Null brane quantization

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Overview

Introduction

• Studying null (Carrollian) branes can have applications in black hole physics, in particular black hole microstate counting, since the black hole horizon is a null surface.

• Considering the difficulties in quantizing branes, null brane quantization as the tensionless limit of this problem, is worthwhile to be studied even only as a mathematical curiosity.

Null brane action

The brane action can be written as

$$S = -\frac{\tilde{T}}{2} \int d^{p+1}\sigma \, \sqrt{-h} \left(h^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \Lambda \right). \tag{1}$$

Taking a tensionless limit, this action will be transformed to an action for the tensionless brane

$$S_{\text{N.B.}} = \frac{\kappa}{2} \int d\tau \int_{\mathcal{N}_p} d^p\sigma \; \mathcal{V}^a \partial_a X^\mu \; \mathcal{V}^b \partial_b X^\nu g_{\mu\nu}, \tag{2}$$

Symmetries

The action is invariant under a diffeomorphism generated by ξ^a.
We have the transformations

$$\begin{aligned} \delta_\xi X^\mu = \mathcal{L}_\xi X^\mu = \xi \cdot \partial X^\mu \\ \delta_\xi g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} = \xi \cdot \partial g_{\mu\nu} \\ \delta_\xi \mathcal{V}^a = \mathcal{L}_\xi \mathcal{V}^a = \xi \cdot \partial \mathcal{V}^a - \mathcal{V}^b \partial_b \xi^a - \frac{1}{2}(\partial_b \xi^b)\mathcal{V}^a \end{aligned} \tag{3}$$

Gauge fixing and residual symmetry

Using the invariance of action under diffeomorphisms, we can fix a gauge by setting

$$\mathcal{V}^a \partial_a = \partial_\tau \tag{4}$$

Using this temporal gauge, the equations of motion are

$$\partial_\tau (g_{\mu\nu} \partial_\tau X^\nu) = 0. \tag{5}$$

There still remains a residual gauge symmetry given by

$$\zeta = \Big(\sum_i \partial_i f^i(\sigma^j)\tau + h(\sigma^j) \Big) \partial_\tau + \sum_i f^i(\sigma^j)\partial_i, \tag{6}$$

Gauge fixing and residual symmetry

We can define the operators $L^{(i)}(f^i)$ (with no sum on i) and $M(h)$ as

$$L^{(i)}(f^i) = f^i \partial_i + (\partial_i f^i) \tau \partial_\tau, \tag{7}$$

$$M(h) = h \partial_\tau, \tag{8}$$

again with no sum on i. The commutation relations are

$$[L^{(i)}(f^i), L^{(j)}(g^j)] = f^i L^{(j)}(\partial_i g^j) - g^j L^{(i)}(\partial_j f^i) \tag{9}$$

$$[L^{(i)}(f^i), M(h)] = M(f^i \partial_i h - h \partial_i f^i). \tag{10}$$

This is a BMS_{p+1} algebra.

Equations of motion and constraints

We consider a toroidal null p-brane in flat p+2-dimensional target spacetime. The equations of motion implies

$$X^{\mu} = x_0^{\mu} + A_i^{\mu}\sigma^{i} + \frac{1}{\kappa}B_0^{\mu}\tau + \sum_{\vec{n}\neq\vec{0}}\left(\frac{1}{|\vec{n}|}\, A_{\vec{n}}^{\mu}\ e^{-i\vec{n}.\vec{\sigma}} + \frac{1}{\kappa}B_{\vec{n}}^{\mu}\tau\ e^{-i\vec{n}.\vec{\sigma}}\right), \quad (11)$$

where $\mu \in \{+, -, i\}$ and $i = 1, ..., p$.

The stress tensor is

$$T^{\alpha}_{\beta} = V^{\alpha}V^{\rho}\partial_{\rho}X^{\mu}\partial_{\beta}X_{\mu} - \frac{1}{2}V^{\lambda}V^{\rho}\partial_{\lambda}X^{\mu}\partial_{\rho}X_{\mu}\delta_{\beta}^{\alpha}. \quad (12)$$

