Anderson localization

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Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
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(Received October 10, 1957)

MARCH 1, 1958

Short History of Anderson Localization

1958 → : Anderson localization of electron wavefunctions in disordered solids

1977 : Nobel Prize in Physics goes to Philip Anderson

1985 → : Growing interest in Anderson localization of classical waves (light, sound, …)

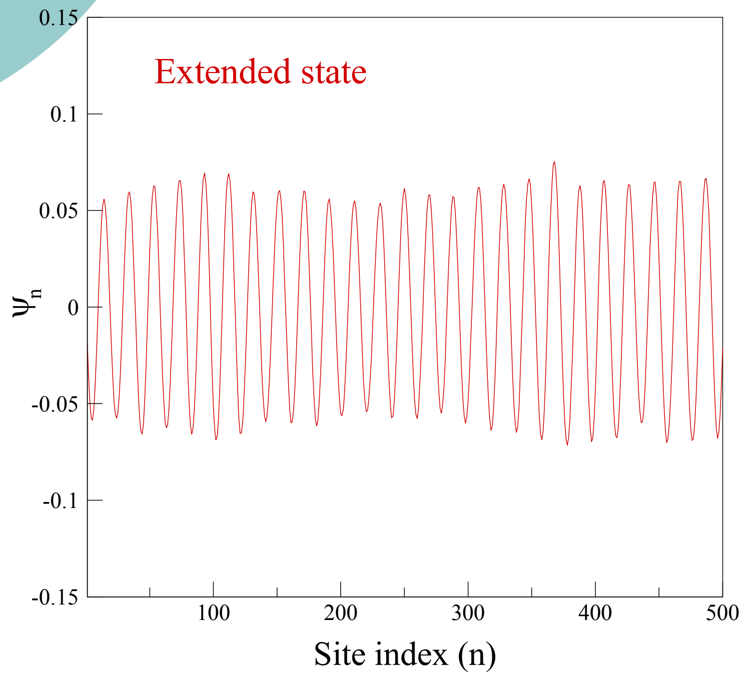
1995 → : Claims of experimental observations of Anderson localization of light, microwaves, ultrasound, …

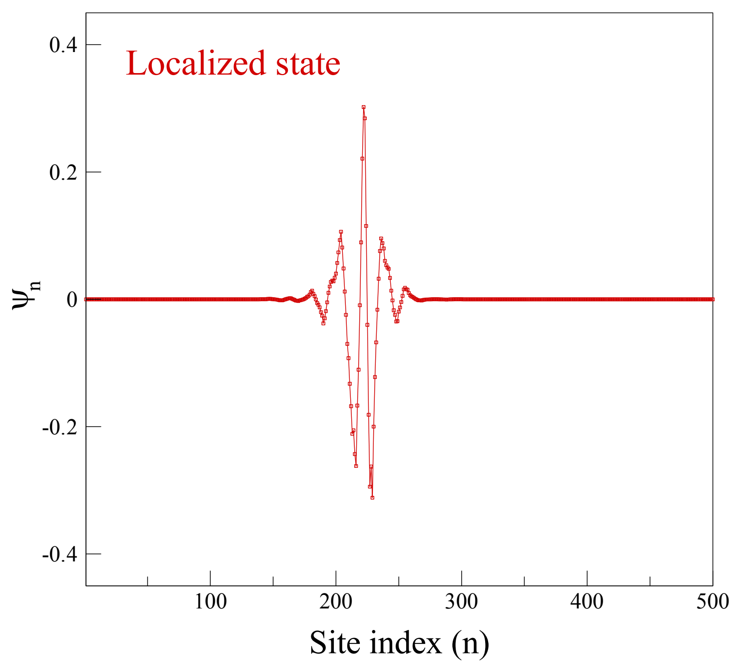
2000 → : Search for Anderson localization of Bose-Einstein condensates in disordered potentials

Publication

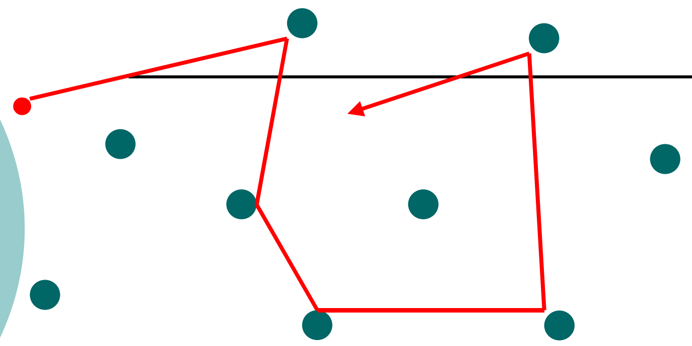
1) *F. Shahbazi, etal. Phys. Rev. Lett. 94, 165505 (2005)*
2) *A. Esmailpour, etal. Phys. Rev. B 74, 024206 (2006)*
3) *A. Bahraminasab, etal. Phys. Rev. B75, 064301 (2007)*
4) *A. Esmailpour, etal. J. Stat. Mech. P09014 (2007)*
5) *N. Abedpour, etal. Phys. Rev. B 76, 195407 (2007)*
6) *R. Seperhrinia, etal. Phys. Rev. B 77, 014203 (2008)*
7) *S. Mehdi Vaez Allaei, etal. J. Stat. Mech. P03016 (2008)*
8) *R. Sepehrinia, etal. Phys. Rev. B. 77, 104202 (2008)*
9) *A. Sheikhan, etal. Phys. Rev. B. (2008) (sub)*
10) *R. Sepehrinia, etal. Phys. Rev. B. (2008)*
11) *A. Bahraminasab, etal. Phys. Rev. B (2008)*
12) *A. Esmailpour, etal. Phys. Rev. B (2008)(sub)*

Localized and extended states in 1D chain



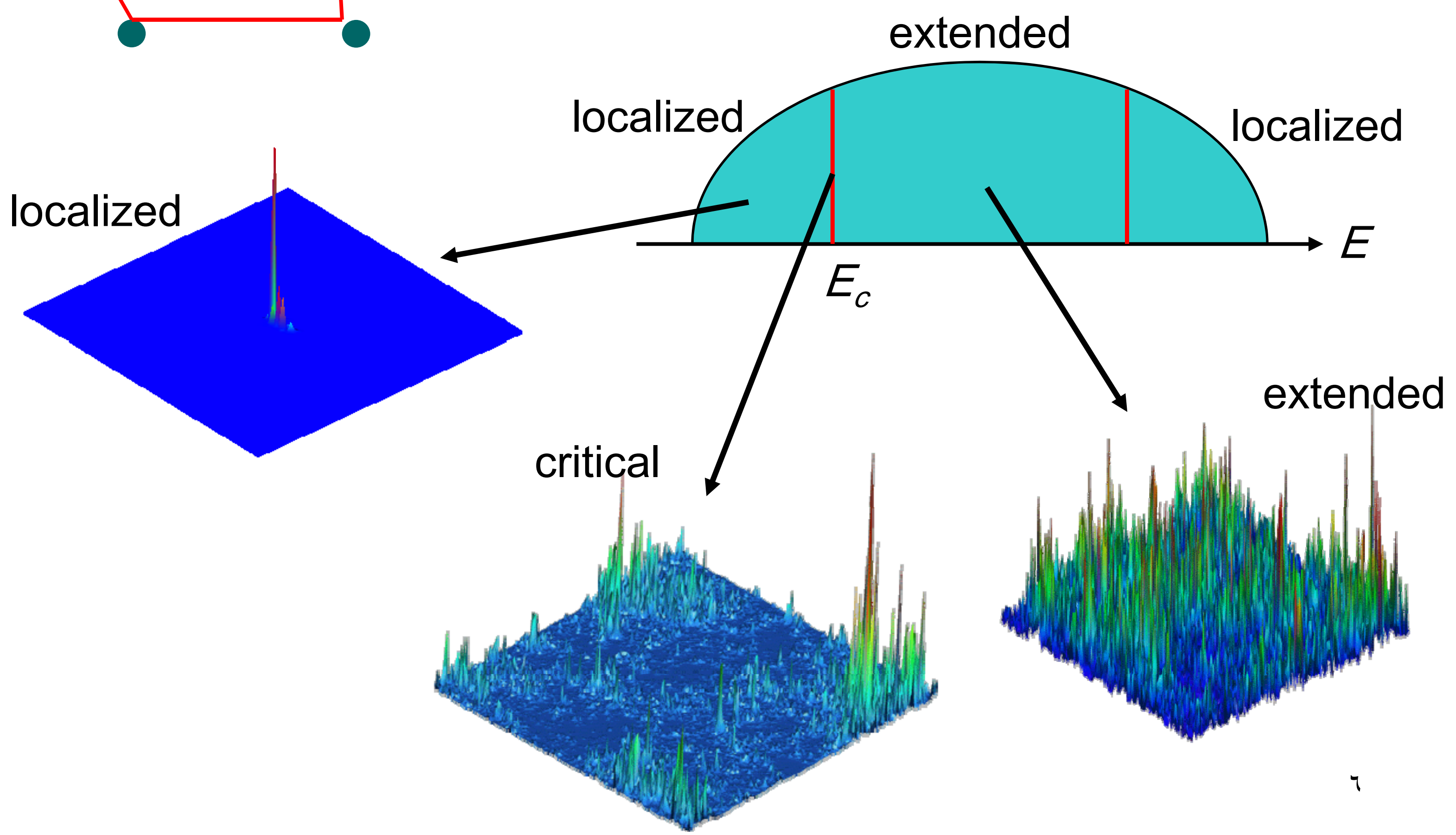


A non-interacting electron moving in random potential

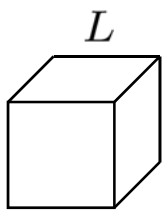


Quantum interference of scattering waves

⇨ Anderson localization of electrons



Scaling theory (1979)



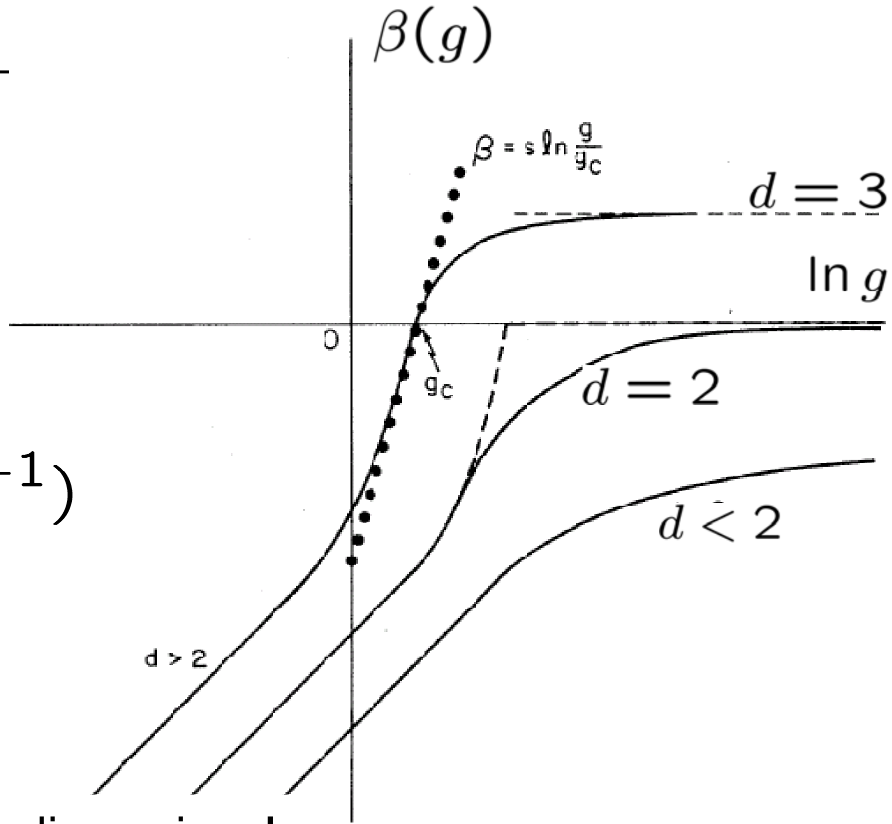
Conductance g changes when system size L is

Metal: $g \propto \dfrac{\text{area}}{\text{length}} = L^{d-2}$

Insulator: $g \propto e^{-L/\xi}$

$$\beta(g) = \frac{d \ln g}{d \ln L} = d - 2 - \mathcal{O}(g^{-1})$$

$$g \gg 1$$



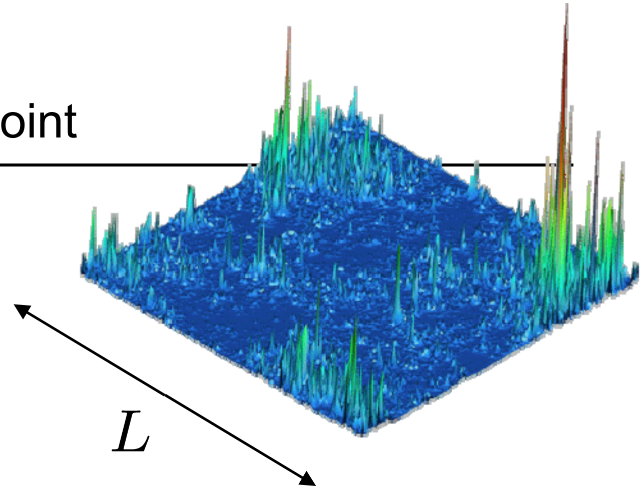
All wave functions are localized below two dimensions!

A metal-insulator transition at g=g_c is continuous (d>2).

But the localization phenomena is Multifractal !

