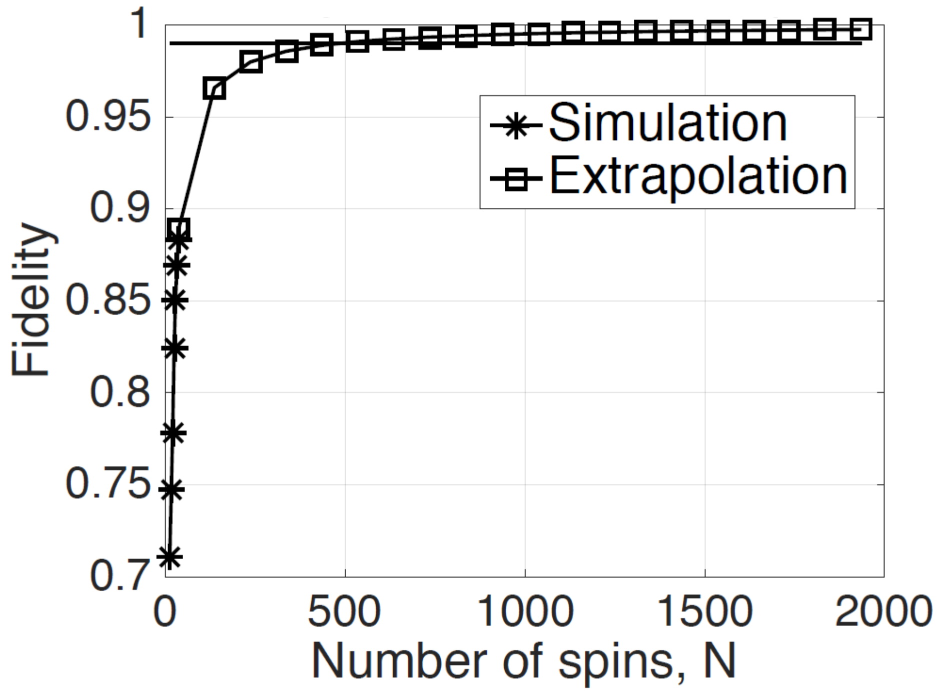


$$\text{Fidelity} = \langle \phi_{ideal} | \rho_q | \phi_{ideal} \rangle,$$

$$0 \leq \text{Fidelity} \leq 1$$

Useful Micro-Macro Entangled States

Based on Indirect Joint Measurement



$$\text{Fidelity} = \langle \phi_{ideal} | \rho_q | \phi_{ideal} \rangle,$$

$$0 \leq \text{Fidelity} \leq 1$$

Entanglement and distillability condition

$$\text{Fidelity} > \frac{1}{2}$$

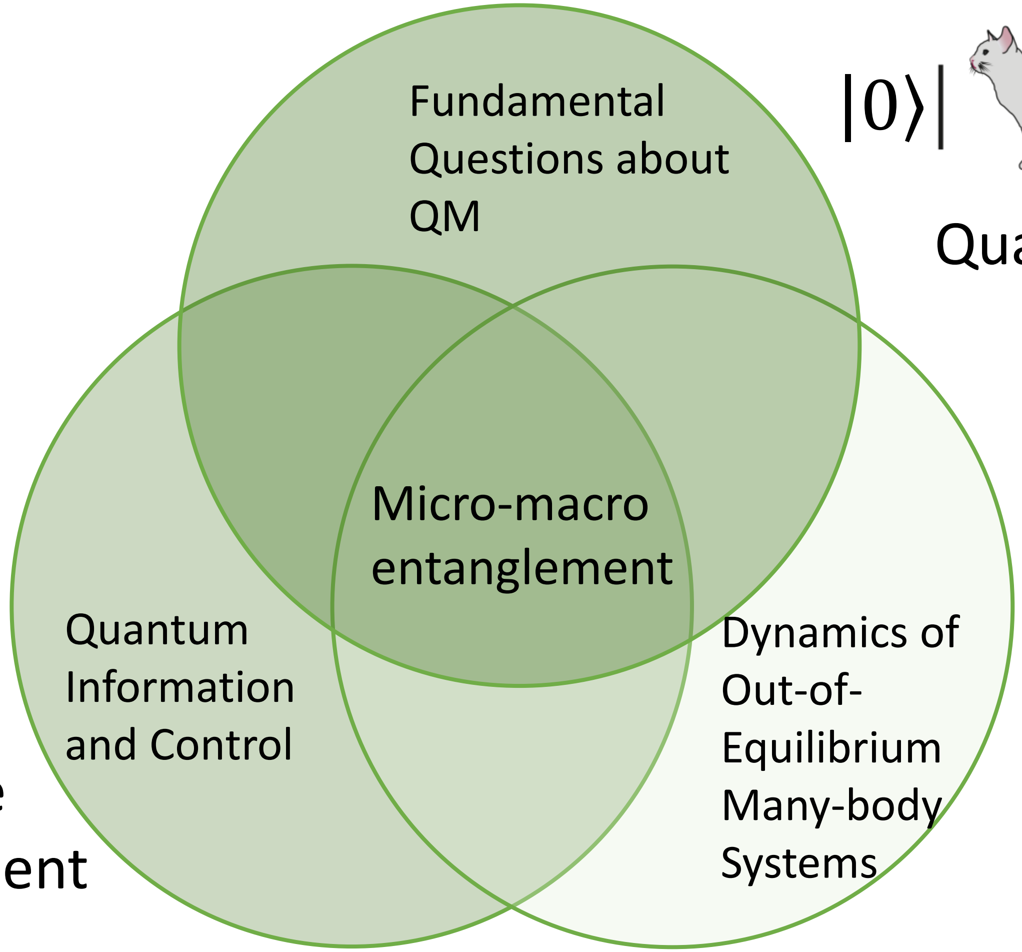
Enough entanglement for violation of Bell inequality

