

The effects of non-helical component of hypermagnetic field on the evolution of the matter-antimatter asymmetry, vorticity, and hypermagnetic field

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- Introduction
- Chiral Vortical Effect (CVE) and Chiral Magnetic Effect (CME)
- Anomalous Magneto Hydro Dynamics (AMHD)
- Non Helical Hyper Magnetic Field (NHHMF)
- Results

Magnetic field:

- The Universe is magnetized on all scales that we have observed so far, planets, stars, galaxies, and galaxy clusters.
- galaxies $\rightarrow B \sim 10^{-6} G$ &, $\lambda_c \sim kpc$.
- Intra-cluster medium (ICM) in clusters $\rightarrow B \sim (1 - 10)10^{-6} G$ &, $\lambda_c \sim (10 - 100)kpc$.
- Intergalactic medium (IGM) $\rightarrow B \sim (10^{-16} - 10^{-9})G$ &, $\lambda_c \sim Mpc$.

Anomalous current:

- **CME:** The chiral magnetic effect refers to the generation of an electric current parallel to the magnetic field in an imbalanced chiral plasma, $J_{\text{cm},r}^{\mu} = \frac{rQ_r^2}{4\pi^2} \mu_r \vec{B}_Y$.
- **CVE:** The chiral vortical effect, generically induced by the rotation of chiral matter, refers to the generation of an electric current parallel to the vorticity field, $\vec{J}_{\text{cv},r} = rQ_r \left(\frac{T^2}{24} + \frac{\mu_r^2}{8\pi^2} \right) \vec{\omega}$.

Anomaly + Magneto Hydro Dynamic (MHD)
⇒ Anomalous Magneto Hydro Dynamic (AMHD).

Anomalous Magnetohydrodynamics:

The energy-momentum tensor $T^{\mu\nu}$ and the total electric current J^μ are given by

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - F^{\nu\sigma}F^\mu{}_\sigma + \tau^{\mu\nu}, \quad (1)$$

$$J^\mu = \rho_{\text{el}}u^\mu + J_{\text{cm}}^\mu + J_{\text{cv}}^\mu + \nu^\mu, \quad (2)$$

$$J_{\text{cm}}^\mu = \sum_{i=l,q} (Q_{R_i} \xi_{B,R_i} + Q_{L_i} \xi_{B,L_i}) B^\mu = c_B B^\mu, \quad (3)$$

$$J_{\text{cv}}^\mu = \sum_{i=l,q} (Q_{R_i} \xi_{v,R_i} + Q_{L_i} \xi_{v,L_i}) \omega^\mu = c_v \omega^\mu \quad (4)$$

$$\nu^\mu = \sigma E^\mu \quad (5)$$

where, $u^\mu = \gamma(1, \vec{v}/R)$ is the four-velocity of the plasma normalized such that $u^\mu u_\mu = 1$. ν^μ and $\tau^{\mu\nu}$ denote the electric diffusion current and viscous stress tensor, respectively. $B^\mu = (\epsilon^{\mu\nu\rho\sigma}/2R^3)u_\nu F_{\rho\sigma}$ is the magnetic field four-vector, $E^\mu = F^{\mu\nu}u_\nu$ is the electric field four-vector, and $\omega^\mu = (\epsilon^{\mu\nu\rho\sigma}/R^3)u_\nu \nabla_\rho u_\sigma$ is the vorticity four-vector, with the totally anti-symmetric Levi-Civita tensor density specified by $\epsilon^{0123} = -\epsilon_{0123} = 1$.

$$\xi_{B,R} = \frac{Q_R \mu_R}{4\pi^2}, \quad (6)$$

$$\xi_{B,L} = -\frac{Q_L \mu_L}{4\pi^2}, \quad (7)$$

$$\xi_{v,R} = \frac{\mu_R^2}{8\pi^2} + \frac{1}{24} T^2, \quad (8)$$

$$\xi_{v,L} = -\frac{\mu_L^2}{8\pi^2} - \frac{1}{24} T^2. \quad (9)$$

The equations of AMHD consist of:

- The Maxwell's equations:

$$\begin{aligned}\nabla_{\mu} F^{\mu\nu} &= J^{\nu}, & F_{\mu\nu} &= \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu} \\ \nabla_{\mu} \tilde{F}^{\mu\nu} &= 0, & \tilde{F}^{\mu\nu} &= \frac{1}{2R^3} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta},\end{aligned}\tag{10}$$

