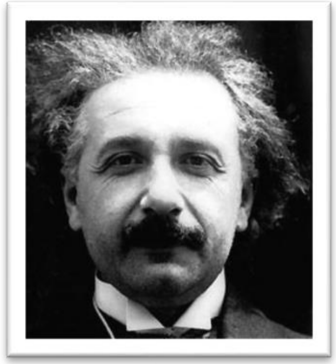
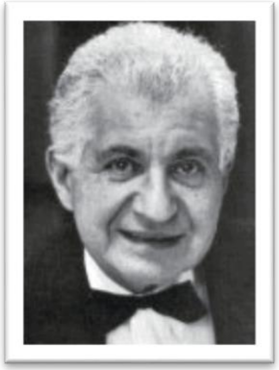
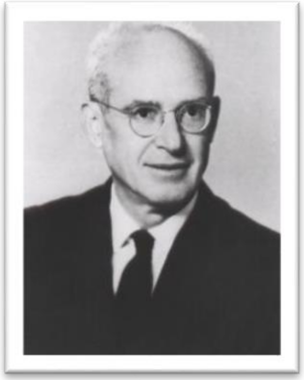
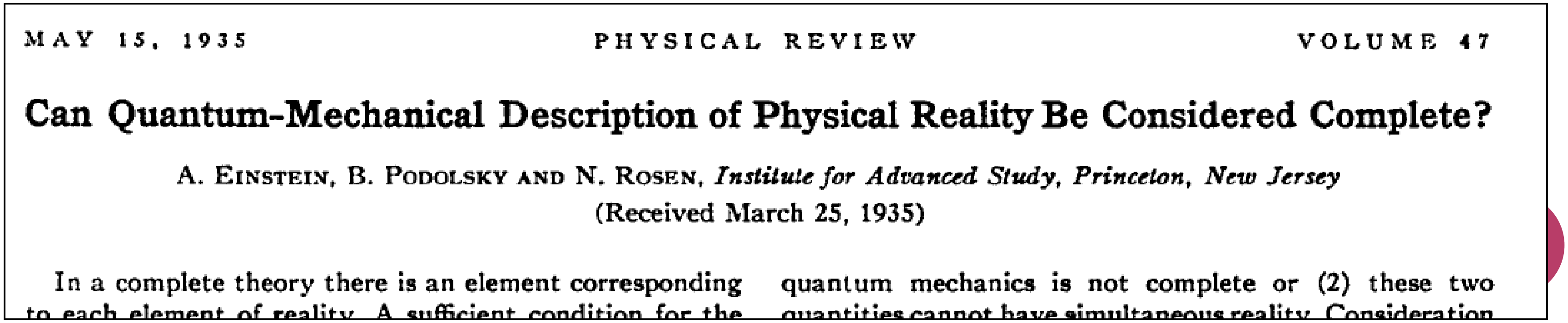
Why is Quantum Physics not more nonlocal than it is?

EPR Paradox:

- **Proposed in 1935:**
  - by A. Einstein, B. Podolsky, N. Rosen (*Phys. Rev. 47, 777*).
- **Original paper can be found at:** http://www.drchinese.com/David/EPR.pdf



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration

Bell's Theorem:

- **Hidden variable theories:** The wave function is not the whole story – some other quantity (or quantities), λ, is needed in addition to ψ, to characterize the state of a system fully.

Theoretical physicists were happily proposing hidden variable theories, until…

1964: John Stewart Bell proved that ***any*** **local** hidden variable theory is ***incompatible*** with quantum mechanics.

III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

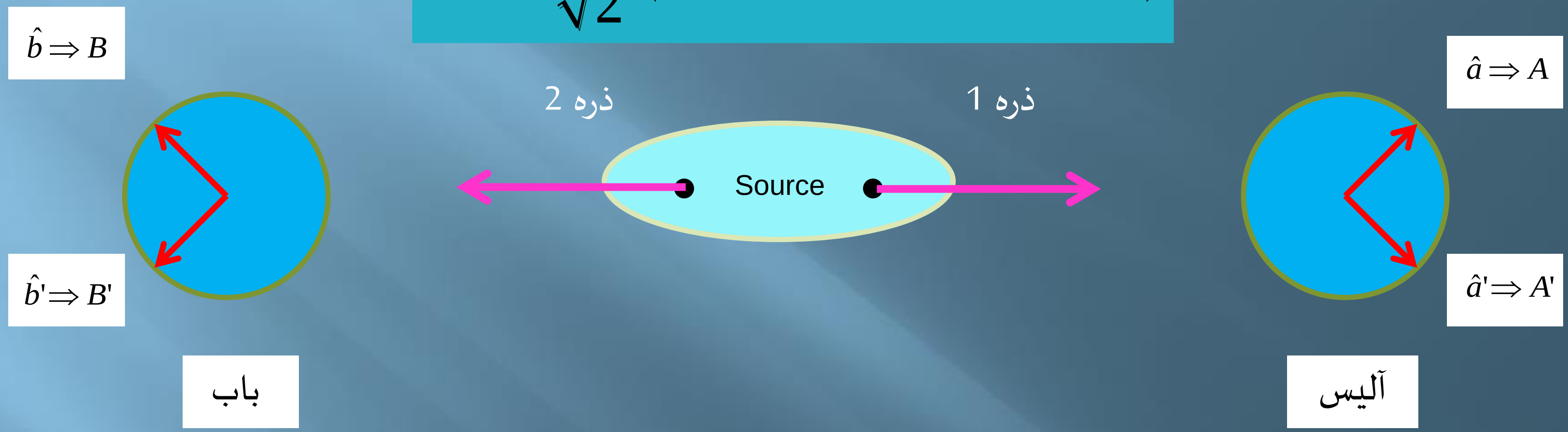
JOHN S. BELL†

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional vari-

The Bell-CHSH inequality

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B\right)$$



$$\vec{\sigma}\cdot\hat{b}|\psi\rangle = B|\psi\rangle$$

$$\vec{\sigma}\cdot\hat{a}|\psi\rangle = A|\psi\rangle$$

$$\vec{\sigma}\cdot\hat{b}'|\psi\rangle = B'|\psi\rangle$$

$$\vec{\sigma}\cdot\hat{a}'|\psi\rangle = A'|\psi\rangle$$

نامساوي بل:

$$|\langle a_1 b_1\rangle + \langle a_1 b_2\rangle + \langle a_2 b_1\rangle - \langle a_2 b_2\rangle| \leq 2$$

ولي از ديد گاه كوانتوم مكانيك ماكزيمم مقدار عبارت فوق

$$|\langle a_1 b_1\rangle + \langle a_1 b_2\rangle + \langle a_2 b_1\rangle - \langle a_2 b_2\rangle|_{QM} \leq 2\sqrt{2}$$

كدام پيشبيني درست است؟؟

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France

(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique, Université Paris-Sud, F-91406 Orsay, France