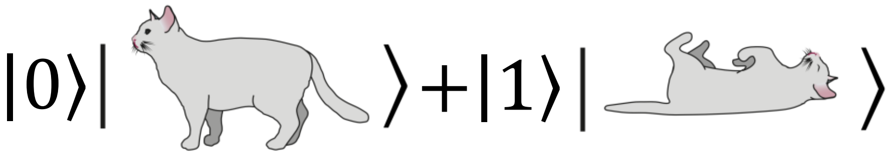
$$\text{Fidelity} > \frac{2 + 3\sqrt{2}}{8} \approx 0.78$$

Fidelity increases with N since for larger MSs,

- Macroscopic distinctness is larger,

Robustness vs Usefulness





Quantum to Classical Transition

- Indirect CNOT Gate
- Indirect Measurement

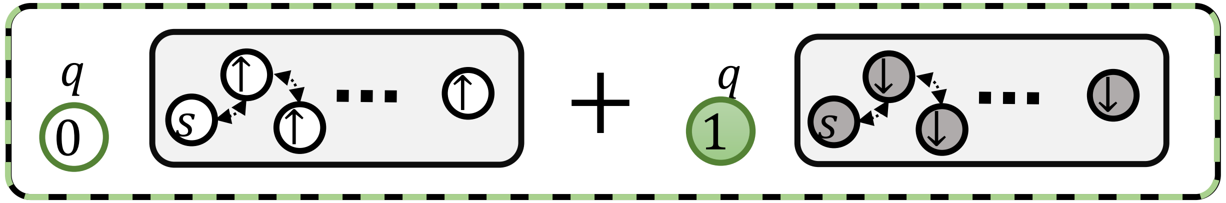
M. S. Mirkamali and D. G. Cory, *In preparation*

Robustness vs Usefulness

Fragility to microscopic noise

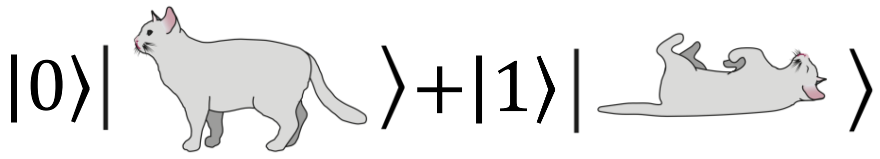
Micro-macro entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle|\psi_0\rangle + |1\rangle\,|\psi_1\rangle)$$



Macroscopic distinctness is robust
Entanglement is fragile

In the extreme case



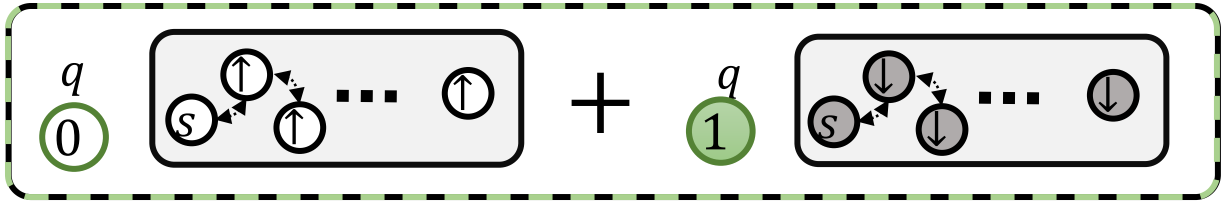
No entanglement can exist!

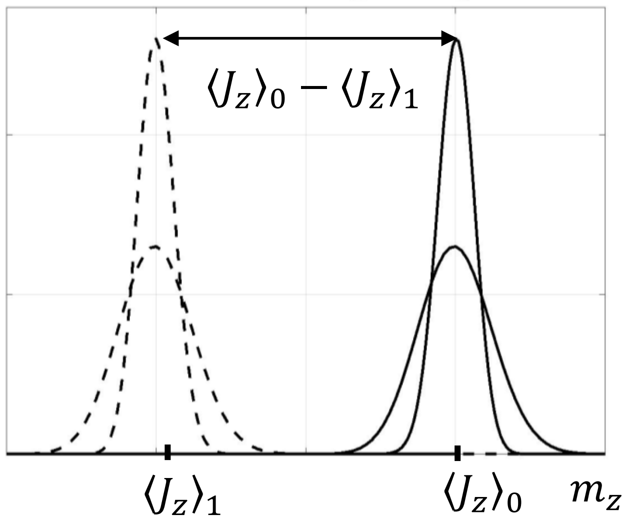
Robustness vs Usefulness

Fragility to microscopic noise

Micro-macro entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle|\psi_0\rangle + |1\rangle\,|\psi_1\rangle)$$