Multifractality: scaling behavior of moments of (critical) wave functions

Critical wave function at a metal-insulator transition point



Multifractal exponents τ_q

In a metal
$|\psi(r)|^2 \sim L^{-d}$

$$L^d\,\overline{|\psi(r)|^{2q}} \sim L^{-\tau_q}$$

$$\tau_q = d(q-1) + \underline{\Delta_q} \qquad D_q = \tau_q/(q-1)$$ fractal dimension

Continuous set of independent and universal critical exponents

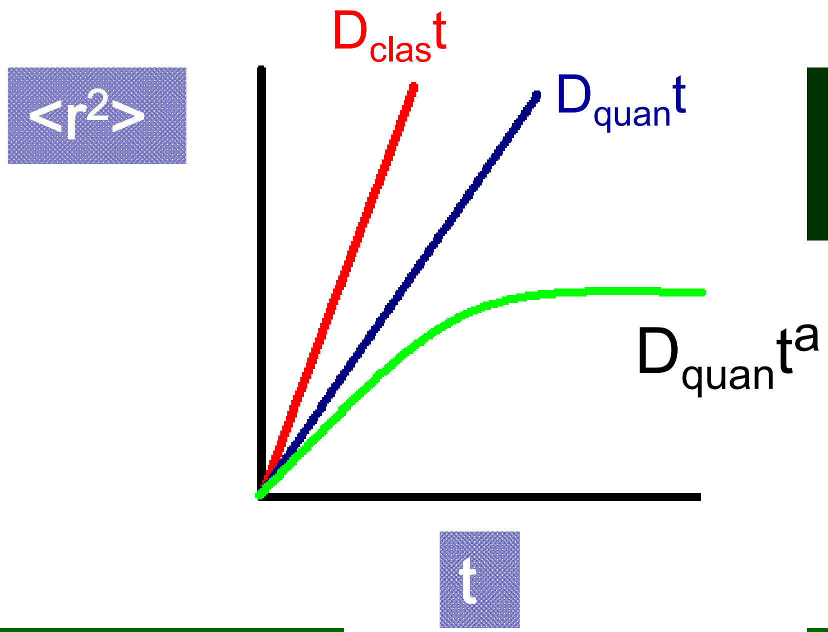
Δ_q : anomalous scaling dimensions

singularity spectrum $\quad f(\alpha) = q\alpha - \tau_q \qquad \alpha = \dfrac{d\tau_q}{dq} \qquad \alpha > 0$

Anderson Localization (1957)

What?

Quantum diffusion in a random potential stops due to interference effects.



Goal of localization theory

a = ?

D_{quan}= f(d,dis)?

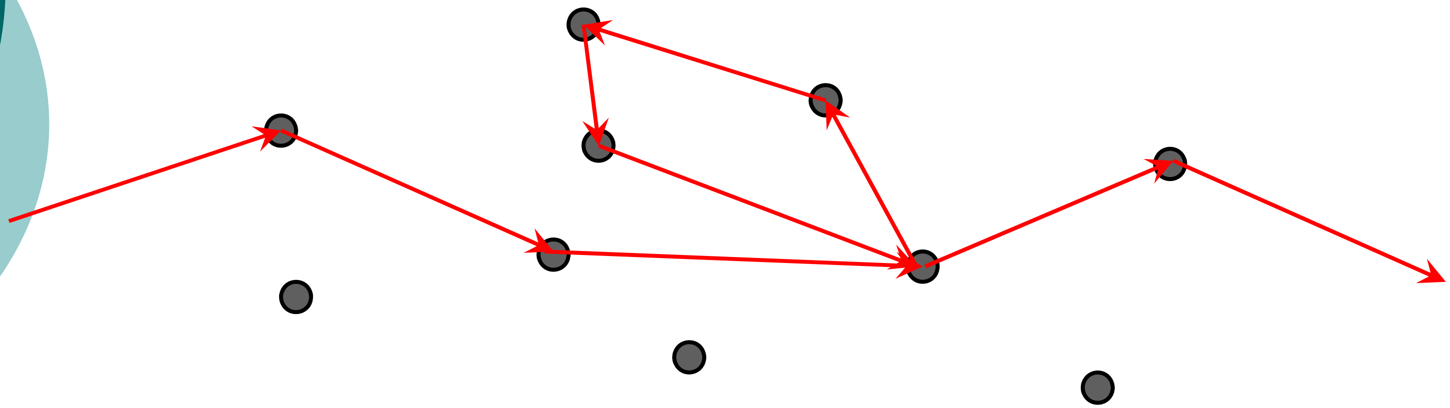
Delocalization

$$\left|\Psi_\alpha(r)\right| \propto 1/\sqrt{V}$$

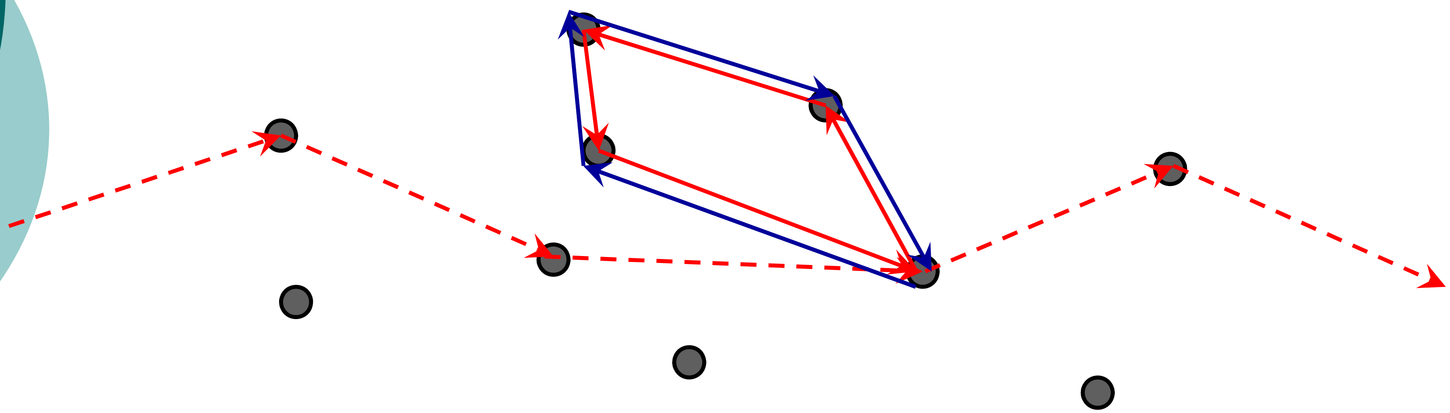
Localization

$$\left|\Psi_\alpha(r)\right| \propto e^{-r/\xi_{loc}}$$

Self-consistent theory of Anderson localization



Self-consistent theory of Anderson localization



The presence of loops increases return probability as compared to 'normal' diffusion

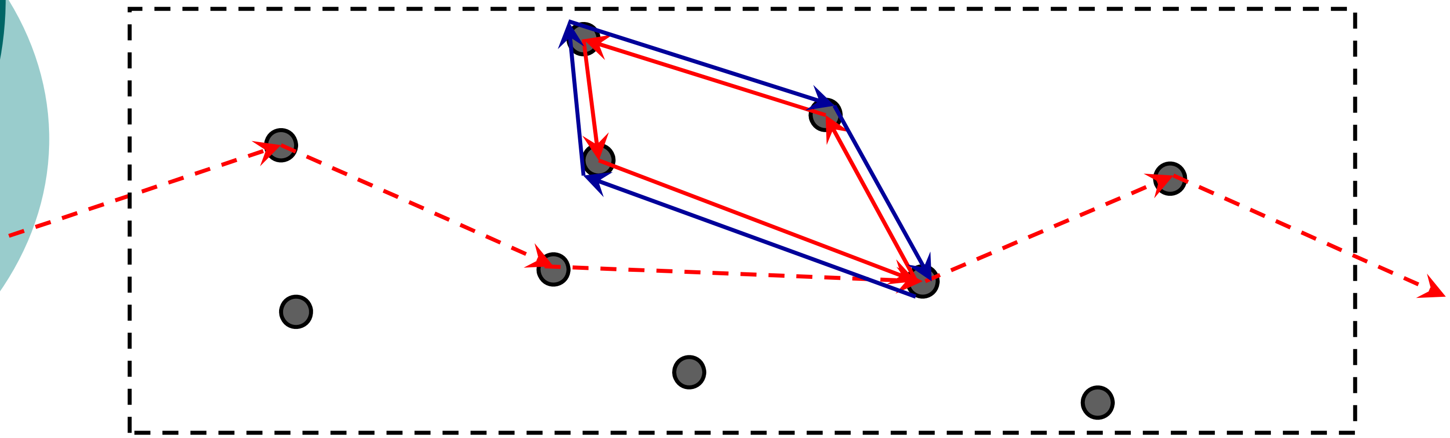
⇩

Diffusion slows down

⇩

Diffusion constant should be renormalized $D_{\mathrm{B}} \rightarrow D < D_{\mathrm{B}}$

Generalization to open media



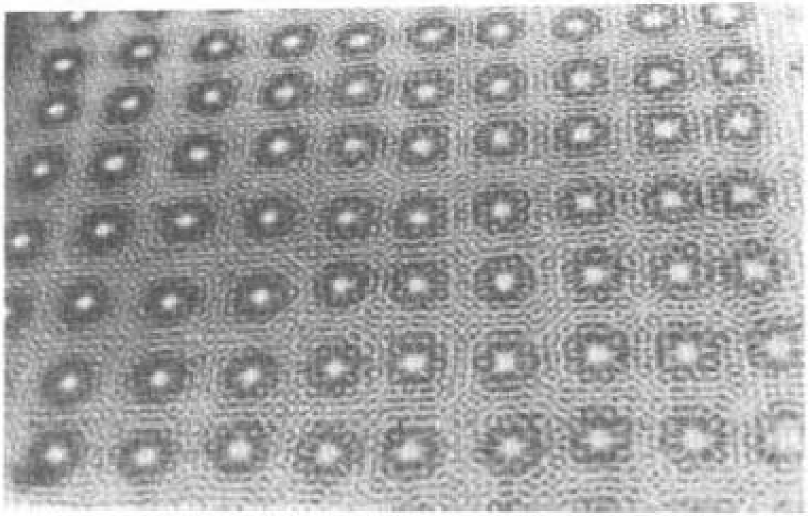
Loops are less probable near the boundaries

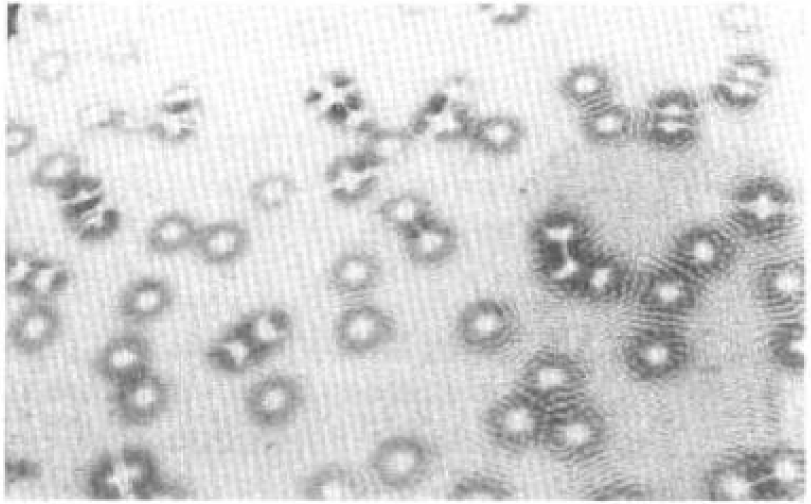
Slowing down of diffusion is spatially heterogeneous

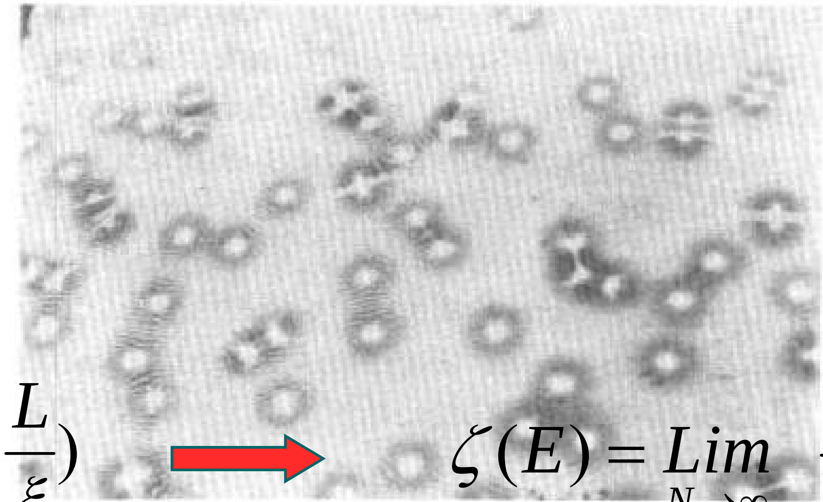
Diffusion constant becomes position-dependent