(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

VOLUME 49, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1982

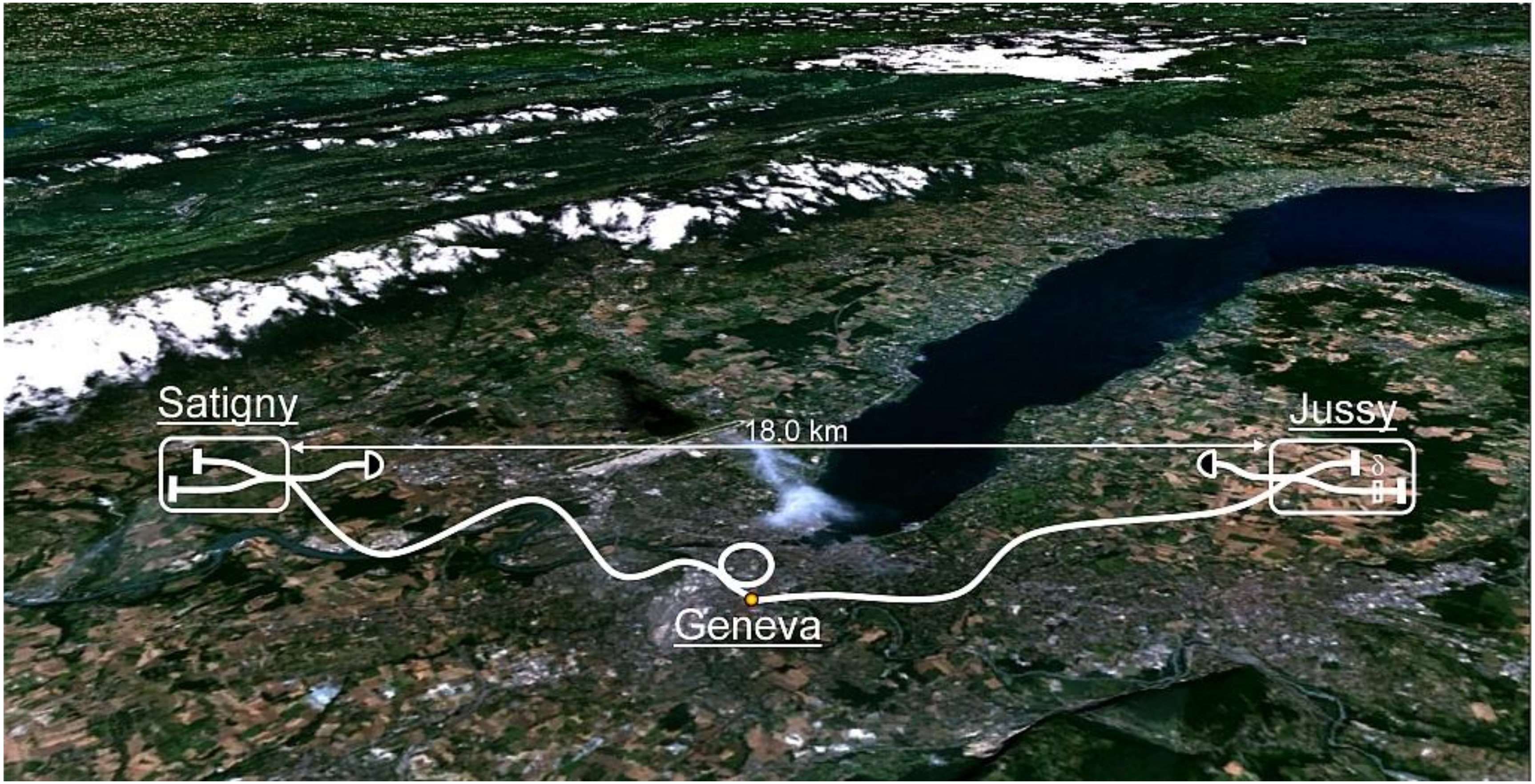
Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[a] and Gérard Roger
Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France

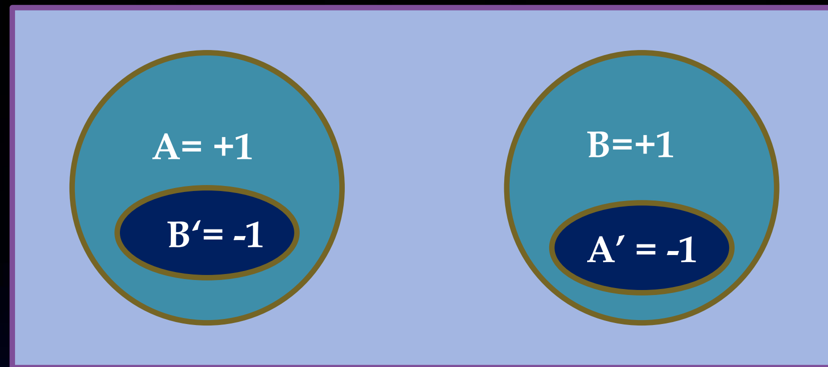
(Received 27 September 1982)

Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

The EPR-experiment



The Hardy argument



$$
\begin{cases}
P(A=+1, B=+1) = q_1 = 0 \\
P(A=-1, B'=-1) = q_2 = 0 \\
P(A'=-1, B=-1) = q_3 = 0 \\
P(A'=-1, B'=-1) = q_4 \neq 0
\end{cases}
$$

$$Max(Hardy) \equiv Max(q_4 - q_1) = Max(P^{H}_{11|11}) =$$

کلاسیک صفر

مکانیک کوانتوم 0/09

Alice measures $A \leftrightarrow x = 0$

Bob measures $B \leftrightarrow y = 0$

Alice measures $A' \leftrightarrow x = 1$

Bob measures $B' \leftrightarrow y = 1$

Alice's result is $1 \leftrightarrow a = 1$

Alice's result is $-1 \leftrightarrow a = 0$

Bob's result is $1 \leftrightarrow b = 1$

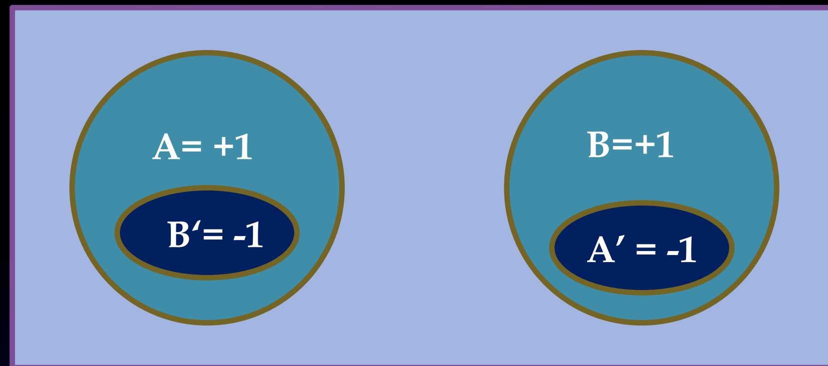
Bob's result is $-1 \leftrightarrow b = 0$

$$\langle xy \rangle = \sum_{a,b=0,1} (-1)^{a+b} P_{ab|xy}$$

$$Max(BI) \equiv K = \left| \langle 00 \rangle + \langle 01 \rangle + \langle 10 \rangle - \langle 11 \rangle \right|$$

$$\langle xy \rangle = \sum_{a,b=0,1} (-1)^{a+b} P_{ab|xy}$$

منطق هاردی



$$\begin{cases} P(A=+1, B=+1) = q_1 = 0 \\ P(A=-1, B'=-1) = q_2 = 0 \\ P(A'=-1, B=-1) = q_3 = 0 \\ P(A'=-1, B'=-1) = q_4 \neq 0 \end{cases} \quad \Rightarrow \quad \begin{cases} P_{00|00} = q_1 = 0 \\ P_{11|01} = 0 \\ P_{11|10} = 0 \\ P_{11|11} = q_4 \neq 0 \end{cases}$$