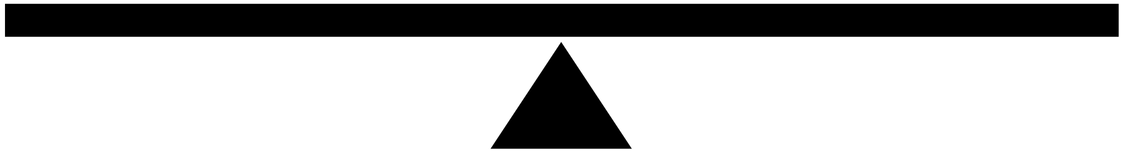




Macroscopic distinctness is robust
Entanglement is fragile

Robustness of **entanglement**
to particle loss/ T_1 relaxation

Usefulness
Macroscopic Distinctness

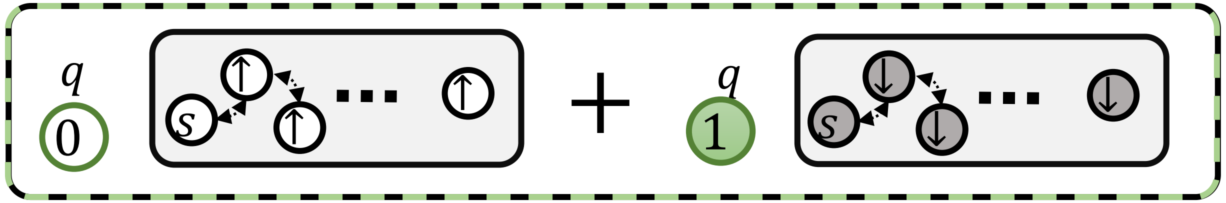


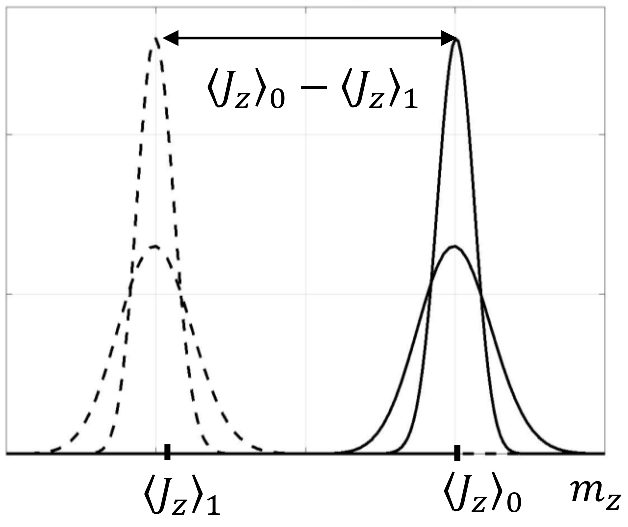
Robustness vs Usefulness

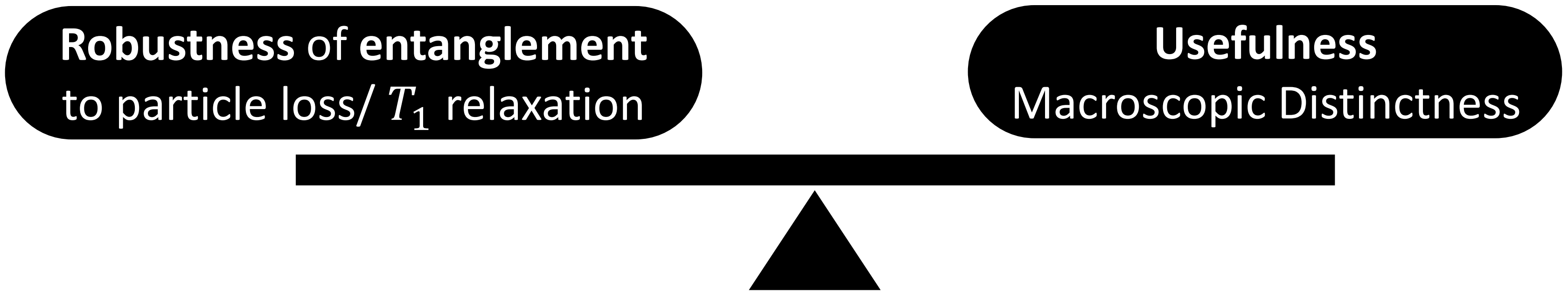
Fragility to microscopic noise

Remaining Entanglement after single particle loss as a function of Macroscopic Distinctness

Measure of Entanglement: $0 \leq$Logarithmic Negativity ≤ 1







Robustness vs Usefulness

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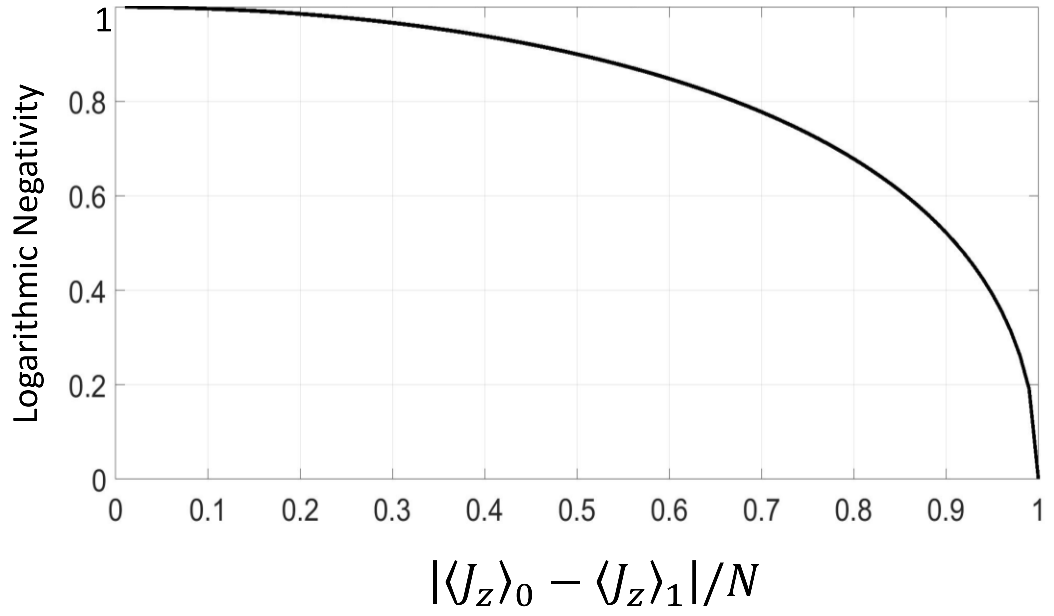
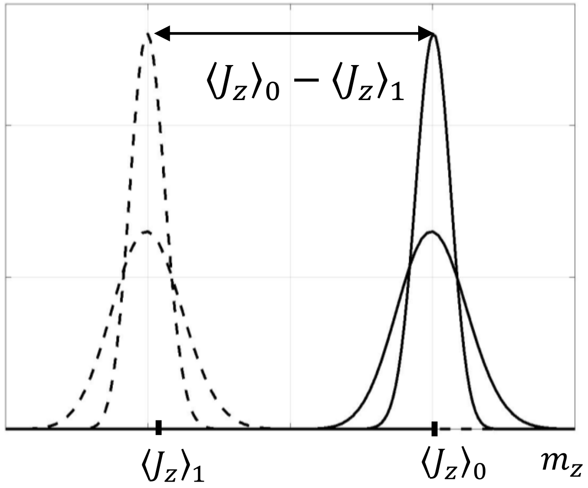
Upper bound on Robustness