Equations of motion and constraints

Therefore we have

$$T^0_{\ i} = \dot{X}.\partial_i X = T_i(\tau, \vec{\sigma}), \tag{13}$$

$$T^0_{\ 0} = -T^i_{\ i} = \frac{1}{2}\dot{X}^2 = T_0. \tag{14}$$

The conserved charge for the transformation (6) is

$$Q = \int d^p\sigma J^0 = \int d^p\sigma\Big(\sum_i T_i f^i + T_0(\sum_i \partial_i f^i \tau + h)\Big), \tag{15}$$

and we have

$$L^i_{\vec{n}} = \int d^p\sigma (T_i + i n_i \tau T_0) e^{i\vec{n}.\vec{\sigma}}, \tag{16}$$

and

$$M_{\vec{n}} = \int d^p\sigma T_0 e^{i\vec{n}.\vec{\sigma}}. \tag{17}$$

Equations of motion and constraints

Given the solution to the equations of motion, we find

$$M_{\vec{n}} = \frac{(2\pi)^p}{2} \sum_{\vec{k}} B_{\mu\vec{k}} B^{\mu}_{\vec{n}-\vec{k}} \tag{18}$$

$$L^i_{\vec{n}} = \frac{(2\pi)^p}{2} \left(2B_{\vec{n}\mu} A^{\mu}_i - i \sum_{\vec{k}\neq 0} \frac{k_i}{|\vec{k}|} A_{\vec{k}\mu} B^{\mu}_{\vec{n}-\vec{k}} \right) \tag{19}$$

Equations of motion and constraints

And the constraints can be written as

$$B_0^\mu B_{0\mu} + \sum_{\vec{k}\neq 0} B_{\mu\vec{k}} B^\mu_{-\vec{k}} = 0, \tag{20a}$$

$$2B_0^\mu B_{\vec{n}\mu} + \sum_{\vec{k}\neq\{0,\vec{n}\}} B_{\mu\vec{k}} B^\mu_{\vec{n}-\vec{k}} = 0, \qquad \vec{n}\neq 0 \tag{20b}$$

$$B_0^\mu A_{\mu i} - i\sum_{\vec{k}\neq 0} \frac{k_i}{|\vec{k}|}\; B^\mu_{-\vec{k}} A_{\mu\vec{k}} = 0, \tag{21a}$$

$$\frac{n_i}{|\vec{n}|} B_0^\mu A_{\mu\vec{n}} + iB^\mu_{\vec{n}} A_{\mu i} + \sum_{\vec{k}\neq\{0,\vec{n}\}} \frac{k_i}{|\vec{k}|}\; B^\mu_{\vec{n}-\vec{k}} A_{\mu\vec{k}} = 0, \qquad \vec{n}\neq 0 \tag{21b}$$

Toroidal null 2-brane in light cone gauge

In light-cone gauge we choose

$$X^+ = x_0^+ + p^+\tau. \tag{22}$$

For simplicity we assume the torus to be an orthogonal torus with radii R_i, then the winding and momentum modes are given by

$$B_0^+ = p^+, \qquad B_{\vec{n}}^+ = 0, \qquad B_0^i = \frac{m_i}{R_i}. \tag{23}$$

$$A_j^i = w^i R_i {\delta^i}_j, \qquad A_i^\pm = 0, \qquad w^i, m_i, \in \mathbb{Z}. \tag{24}$$

Toroidal null 2-brane in light cone gauge

In light cone gauge and for null 2-brane, the constraints can be written as

$$\mathbf{M}^{\,2} := 2p^{+}\mathbf{B}_0^{-} = \sum_i \frac{m_i^2}{R_i^2} + \frac{1}{2}\sum_{\vec{k}\neq 0} \mathbf{B}^i_{\vec{k}}\mathbf{B}^i_{-\vec{k}}, \tag{25a}$$

$$\mathbf{L}_i := m_i w^{\,i} - i\sum_{\vec{k}\neq 0} \frac{k_i}{|\vec{k}|}\mathbf{A}^j_{\vec{k}}\mathbf{B}^j_{-\vec{k}} = 0, \qquad i = 1, 2, \tag{25b}$$

$$\mathbf{K}_{\vec{n}} := i\epsilon^{ij} n_i w^{\,j} R_j \mathbf{B}^j_{\vec{n}} + \sum_{\vec{k}\neq 0,\vec{n}} \frac{\epsilon^{ij}(n_i - k_i)k_j}{|\vec{k}|}\, \mathbf{A}^l_{\vec{k}}\mathbf{B}^l_{\vec{n}-\vec{k}} = 0, \tag{25c}$$