$$D_{\mathrm{B}} \rightarrow D(\mathbf{r}) < D_{\mathrm{B}}$$

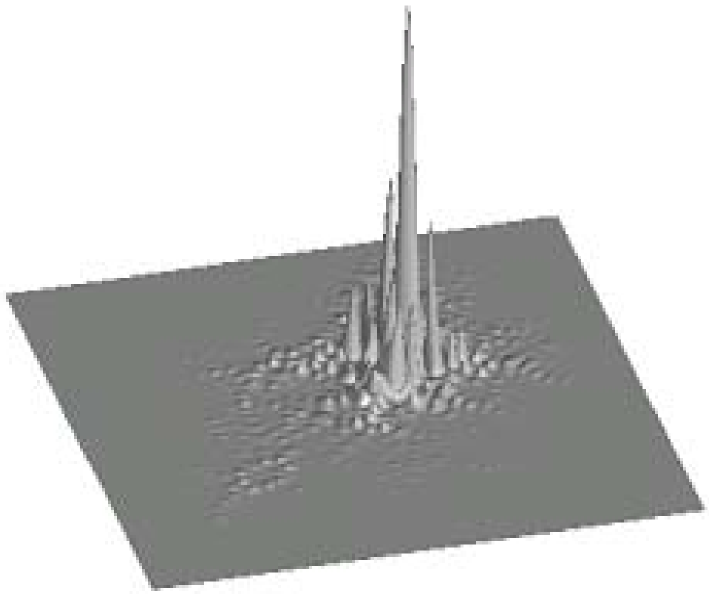
Classical localization

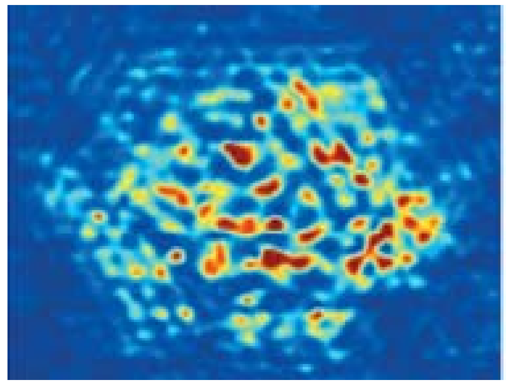


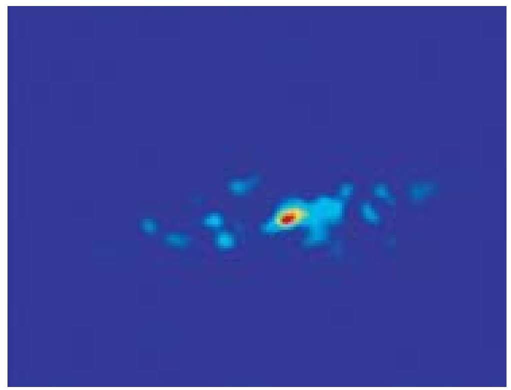




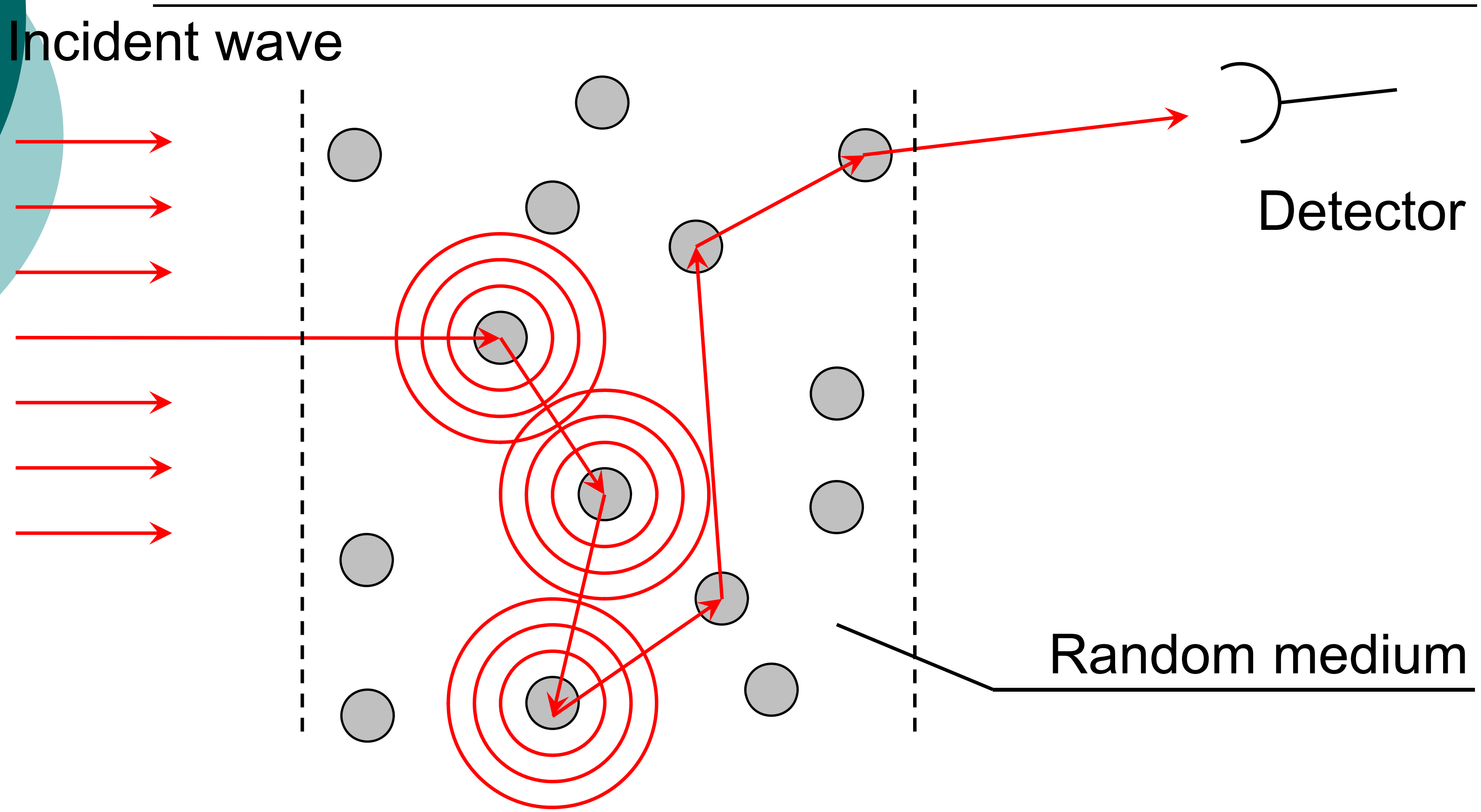
$$\frac{\psi_N}{\psi_1} = \exp(-\frac{L}{\xi}) \quad \Rightarrow \quad \zeta(E) = \lim_{N\to\infty} \frac{1}{N} \ln\left|\frac{\psi_N}{\psi_1}\right|$$