- Difference in the means matters but the standard deviation does not



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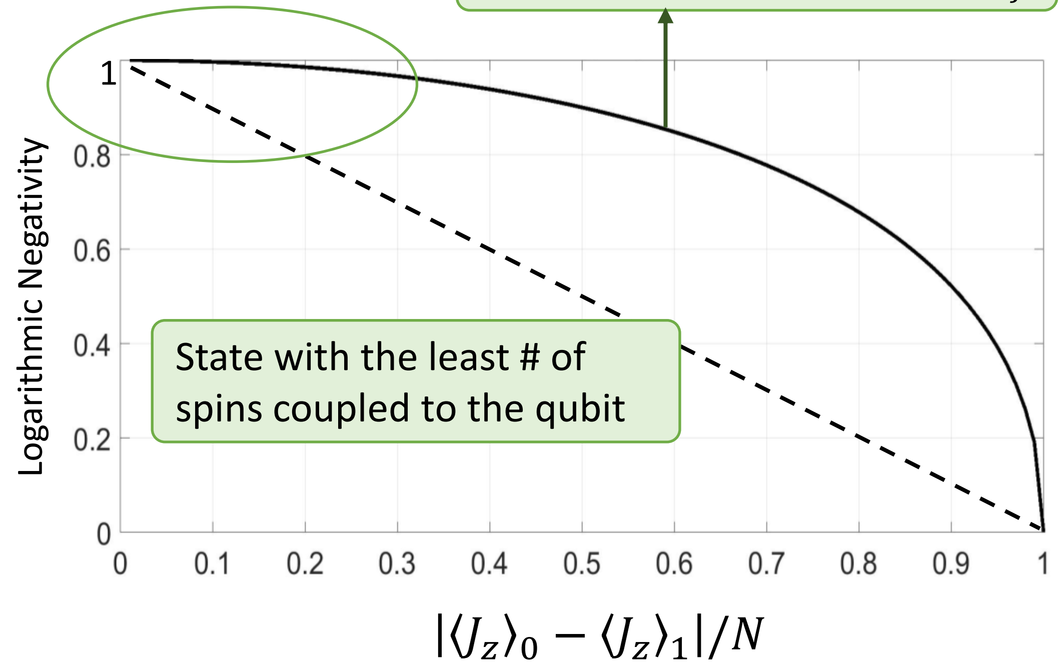
Robustness vs Usefulness

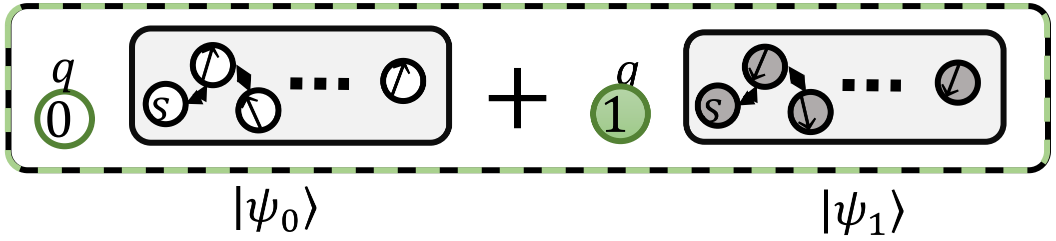
Remaining Entanglement after single particle loss as a function of Macroscopic Distinctness

Measure of Entanglement: Logarithmic Negativity

Upper bound
Symmetric states, $\langle\sigma_z\rangle_i = \langle\sigma_z\rangle_j$



State with the least # of spins coupled to the qubit



- Symmetric states are the most robust states
- Second order initial drop for symmetric states

An Interesting Open Question

How can one verify and quantify micro-macro entanglement?

