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# Spintronics: Physics and applications (II)

**Malek Zareyan**

**IPM, 15 TiR 1387**

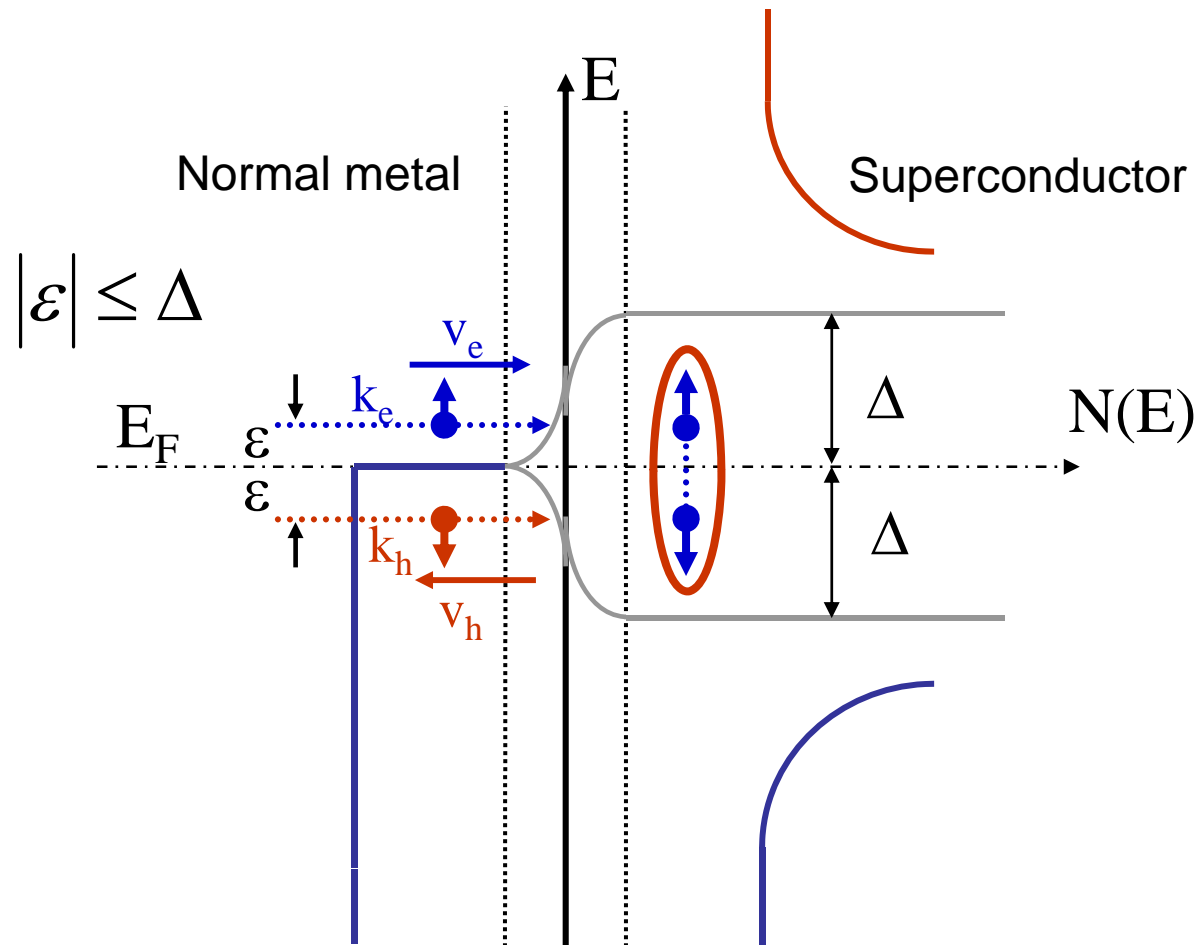
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وزارت علوم، تحقیقات و فناوری  
مرکز تحصیلات تکمیلی در علوم پایه  
گاوازنگ - زنجان



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# Andreev reflection in NS junctions



[A. F. Andreev, Sov. JETP **19**, 1228(1964)]

# Andreev reflection in NS junctions

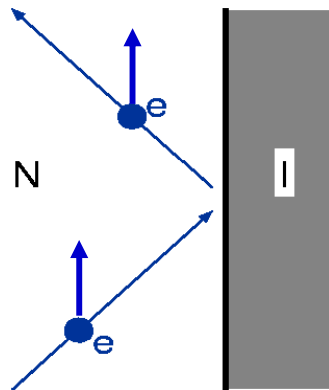
- *Conversion of normal current to supercurrent: charge transfer by  $2e$*
- *Momentum is changed by  $\sim 2\varepsilon/v_F \ll p_F$  negligibly small for degenerate N*

$$\Delta \ll E_F \quad \vec{k}_e \cong \vec{k}_h \quad \text{AR is Retro Reflection}$$

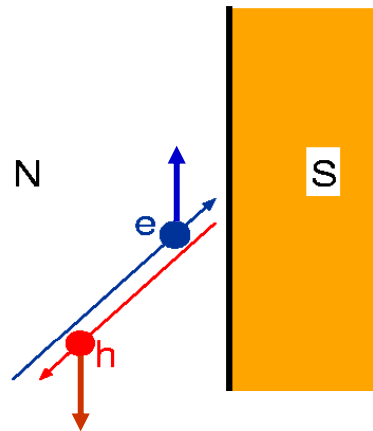
- *Spin is inverted*  
*(has significant consequences if N is a ferromagnet)*

[G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982)]

# Andreev reflection in NS junctions



Specular Normal Reflection



Retro Andreev Reflection

## Andreev Conductance:

finite  $G_{NS}$  of NS junction at voltages below the superconducting gap  $\Delta/e$ , for ideal interface at zero voltage

$$G_{NS} = 2_{AR} G_{NN}$$

[G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982)]

# Superconducting proximity effect

*Superconducting pair potential (order parameter)*

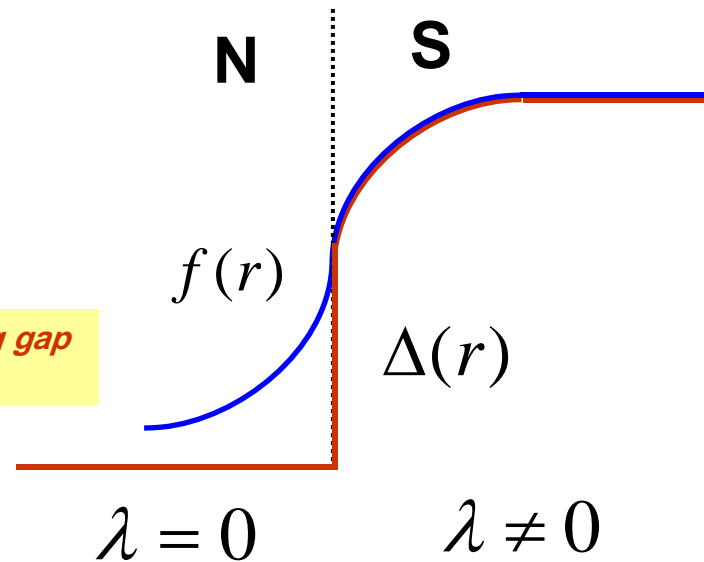
$$f \propto \Psi$$

$$\Delta(r) = \lambda f(r)$$

$$\lambda = gN_0$$

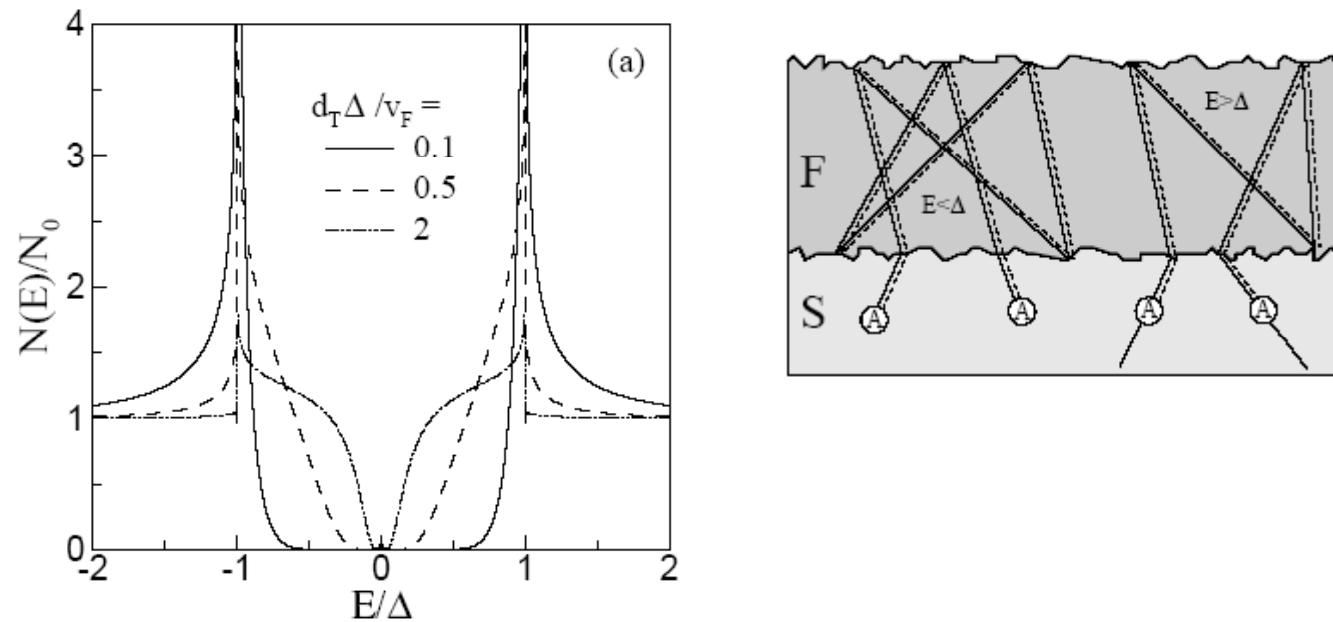
*Electron-phonon coupling constant*

*Superconducting gap function*



[G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982)]