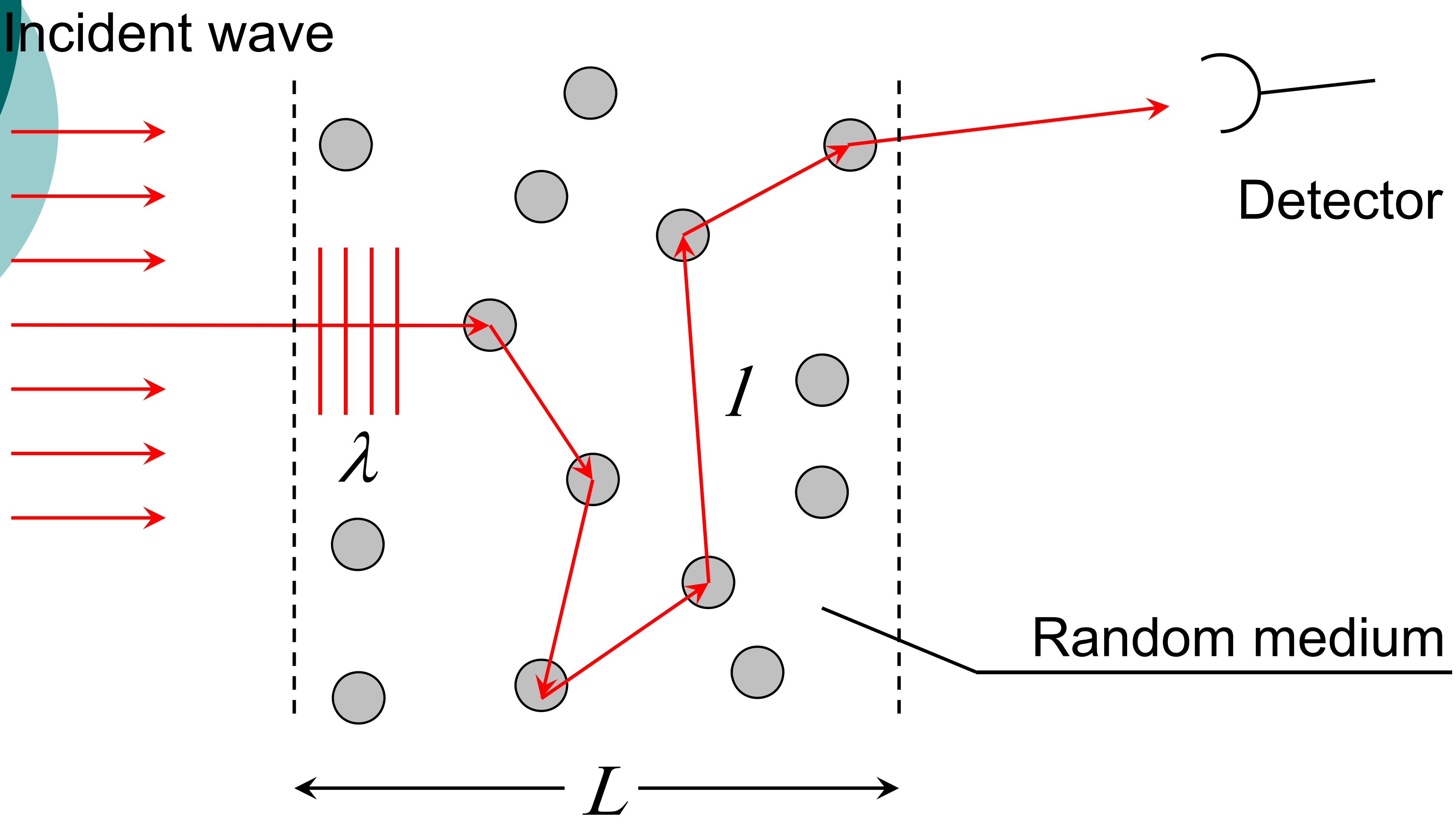




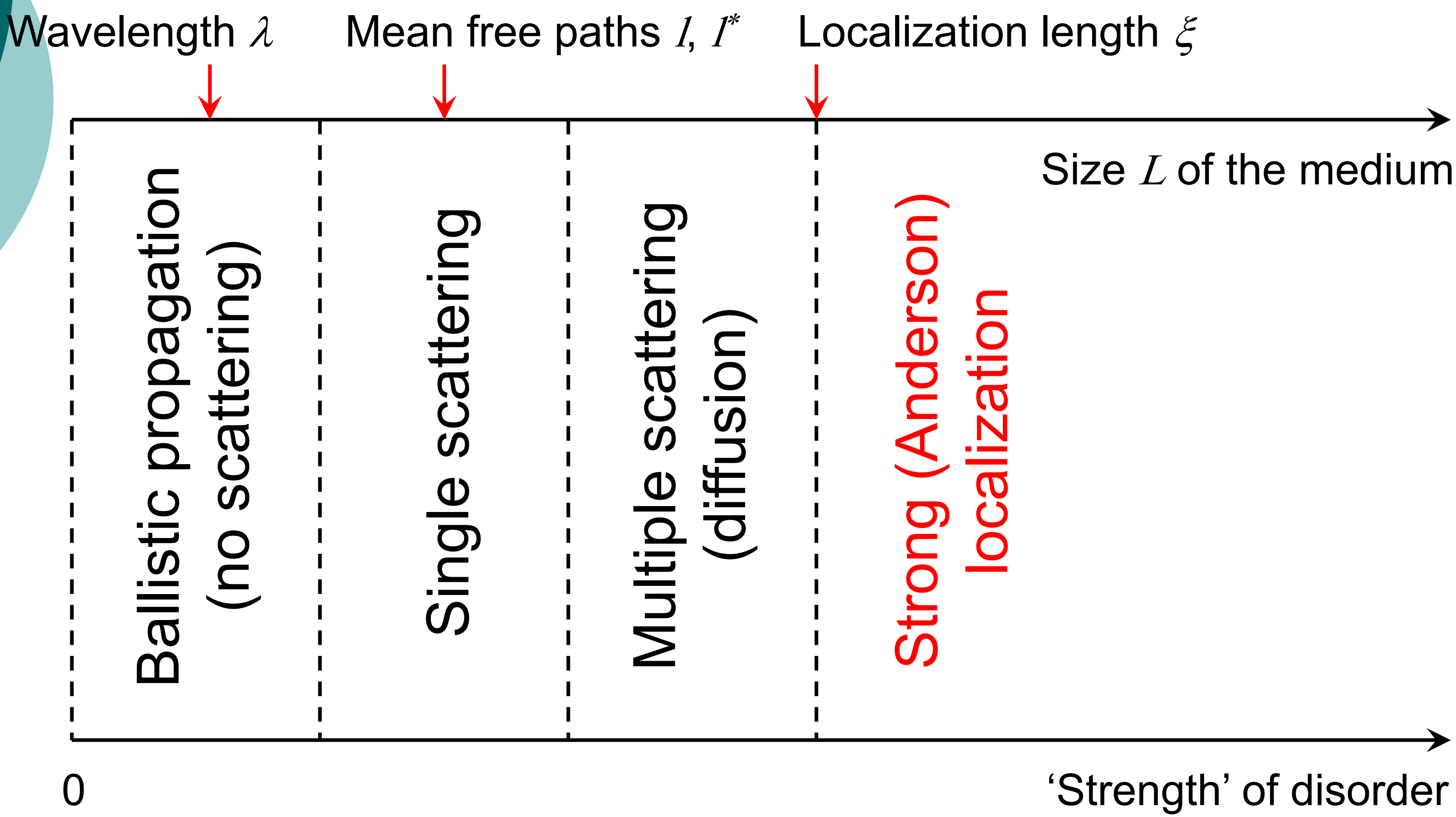
Multiple Scattering of Waves



Multiple Scattering of Waves

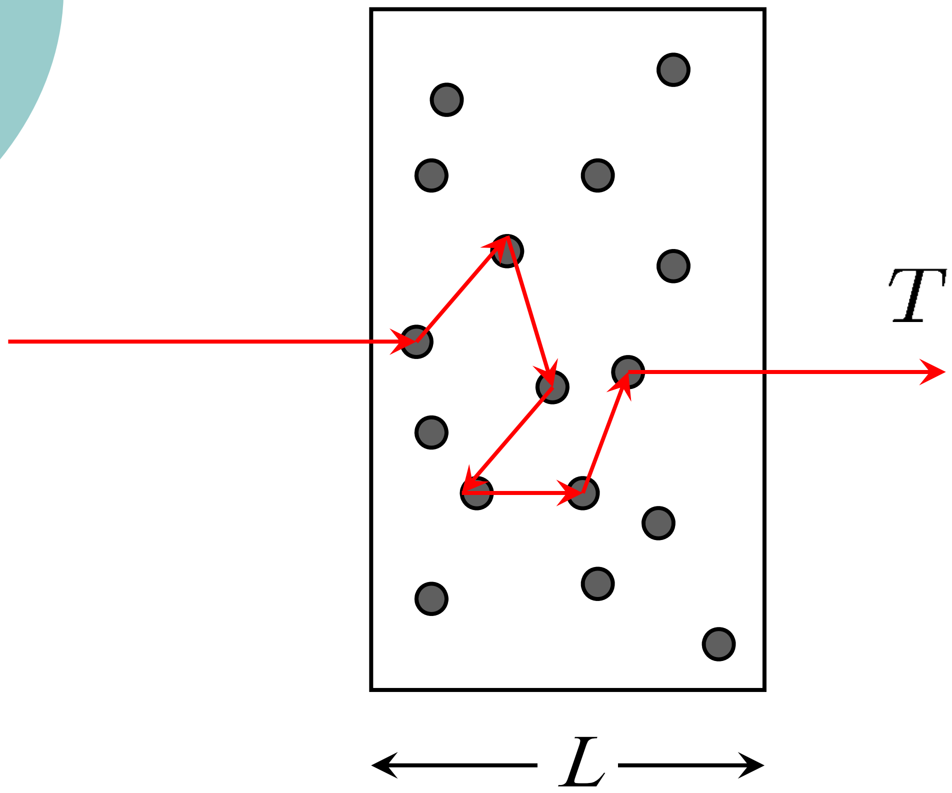


From Single Scattering to Anderson Localization



Experimental Evidence

Exponential scaling of average transmission with L



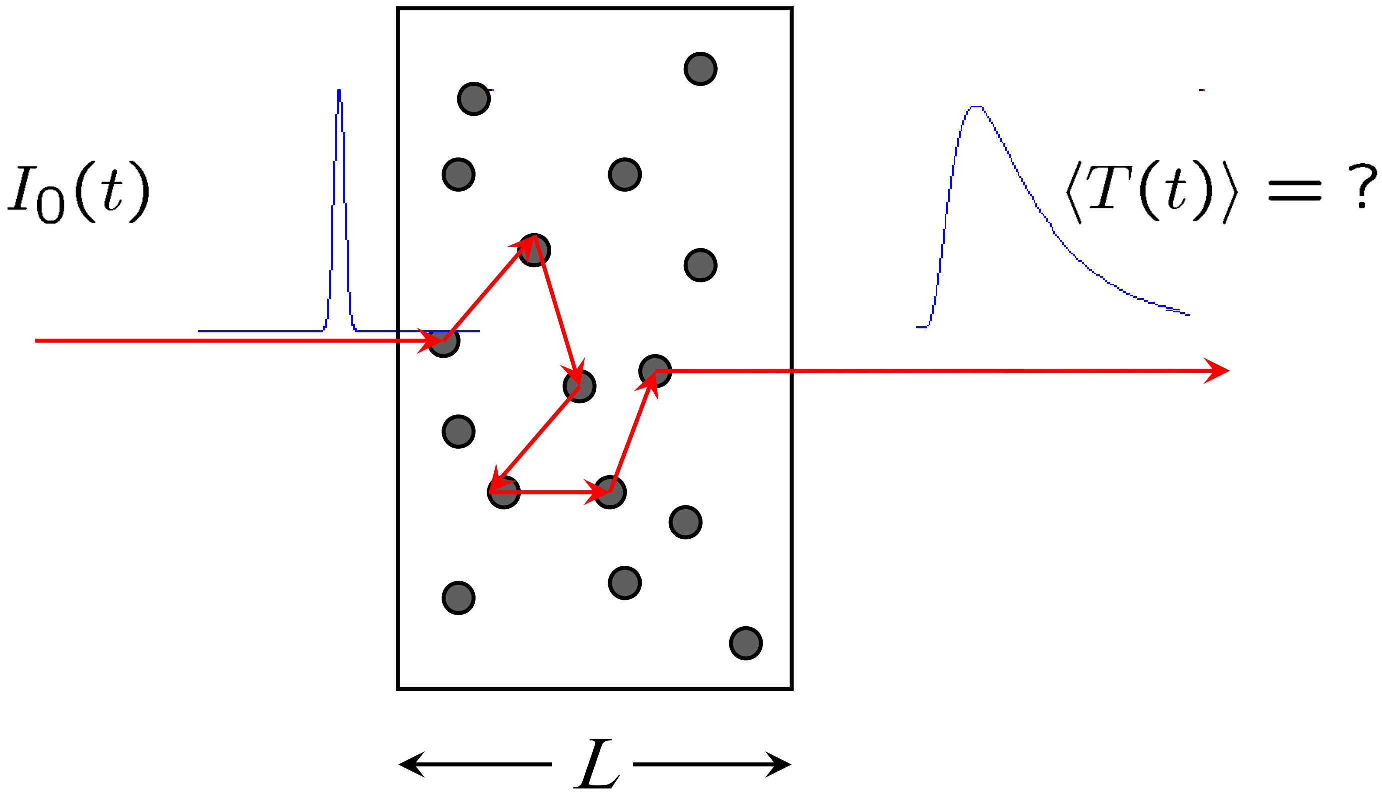
Diffusive regime:

$$\langle T \rangle \propto \frac{\ell}{L}$$

Localized regime:

$$\langle T \rangle \propto \exp\left(-\frac{L}{\xi}\right)$$

What if We Look at the Dynamics?



Outline

1) Non-perturbative method in 1D

   a) random mass oscillators

   b) Disordered Schrödinger equation

   c) Magnon modes of spin glass

2) Perturbative method

   a) Mapping the quenched random differential equation to a Martin-Siggia-Rose action (elastic wave)

3) Numerical method

   a) Red-Shift of spectral density (light localization)

4) Acoustic wave in random Dimer media

Part – I

a) Random Mass (1D)

$$\frac{\partial^2}{\partial x^2}\psi + m(x)\omega^2\psi = 0.$$

Random density $m(x) = \bar{m} + c(x)$ ($\bar{m}$ is mean density)

$c(x)$ is Gaussian uncorrelated random function

$$< c(x) >= 0,$$

$$< c(x)c(x') >= \sigma^2\delta(x - x').$$

Random Mass

The log-derivative of ψ, $\quad f(x) = \psi'(x)/\psi(x)$

$$f' + f^2 + \omega^2 m(x) = 0. \qquad f' = df/dx.$$

$$D^{(1)} = -f^2 - \omega^2 \bar{m}$$

$$D^{(2)} = \omega^2 \sigma.$$

Random Mass

The probability distribution function (PDF) of $f(x)$, i.e. $P(\xi, x)$,

Fokker-Planck equation

$$\frac{\partial}{\partial x} P(\xi, x) = \frac{\partial}{\partial \xi}(\xi^2 + \omega^2 \bar{m} + \frac{\sigma^2 \omega^4}{2} \frac{\partial}{\partial \xi}) P(\xi, x).$$

$P(\xi, x) d\xi$ is the probability of finding $f(x)$ between ξ and $\xi + d\xi$

$$P(\xi, x) = \langle \delta(f(x) - \xi) \rangle$$

Random Mass

The homogeneous solution (or at $x \to \infty$) of Fokker-Planck equation

$$(\xi^2 + \omega^2 \bar{m} + \frac{\sigma^2 \omega^4}{2} \frac{\partial}{\partial \xi}) p(\xi) = p_0.$$

$$p(\xi) = \lim_{x\to\infty} P(\xi, x).$$

$$p(\xi) = \frac{2p_0}{\sigma^2\omega^4} \exp\left\{ -\frac{2}{\sigma^2\omega^4} (\frac{\xi^3}{3} + \omega^2 \bar{m} \xi) \right\} \times \int_{-\infty}^{\xi} d\eta \, \exp\left\{ \frac{2}{\sigma^2\omega^4} (\frac{\eta^3}{3} + \omega^2 \bar{m} \eta) \right\}.$$

Random Mass (gerenelized PDF)

$\mathcal{P}(\xi, x)$, which is defined as $\mathcal{P}(\xi, x) = \left\langle \delta(f(x) - \xi) \left| \frac{\partial f}{\partial \omega^2} \right| \right\rangle$

Dos:

$$\rho(\omega^2) = \lim_{L \to \infty} \frac{1}{L} \mathcal{P}(\xi_L, L).$$

sign of $\partial f / \partial \omega^2$

$$\frac{\partial}{\partial x} \frac{\partial f}{\partial \omega^2} + 2f \frac{\partial f}{\partial \omega^2} + m(x) = 0.$$

$$\frac{\partial f}{\partial \omega^2} = -\int_0^x dy \frac{\psi^2(y)}{\psi^2(x)} m(y).$$

has negative sign if $\sigma / \bar{m} \ll 1$

positivity condition of mass

Random Mass

$$\mathcal{P}(\xi, x) = - < \frac{\partial f}{\partial \omega^2} \delta(f(x) - \xi) > .$$

$$\frac{\partial \mathcal{P}(\xi, x)}{\partial x} = \bar{m} P(\xi, x) + \omega^2 \sigma^2 \frac{\partial P(\xi, x)}{\partial \xi} - \\ [\ (\xi^2 + \omega^2 \bar{m}) \frac{\partial}{\partial \xi} + \frac{\omega^4 \sigma^2}{2} \frac{\partial^2}{\partial \xi^2}] \mathcal{P}(\xi, x).$$

$$\rho(\omega^2) = \bar{m} \int p(-\xi) p(\xi) d\xi + \omega^2 \sigma^2 \int p(-\xi) \frac{\partial p}{\partial \xi} d\xi.$$

Random Mass

$$p(\xi) = \frac{2p_0}{\sigma^2\omega^4} \exp\left\{-\frac{2}{\sigma^2\omega^4}(\frac{\xi^3}{3} + \omega^2\bar{m}\xi)\right\}$$
$$\times \int_{-\infty}^{\xi} d\eta \, \exp\left\{\frac{2}{\sigma^2\omega^4}(\frac{\eta^3}{3} + \omega^2\bar{m}\eta)\right\}.$$

$$\partial p_0/\partial\omega^2 = \rho(\omega^2). \qquad p_0 = \mathcal{N}(\omega^2)$$

$$\frac{1}{\mathcal{N}(\omega^2)} = \frac{\sqrt{2\pi}}{(\omega^2\sigma)^{\frac{2}{3}}} \int_0^{\infty} \frac{dx}{\sqrt{x}} exp\left[-\frac{1}{6}x^3 - \frac{2\bar{m}}{\omega^{\frac{2}{3}}\sigma^{\frac{4}{3}}}x\right].$$

In the limit $\sigma \to 0$ $\mathcal{N}(\omega^2) = 1/\pi\sqrt{\bar{m}\omega^2}$

Lyapunov Exponents

$$f = \partial \ln(\psi)/\partial x,$$

$$\gamma = \lim_{x\to\infty} \langle \frac{\partial}{\partial x} \ln(\psi) \rangle = \langle f \rangle_\infty.$$

$$\gamma = \int \xi p(\xi) d\xi.$$

$$\gamma = \frac{\omega^2}{2} \left(\frac{I_+}{I_-} \right),$$

$$I_\pm = \int_0^\infty du u^{(\pm)\frac{1}{2}} \exp\left\{ -\frac{\omega^2}{6\sigma^2} u^3 - \frac{2\bar{m}}{\sigma^2} u \right\}.$$

Lyapunov Exponents

in limit of $\omega \to 0$,

$$\gamma = \frac{\sigma^2 \omega^2}{8\bar{m}},$$

$\omega \to \infty$

$$\gamma = \frac{6^{\frac{1}{3}}\sqrt{\pi}}{2 \times \Gamma(\frac{1}{6})} \sigma^{2/3} \omega^{4/3}.$$

localization length scales as $l \sim \omega^{-4/3}$

b) Disordered Schrodinger Equation (1D)

$$H = -\frac{1}{2}\Delta + V(x)$$

potential $V(x)$ is a zero mean, Gaussian random white noise

$$\langle V(x)V(x')\rangle = \delta(x - x').$$

$$\gamma = \frac{1}{2E}\left(\frac{I_+}{I_-}\right), \quad I_\pm = \int_0^\infty du u^{(\pm)\frac{1}{2}} \exp\left\{-\frac{1}{24E^3}u^3 - u\right\}.$$

In the limit $(E \to \infty)$ $\quad \gamma \sim 1/4E$

c) HARMONIC MAGNON MODES HEISENBERG--MATTIS SPIN GLASSES

$$H = \sum_{i,j} J_{ij} S_i \cdot S_j .$$

$$P(J_{ij}) = 1/2\delta(J_{ij} - J) + 1/2\delta(J_{ij} + J)$$

coupling J is a constant $\qquad S^{\pm} = S_x \pm iS_y)$

$$H = \sum_{i,j} J_{ij} \left[\frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z \right] .$$

HEISENBERG--MATTIS SPIN GLASSES

$$i\hbar\frac{\partial S_i^+}{\partial t} = \sum_{j\neq i} J_{ij}\left\{\frac{1}{2}[S_i^+, S_i^+S_j^- + S_i^-S_j^+] + [S_i^+, S_i^zS_j^z]\right\}.$$

$$[S_i^+, S_j^-] = 2\delta_{ij}S_i^z \text{ and } [S_i^z, S_j^\pm] = \pm\delta_{ij}S_i^\pm$$

$$i\hbar\frac{\partial S_i^+}{\partial t} = \sum_{j\neq i} J_{ij}(S_i^zS_j^+ - S_j^zS_i^+).$$

$$i\hbar\frac{\partial S_i^+}{\partial t} = -(J_{i,i+1}S_{i+1}^z + J_{i,i-1}S_{i-1}^z)S_i^+ + J_{i,i+1}S_i^zS_{i+1}^+ + J_{i,i-1}S_i^zS_{i-1}^+.$$

HEISENBERG--MATTIS SPIN GLASSES

Heisenberg–Mattis spin glass.