✓ **Macroscopic Distinctness**

Measurement of the qubit’s state Followed by measurement of the MSS along the quantization access
Showing correlation between the outcomes

? Bipartite Entanglement

Distinguishing between entanglement and classical correlation

$$\frac{1}{\sqrt{2}}\left(|0\rangle_q|\uparrow\rangle^{\otimes N} + |1\rangle_q|\psi_1\rangle\right)$$

$$\frac{1}{2}\left(|0\rangle\langle 0|_q \otimes \rho_0 + |1\rangle\langle 1|_q \otimes \rho_1\right)$$

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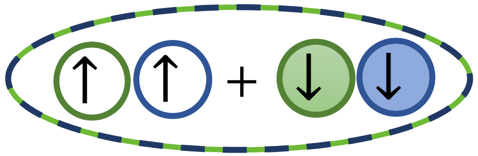

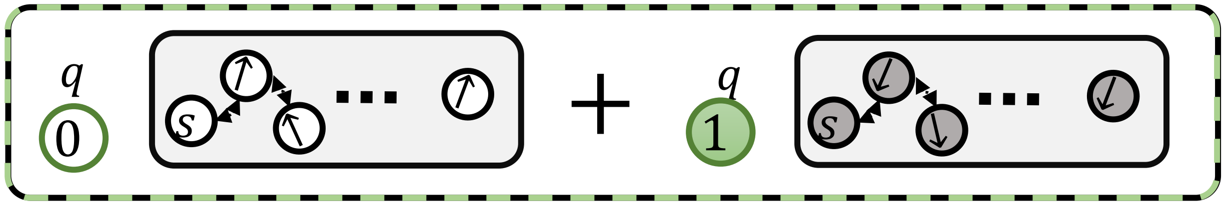

Verification: Bell inequality and in general Entanglement witnesses

Quantification: State tomography and measure like concurrence/negativity

What entanglement witnesses? Needs to be compatible with limitations (Available collective measurement)

State tomography is not a choice! (Negativity can't be measured)

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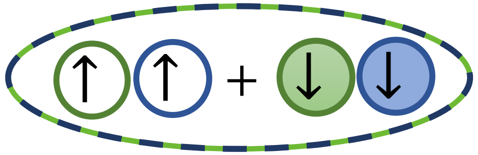
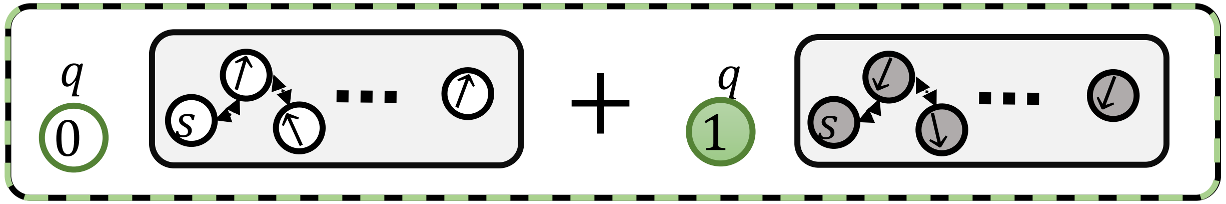
? Bipartite Entanglement





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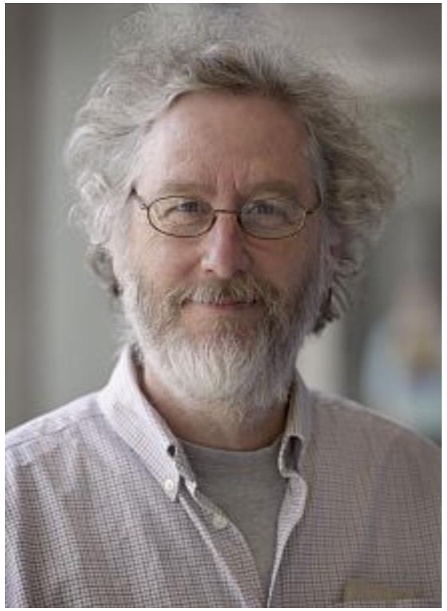
Does our quantum control approach provide a solution?

Entanglement of the target qubits $\overset{?}{\Rightarrow}$ micro-macro entanglement

UNIVERSITY OF WATERLOO
FACULTY OF SCIENCE
Department of Physics & Astronomy

IQC Institute *for* Quantum Computing

Thank you



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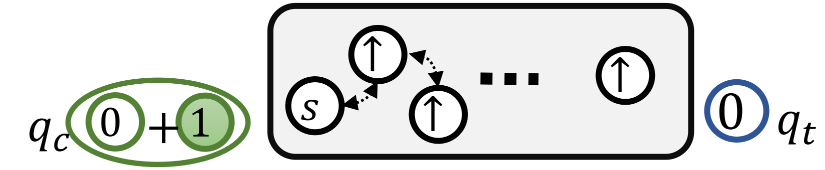
Entangling by Indirect CNOT gate

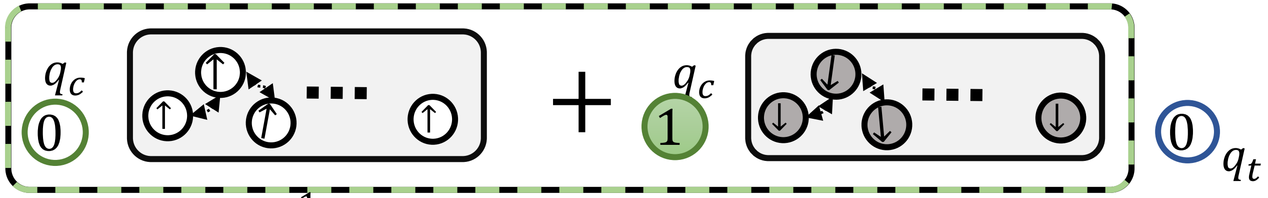
1. Preparation



$|\psi_{q,\mathrm{MS}}\rangle = |+\rangle_{q_c} \otimes \quad |\psi_{in}\rangle_{\mathrm{MS}} \otimes \quad |0\rangle_{q_t}$