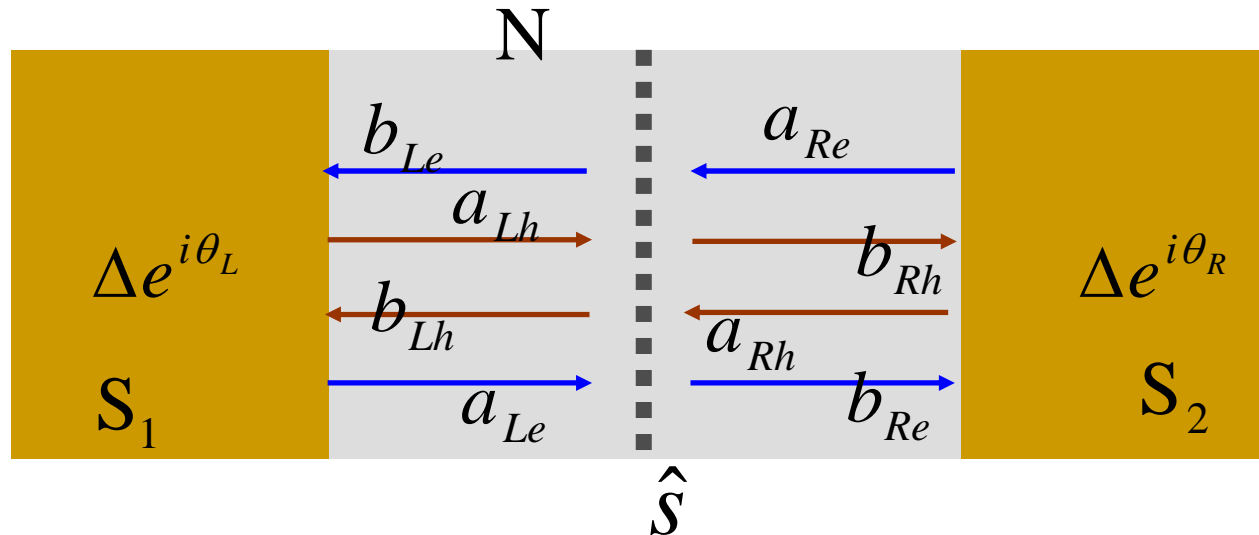
# Superconducting proximity effect



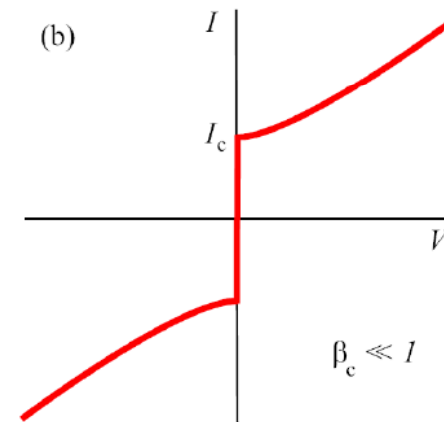
**N metal acquires characteristics of superconducting states**

[E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982)]

# Josephson effect in SNS junctions



$$I(\varphi) = I_c \sin \varphi$$



# Bogoliubov-deGennes equations

Scattering formalism of Andreev reflection:  
Blonder-Tinkham-Klapwijk (BTK formula)

*electron-hole conversion:  
superconducting correlations*

*electron wave function*

$$\begin{pmatrix} \hat{H} & \Delta e^{i\theta} \\ \Delta e^{-i\theta} & -\hat{H}^* \end{pmatrix} \begin{pmatrix} \psi_e(\mathbf{r}) \\ \psi_h(\mathbf{r}) \end{pmatrix} = E \begin{pmatrix} \psi_e(\mathbf{r}) \\ \psi_h(\mathbf{r}) \end{pmatrix}$$

*Diagonal normal  
state Hamiltonian*

*hole wave function*

$$\hat{H} = \hat{H}_0 - E_F, \quad \hat{H} = -(\hbar^2/2m)(\nabla + ie\mathbf{A}(\mathbf{r})/\hbar c)^2 + U(\mathbf{r})$$

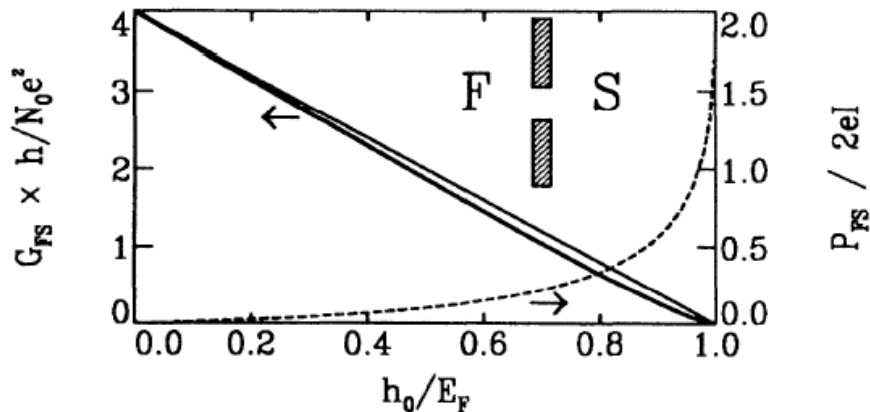
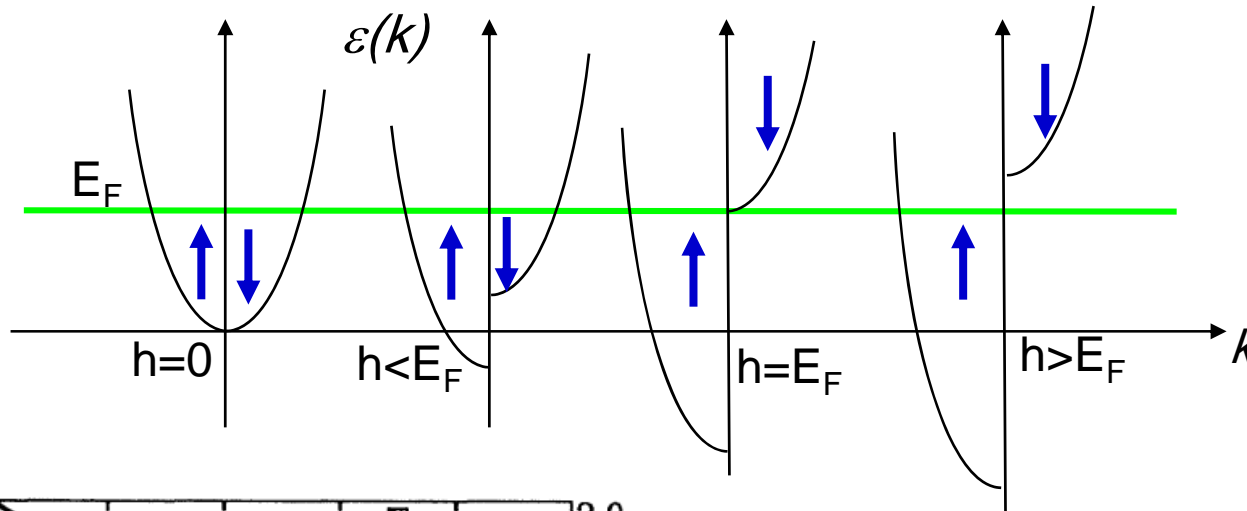
$$\Delta e^{i\theta}(\mathbf{r}) \propto \psi_e(\mathbf{r})\psi_h(\mathbf{r})$$