Now assuming to have small spin wave amplitude

$$S^x, S^y \ll S \qquad S_i^z \approx \zeta_i S,$$

where S is the length of spin vectors and ζ_i equates ± 1.

$$(2 - \zeta_i \Omega)\mu_i = \mu_{i+1} + \mu_{i-1}.$$

$$\mu_i = \zeta_i S_i^+$$

zero average mass

$$\omega^2 \equiv \Omega$$

$$\gamma = \frac{6^{\frac{1}{3}}\sqrt{\pi}}{2 \times \Gamma(\frac{1}{6})}(\sigma\Omega)^{2/3};$$

Part – (II-a)

Elastic wave localization

Martin-Siggia-Rose action

II-a) The Model (Scalar Field)

The scalar wave equation:

$$\frac{\partial^2}{\partial t^2}\psi(\mathbf{x},t) - \boldsymbol{\nabla}\cdot[\lambda(\mathbf{x})\boldsymbol{\nabla}\psi(\mathbf{x},t)] = 0 \ , \tag{2}$$

where $\psi(\mathbf{x},t)$ is the wave amplitude, and $\lambda(\mathbf{x}) = e(\mathbf{x})/m$ the ratio of the elastic stiffness $e(\mathbf{x})$ and the medium's mean density m. We then write λ as,

$$\lambda(\mathbf{x}) = \lambda_0 + \eta(\mathbf{x}) \ , \tag{3}$$

where $\lambda_0 = \langle\lambda(\mathbf{x})\rangle$. In the present paper $\eta(\mathbf{x})$ is assumed to be a Gaussian random process with a zero mean and the covariance,

$$\langle\eta(\mathbf{x})\eta(\mathbf{x}')\rangle = 2C(|\mathbf{x}-\mathbf{x}'|) = 2D_0\delta^d(\mathbf{x}-\mathbf{x}') + 2D_\rho|\mathbf{x}-\mathbf{x}'|^{2\rho-d}.$$

Propagation of Wave Component with Frequency ω

$$\nabla^2 \psi(\mathbf{x}, \omega) + \frac{w^2}{\lambda_0} \psi(\mathbf{x}, \omega) + \nabla \cdot \left(\frac{\eta(\mathbf{x})}{\lambda_0} \nabla \psi(\mathbf{x}, \omega)\right) = 0$$

The Martin-Siggia-Rose Action

$$S_e[\psi_I, \psi_R, \tilde{\psi}, \chi, \chi^*] =$$
$$\int d\mathbf{x}d\mathbf{x}' \Big[(i\tilde{\psi}_I(\mathbf{x}')(\nabla^2 + \frac{\omega^2}{\lambda_0})\psi_I(\mathbf{x}) + i\tilde{\psi}_R(\mathbf{x}')(\nabla^2 + \frac{\omega^2}{\lambda_0})\psi_R(\mathbf{x})$$
$$+ \chi^*(\mathbf{x}')(\nabla^2 + \frac{\omega^2}{\lambda_0})\chi(\mathbf{x}))\delta(\mathbf{x} - \mathbf{x}')$$
$$+ (i\boldsymbol{\nabla}\tilde{\psi}_I\boldsymbol{\nabla}\psi_I + i\boldsymbol{\nabla}\tilde{\psi}_R\boldsymbol{\nabla}\psi_R + \boldsymbol{\nabla}\chi\boldsymbol{\nabla}\chi)\frac{K(\mathbf{x} - \mathbf{x}')}{\lambda_0^2}\Big(i\boldsymbol{\nabla}\tilde{\psi}_I\boldsymbol{\nabla}\psi_I$$
$$+ i\boldsymbol{\nabla}\tilde{\psi}_R\boldsymbol{\nabla}\psi_R + \boldsymbol{\nabla}\chi\boldsymbol{\nabla}\chi\Big)\Big]$$

- Two coupling constants:

  $g_0 = D_0/\lambda_0^2$, $\quad g_\rho = D\rho/\lambda_\rho^2$

- RG analysis to one-loop order in the limit, $\omega^2/\lambda_0 \to 0$, to determine the two beta functions.

Diagrammatic Representation and One-Loop Corrections

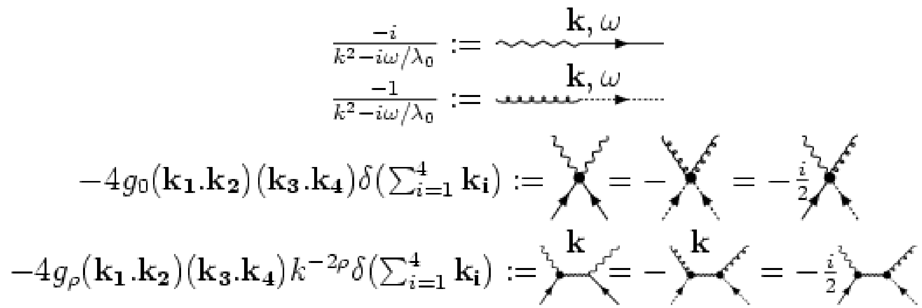


FIG. 2. Diagramatic representations of the propagators and vortices in the effective action S_c.

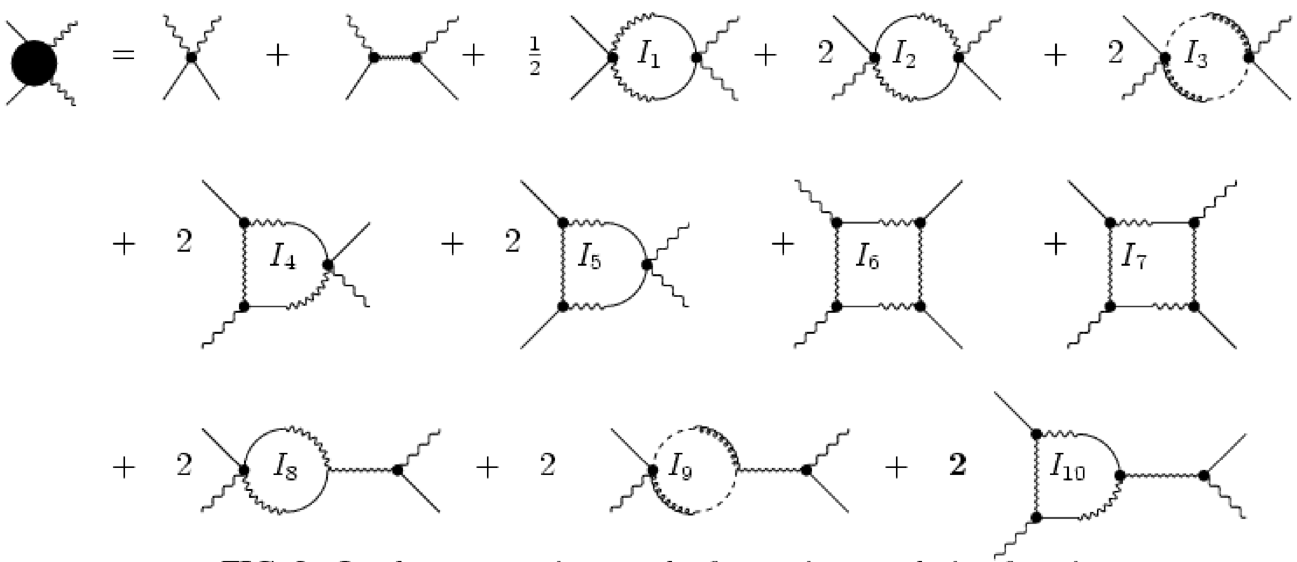


FIG. 3. One-loop corrections to the four-point correlation function.

The Beta Functions

The functions $\beta(\tilde{g}_0)$ and $\beta(\tilde{g}_\rho)$ are then given by,

$$\beta(\tilde{g}_0) = \frac{\partial \tilde{g}_0}{\partial \ln l} = -d\tilde{g}_0 + 8\tilde{g}_0^2 + 10\tilde{g}_\rho^2 + 20\tilde{g}_0\tilde{g}_\rho \ ,$$

$$\beta(\tilde{g}_\rho) = \frac{\partial \tilde{g}_\rho}{\partial \ln l} = (2\rho - d)\tilde{g}_\rho + 12\tilde{g}_0\tilde{g}_\rho + 16\tilde{g}_\rho^2 \ ,$$

where $l > 1$ is the re-scaling parameter, and

$$\tilde{g}_0 = k_d \left[\frac{d+5}{2d(d+2)} \right] g_0 \ ,$$

$$\tilde{g}_\rho = k_d \left[\frac{d+5}{2d(d+2)} \right] g_\rho \ ,$$

Phase Space and Fixed Points

Three sets of fixed points for $0 < \rho < d/2$:

Trivial FP (Gaussian) at $g_0^* = g_\rho^* = 0$ (stable)

Non-trivial FPs, one at $g_0^* = d/8$, $g_\rho^* = 0$, and the other at

$$g_0^* = -\frac{4}{41}\left[d + \frac{5}{16}(2\rho - d)\right] - \frac{4}{41}\sqrt{\left[d + \frac{5}{16}(2\rho - d)\right]^2 + \frac{205}{256}(2\rho - d)^2}\,,$$

$$g_\rho^* = \frac{3}{4}g_0^* + \frac{1}{16}(d - 2\rho)\,,$$

Stable in one eigendirection, but unstable in the other eigendirection.

Phase Space (Continued)

- Thus, a system with uncorrelated disorder is unstable against disorder with long-range correlations towards a new FP.

- Thus, with increasing disorder, extended $\rightarrow$ localized

Phase Space (Continued)

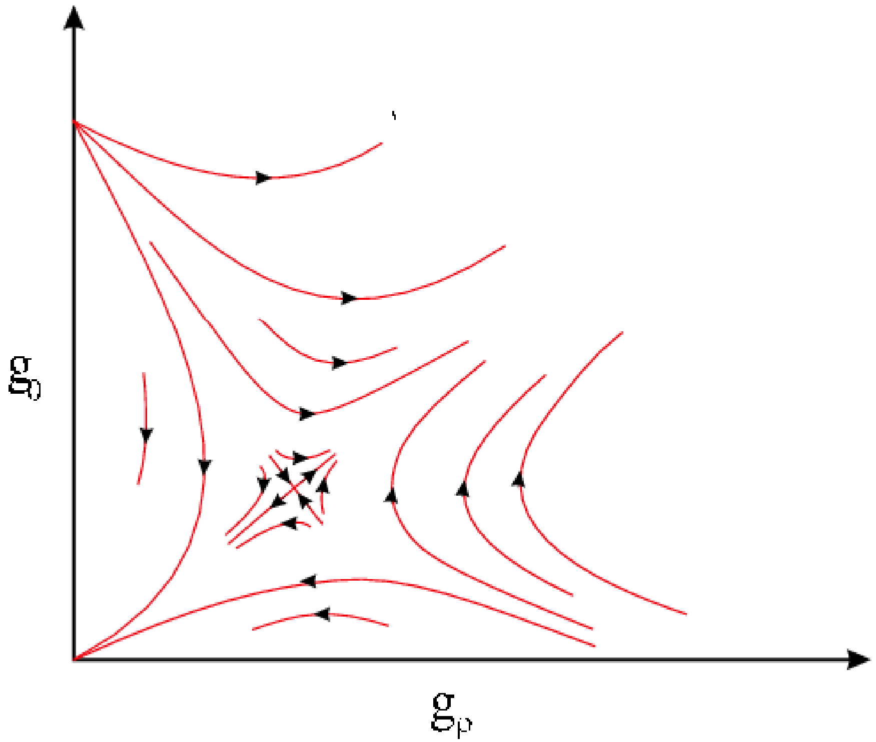


FIG. 5. Flows in the coupling constants space for $0 < \rho < \frac{1}{2}d$.

Phase Space (Continued)

~~Two sets of fixed points for $\rho > d/2$:~~

- Gaussian FP, stable on the g_0 axis, but not on the g_ρ axis
- Non-trivial FP at $g_0^* = d/8$, $g_\rho^* = 0$, unstable in all directions.
- Thus, power-law disorder relevant, but no new FP.