- The energy-momentum conservation

$$\nabla_{\mu} T^{\mu\nu} = 0,\tag{11}$$

- Anomaly equations:

$$\nabla_{\mu} j_{R,L}^{\mu} = C_{R,L} F_{\mu\nu} \tilde{F}^{\mu\nu}\tag{12}$$

$$\begin{aligned}j_R^{\mu} &= n_R u^{\mu} + \xi_{B,R} B^{\mu} + \xi_{v,R} \omega^{\mu} + V_R^{\mu} \simeq n_R u^{\mu}, \\ j_L^{\mu} &= n_L u^{\mu} + \xi_{B,L} B^{\mu} + \xi_{v,L} \omega^{\mu} + V_L^{\mu} \simeq n_L u^{\mu},\end{aligned}\tag{13}$$

Maxwell's equations: $\nabla_\mu F^{\mu\nu} = J^\nu$, $\nabla_\mu \tilde{F}^{\mu\nu} = 0$

$$\frac{1}{R} \vec{\nabla} \cdot \vec{E}_Y = 0, \quad \frac{1}{R} \vec{\nabla} \cdot \vec{B}_Y = 0, \quad (14)$$

$$\frac{1}{R} \vec{\nabla} \times \vec{E}_Y + \left(\frac{\partial \vec{B}_Y}{\partial t} + 2H \vec{B}_Y \right) = 0, \quad (15)$$

$$\frac{1}{R} \vec{\nabla} \times \vec{B}_Y - \left(\frac{\partial \vec{E}_Y}{\partial t} + 2H \vec{E}_Y \right) = \vec{J} \quad (16)$$

$$\vec{J} = \underbrace{\sigma (\vec{E}_Y + \vec{v} \times \vec{B}_Y)}_{\vec{J}_{\text{Ohm}}} + \underbrace{c_B \vec{B}_Y}_{\vec{J}_{\text{cm}}} + \underbrace{c_v \vec{\omega}}_{\vec{J}_{\text{cv}}}, \quad (17)$$

$$\vec{E}_Y = \frac{1}{\sigma R} \vec{\nabla} \times \vec{B}_Y - \frac{c_v}{\sigma} \vec{\omega} - \frac{c_B}{\sigma} \vec{B}_Y - \vec{v} \times \vec{B}_Y. \quad (18)$$

$$\frac{\partial \vec{B}_Y}{\partial t} = \frac{1}{\sigma R^2} \nabla^2 \vec{B}_Y + \frac{c_v}{\sigma R} \vec{\nabla} \times \vec{\omega} + \frac{c_B}{\sigma R} \vec{\nabla} \times \vec{B}_Y + \frac{1}{R} \vec{\nabla} \times (\vec{v} \times \vec{B}_Y) - \frac{\vec{B}_Y}{t}, \quad (19)$$

CVE and CME coefficients:

$$c_v(t) = \sum_{i=1}^{n_G} \left[\frac{g'}{48} \left(-Y_R T_{R_i}^2 + Y_L T_{L_i}^2 N_w - Y_{d_R} T_{d_{R_i}}^2 N_c - Y_{u_R} T_{u_{R_i}}^2 N_c + Y_Q T_{Q_i}^2 N_c N_w \right) + \frac{g'}{16\pi^2} \left(-Y_R \mu_{R_i}^2 + Y_L \mu_{L_i}^2 N_w - Y_{d_R} \mu_{d_{R_i}}^2 N_c - Y_{u_R} \mu_{u_{R_i}}^2 N_c + Y_Q \mu_{Q_i}^2 N_c N_w \right) \right], \quad (20)$$

$$c_B(t) = -\frac{g'^2}{8\pi^2} \sum_{i=1}^{n_G} \left[-\left(\frac{1}{2}\right) Y_R^2 \mu_{R_i} - \left(\frac{-1}{2}\right) Y_L^2 \mu_{L_i} N_w - \left(\frac{1}{2}\right) Y_{d_R}^2 \mu_{d_{R_i}} N_c - \left(\frac{1}{2}\right) Y_{u_R}^2 \mu_{u_{R_i}} N_c - \left(\frac{-1}{2}\right) Y_Q^2 \mu_{Q_i} N_c N_w \right], \quad (21)$$