*Superconducting pair potential (order parameter): self-consistency relation*



# Suppression of Andreev conductance in FS junctions

*Spin is inverted (has significant consequences if N is a ferromagnet)*

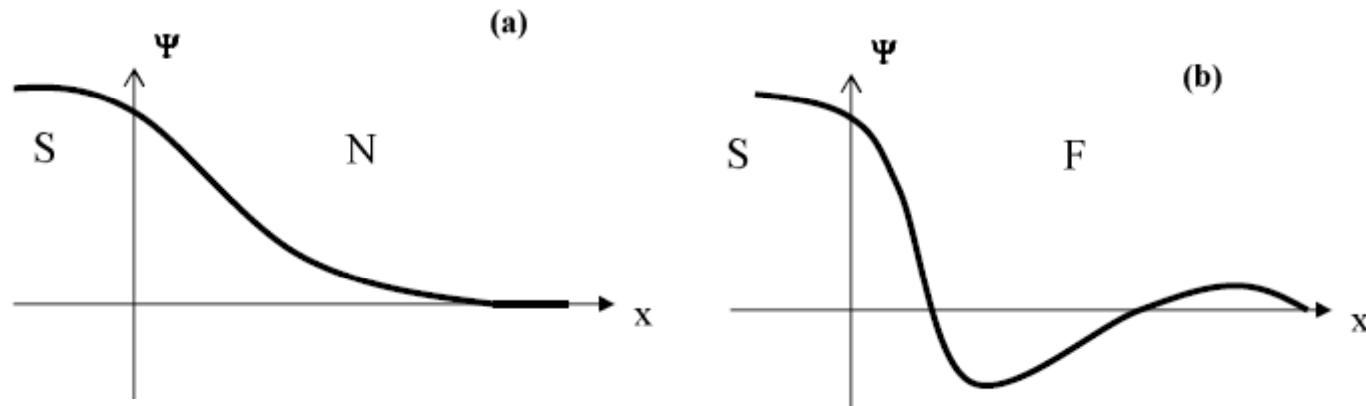


Ferromagnetism and singlet-spin superconductivity are exclusive phenomena

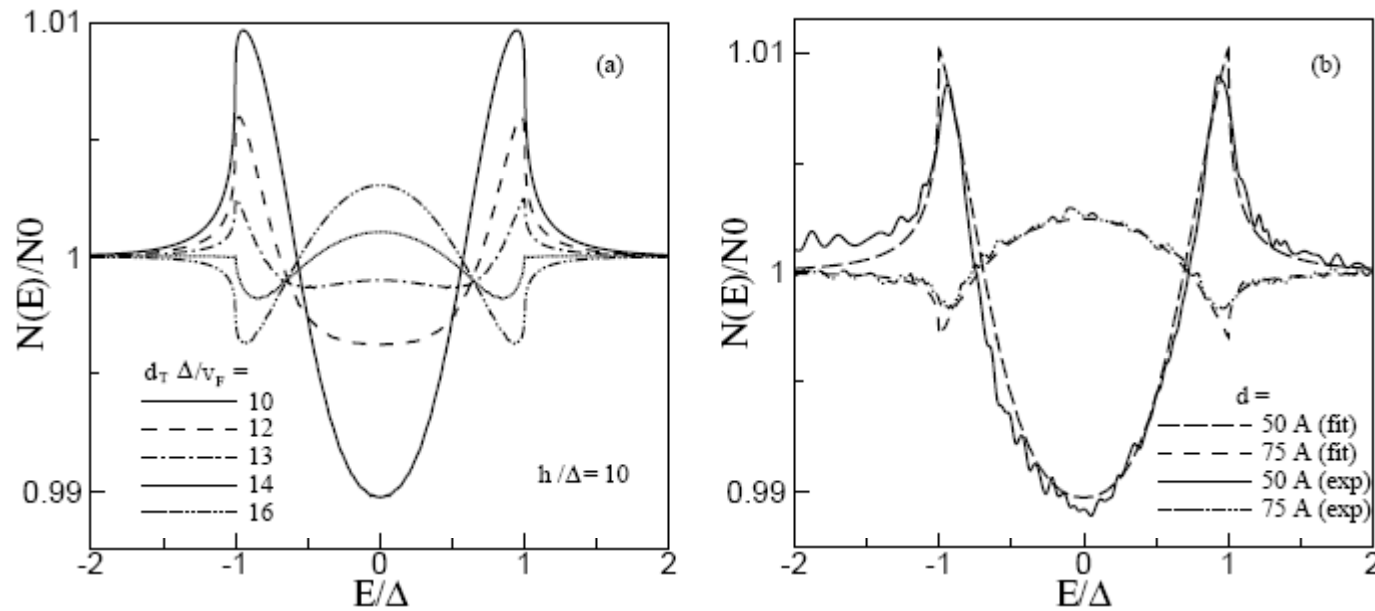
Their interplay in FS structures very active field:  
*π*-SFS junction qubits, long range triplet effects, entanglement,....

[M. J. M. de Jong and C. W. J. Beenakker Phys. Rev. Lett. **74**,1657 (1995);.....]

# Oscillatory proximity effect in FS junctions

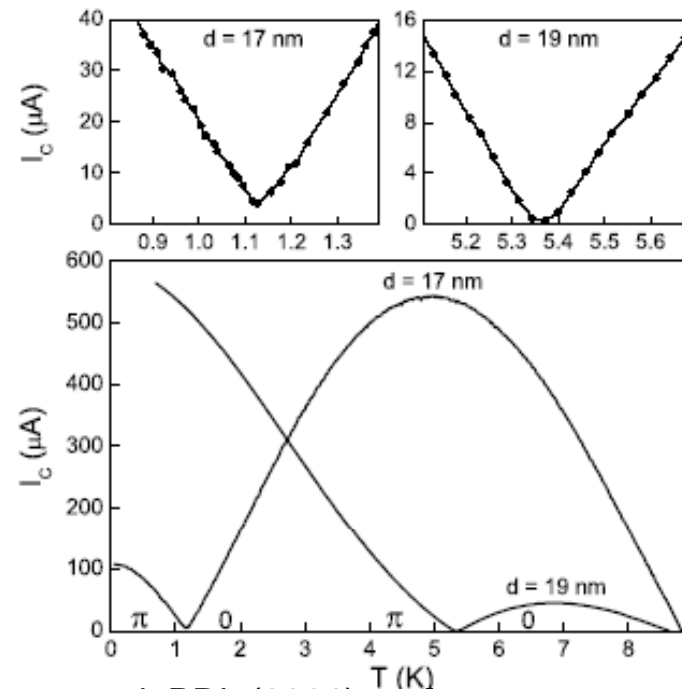
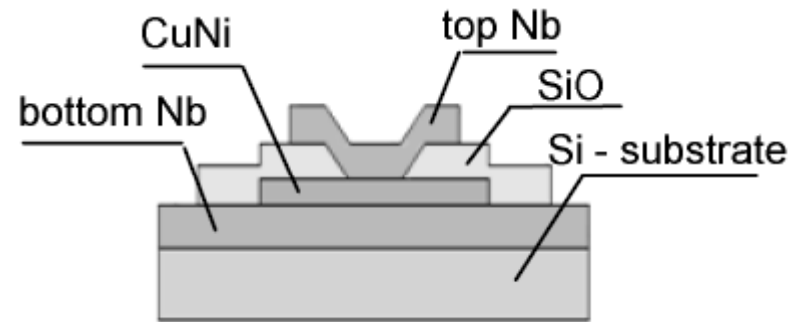
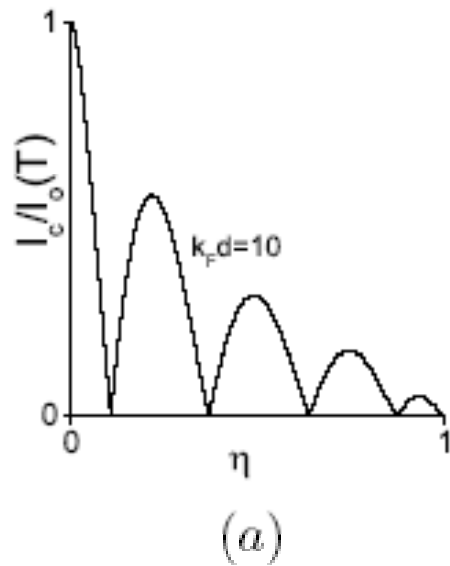


# DOS oscillations in FS junctions



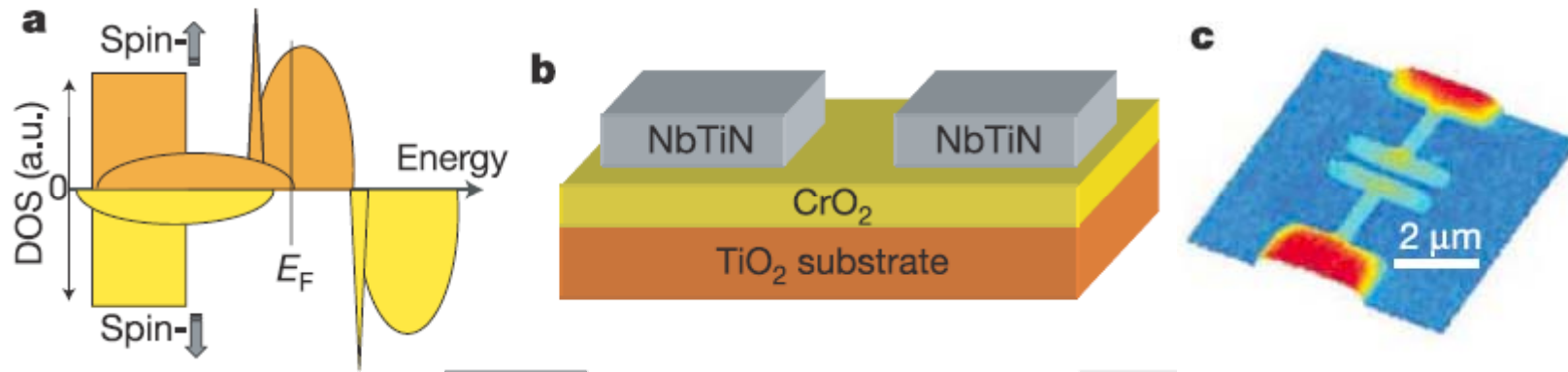
[M. Zareyan et al, PRL (2000); Kontos et al, PRL (2000); ... ]

# 0- $\pi$ transitions



[Ryazanov et al, PRL (2001); ...]