Phase Space (Continued)

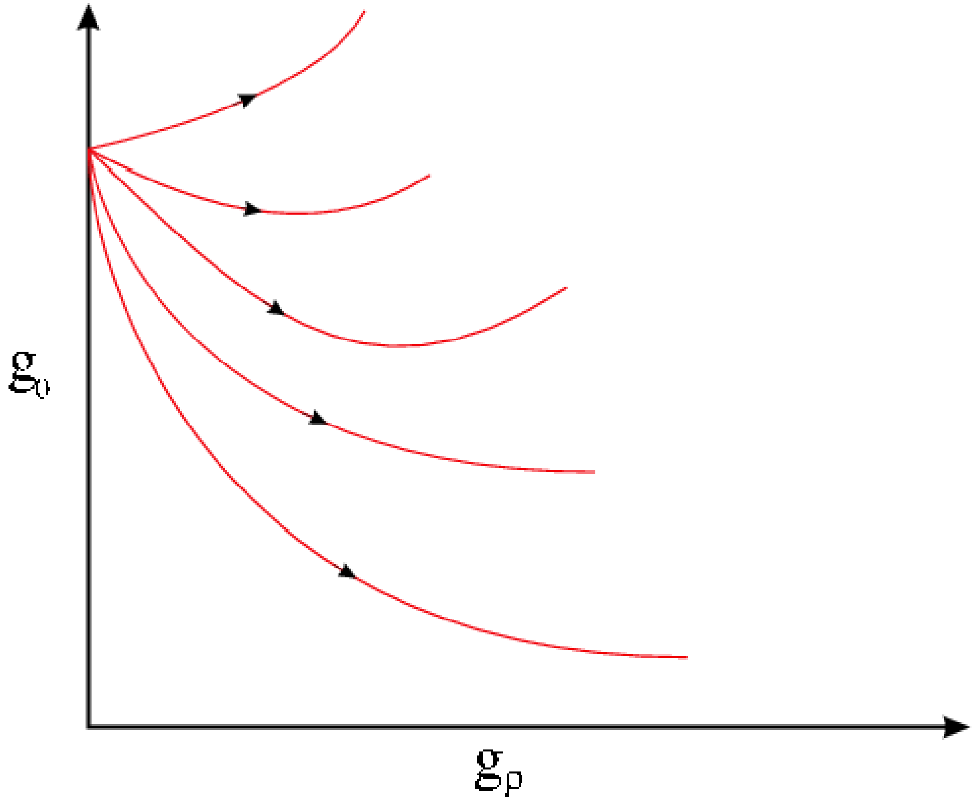
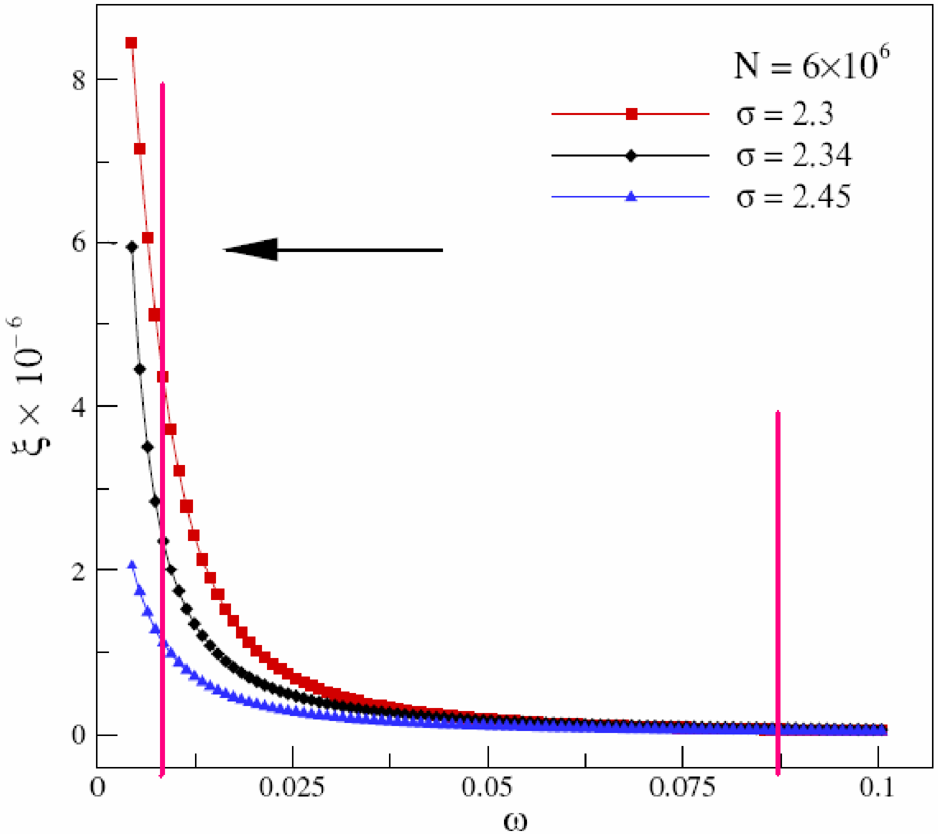


FIG. 6. Flows in the coupling constants space for $\rho > \frac{1}{2}d$.

Frequency-Dependence of Localization Length



Localization length ξ as a function of the frequency ω for $\sigma < \sigma_c \simeq 2.4$ and $\sigma > \sigma_c$.

The system size is $N = 6 \times 10^6$. The results represent averages over 6000 realizations.

Wave Front

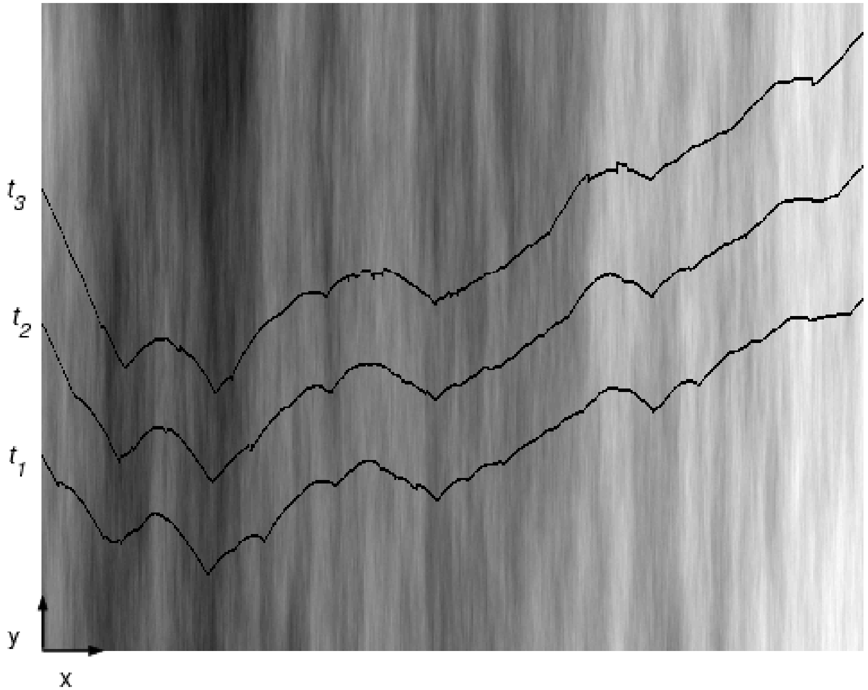


FIG. 7. The wave front in a 2D anisotropic system at (dimensionless) times, $t_1 = 328$, $t_2 = 384$, and $t_3 = 440$, with $\rho = 1.5$.

Roughness of Wave Front: Self-Affine Fronts

- Computing the correlation function

$$C(r) = < [d(x) - d(x + r)]^2 >$$

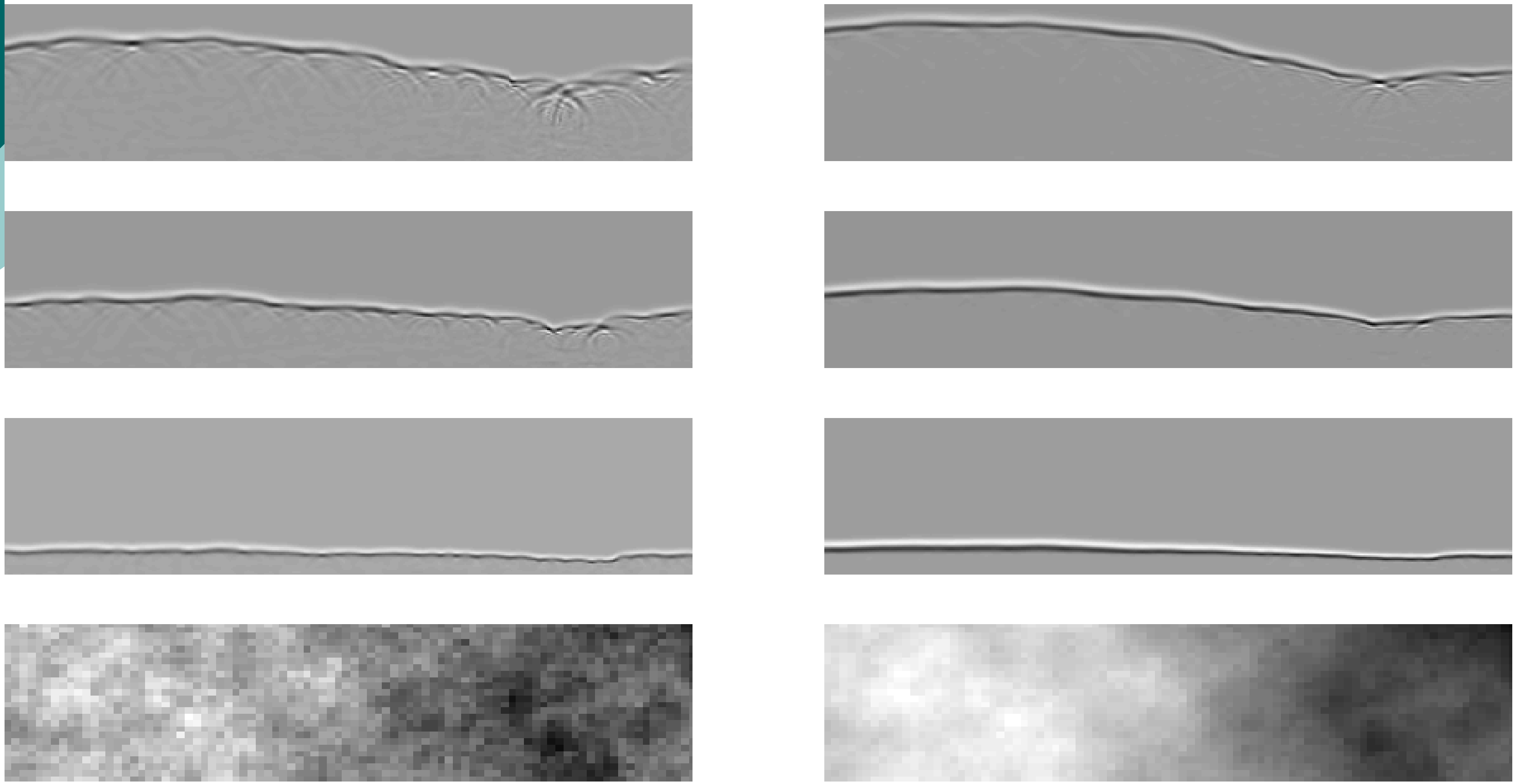
d(x) = distance from the source along the propagation direction

$$C(r) \sim r^{2\alpha}$$

$$\alpha = H = \rho - 1$$

S. M. Vaez Allaei and M. Sahimi, PRL **96**, 075507 (2006).

The Shape of Wave Front and its Evolution



H=0.3 *H=0.75*

Part – III

Numerical Method
Red-Shift of Spectral Density
(Light Localization)

Transfer Matrix
(random mass)

$$\psi_{i+1} + \psi_{i-1} - (2 - m_i\omega^2)\psi_i = 0 \ ,$$

$$\begin{pmatrix} \psi_{i+1} \\ \psi_i \end{pmatrix} = \begin{pmatrix} 2 - m_i\omega^2 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_i \\ \psi_{i-1} \end{pmatrix}$$

with initial normalized vector, $\mathbf{v} = (1/\sqrt{2}, 1/\sqrt{2})^{\mathrm{T}}$.

Oseledec theorem:

To ensure the stability of the numerical method, after multiplying the transfer matrices M times, we checked the length of the resulting vector, normalized it again, and then continued with the new vector

Transfer Matrix (random mass)

The Lyapunov exponent is then expressed in terms of vector lengths d_α obtained after N normalization of v.

$$\gamma = \frac{1}{MN}\sum_{\alpha=1}^{N} \ln(d_\alpha).$$

error in estimating γ is given by,

$$\frac{\Delta\gamma}{\gamma} = \frac{1}{\sqrt{N}} \frac{\sqrt{\langle(\ln d_\alpha)^2\rangle - \langle \ln d_\alpha\rangle^2}}{\langle \ln d_\alpha\rangle}$$

γ is a self-averaged quantity, and the error of its estimates approaches zero as $1/\sqrt{N}$, as N increases.

Dynamics of Pulse in Random Media
"3D Light localization"

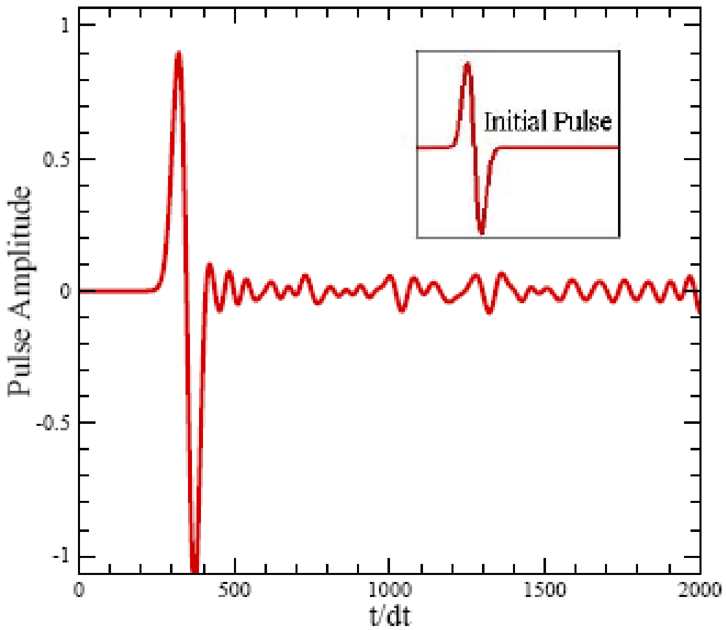


FIG. 1. The Observed pulse in $Z = 500d$, where d is the lattice spacing and the initial pulse at $Z = 0$ (inset).