Quantization

$$\begin{aligned}[\mathbf{A}^i_{\vec{n}}, \mathbf{A}^j_{\vec{m}}] &= 0 = [\mathbf{B}^i_{\vec{n}}, \mathbf{B}^j_{\vec{m}}] \\ [\mathbf{A}^i_{\vec{n}}, \mathbf{B}^j_{\vec{m}}] &= i|\vec{n}|\ \delta_{\vec{m}+\vec{n},0}\delta^{ij}.\end{aligned} \tag{26}$$

We define a new set of "normalized oscillators",

$$\mathbf{C}^i_{\vec{k}} := \sqrt{\frac{\kappa}{2|\vec{k}|}}(\mathbf{A}^i_{\vec{k}} + \frac{i}{\kappa}\mathbf{B}^i_{\vec{k}}), \qquad (\mathbf{C}^i_{\vec{k}})^\dagger := \sqrt{\frac{\kappa}{2|\vec{k}|}}(\mathbf{A}^i{}_{-\vec{k}} - \frac{i}{\kappa}\mathbf{B}^i{}_{-\vec{k}}), \qquad \vec{k} \neq 0 \tag{27}$$

with commutators

$$[\mathbf{C}^i_{\vec{k}}, \mathbf{C}^j_{\vec{l}}] = 0 = [(\mathbf{C}^i_{\vec{k}})^\dagger, (\mathbf{C}^j_{\vec{l}})^\dagger], \qquad [\mathbf{C}^i_{\vec{k}}, (\mathbf{C}^j_{\vec{l}})^\dagger] = \delta_{\vec{k},\vec{l}}\ \delta^{ij} \tag{28}$$

Quantization

In terms of the "creation-annihilaton" operators, we can write

$$\mathbf{M}^2 = \sum_i \frac{m_i^2}{R_i^2} + \frac{1}{\kappa}\sum_{\vec{k}} |\vec{k}|(\mathbf{N}_{\vec{k}} - \mathbf{X}_{\vec{k}} - \mathbf{X}_{\vec{k}}^{\dagger}) + A \tag{29a}$$

$$\mathbf{L}_i = m_i w^i + \frac{1}{\kappa} \sum_{\vec{k}} k_i \mathbf{N}_{\vec{k}} \tag{29b}$$

where

$$\mathbf{N}_{\vec{k}} = \sum_i \mathbf{C}_{\vec{k}}^{i\dagger}\mathbf{C}_{\vec{k}}^{i}, \qquad \mathbf{X}_{\vec{k}} = \sum_i \mathbf{C}_{\vec{k}}^{i}\mathbf{C}^{i}_{-\vec{k}}. \tag{30}$$

and A is some normal ordering constant.

Quantization

The $\mathbf{K}_{\vec{n}}$ constraint can be written as

$$\mathbf{K}_{\vec{n}} = i\frac{\sqrt{2\kappa}}{\sqrt{|\vec{n}|}}\epsilon^{ij}n_i\omega_j R_j(\mathbf{C}^j_{\vec{n}} - \mathbf{C}^{j\dagger}_{-\vec{n}})$$
$$+ \sum_{\vec{k}\neq\vec{0},\vec{n}} \epsilon^{ij}(n_i - k_i)k_j\sqrt{\frac{|\vec{k}|}{|\vec{n}-\vec{k}|}}(\mathbf{C}^l_{\vec{k}}\mathbf{C}^l_{\vec{n}-\vec{k}} - \mathbf{C}^{l\dagger}_{-\vec{k}}\mathbf{C}^{l\dagger}_{\vec{k}-\vec{n}} + \mathbf{C}^{l\dagger}_{-\vec{k}}\mathbf{C}^l_{\vec{n}-\vec{k}} - \mathbf{C}^l_{\vec{k}}\mathbf{C}^{l\dagger}_{\vec{k}-\vec{n}}) \tag{31}$$

With the above $(\mathbf{M}^2)^\dagger = \mathbf{M}^2, (\mathbf{L}_i)^\dagger = \mathbf{L}_i, (\mathbf{K}_{\vec{n}})^\dagger = \mathbf{K}_{-\vec{n}}$.

Algebra of Constraints

$$
\begin{aligned}
[\mathbf{M}^2, \mathbf{L}_i] &= 0, \qquad [\mathbf{M}^2, \mathbf{K}_{\vec{n}}] = 0, \\
[\mathbf{L}_i, \mathbf{L}_j] &= 0, \qquad [\mathbf{L}_i, \mathbf{K}_{\vec{n}}] = -n_i \mathbf{K}_{\vec{n}} \\
[\mathbf{K}_{\vec{m}}, \mathbf{K}_{\vec{n}}] &= i\epsilon_{ij} m_i n_j \mathbf{K}_{\vec{m}+\vec{n}}
\end{aligned}
\tag{32}
$$

The algebra of $\mathbf{K}_{\vec{n}}$ is the *area preserving diffeomorphism* algebra on T^2, SDiff(T^2).