$$Y_L = -1, \quad Y_R = -2, \quad Y_Q = \frac{1}{3}, \quad Y_{u_R} = \frac{4}{3}, \quad Y_{d_R} = -\frac{2}{3}. \quad (22)$$

$$c_v(t) = \frac{g'}{8\pi^2} \left(\mu_{e_R}^2 - \mu_{e_L}^2 \right), \quad c_B(t) = -\frac{g'^2}{8\pi^2} \left(-2\mu_{e_R} + \mu_{e_L} - \frac{3}{4}\mu_B \right), \quad (23)$$

Continuity and Navier-Stokes equations: $\nabla_{\mu} T^{\mu\nu} = 0$

$$\frac{\partial \rho}{\partial t} + \frac{1}{R} \vec{\nabla} \cdot [(\rho + p) \vec{v}] + 3H(\rho + p) = 0, \quad (24)$$

$$\left[\frac{\partial}{\partial t} + \frac{1}{R} (\vec{v} \cdot \vec{\nabla}) + H \right] \vec{v} + \frac{\vec{v}}{\rho + p} \frac{\partial p}{\partial t} =$$

$$- \frac{1}{R} \frac{\vec{\nabla} p}{\rho + p} + \frac{\vec{J} \times \vec{B}_Y}{\rho + p} + \frac{\nu}{R^2} \left[\nabla^2 \vec{v} + \frac{1}{3} \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) \right], \quad (25)$$

$$\Rightarrow \frac{\partial \vec{v}}{\partial t} = \frac{\vec{J} \times \vec{B}_Y}{\rho + p} + \frac{\nu}{R^2} \nabla^2 \vec{v}, \quad (26)$$

$$\nu \simeq 1/(5\alpha_Y T)$$

Anomaly equations: $\nabla_{\mu} j_{R,L}^{\mu} = C_{R,L} F_{\mu\nu} \tilde{F}^{\mu\nu}$,

$$\Rightarrow \partial_t \eta_{R,L} = \frac{-4C_{R,L}}{s} \langle \vec{E}_Y \cdot \vec{B}_Y \rangle, \quad (27)$$

where $\eta_i = (n_i - \bar{n}_i)/s$, $s = (2\pi^2 g^* T^3)/45$ is entropy, and $g^* = 106.75$ is the number of relativistic degrees of freedom.

The non-helical hypermagnetic field, vorticity and asymmetry

Since $\vec{\nabla} \cdot \vec{B}_Y = 0$, the hypermagnetic field can be written as $\vec{B}_Y = (1/R)\vec{\nabla} \times \vec{A}_Y$, where \vec{A}_Y is the vector potential of the hypermagnetic field. For incompressible fluid, $\vec{\nabla} \cdot \vec{v} = 0$, in analogy with the hypermagnetic field, the velocity field can be written as $\vec{v} = (1/R)\vec{\nabla} \times \vec{S}$, where \vec{S} is the vector potential of the velocity field. Now we choose the configurations for our hypermagnetic field and the velocity field by using the following orthonormal basis $\{\hat{a}(z, k) = (\cos kz, -\sin kz, 0)$, $\hat{b}(z, k) = (\sin kz, \cos kz, 0)$, $\hat{z}\}$.

$$\vec{B}_Y(t, z) = B_z(t)\hat{z} + B_a(t)\hat{a}(z, k) + B_b(t)\hat{b}(z, k), \quad (28)$$

$$\vec{v}(t, z) = v_a(t)\hat{a}(z, k) + v_b(t)\hat{b}(z, k), \quad (29)$$

$$\frac{1}{R}\vec{\nabla} \times (\vec{v} \times \vec{B}_Y) \neq 0, \quad \text{and} \quad \frac{\vec{J} \times \vec{B}_Y}{\rho + p} \neq 0 \quad (30)$$