2. Coherent Magnification

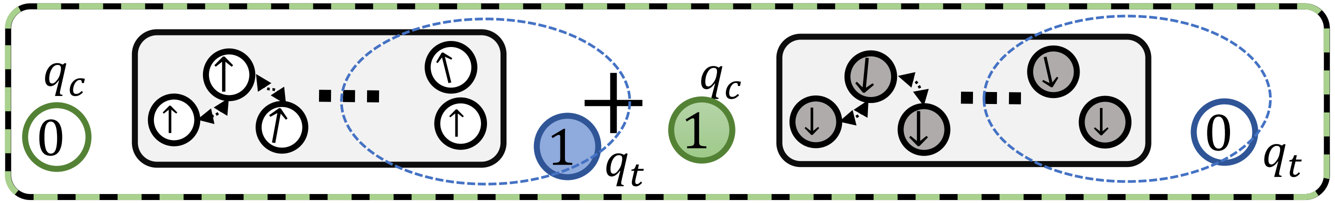
Creating **micro-macro entanglement**:
Spins in MS coherently learn the state of the qubit q_L



$\frac{1}{\sqrt{2}}\left(|0\rangle_{q_c}|\psi_0\rangle_{\mathrm{MS}} + |1\rangle_{q_c}|\psi_1\rangle_{\mathrm{MS}}\right) \otimes \quad |0\rangle_{q_t}$

3. Local Interaction with q_t

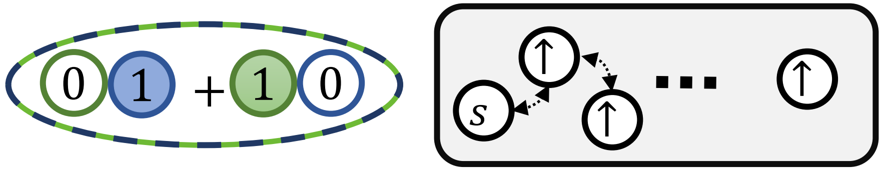
Entangling with the qubit q_t:
q_t coherently learn the state of the qubit q_c



$1/\sqrt{2}\left(|0\rangle_{q_c}|\psi_0\rangle_{\mathrm{MS}}|1\rangle_{q_t} + |1\rangle_{q_c}|\psi_1\rangle_{\mathrm{MS}}|0\rangle_{q_t}\right)$

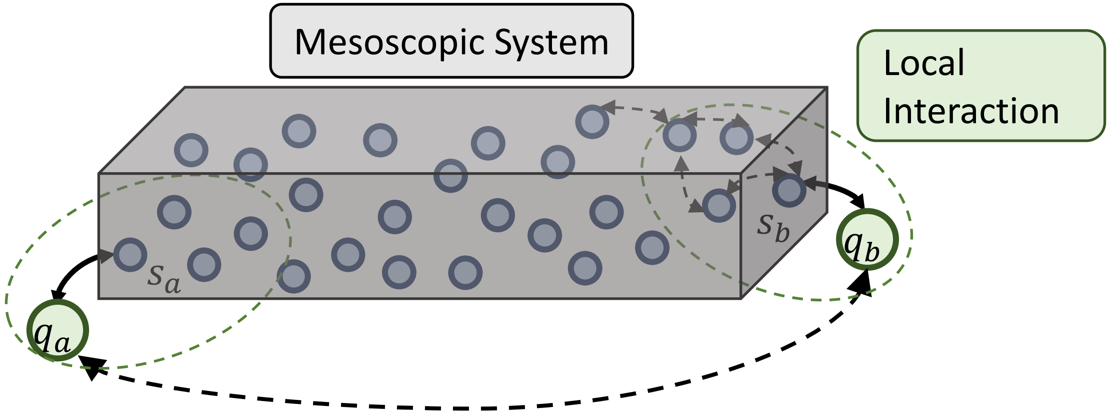
4. Disentangling the MS

Quantum Eraser



$\frac{1}{\sqrt{2}}\left(|0\rangle_{q_c}|1\rangle_{q_t} + |1\rangle_{q_c}|0\rangle_{q_t}\right) \otimes \quad |\psi_{in}\rangle_{\mathrm{MS}}$