# Supercurrent through half-metal ferromagnetic layer

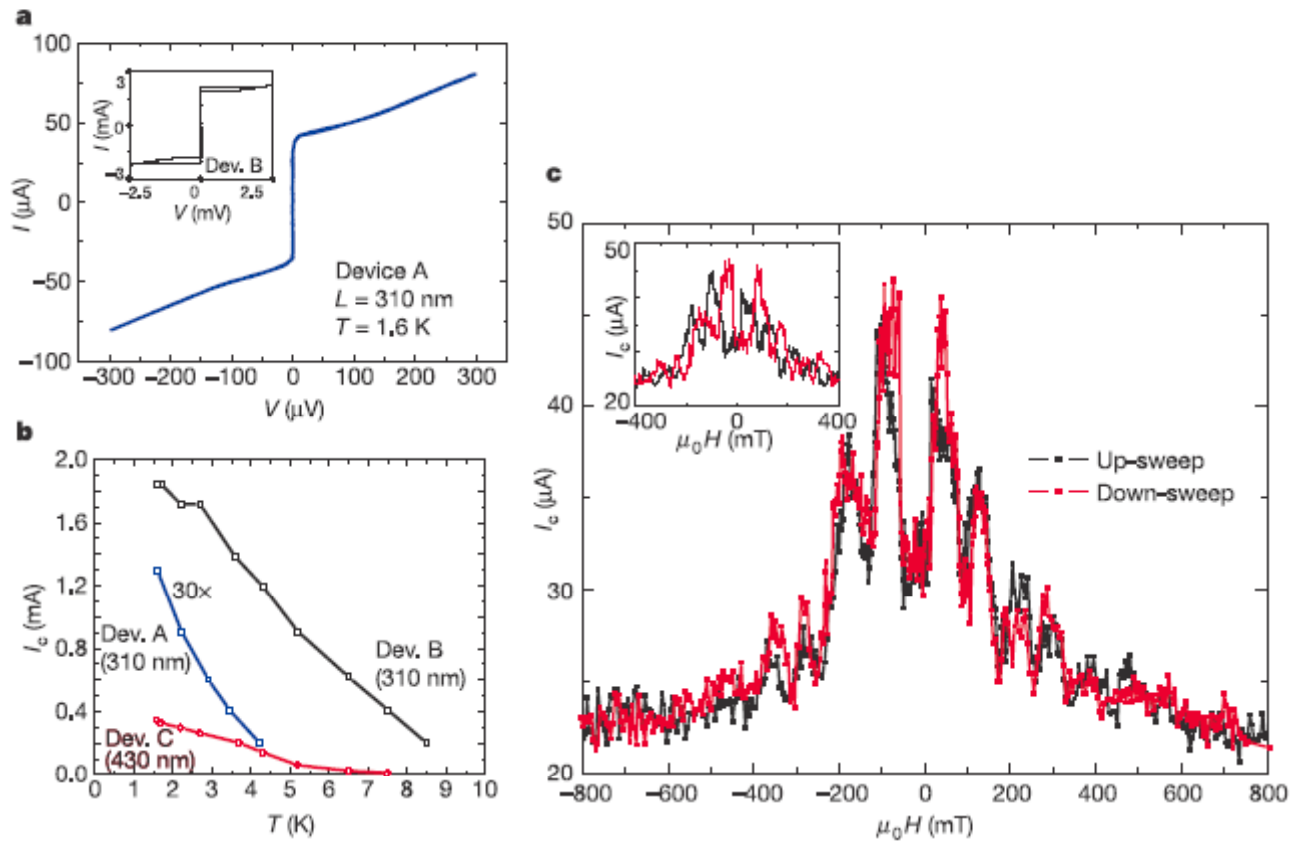


$$\xi_F \propto 1/E_{exh} \text{ or } 1/\sqrt{E_{exh}}$$

In CrO<sub>2</sub> ( $E_{exh} \sim 2$  eV) superconductivity should be destroyed over interatomic distance

[ R. S. Keizer et al, Nature **439**, 825 (2006)]

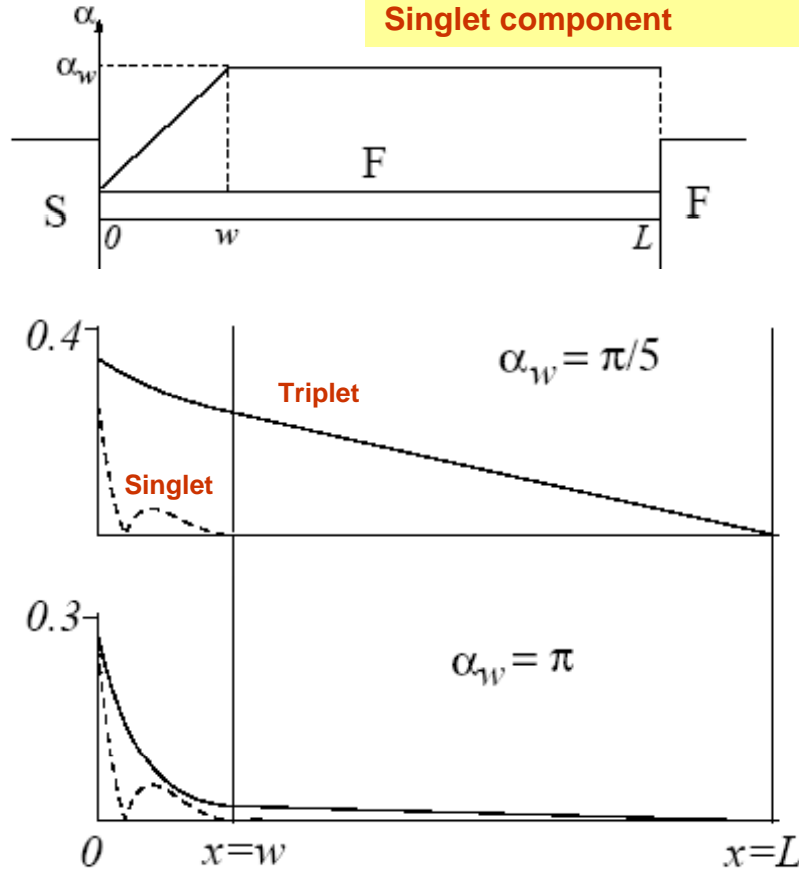
# Supercurrent through half-metal ferromagnetic layer



[ R. S. Keizer et al, Nature **439**, 825 (2006)]

# Long range proximity in Ferromagnets

Inhomogeneous noncollinear configuration of magnetization:  
Induced superconductivity has Triplet component as well as Singlet component



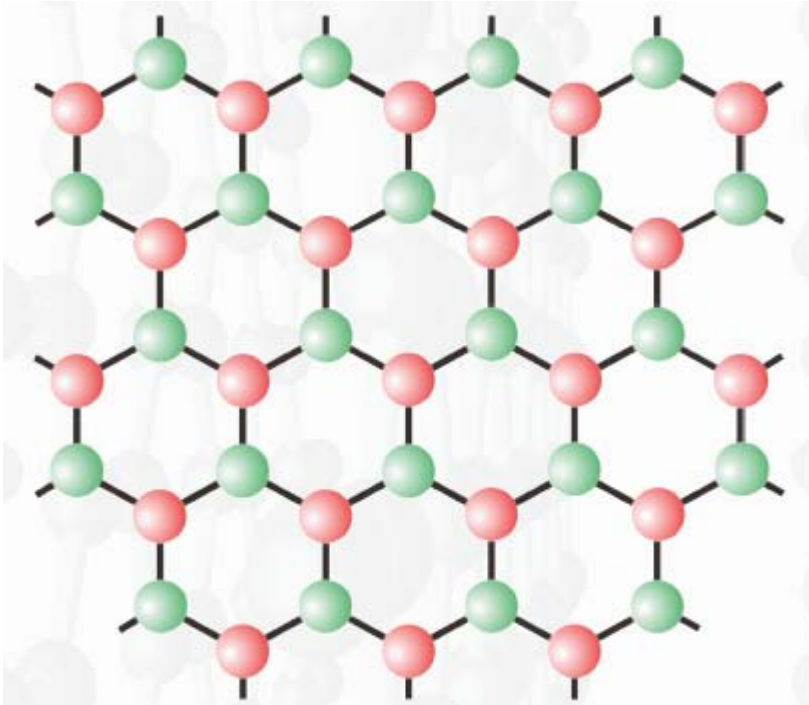
$$\xi_F \propto 1/E_{exh} \text{ or } 1/\sqrt{E_{exh}}$$

$$\xi_N \propto 1/T \text{ or } 1/\sqrt{T}$$

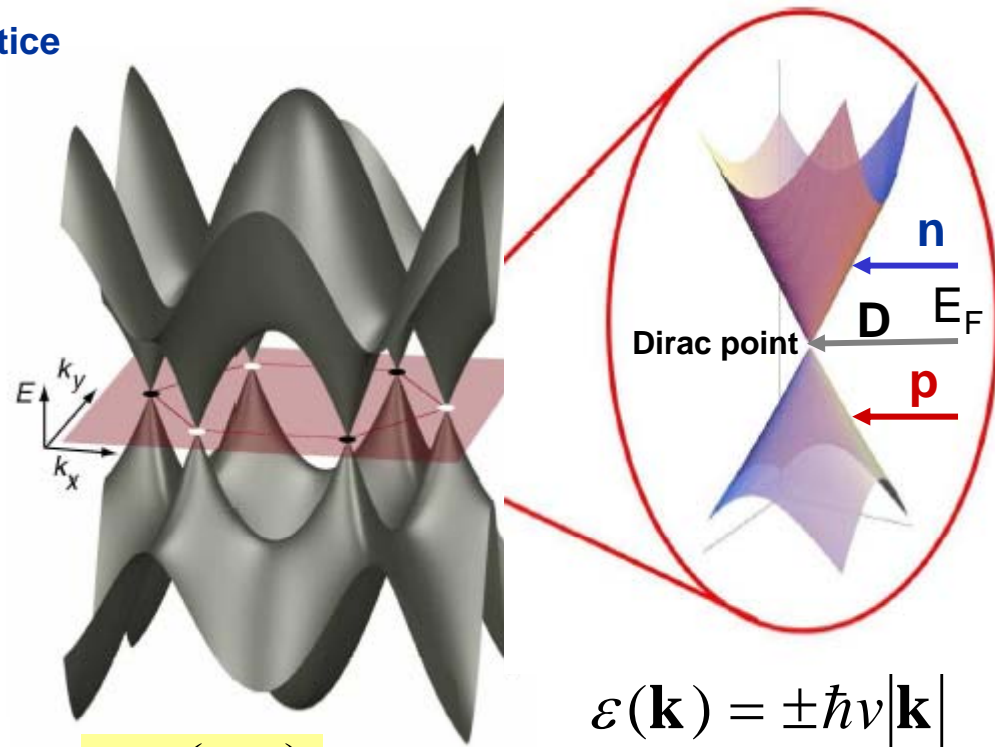
[ F. S. Bergeret et al, PRL **86**, 4096 (2001)]

# Graphene

Isolated single one atom thick honeycomb lattice of carbon atoms: Two (A and B) sublattices



4-component spinor in two (A,B) sublattices space (pseudospin) and two (-,+)-valley space



$$\varepsilon(\mathbf{k}) = \pm \hbar v |\mathbf{k}|$$

$$\hat{u} = \begin{pmatrix} \psi_A^+ \\ \psi_B^+ \\ \psi_A^- \\ \psi_B^- \end{pmatrix}$$

Carrier density and its type (n or p or D) tuned by gate voltage or doping of substrate

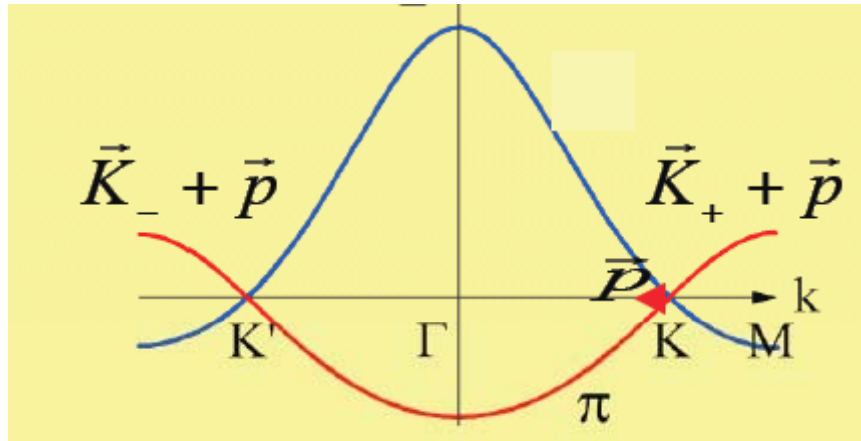
[Novosolov et al, Nature **438**, 197 (2005); Zhang et al, Nature **438**, 201 (2005); ...]