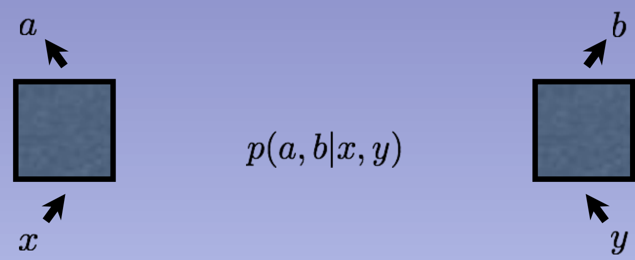
ورودی ها

$$x = 0 \leftrightarrow A \;, x = 1 \leftrightarrow A'$$

$$y = 0 \leftrightarrow B \;, y = 1 \leftrightarrow B'$$

خروجی ها

$$a, b \in \{0, +1\} \leftrightarrow \{+1, -1\}$$



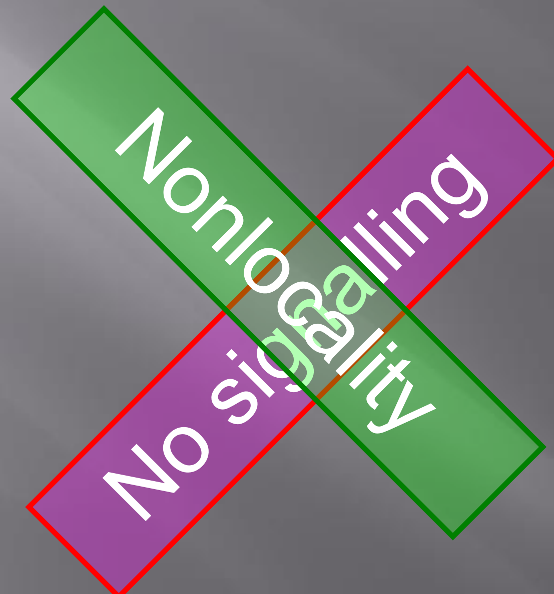
Local Correlations $\mathcal{L}$

$$p(a,b|x,y) = \sum_{\lambda} p(\lambda)\; p(a|x,\lambda)\; p(b|y,\lambda)$$

Quantum Correlations $\mathcal{Q}$

$$\mathcal{L} \subset \mathcal{Q}$$

$$p(a,b|x,y) = \mathrm{tr}\left(\rho_{AB} \Pi_a^x \otimes \Pi_b^y\right)$$


Nonlocality
No signalling

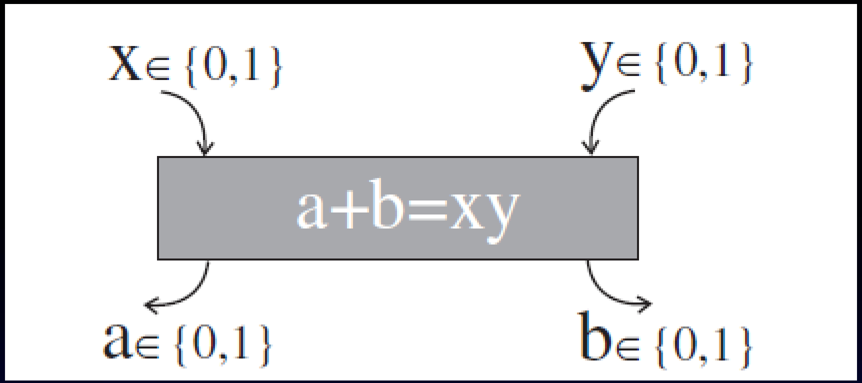
Is QM the unique theory combining them?

Is quantum physics, for instance, the most general theory that allows violations of Bell inequalities, while satisfying no-signaling?

Why not can quantum theory be more non-local?

PR–Box

$$+1 \to 0$$
$$-1 \to +1$$



ورودی های آلیس x

ورودی های باب y

خروجی هایشان a و b

احتمال توامان بدست آوردن یک جفت خروجی برای یک جفت ورودی $P_{ab|xy}$

$x=0$, $y=0$	⇒	$a=0$, $b=0$	یا	$a=1$, $b=1$	$x=1$, $y=0$	⇒	$a=0$, $b=0$	یا	$a=1$, $b=1$
$x=0$, $y=1$	⇒	$a=0$, $b=0$	یا	$a=1$, $b=1$	$x=1$, $y=1$	⇒	$a=0$, $b=1$	یا	$a=1$, $b=0$

و از موارد زیر که قبلاً داشتیم :

برای **نامساوی بل** :