Available Control



Each Target Qubit
- Initialization
- Control
- Measurement

- Static, Do not move

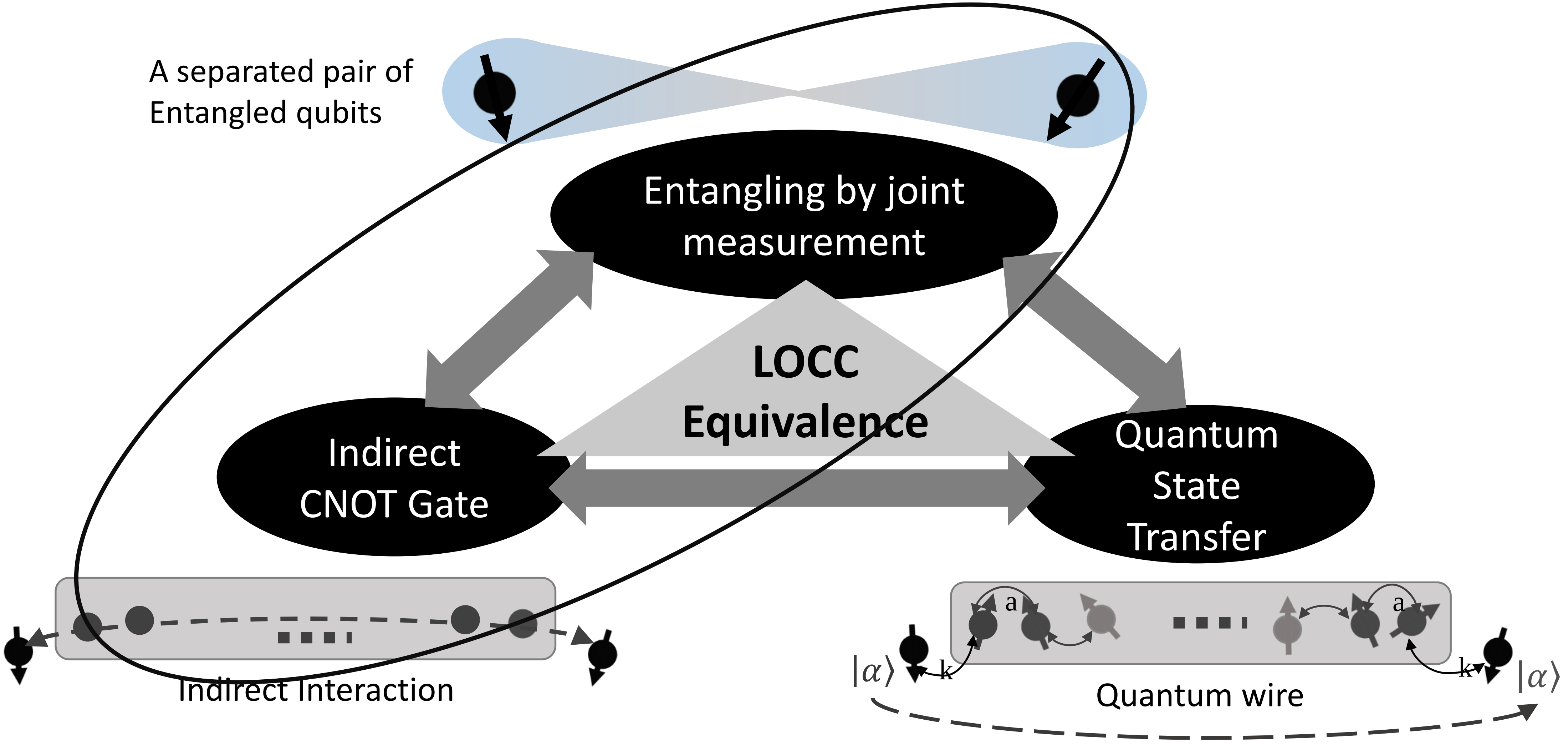
Mesoscopic System
- Initialization (high polarization)
- Collective Control
- Global Measurement
- Naturally Available Internal Interaction among the Spins

- No Control or Measurement of Individual Spins
- No Tuning of Internal Interaction

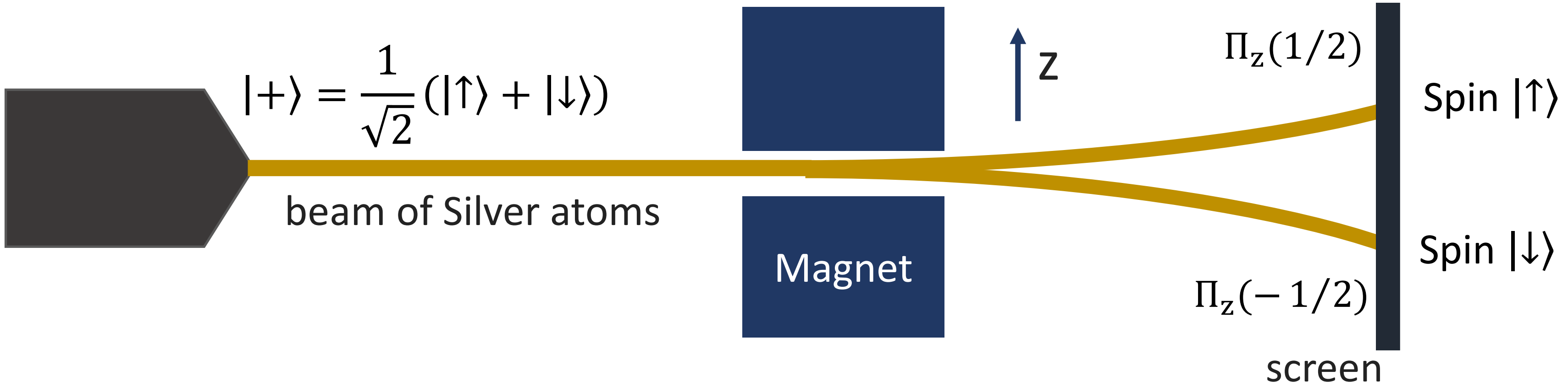
Qubit-MS Interaction
- Local Interaction
- Universal control over each qubit and a nearby spin from the MS

✓ General Mesoscopic system
✓ Mesoscopic **Spin** System

Mesoscopic Systems as Control Elements



Entangling by Joint Measurement



Projective Measurement → **State preparation**

q_a: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

q_b: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Hamming W. Measurement

$$\begin{cases} |h_0\rangle = |00\rangle & \text{p=1/4} \\ |h_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) & \text{p=1/2} \\ |h_2\rangle = |11\rangle & \text{p=1/4} \end{cases}$$

Single excitation

Triplet zero

q_a: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

q_b: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Parity Measurement

$$\begin{cases} |e\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) & \text{p=1/2} \\ |o\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) & \text{p=1/2} \end{cases}$$