Red-Shift of Spectral Density

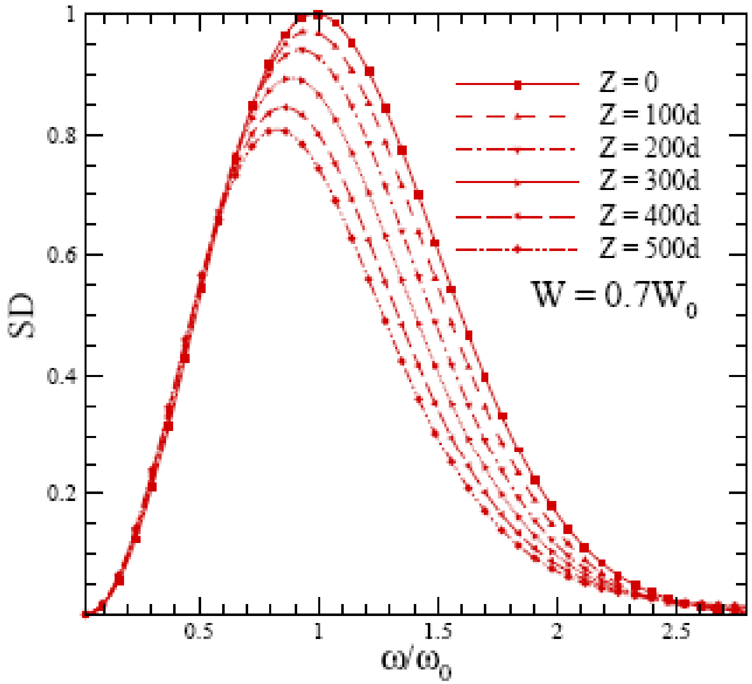


FIG. 2. The initial spectral density of pulse at $Z = 0$ and the observed ones (others) in different location from the source. Here ω_0 is the maximum frequency of spectrum. There is a red-shift in the spectral density due to the localization of different frequencies in different length scales.

$$\psi(\omega, Z) \sim \exp\{-Z/\xi(\omega)\}$$

$$SD(Z) \sim \exp\{-2Z/\xi(\omega)\}.$$

$$\xi^{-1}(\omega) = -\lim_{Z\to\infty}(2Z)^{-1} ln\left(\frac{SD(\omega, Z)}{SD(\omega, 0)}\right).$$

Localized Behavior

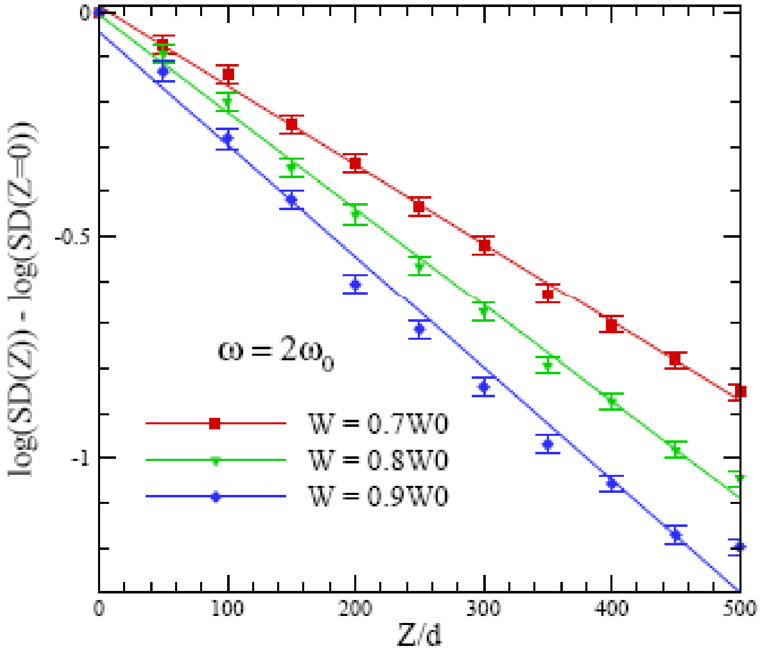


FIG. 3. Semi-Log plot of amplitude difference of spectral density verses distance Z, for different disordered strengths,(for frequency $\omega = 2\omega_0$, where ω_0 is the frequency that the spectral density of the incident pulse has maximum).

Localization length

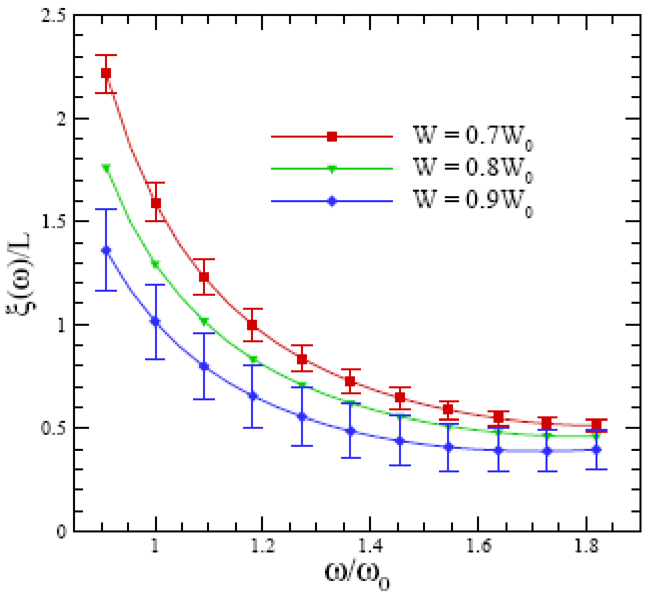


FIG. 4. The frequency dependence of the localization length for different for different disordered strengths. The modes with $\xi(\omega)/L > 1$ and $\xi(\omega)/L < 1$ are delocalized and localized, respectively.

Black-Body radiation

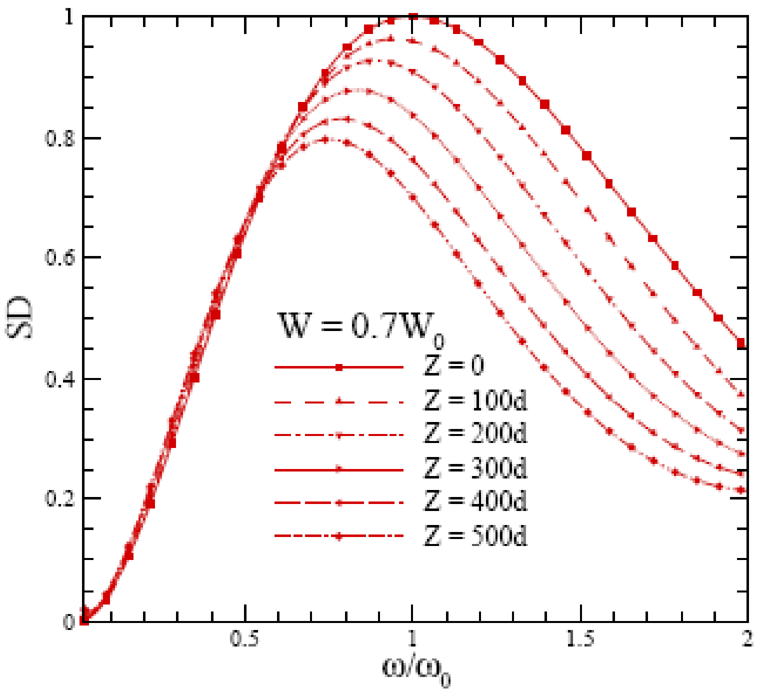


FIG. 5. Spectral density of black body radiation and its shape deformation by propagating in for instance, Z direction.

Red-shift vs Distance from the source.

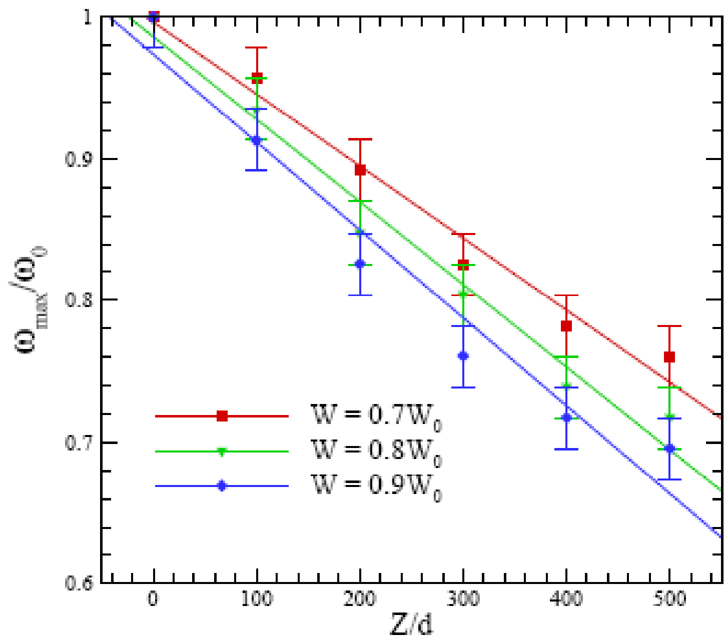


FIG. 6. Peak frequency of the spectral density versus the location of observation for different noise strengths.

For small disorder strength $\Delta\omega/\omega_0 \propto Z/d,$

Red-shift vs strength of disorder

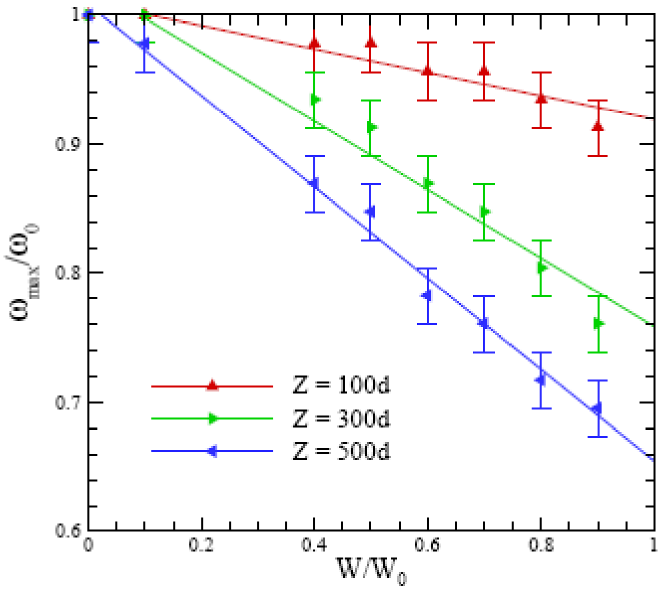


FIG. 7. Ration of peak frequency of the spectral density versus the disorder strength at different location.

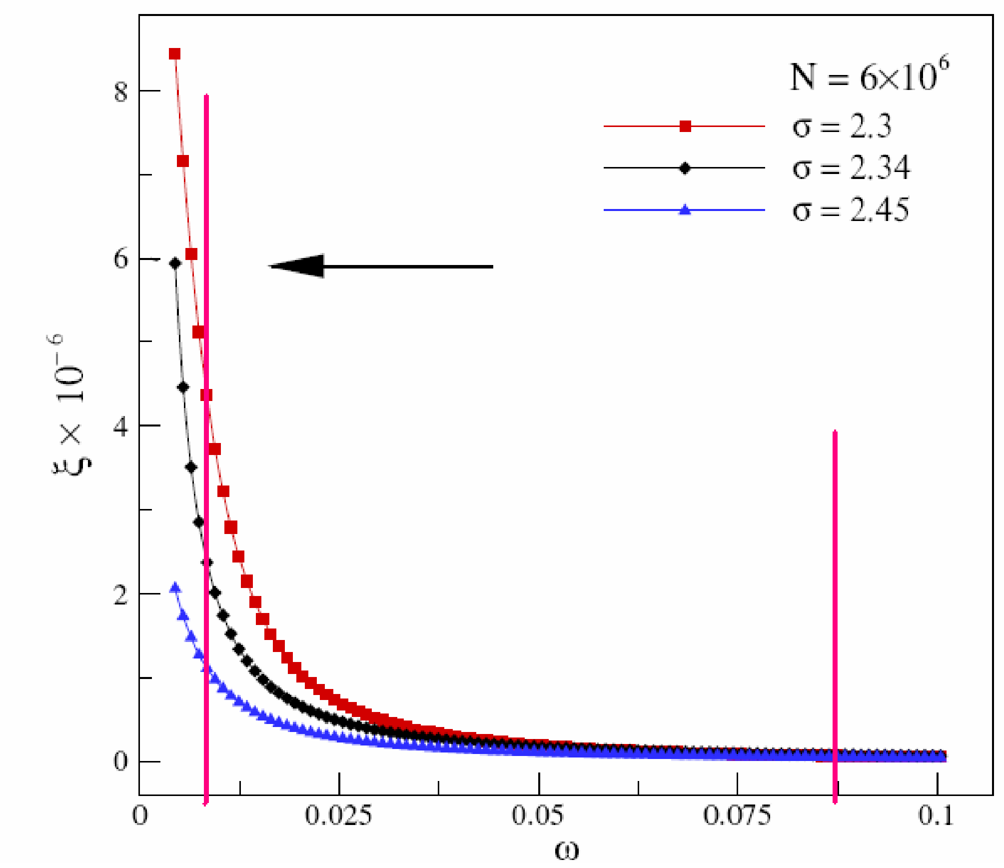
$$\Delta\omega/\omega_0 \propto W/W_0.$$