$$A, B \to 0$$
$$A', B' \to +1$$

نتایج اندازه گیری آلیس و باب

$$\langle xy \rangle = \sum_{a,b=0,1} (-1)^{a+b} P_{ab|xy}$$

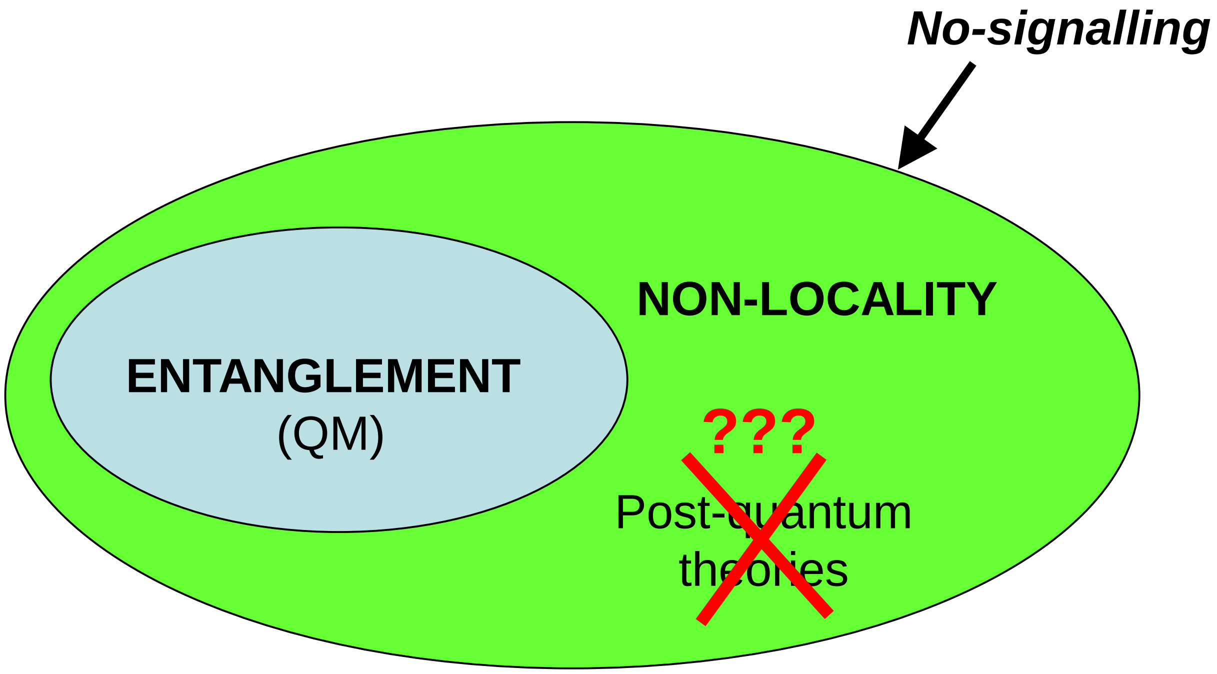
$$Max(BI) \equiv K = \left| \langle 00 \rangle + \langle 01 \rangle + \langle 10 \rangle - \langle 11 \rangle \right|$$

$$\langle 00 \rangle = P_{00|00} + P_{11|00} - P_{01|00} - P_{10|00} = \frac{1}{2} + \frac{1}{2} - 0 - 0 = +1$$
$$\langle 01 \rangle = P_{00|01} + P_{11|01} - P_{01|01} - P_{10|01} = \frac{1}{2} + \frac{1}{2} - 0 - 0 = +1$$
$$\langle 10 \rangle = P_{00|10} + P_{11|10} - P_{01|10} - P_{10|10} = \frac{1}{2} + \frac{1}{2} - 0 - 0 = +1$$
$$\langle 11 \rangle = P_{00|11} + P_{11|11} - P_{01|11} - P_{10|11} = 0 + 0 - \frac{1}{2} - \frac{1}{2} = -1$$

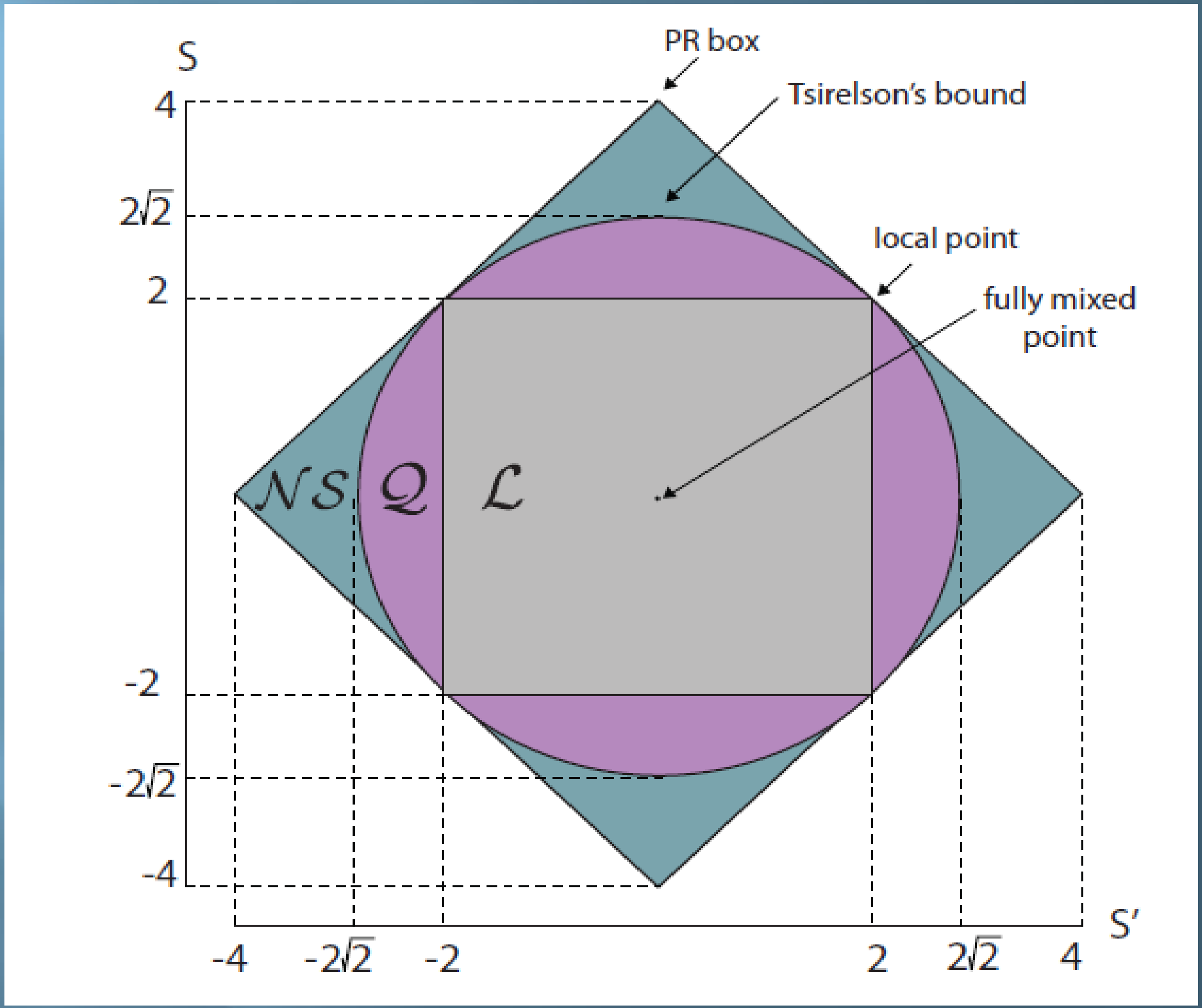
$$K_{PR-Box} = 4$$

Max (Hardy Nonlocality) = 0	LHVT	Max BI =2
Max (Hardy Nonlocality)=0.09	QM	Max BI=$2\sqrt{2}$
Max (Hardy Nonlocality)=0.5	NS	MAX BI=4

Entanglement versus non-locality



Why is quantum non-locality limited ?


S
PR box
Tsirelson's bound
4
2√2
local point
2
fully mixed
point
$\mathcal{NS}$
$\mathcal{Q}$
$\mathcal{L}$
-2
-2√2
-4
S'
-4
-2√2
-2
2
2√2
4

QM = NS+ ?