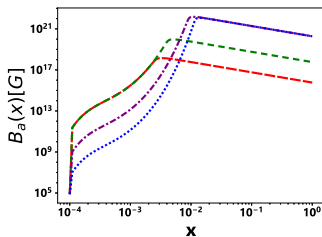
$$\begin{aligned} \langle \vec{E}_Y \cdot \vec{B}_Y \rangle &= -\frac{c_B}{\sigma} [B_z^2(t) + B_a^2(t) + B_b^2(t)] + \frac{k'}{\sigma} [B_a^2(t) + B_b^2(t)] \\ &\quad - \frac{c_v}{\sigma} k' [v_a(t)B_a(t) + v_b(t)B_b(t)]. \end{aligned} \quad (31)$$

$$\begin{aligned}\frac{\partial B_a(t)}{\partial t} &= k' [v_b(t)B_z(t)] + \left[-\frac{k'^2}{\sigma} + \frac{k'c_B}{\sigma} \right] B_a(t) + \frac{k'^2 c_v}{\sigma} v_a(t) - \frac{B_a(t)}{t}, \\ \frac{\partial B_b(t)}{\partial t} &= -k' [v_a(t)B_z(t)] + \left[-\frac{k'^2}{\sigma} + \frac{k'c_B}{\sigma} \right] B_b(t) + \frac{k'^2 c_v}{\sigma} v_b(t) - \frac{B_b(t)}{t}, \\ \frac{\partial B_z(t)}{\partial t} &= -\frac{B_z(t)}{t}, \\ \frac{\partial v_a(t)}{\partial t} &= \frac{k'}{\rho + p} [B_b(t)B_z(t)] - k'^2 \nu v_a(t), \\ \frac{\partial v_b(t)}{\partial t} &= -\frac{k'}{\rho + p} [B_a(t)B_z(t)] - k'^2 \nu v_b(t).\end{aligned}\tag{32}$$

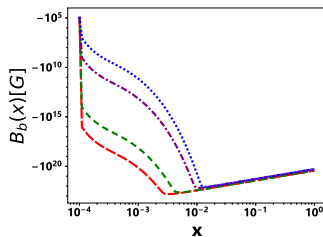
$$\begin{aligned}\frac{d\eta_{eR}}{dt} &= \frac{g'^2}{4\pi^2 s} \langle \vec{E}_Y \cdot \vec{B}_Y \rangle + \left(\frac{\Gamma_0}{t_{EW}} \right) \left(\frac{1-x}{\sqrt{x}} \right) (\eta_{eL} - \eta_{eR}), \\ \frac{d\eta_{eL}}{dt} &= \frac{d\eta_{\nu_e}^L}{dt} = -\frac{g'^2}{16\pi^2 s} \langle \vec{E}_Y \cdot \vec{B}_Y \rangle + \left(\frac{\Gamma_0}{2t_{EW}} \right) \left(\frac{1-x}{\sqrt{x}} \right) (\eta_{eR} - \eta_{eL}), \\ \frac{1}{3} \frac{d\eta_B}{dt} &= \frac{d\eta_{eR}}{dt} + 2 \frac{d\eta_{eL}}{dt},\end{aligned}\quad (33)$$

where the variable $x = \frac{t}{t_{EW}} = \left(\frac{T_{EW}}{T} \right)^2$, $t_{EW} = \frac{M_0}{2T_{EW}^2}$, $M_0 = M_{Pl}/1.66\sqrt{g^*}$, and M_{Pl} is the Plank mass, and $\Gamma_0 = 121$.

First scenario:



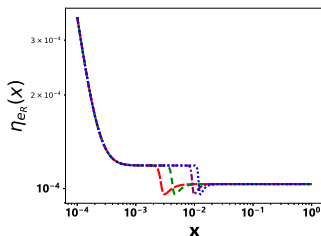
(c)



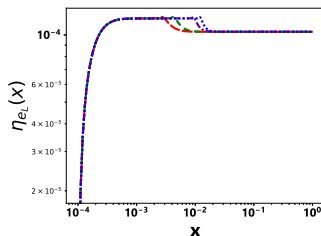
(d)

Figure: The time plots of the helical components $B_a(x)$ and $B_b(x)$, with the initial conditions $B_a^{(0)} = B_b^{(0)} = 0$, $\eta_{eR}^{(0)} = 3.56 \times 10^{-4}$, and $\eta_{eL}^{(0)} = \eta_B^{(0)} = 0$. Large (red) dashed line is for $v_a^{(0)} = 10^{-7}$, $v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{19}$ G, dashed (green) line for $v_a^{(0)} = 10^{-7}$, $v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{17}$ G, dotted-dashed (violet) line for $v_a^{(0)} = v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{19}$ G, and dotted (blue) line for $v_a^{(0)} = v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{17}$ G.