Even Parity

Odd Parity

Indirect Joint Measurement

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\begin{cases} |h_0\rangle = |00\rangle & \text{p=1/4} \\ |h_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) & \text{p=1/2} \\ |h_2\rangle = |11\rangle & \text{p=1/4} \end{cases}$$

Single excitation

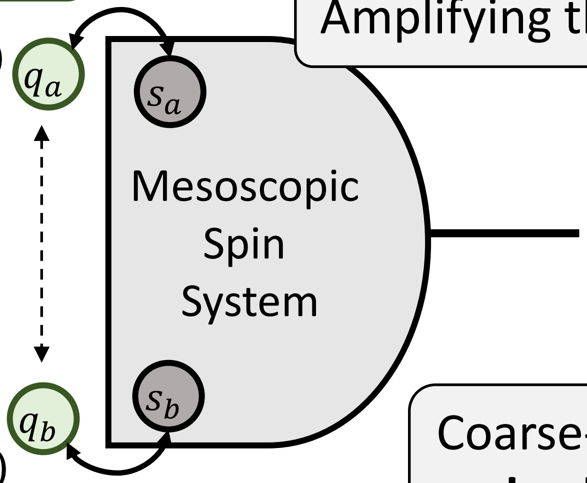
Triplet zero

Needs to detect **a single qubit flip**.

Local Interaction

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Amplifying the joint state



$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\begin{cases} |h_0\rangle = |00\rangle & \text{p=1/4} \\ |h_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) & \text{p=1/2} \\ |h_2\rangle = |11\rangle & \text{p=1/4} \end{cases}$$

Single excitation

Coarse-Grained Collective Measurement
only able to detect many spin flips

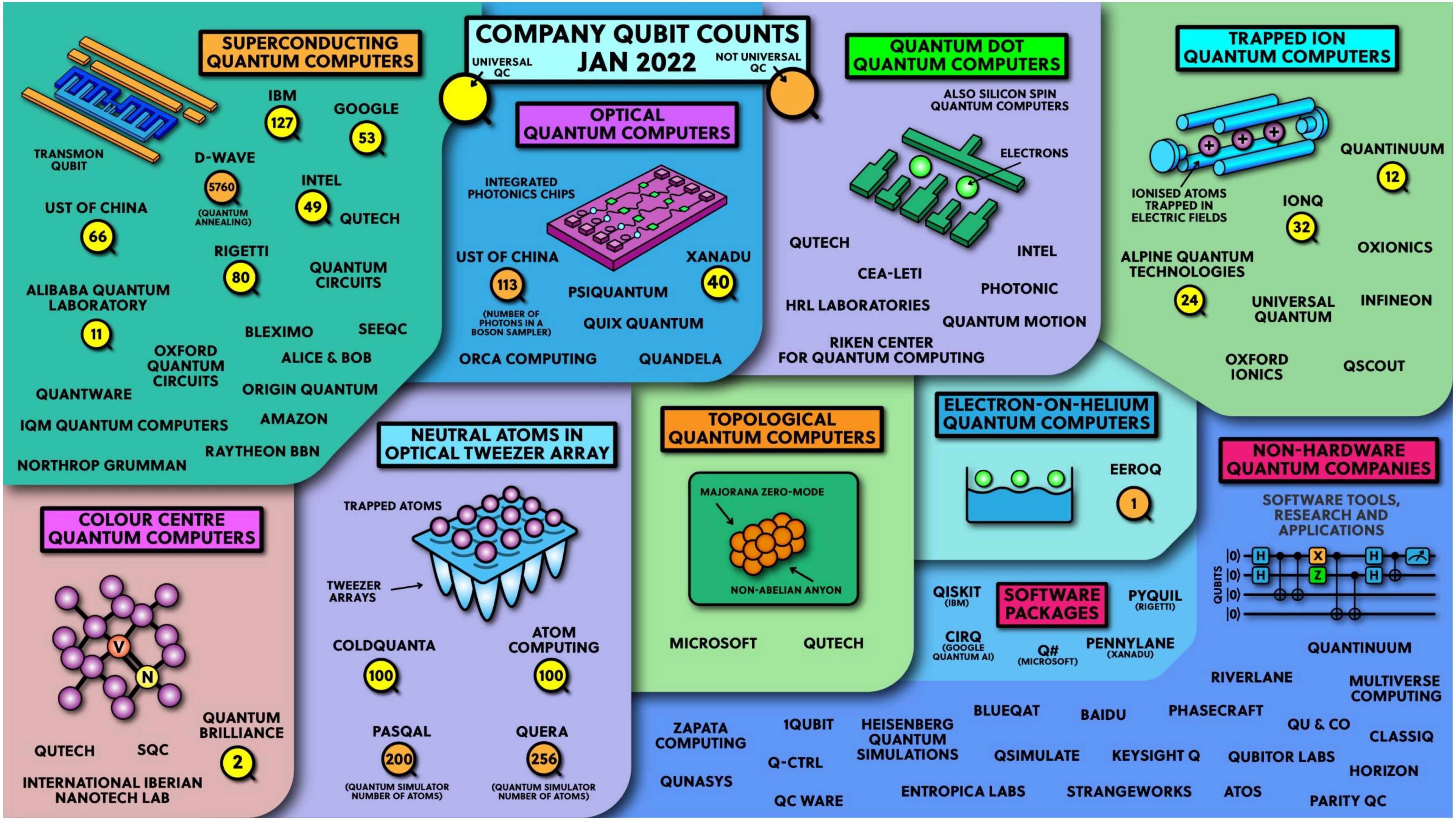


Photo by Dominic Walliman, https://dominicwalliman.com/