اصل علیت اطلاعات

اصل علیت اطلاعات بیان می کند :

« اگر آلیس m بیت کلاسیکی را به باب انتقال دهد، کل اطّلاعاتی که باب بـالقوه از داده هـای آلـیس بدست می آورد، بزرگتر از m نیست. »

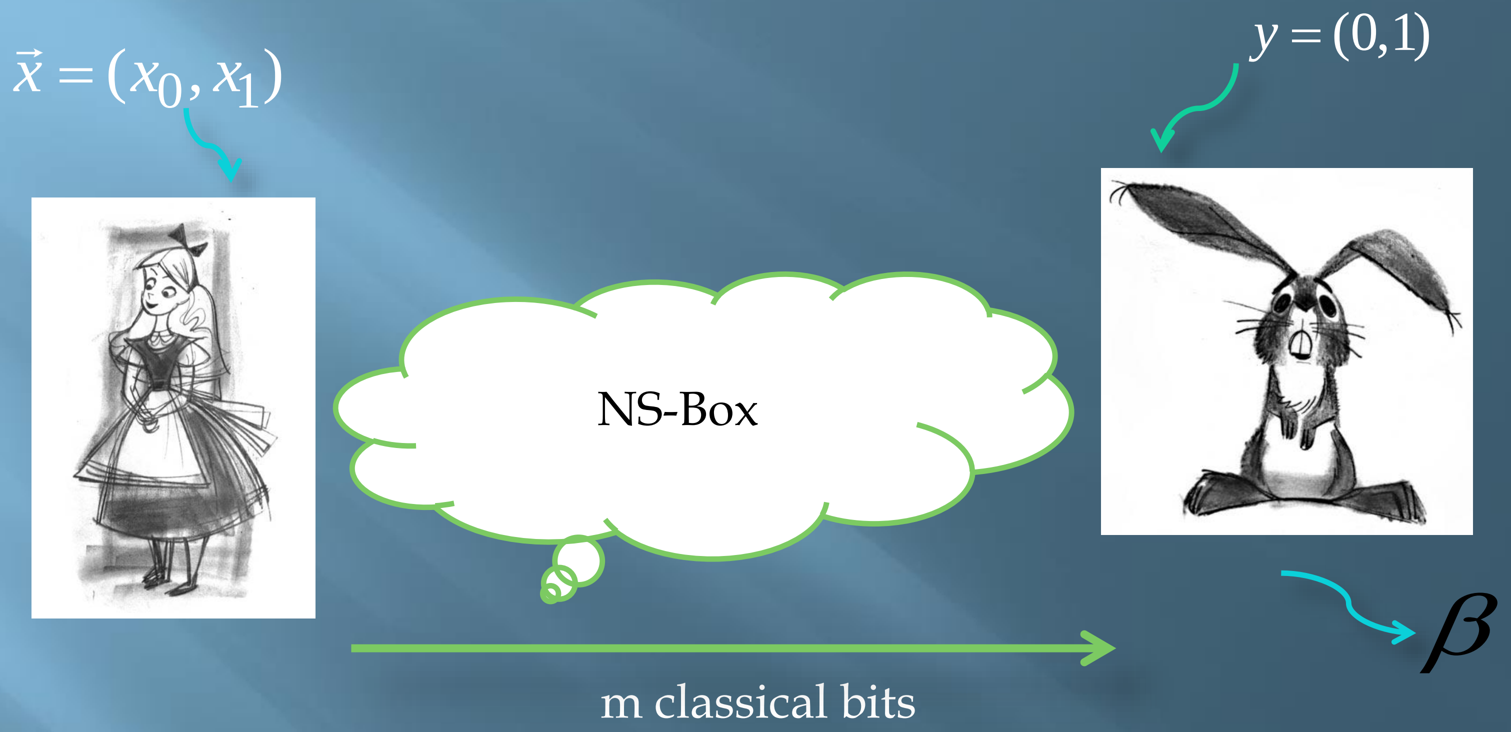
برای حالت $m=0$ ، اصل علیت اطلاعات به اصل عدم ارسال آنی اطلاعات (N-S) کاهش می یابد.

پس N-S حالت خاصی از علیت اطلاعات است

همان گونه که نامساوی بل، محدودیتی روی میزان همبستگی نتایج دو سیستم مجزا شده (Local) ، برقرار می کند، که توسط کلاسیک تایید و توسط مکانیک کوانتوم نقض می شود؛

اصل علیت اطلاعات نیز محدودیتی روی نتایج دو سیستم و با منابع به اشتراک گذاشته ی No-Signaling برقرار می کند که توسط کلاسیک و مکانیک کوانتوم تایید و توسط PR-Box نقض می گردد

Information Causality



$$I \leq m$$

$$I(X:Y) = H(X) + H(Y) - H(XY) \qquad H(X) = -\sum_x p(x)\log_2 p(x)$$

- I(x:y) is the classical mutual information

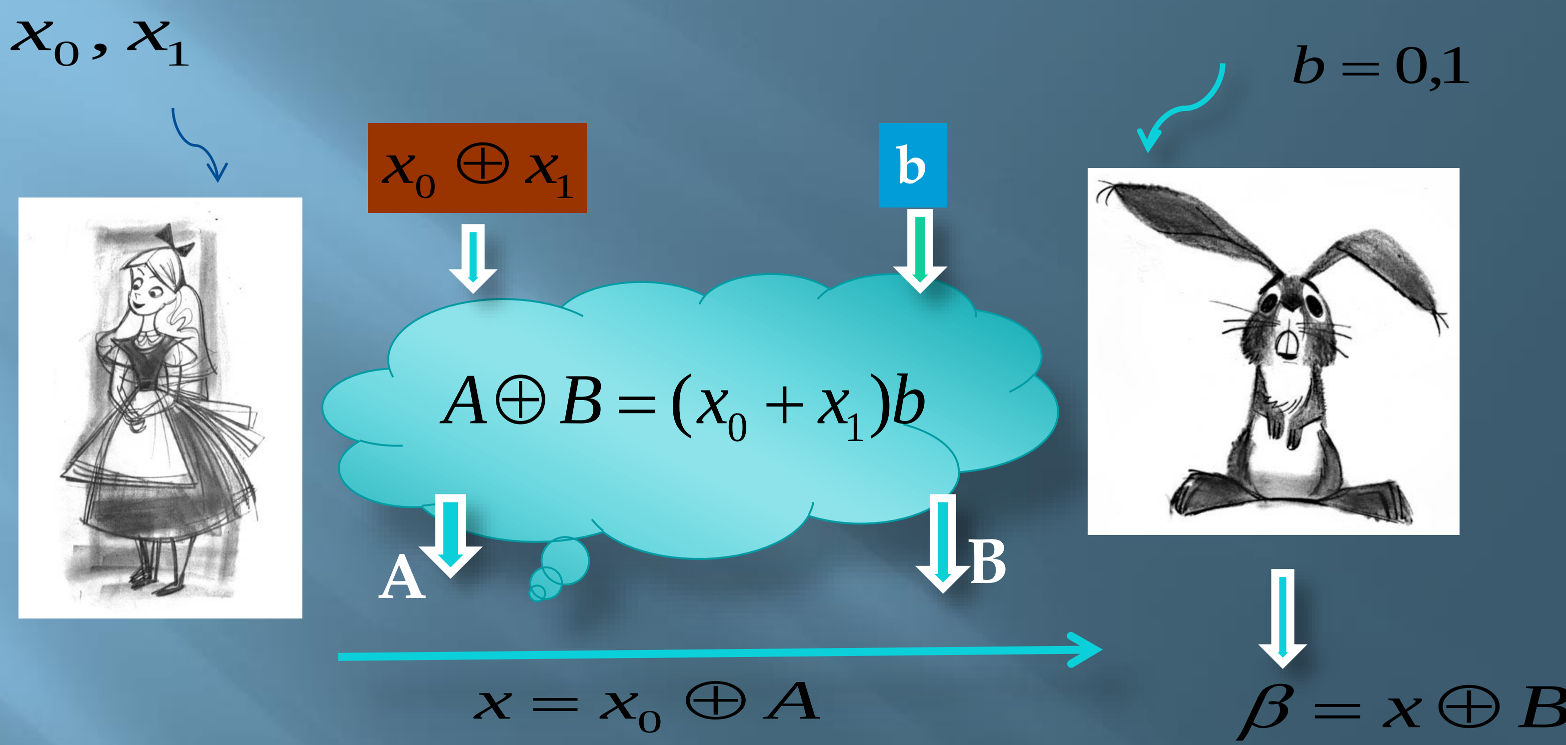
- The **Information causality principle** states