First scenario:



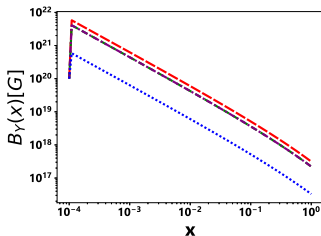
(a)



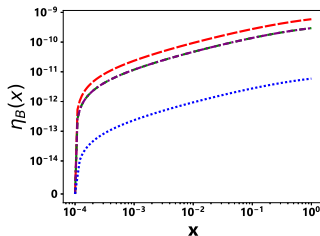
(b)

Figure: The time plots of the the right-handed electron asymmetry $\eta_{eR}(x)$, the left-handed electron asymmetry $\eta_{eL}(x)$ with the initial conditions $B_a^{(0)} = B_b^{(0)} = 0$, $\eta_{eR}^{(0)} = 3.56 \times 10^{-4}$, and $\eta_{eL}^{(0)} = \eta_B^{(0)} = 0$. Large (red) dashed line is for $v_a^{(0)} = 10^{-7}$, $v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{19} \text{G}$, dashed (green) line for $v_a^{(0)} = 10^{-7}$, $v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{17} \text{G}$, dotted-dashed (violet) line for $v_a^{(0)} = v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{19} \text{G}$, and dotted (blue) line for $v_a^{(0)} = v_b^{(0)} = 10^{-14}$, and $B_z^{(0)} = 10^{17} \text{G}$.

Second scenario:



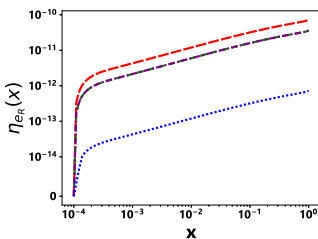
(a)



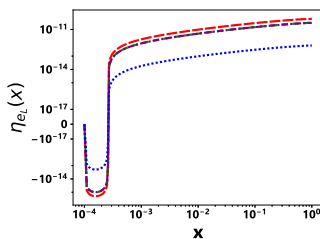
(b)

Figure: The time plots of the hypermagnetic field amplitude $B_Y(x)$, the baryon asymmetry $\eta_B(x)$, in the presence of the viscosity, with the initial conditions $B_z^{(0)} = 10^{20} \text{G}$, $B_a^{(0)} = B_b^{(0)} = 0$, and $\eta_{eR}^{(0)} = \eta_{eL}^{(0)} = \eta_B^{(0)} = 0$. Large dashed (red) line is for, $v_a^{(0)} = v_b^{(0)} = 10^{-2}$, dashed (green) line for $v_a^{(0)} = 10^{-2}$, $v_b^{(0)} = 0$, dot-dashed (violet) line for $v_a^{(0)} = 0$, $v_b^{(0)} = 10^{-2}$, and dotted (blue) line for $v_a^{(0)} = v_b^{(0)} = 10^{-3}$.

Second scenario:



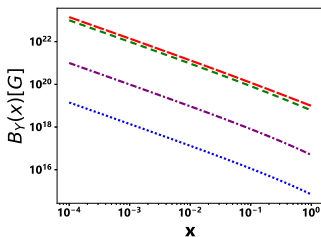
(a)



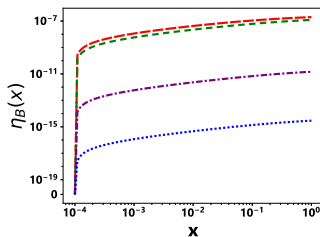
(b)

Figure: The time plots of the the right-handed electron asymmetry $\eta_{eR}(x)$, the left-handed electron asymmetry $\eta_{eL}(x) \simeq \eta_{\nu_L}(x)$, in the presence of the viscosity, with the initial conditions $B_z^{(0)} = 10^{20} \text{G}$, $B_a^{(0)} = B_b^{(0)} = 0$, and $\eta_{eR}^{(0)} = \eta_{eL}^{(0)} = \eta_B^{(0)} = 0$. Large dashed (red) line is for, $v_a^{(0)} = v_b^{(0)} = 10^{-2}$, dashed (green) line for $v_a^{(0)} = 10^{-2}$, $v_b^{(0)} = 0$, dot-dashed (violet) line for $v_a^{(0)} = 0$, $v_b^{(0)} = 10^{-2}$, and dotted (blue) line for $v_a^{(0)} = v_b^{(0)} = 10^{-3}$.