# PseudoRelativistic chirality

4-component spinor in two (A,B) sublattices space  
(pseudospin) and two (-,+)-valley space

Low excitation Dirac Hamiltonian



$$\hat{H} = v \begin{pmatrix} \sigma \cdot \vec{p} & 0 \\ 0 & -\sigma \cdot \vec{p} \end{pmatrix}$$

$$\hat{u} = \begin{pmatrix} \psi_A^+ \\ \psi_B^+ \\ \psi_A^- \\ \psi_B^- \end{pmatrix}$$

$$\hat{H}\hat{u} = \varepsilon\hat{u}$$

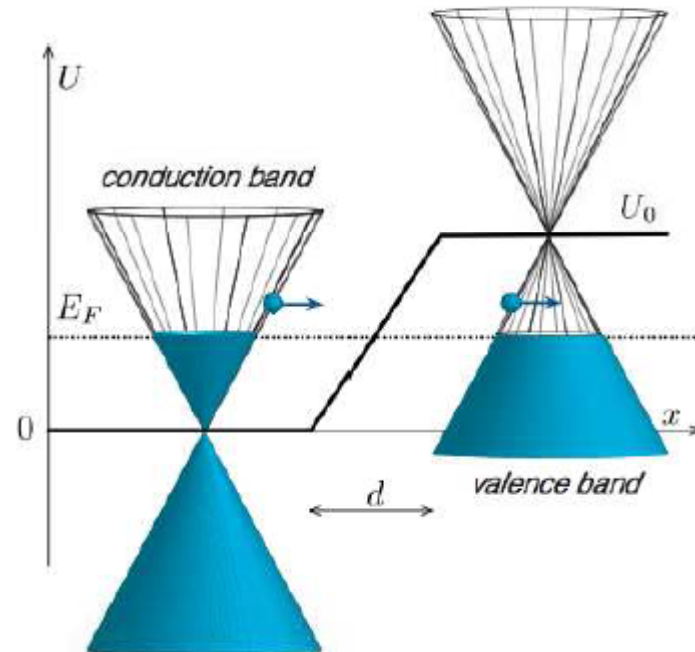
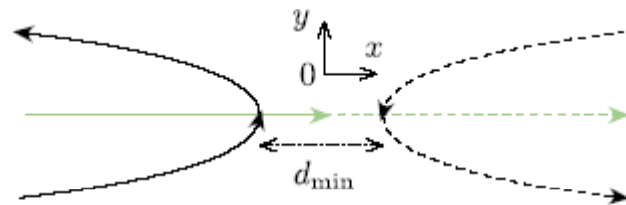
Chirality in valley  $K_+$

$$\sigma \cdot \hat{n} = +1 \quad \text{for conduction band}$$

$$\sigma \cdot \hat{n} = -1 \quad \text{for valance band}$$

[M. I. Katsnelson, K. S. Novosolov A. K. Geim, Nature Phys. **2**, 620 (2006);.....]

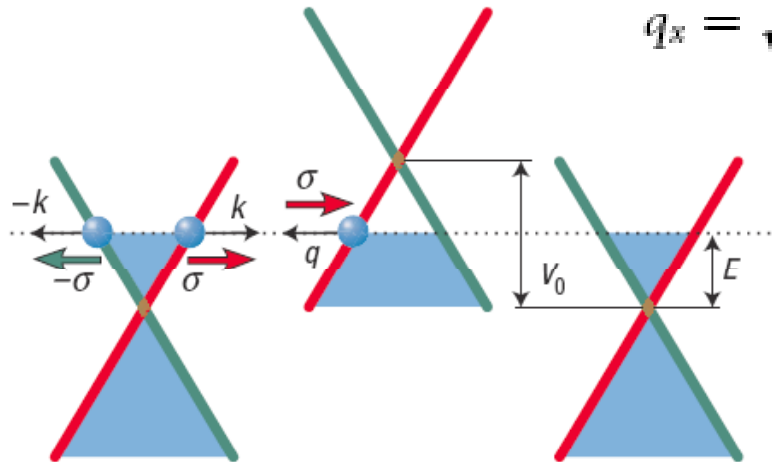
# Klein tunnelling



[M. I. Katsnelson, K. S. Novosolov A. K. Geiml, Nature Phys. **2**, 620 (2006);.....]

# Klein tunnelling

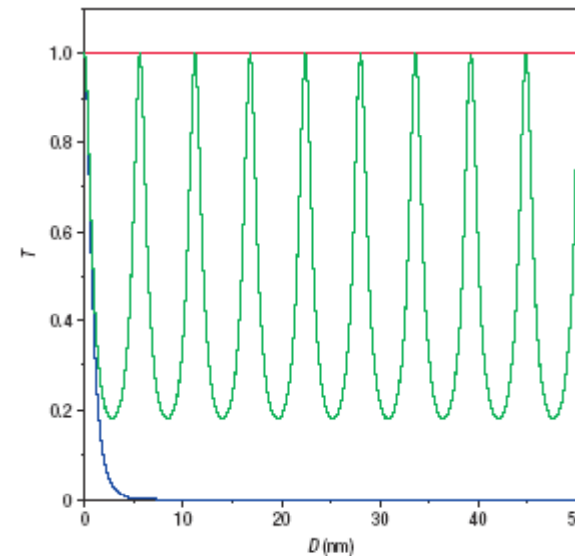
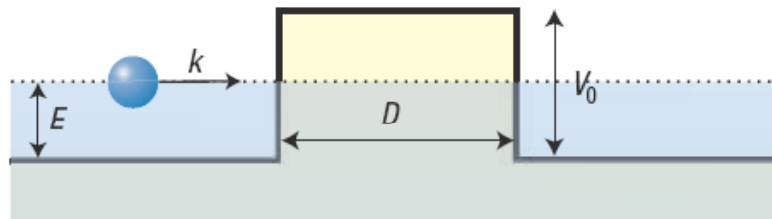
**a**



$$q_x = \sqrt{(E - V_0)^2 / \hbar^2 v_F^2 - k_y^2}, \quad \theta = \tan^{-1}(k_y / q_x)$$

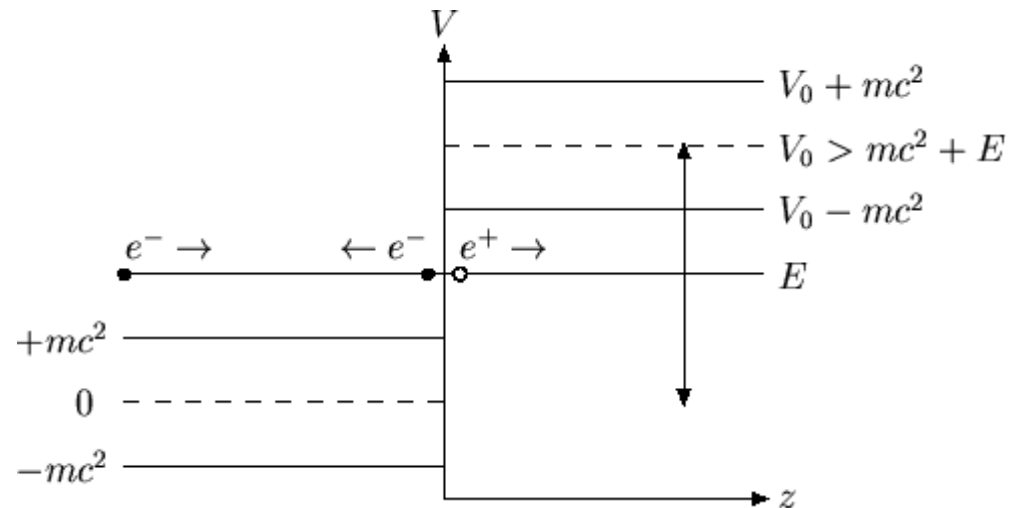
$$T = \frac{\cos^2 \phi}{1 - \cos^2(q_x D) \sin^2 \phi}$$

**b**



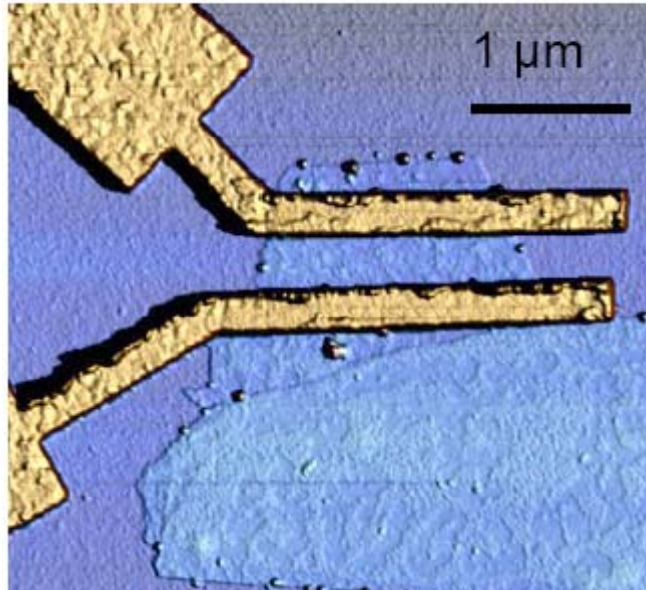
[M. I. Katsnelson, K. S. Novosolov A. K. Geim, Nature Phys. **2**, 620 (2006);.....]