$$J = \sum_{y=1}^{N} I(x_y : b_y) \le m$$

- Information Causality can be violated using any correlations which violate Tsirelson's bound for the CHSH.

$$p > \frac{2+\sqrt{2}}{4} \quad \Rightarrow \quad J > m$$

PR-box violates IC condition



$$\beta = x \oplus B = x_0 \oplus A \oplus B = x_0 \oplus (x_0 \oplus x_1)b$$

- It was proved that both classical and quantum correlations satisfy IC condition.
- The IC condition is violated for all boxes for which

where

$$E_j = 2P_j - 1 \qquad E_1^{\ 2} + E_2^{\ 2} > 1$$

$$P_1 = \frac{1}{2}[p_{00|00} + p_{11|00} + p_{00|10} + p_{11|10}]$$

$$P_2 = \frac{1}{2}[p_{00|01} + p_{11|01} + p_{01|11} + p_{10\backslash 11}]$$

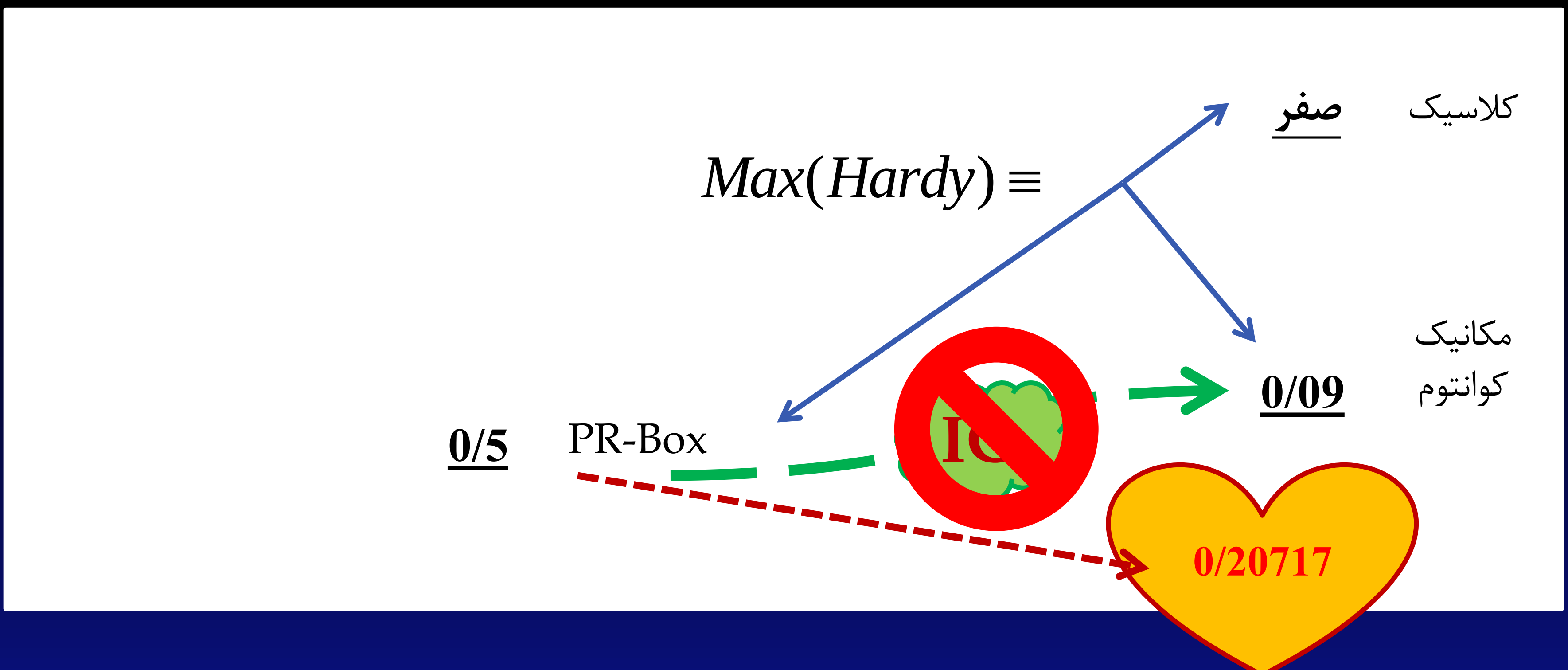
$$E_1^{\,2} + E_2^{\,2} \leq 1 \Rightarrow IC$$

LHVT ⇒ IC

QM ⇒ IC

PR-BOX ⇒ ~IC

$$
\begin{cases}
Max\ (BI)_{Classic} = 2 \\
Max\ (BI)_{QM} = 2\sqrt{2} \\
Max\ (BI)_{Algebra} = 4
\end{cases}
$$