Third scenario:



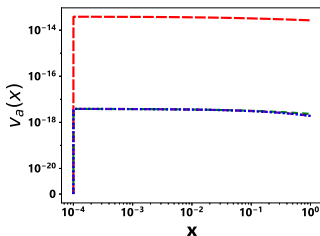
(a)



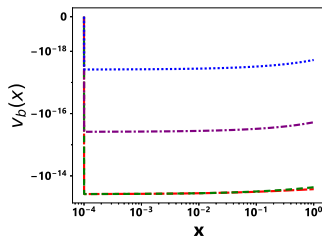
(b)

Figure: The time plots of the the hypermagnetic field amplitude $B_Y(x)$, the baryon asymmetry $\eta_B(x)$, with the initial conditions $B_z^{(0)} = 10^{17} \text{G}$, and $\eta_{eR}^{(0)} = \eta_{eL}^{(0)} = \eta_B^{(0)} = v_a^{(0)} = v_b^{(0)} = 0$. Large (red) dashed line is for $B_a^{(0)} = B_b^{(0)} = 10^{23} \text{G}$, dashed (green) line for $B_a^{(0)} = 10^{23} \text{G}$, $B_b^{(0)} = 10^{19} \text{G}$, dotted-dashed (violet) line for $B_a^{(0)} = 10^{21} \text{G}$, $B_b^{(0)} = 10^{19} \text{G}$, and dotted (blue) line for $B_a^{(0)} = B_b^{(0)} = 10^{19} \text{G}$.

Third scenario:



(a)



(b)

Figure: The time plots of the velocity fields amplitude $v_a(x)$ and $v_b(x)$, with the initial conditions $B_z^{(0)} = 10^{17} \text{G}$, and $\eta_{eR}^{(0)} = \eta_{eL}^{(0)} = \eta_B^{(0)} = v_a^{(0)} = v_b^{(0)} = 0$. Large (red) dashed line is for $B_a^{(0)} = B_b^{(0)} = 10^{23} \text{G}$, dashed (green) line for $B_a^{(0)} = 10^{23} \text{G}$, $B_b^{(0)} = 10^{19} \text{G}$, dotted-dashed (violet) line for $B_a^{(0)} = 10^{21} \text{G}$, $B_b^{(0)} = 10^{19} \text{G}$, and dotted (blue) line for $B_a^{(0)} = B_b^{(0)} = 10^{19} \text{G}$.

$$\begin{aligned} B_A &= B_Y \cos \theta_W \simeq 0.88 B_Y \\ B_Y(T_{EW}) &\sim 10^{17-20} G \end{aligned} \quad (34)$$

$$\begin{aligned} \eta_B &= (n_B - \bar{n}_B)/s \\ s &= 2\pi^2 g^* T^3/45, & g^*(T > T_{EW}) &= 106.75 \\ g_0^* &= 43/11 \\ \eta_B(T_0) &= 27.3 \times \eta_B(T_{EW}) \\ 10^{-15} &< \eta_B(T_{EW}) < 10^{-8} \\ \eta_{BBN} &= 5.8 \pm 0.27 \times 10^{-10} \\ \eta_{CMB} &= 6.16 \pm 0.15 \times 10^{-10} \end{aligned} \quad (35)$$

- In this study, the hypermagnetic field configurations which include both helical and non-helical components, are considered. It is shown that in the presence of weak vorticity and a large right-handed electron asymmetry, the helicity can be generated and amplified for an initially non-helical hypermagnetic field. The vorticity also grows, even in the presence of the viscosity, in contrast to the case in which a fully-helical hypermagnetic field is assumed. In a different scenario it is also shown that in the presence of a strong non-helical hypermagnetic field and large vorticity, helicity and baryon asymmetry can be generated and amplified.

Thank you for your attention