# Klein paradox in relativistic quantum theory



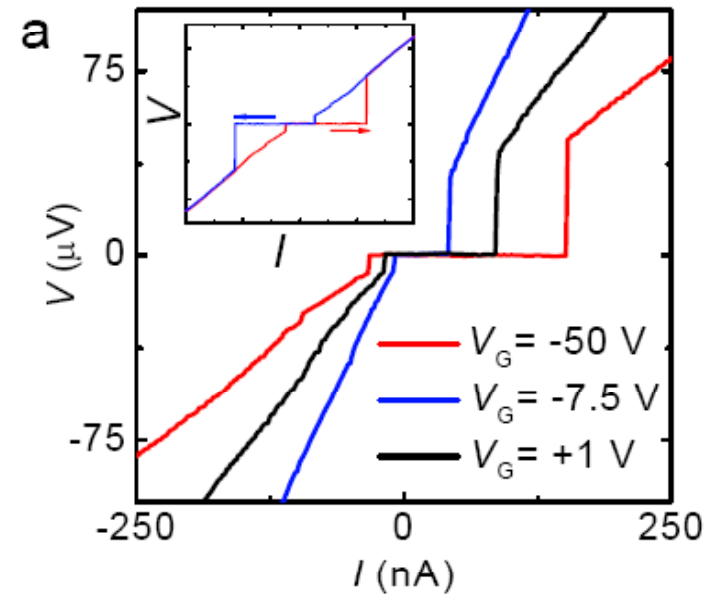
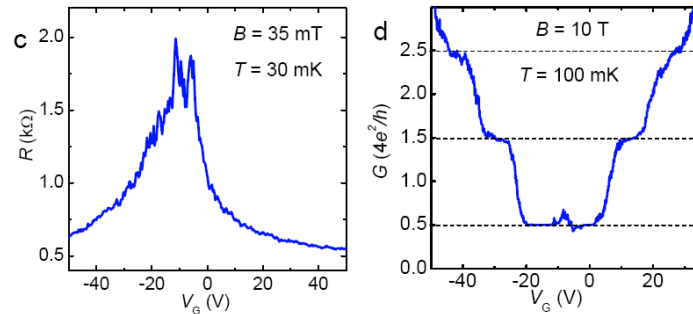
[ O. Klein, Z. Phys. **53**, 157 (1929)]

# Bipolar supercurrent in graphene Josephson contact



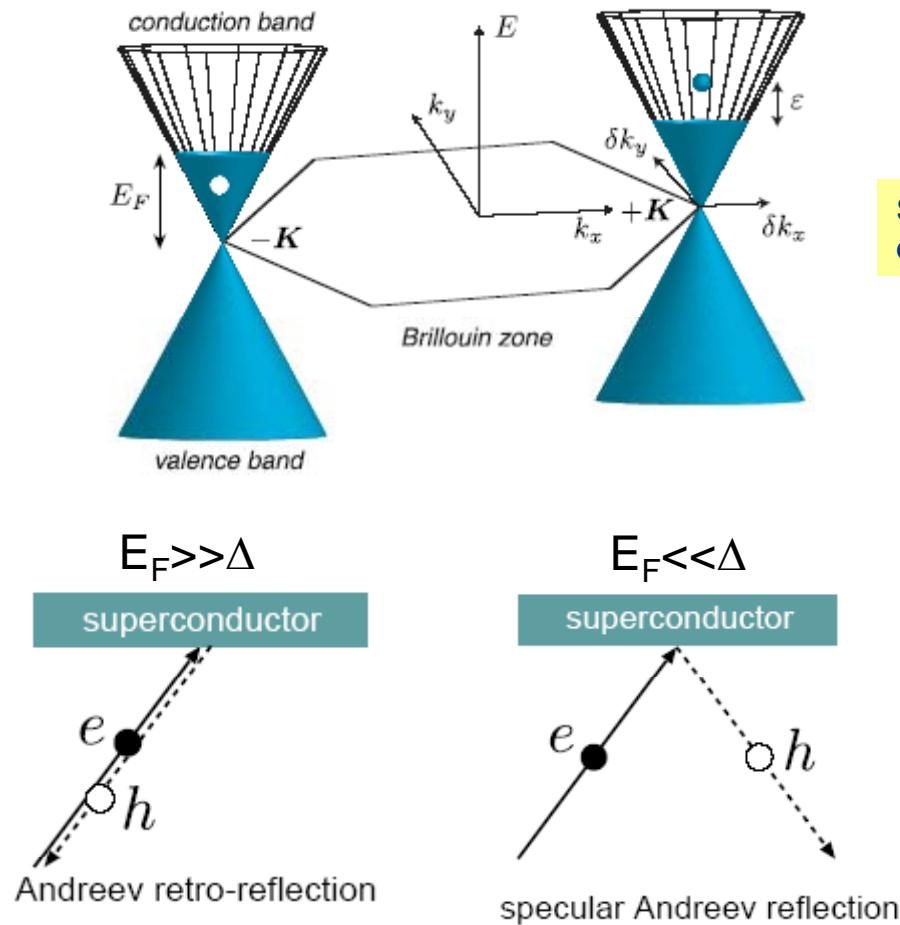
Ti/AI (10/70nm) superconducting bilayer ( $T_c \sim 1.3$  K)

strong  $V_G$ -dependence + half-IQH : single layer graphene

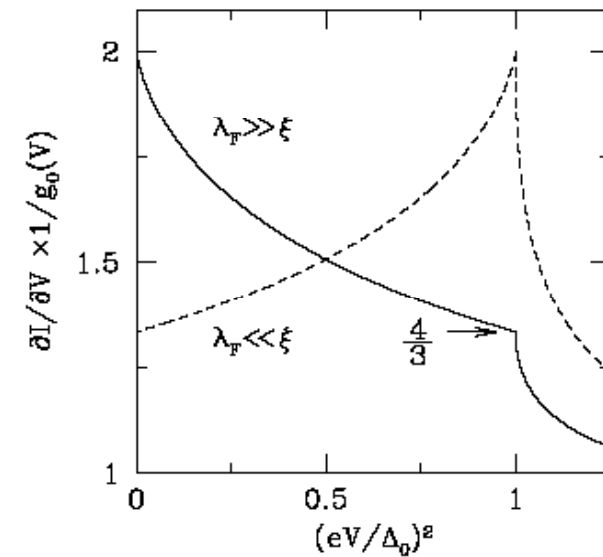


[Heersche et al, Nature **446**, 56-59 (2007);... ]

# Specular Andreev reflection in graphene NS junctions



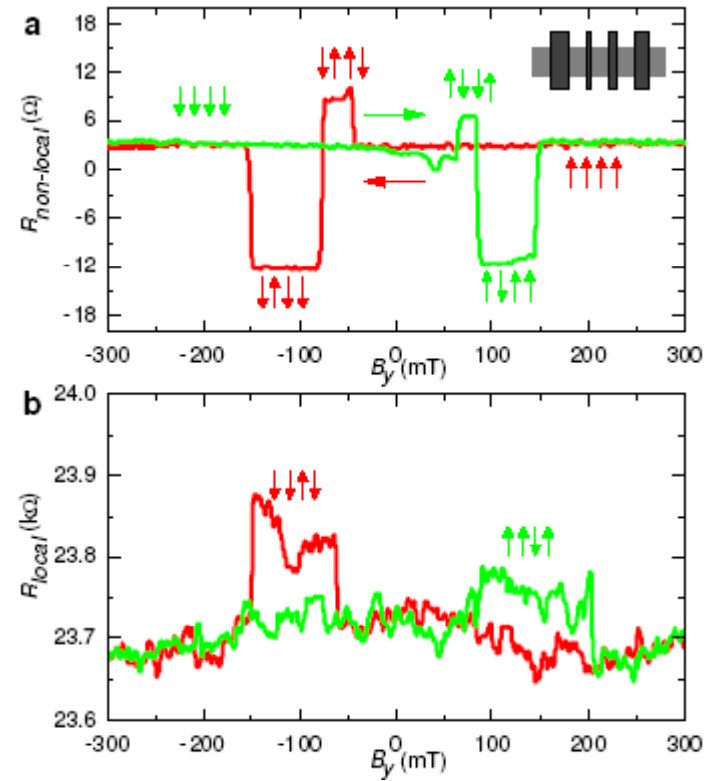
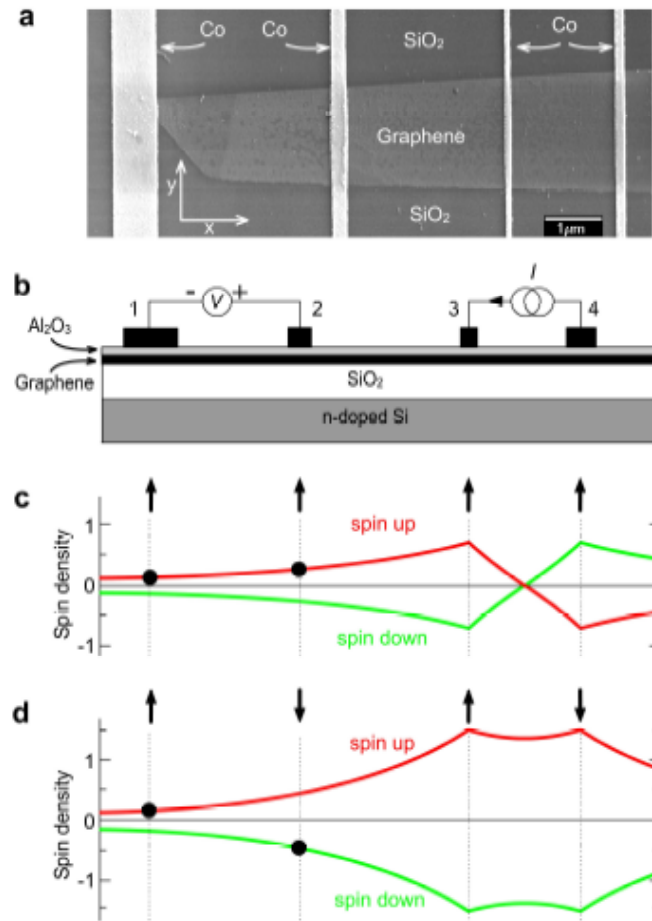
Superconducting correlation of massless Dirac electrons with **opposite spin and different valley**