Max (BI) = $2\sqrt{2}$ NS+IC ≡ QM

Max HN= 0.5 NS > QM

Max HN= 0.20717 NS+IC > QM

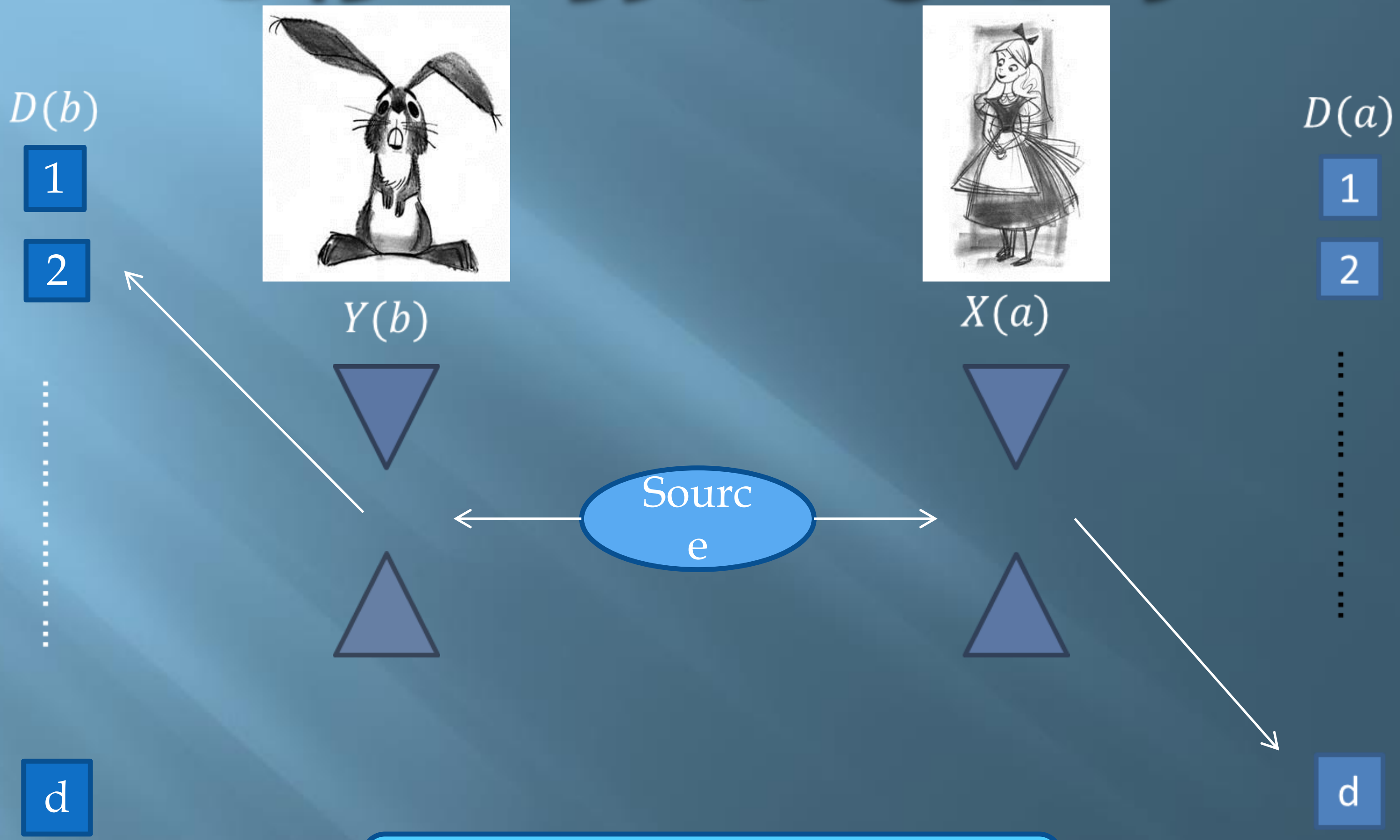
Max HN = 0.09 NS+IC+?+? = QM

QM = NS + IC + ?

Macroscopic Locality (ML)

principle

آزمایش میکروسکوپی



Microscopic correlation P(a,b)

for any X ≠X'

$$\sum_{a\in X} P(a,b) = \sum_{a\in X'} P(a,b) = \mathrm{P(b)}$$

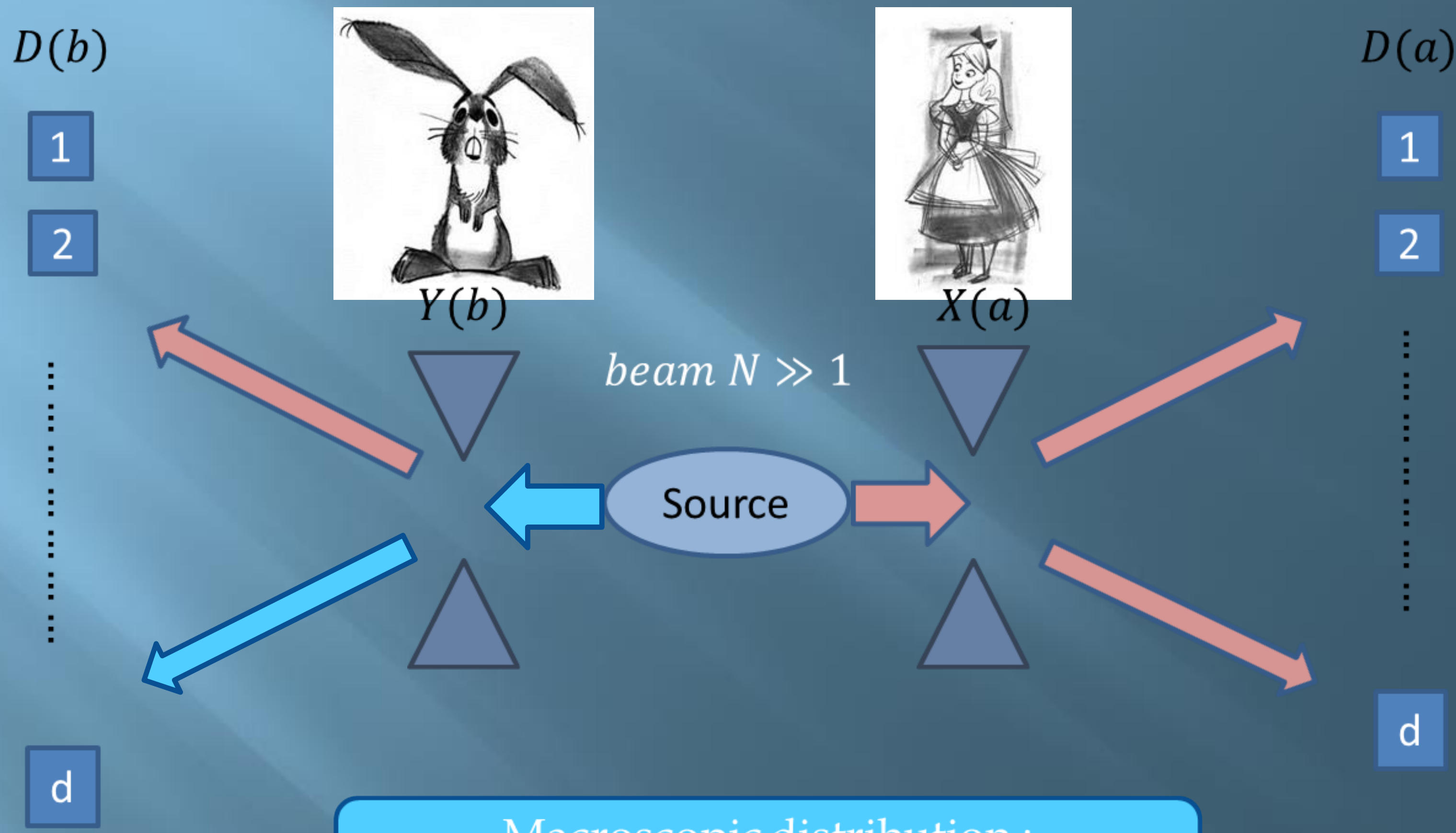
For any Y ≠Y'