$\mathbf{K}_{\vec{n}}$ commutes with the area operator, therefore it's action preserves area.

Hilbert space

We require our physical states to satisfy

$$\langle\tilde{\Psi}|\mathbf{L}_i|\Psi\rangle = 0, \qquad \langle\tilde{\Psi}|\mathbf{K}_{\vec{n}}|\Psi\rangle = 0, \qquad \forall\ |\Psi\rangle, |\tilde{\Psi}\rangle \in \mathcal{H}_{\text{phys}}. \tag{33}$$

Our zero-excitation states are defined by

$$\mathbf{C}^i_{\vec{n}}|0; m_i, w^i\rangle = 0 \qquad \forall \vec{n} \neq 0. \tag{34}$$

A generic excited state is a sum of monomials. A monomial is an excited state like

$$|\alpha\rangle = \prod_{i,\vec{p}} (\mathbf{C}^{i\dagger}_{\vec{p}})^{\alpha^i_{\vec{p}}}|0; m_i, w^i\rangle. \tag{35}$$

where $\alpha^i_{\vec{p}}$ are integer numbers.

Classification of physical Hilbert space

We have 3 classes of physical states considering the action of $\mathbf{L}_i$ on these states.

• States which are zero-eigenstates of $\mathbf{L}_i$ for both $i = 1, 2$, which we call Class 1 states.

• States which are zero-eigenstates of $\mathbf{L}_i$ for only one of $i = 1$ or $i = 2$ which we call Class 1.5P.

• States which are not zero-eigenstates of $\mathbf{L}_i$ which we call Class 2P.

$\mathbf{L}_i$ and $\mathbf{K}_{\vec{n}}$ constraint for Class 1

Class 1 states by definition satsify the $\mathbf{L}_i$ constraint.

Regarding the $\mathbf{K}_{\vec{n}}$ constraint, we can also see it will be automatically satisfied since

$$\mathbf{L}_i \mathbf{K}_{\vec{n}} |0, 0\rangle = -n_i \mathbf{K}_{\vec{n}} |0, 0\rangle \tag{36}$$

which follows from $[\mathbf{K}_{\vec{n}}, \mathbf{L}_i] = n_i \mathbf{K}_{\vec{n}}$.

Therefore $\langle 0', 0' | \mathbf{K}_{\vec{n}} | 0, 0 \rangle = 0$.

Class 1 states form a complete physical Hilbert space by themselves.

Class 2P of the null string

Class-2P states are a subset of non-zero eigenstates of **L** *that satisfy physicality condition.*

By definition of being in Class-2,

$$\mathbf{L}|\psi\rangle = |\psi^c\rangle, \tag{37}$$

where $|\psi^c\rangle$ is an unphysical state.
All states in the Class-2P, physical or unphysical can be written as a superposition of nonzero eigenstates of **L**

$$|\psi\rangle = \sum_{\ell} \psi_\ell |\ell\rangle, \qquad |\psi^c\rangle = \sum_{\ell} \psi^c_\ell |\ell\rangle, \qquad \psi^c_\ell = \ell \psi_\ell. \tag{38}$$

Eq. (37) implies

$$\langle\psi|\mathbf{L}\ \mathbf{L}|\psi\rangle = \langle\psi|\mathbf{L}^2|\psi\rangle = \langle\psi^c|\psi^c\rangle \neq 0 \quad \Longrightarrow \quad \mathbf{L}^2|\psi\rangle \in \mathcal{H}_{\text{phys}} \,. \tag{39}$$

Similarly, one learns that $\mathbf{L}|\psi^c\rangle \in \mathcal{H}_{\text{phys}}$ and $\mathbf{L}^2|\psi^c\rangle \in \mathcal{H}^c_{\text{phys}}$, and

$$\sum_\ell \ell|\psi_\ell|^2 = 0, \qquad \sum_\ell \frac{1}{\ell}|\psi_\ell|^2 = 0, \tag{40}$$

where we used orthonormality of $|\ell\rangle$ states.