**GS  
Junction**

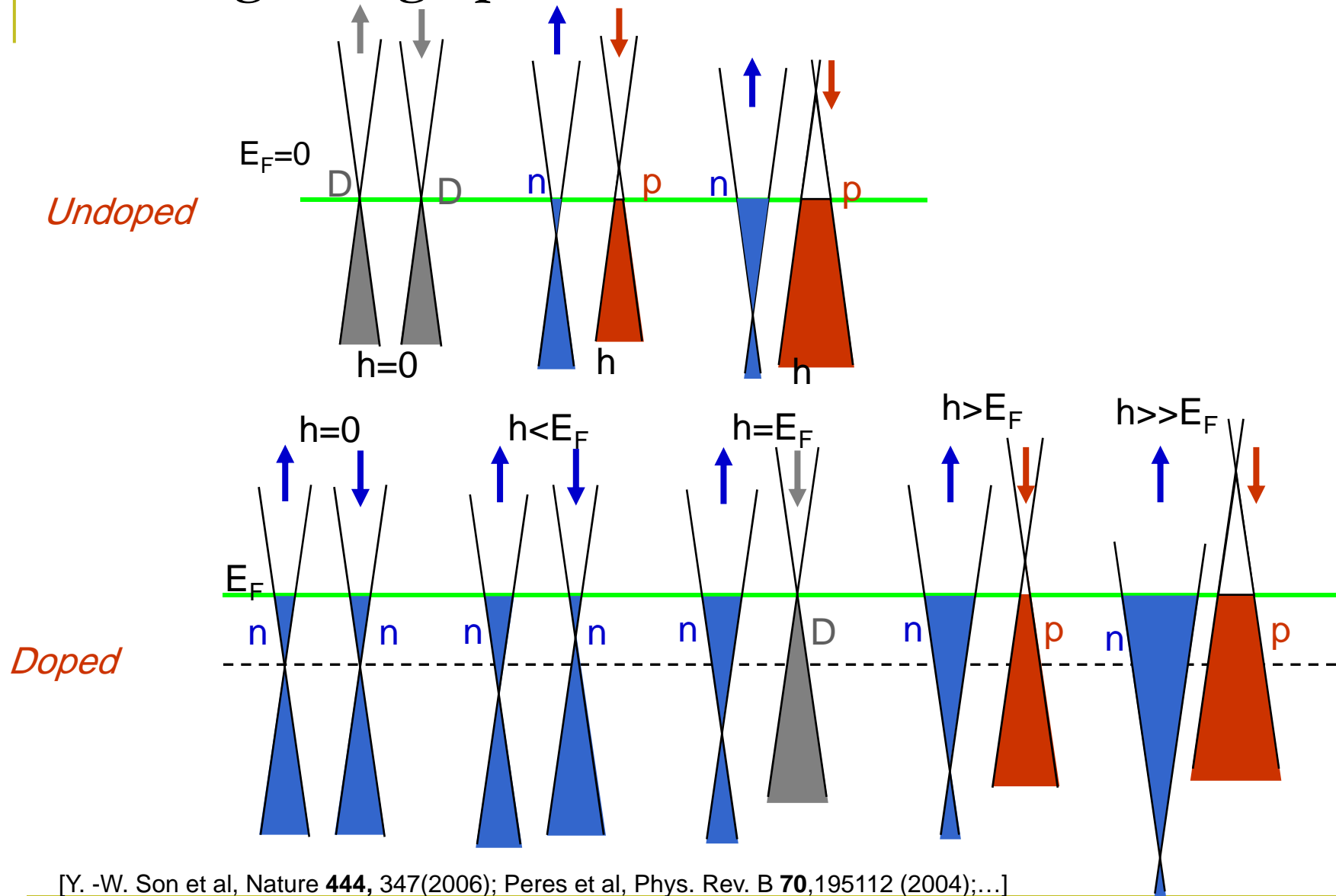
[C. W. J. Beenakker Phys. Rev. Lett. **97**,067007 (2006)]

# Graphene Spin-Valve



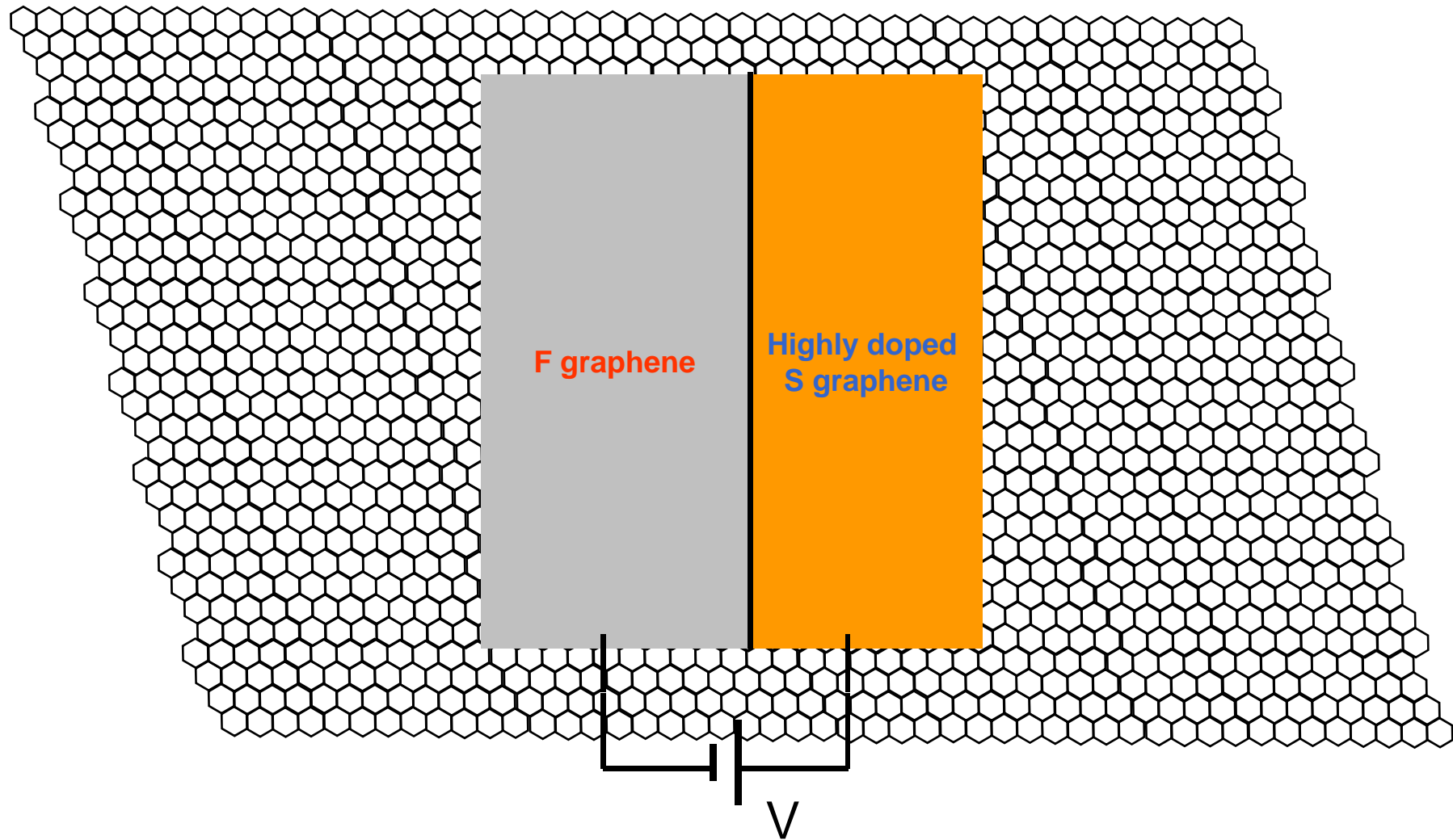
[N. Tomboros et al, Nature **448**, 571(2007);.....]

# Ferromagnetic graphene





# Graphene FS junction



# Dirac-Bogoliubov-deGennes equations

$$\begin{pmatrix}
 v_F \hat{\sigma} \cdot \vec{p} - sh\hat{1} - E_F \hat{1} & \Delta \hat{1} \\
 \Delta^* \hat{1} & -(v_F \hat{\sigma} \cdot \vec{p} - \bar{s}h\hat{1} - E_F \hat{1})
 \end{pmatrix}
 \begin{pmatrix}
 \hat{u}_s \\
 \hat{v}_{\bar{s}}
 \end{pmatrix}
 = \epsilon_s
 \begin{pmatrix}
 \hat{u}_s \\
 \hat{v}_{\bar{s}}
 \end{pmatrix}$$

$s = \bar{s} = \pm 1$

*Electron-hole conversion:  
superconducting correlations*

*electron 4-component spinor*

*Diagonal Dirac Hamiltonian with  
exchange field*

*hole 4-component spinor*

**Superconducting correlation of massless Dirac electrons  
with opposite spin and different valley**

# Variety of Andreev processes

$$r_s = \frac{\sec \beta \sqrt{\cos \phi_s \cos \phi'_s}}{\cos[(\phi_s - \phi'_s)/2] + i \tan \beta \cos[(\phi_s + \phi'_s)/2]}$$