Measured Temperature at distance Z (in weak disorder limit)

$$(T_0 - T(Z))/T_0 \propto Z$$

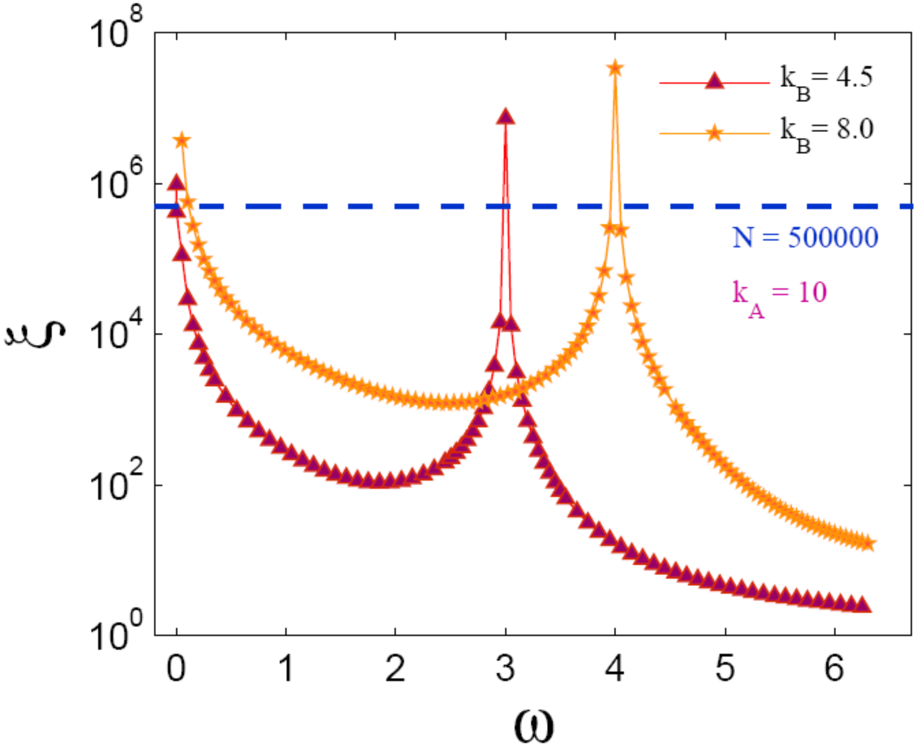
As an example, suppose that the source at $Z = 0$ is at temperature 1000 K. At a distance 92 μm (which is about 500 characteristic length of the disorder), the apparent measured temperature of the source would be about 900 ± 50 K. This difference in the actual and apparent temperatures is obtained for the disorder strength $\sigma = 0.3$.

Acoustic waves in the random dimer media

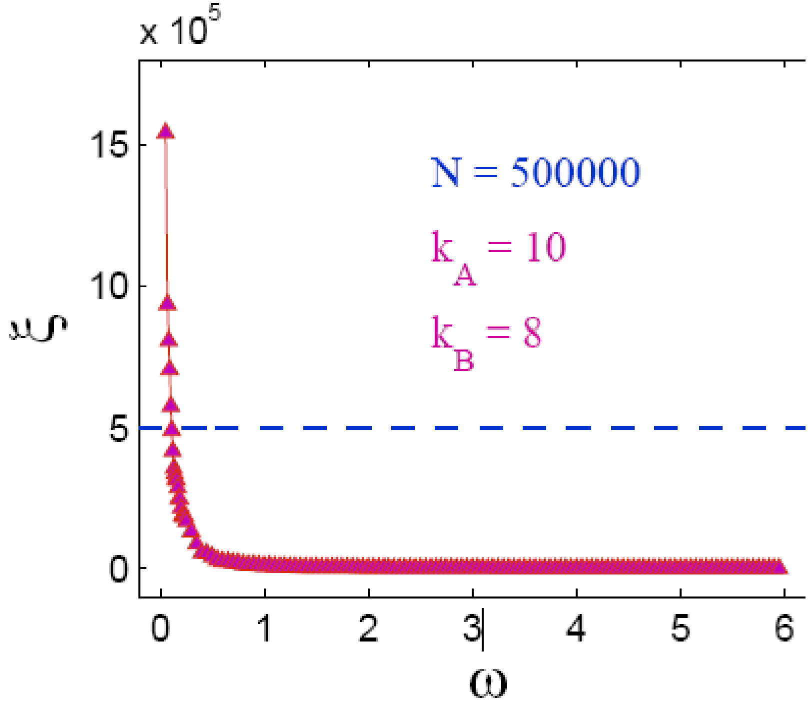


Random Dimer

$$k_A, k_A, k_A, k_A, k_A, k_B, k_B, k_A, k_A, k_A, k_B, k_B, \cdots$$

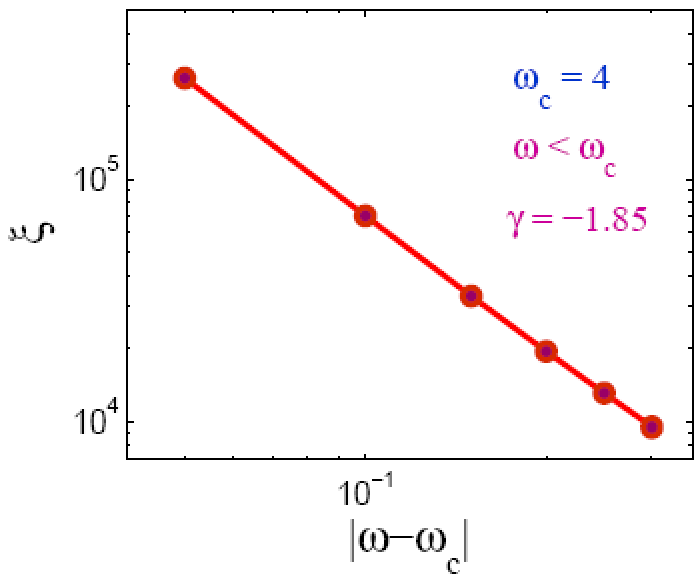


$k_B = 4.5$
$k_B = 8.0$
N = 500000
$k_A = 10$
ξ
ω



x 10^5
N = 500000
$k_A = 10$
$k_B = 8$
ξ
ω

Critical exponents



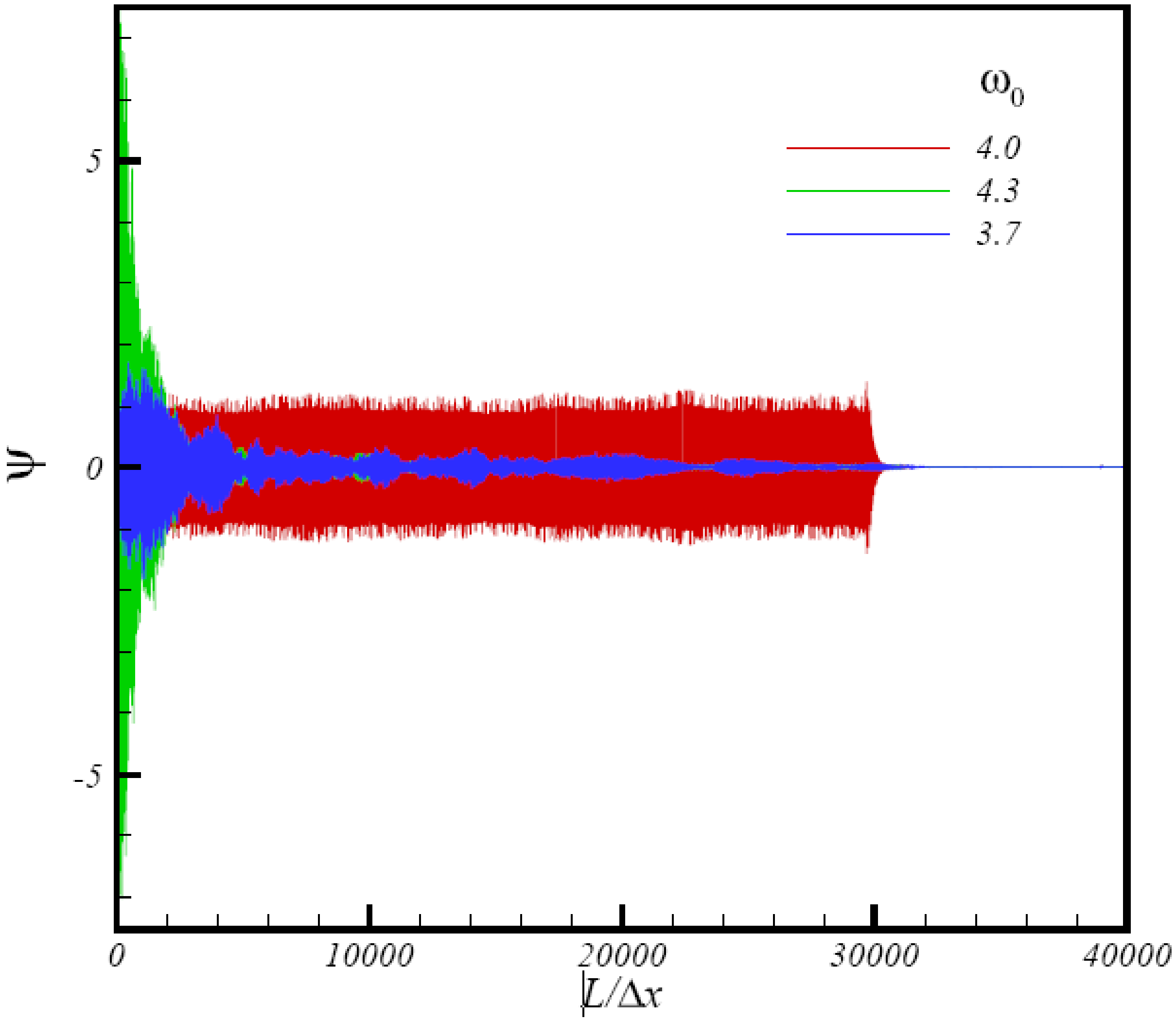
$$\xi \propto | \omega - \omega_c |^{-\nu}$$

$\omega > \omega_c \qquad \nu \simeq 2.20$

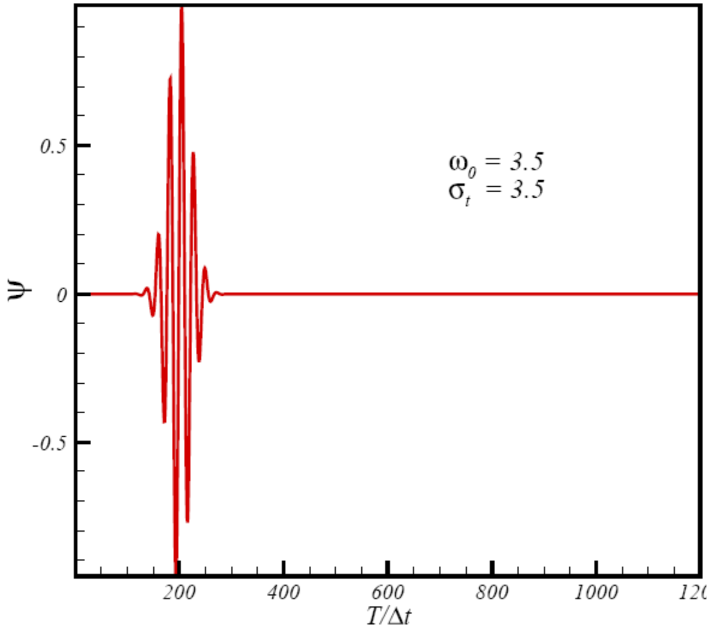
$\omega < \omega_c \qquad \nu \simeq 1.85$

$\omega \rightarrow 0^+ \qquad \nu \simeq 1.94$

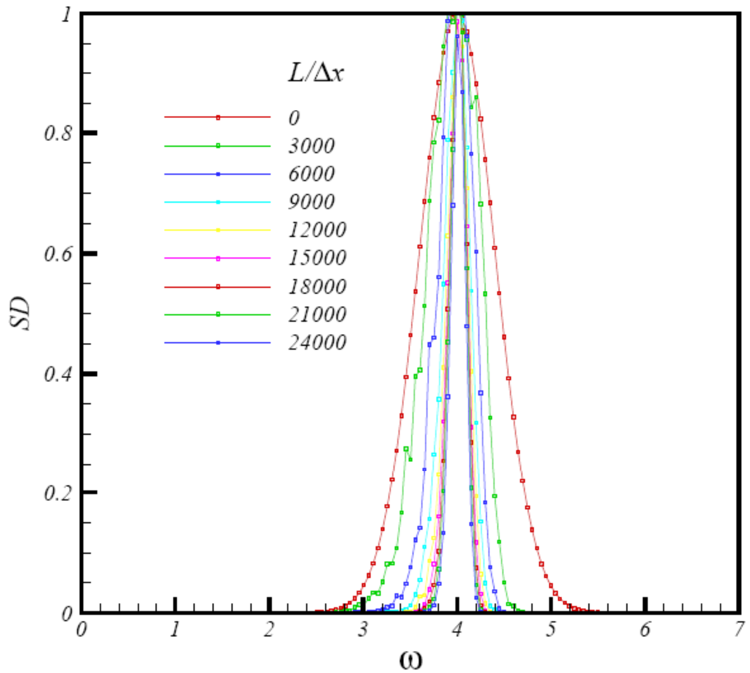
$$\omega_c^2 = 2k_B$$



ω0
4.0
4.3
3.7
5
0
-5
Ψ
0
10000
20000
30000
40000
L/Δx



$\omega_0 = 3.5$
$\sigma_t = 3.5$
0.5
0
-0.5
ψ
200
400
600
800
1000
$T/\Delta t$



$L/\Delta x$
0
3000
6000
9000
12000
15000
18000
21000
24000
SD
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
ω

Thanks