$$\sum_{b\in Y} P(a,b) = \sum_{b\in Y'} P(a,b) = \mathrm{P(a)}$$

همچنین، مجموعه ای از توزیع های احتمال مارجینال **P(a,b)** در حالت عمومی مدل متغییر های پنهان موضعی را نخواهد پذیرفت، بدان معنا که در چندین مورد، یک توزیع احتمال مشترک برای تمام **2S** اندازه گیری ممکن وجود نخواهد داشت ، آنچنان که:

$$P(a,b) = \sum_{c} P(\ldots, c_{x-1}, a, c_{x+1}, \ldots, c_{y-1}, b, c_{y+1}, \ldots)$$

آزمایش ماکروسکوپی



Macroscopic distribution :
$P\,(I_\chi , I_\gamma)\; dI_\chi\, dI_\gamma$

$$I_X = (I_X^1, I_X^2, \ldots, I_X^d)$$

$$I_Y = (I_Y^1, I_Y^2, \ldots, I_Y^d)$$

از آنجا که فیزیک کلاسیک یک تئوری موضعی است ، پس یک مدل متغییر های پنهان موضعی می باشد. و این لازمه به وجود آمدن یک چگالی احتمال کلی

$$P(I_1, I_2, \ldots, I_{2S})$$

است. می توان ساختار موضعیت ماکروسکوپی را به صورت زیر نوشت:

$$P(a,b) = \int \Big(\prod_{Z \neq X,Y} dI_Z \Big) P(I_1, I_2, \ldots, I_{2S})$$

- $I_\chi=(I_\chi{}^1,I_\chi{}^2,\dots,I_\chi^d)$ خروجی آلیس

$\mathcal{y}=(I\mathcal{y}^1,I\mathcal{y}^2,\dots,I\mathcal{y}^d)I$ خروجی باب

$P(I_\chi,I\mathcal{y})dI_\chi dI\mathcal{y}$ توزیع احتمال

ساختار موضعیت ماکروسکوپی

$$P(I_X, I_Y) = \int \left(\prod_{Z \neq X,Y} dI_Z \right) P(\mathrm{I}_1, \mathrm{I}_2, \ldots, \mathrm{I}_{2s})$$

$$BI_{PR} = 4 \xrightarrow{\text{IC}} BI_{QM} = 2\sqrt{2}$$

$$BI_{PR} = 4 \xrightarrow{\text{ML}} BI_{QM} = 2\sqrt{2}$$

$$ML \subset IC \; or \; IC \subset ML \; ?$$

$$H_{PR} = 0.5 \xrightarrow{\text{IC}} 0.21 > H_{QM}$$

$$H_{PR} = 0.5 \xrightarrow{ML} 0.21 > H_{QM}$$

QM = NS+ IC+ML+?

Local Orthogonality Principle

Event: $e = (a_1 \dots a_n | x_1 \dots x_n)$

Two events are "orthogonal" if they can be locally distinguished by one party

$$(a_1 \dots a_n | x_1 \dots x_n) \perp (b_1 \dots b_n | y_1 \dots y_n)$$

if $\exists j \in [1, n] \quad x_j = y_j$ and $a_j \neq b_j$

A set of events is "exclusive" if they are pairwise orthogonal

The sum of the probabilities of pairwise orthogonal events is upper-bounded by "1"

$$\sum_{e \in S} P(e) \leq 1$$

The sum of the probabilities of pairwise orthogonal events is upper-bounded by "1"

$$P(a_1 \dots a_i \dots a_n | x_1 \dots x_i \dots x_n) + P(b_1 \dots \bar{a}_i \dots b_n | y_1 \dots x_i \dots y_n) =$$
$$\mathrm{tr}(\rho P_{a_1}^{x_1} \otimes \dots \otimes P_{a_i}^{x_i} \otimes \dots \otimes P_{a_n}^{x_n}) + \mathrm{tr}(\rho P_{b_1}^{y_1} \otimes \dots \otimes P_{\bar{a}_i}^{x_i} \otimes \dots \otimes P_{b_n}^{y_n}) =$$
$$\mathrm{tr}(\rho P') \leq 1$$

Quantum correlations satisfy LO constraints

The maximum probability of success of the Hardy's non locality

QM $\rightarrow$ 0.09

NS$\rightarrow$ 0.5

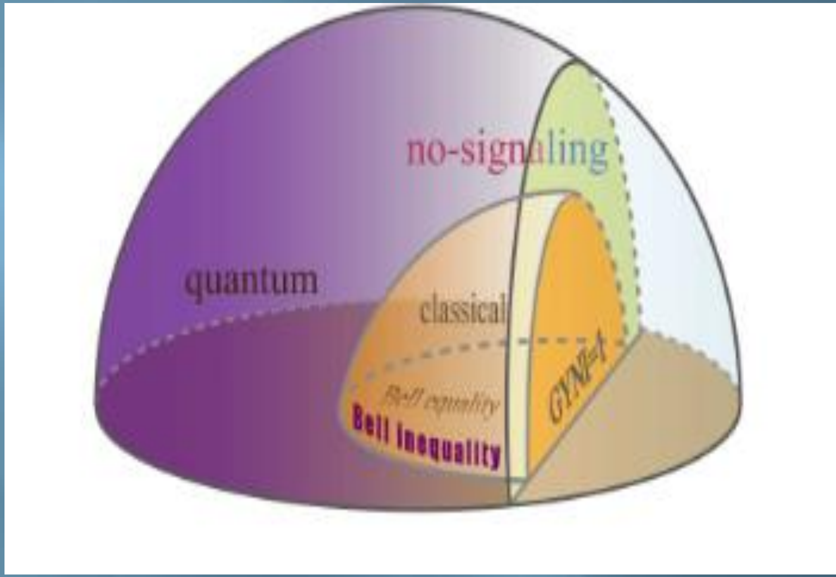
NS + IC $\rightarrow$ 0.20717 > QM

NS + LO $\rightarrow$ 0.0177 > QM

On the other hand, IC or any other bi-partite principle insufficient for

witnessing all multipartite post-quantum correlations.

Guess Your Neighbor's Input



$$\omega = \frac{1}{4}\left[P(000|000) + P(110|011) + P(011|101) + P(101|110)\right] \le \frac{1}{4},$$

$$\omega_q = \omega_c \,.$$

$$\omega_{ns} \le 2\omega_c \,,$$

WHAT IS QUANTUM MECHANICS?