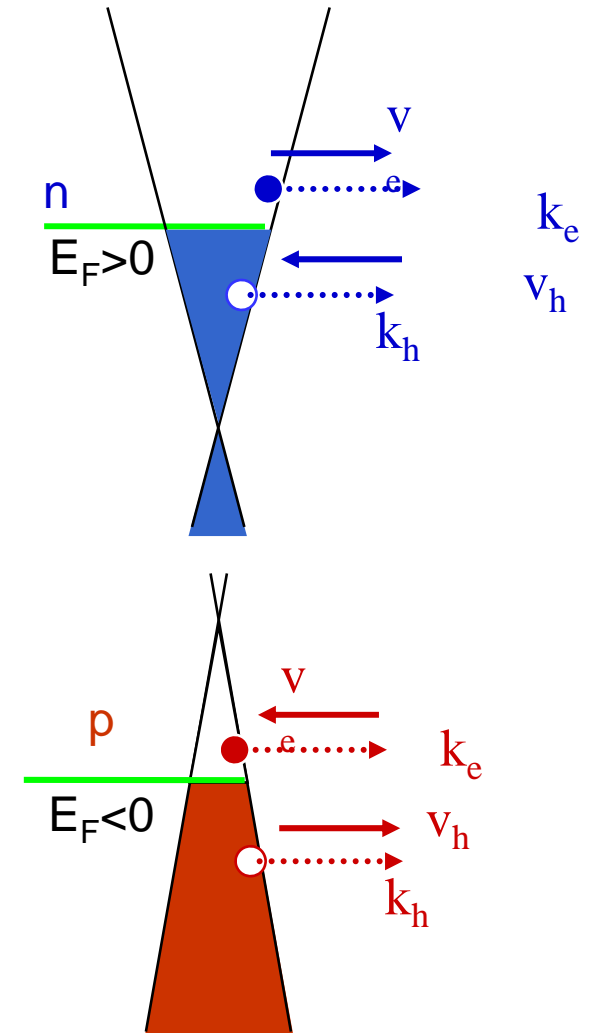
$$\beta = \cos^{-1}(\varepsilon / \Delta)$$

$$r_{As} = \frac{-\sin[(\phi_s + \phi'_s)/2] + i \tan \beta \cos[(\phi_s - \phi'_s)/2]}{\cos[(\phi_s - \phi'_s)/2] + i \tan \beta \cos[(\phi_s + \phi'_s)/2]}$$

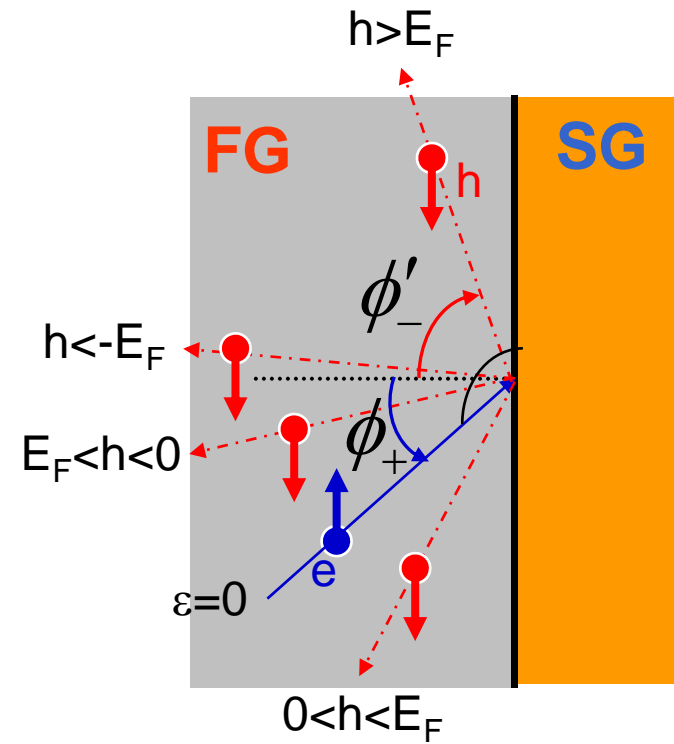
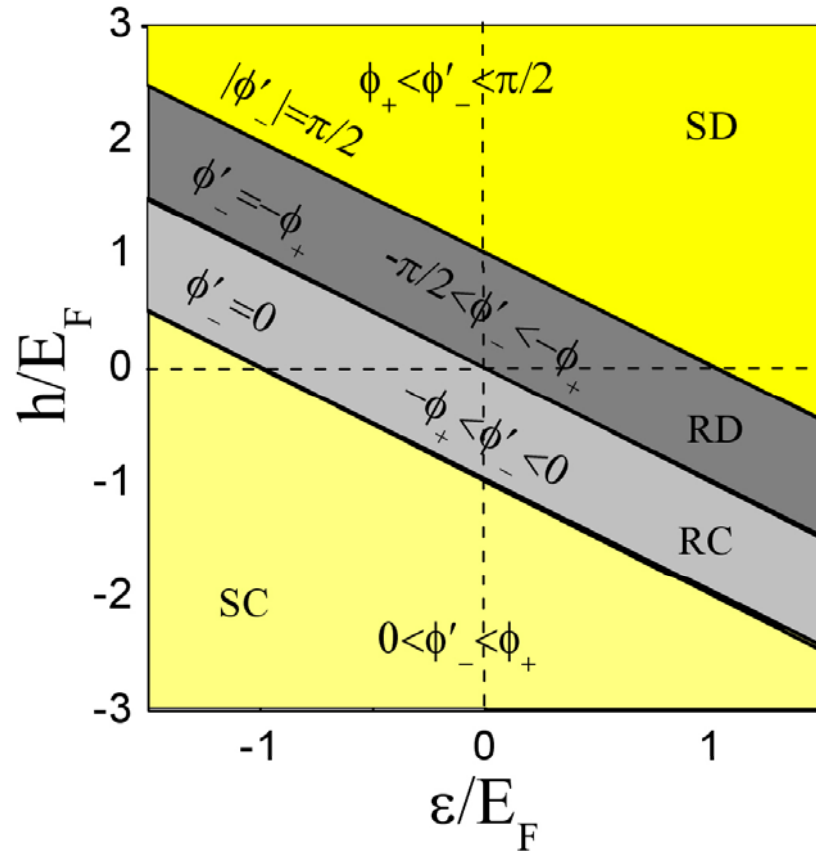
$$\varepsilon_s = \left| E_F \pm \hbar v_F \sqrt{k_x^2 + k_y^2} + sh \right|, \quad \vec{v}_s = \frac{1}{\hbar} \frac{\partial \varepsilon_s}{\partial \vec{k}}$$

$$k'_{y-} = k_{y+}$$

$$k'_{x-} = \sqrt{(\varepsilon + h + E_F)^2 - (\varepsilon + h + E_F)^2 \sin^2 \phi_+}$$



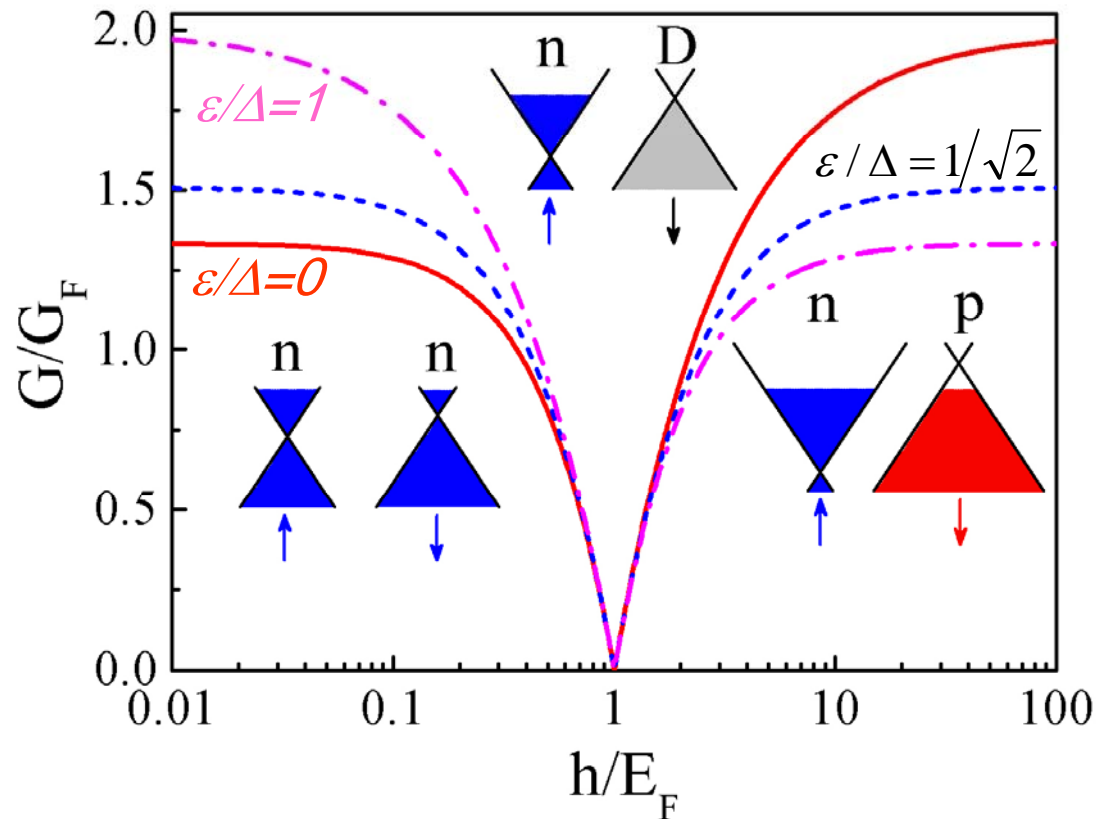
# Map of Andreev reflection angle



[M. Z. , H. Mohammadpour, A. G. Moghaddam, arXiv: 0804.2774v1(2008)]

# Andreev-Klein conductance of FS contacts

$$G = \sum_{s=\pm} G_s \int d\phi_s \cos \phi_s (1 - r_s + r_{As}), \quad G_F = \sum_{s=\pm} G_s$$



[M. Z. , H. Mohammadpour, A. G. Moghaddam, arXiv: 0804.2774v1(2008)]

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# Conclusion

- AR from FS interface is fundamentally different for  $\hbar > E_F$ : associated with **spin Klein tunnelling** of chiral electrons through exchange field p-n barrier: **Andreev-Klein reflection**
- It can be **retro** or **specular** in both **convergent** way and **divergent** way
- Exchange field induced enhancement of Andreev conductance: **ferromagnetic ordering amplifies spin-singlet superconducting correlations!!!**