Class 2P of the null string

Since

$$\mathbf{L}|\psi\rangle \in \mathcal{H}^c_{\text{phys}}, \qquad \mathbf{L}|\psi^c\rangle \in \mathcal{H}_{\text{phys}}, \qquad \mathbf{L}^2|\psi\rangle \in \mathcal{H}_{\text{phys}}, \tag{41}$$

there is a $\mathbb{Z}_2$ mapping between $\mathcal{H}_{phys}$ and $\mathcal{H}^c_{phys}$ and hence one may identify states in $\mathcal{H}_{phys}$ by modding $\mathcal{H}$ by this $\mathbb{Z}_2$.

Any state in $\mathcal{H}_{\text{phys}}$ or $\mathcal{H}^c_{\text{phys}}$ may be expanded in terms of eigenstates of $\mathbf{L}^2$. Recalling that the physicality condition, $\langle\phi|\mathbf{L}|\psi\rangle = 0$ for every physical state, is bilinear in $|\psi\rangle, |\phi\rangle$, this condition may be imposed separately for each eigenstate of $\mathbf{L}^2$.

Class 2P of the null string

We can write the eigenstates of $\mathbf{L}^2$ as

$$\mathbf{L}|\ell,\pm\rangle = \pm\ell|\ell,\pm\rangle, \qquad \ell > 0. \tag{42}$$

Next, let us define

$$|\ell\rangle_\pm := \frac{1}{\sqrt{2}}(|\ell,+\rangle \pm |\ell,-\rangle), \qquad \ell > 0. \tag{43}$$

One may readily check that

$$\begin{gathered}\mathbf{L}|\ell\rangle_\pm = \ell|\ell\rangle_\mp, \qquad \mathbf{L}^2|\ell\rangle_\pm = \ell^2|\ell\rangle_\pm, \qquad \ell > 0. \\ {}_\pm\langle\ell|\tilde{\ell}\rangle_\pm = \delta_{\ell,\tilde{\ell}}, \qquad {}_\pm\langle\ell|\tilde{\ell}\rangle_\mp = 0\end{gathered} \tag{44}$$

Class 2P of the null string

The above provides the key to our construction: We can take $\mathcal{H}_{\text{phys}}$ to be spanned by $|\ell\rangle_+$ (or $|\ell\rangle_-$) and $\mathcal{H}^c_{\text{phys}}$ by $|\ell\rangle_-$ (or $|\ell\rangle_+$).

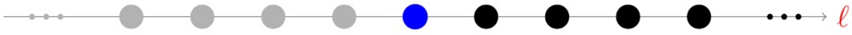


Figure: Depiction of **L** spectrum, ℓ. The origin (blue circle) corresponds to $\ell = 0$ Class-1 physical states. The black circles in $\ell > 0$ correspond to Class-2P physical states. The gray circles ($\ell < 0$) are modded out by the $\mathbb{Z}_2$ symmetry which maps $+\ell$ to $-\ell$; $\ell < 0$ correspond to unphysical Hilbert space $\mathcal{H}^c_{\text{phys}}$. Union of Class-1 and Class-2P ($\ell \geq 0$) specifies the largest $\mathcal{H}_{\text{phys}}$. Each dot corresponds to an infinite set of states which correspond states of different mass from zero to infinity.

$\mathbf{L}_i$ constraint for Class 2P

States of the form

$$|\ell_1,\ell_2\rangle_{s_1,s_2} = \frac{1}{2}(|\ell_1,\ell_2\rangle + s_1\,|-\ell_1,\ell_2\rangle + s_2\,|\ell_1,-\ell_2\rangle + s_1 s_2\,|-\ell_1,-\ell_2\rangle), \quad (45)$$

with s_1, s_2 taking $\pm$ values, are eigenstates of $\mathbf{L}_i^2$ with eigenvalues ℓ_i^2 and

$$\mathbf{L}_1|\ell_1,\ell_2\rangle_{s_1,s_2} = \ell_1|\ell_1,\ell_2\rangle_{-s_1,s_2}, \qquad \mathbf{L}_2|\ell_1,\ell_2\rangle_{s_1,s_2} = \ell_2|\ell_1,\ell_2\rangle_{s_1,-s_2}, \quad (46)$$

and $|\ell_1,\ell_2\rangle_{s_1,s_2}$ states with different s_1, s_2 are orthogonal to each other,

$$_{s_1,s_2}\langle\ell_1,\ell_2|\tilde{\ell}_1,\tilde{\ell}_2\rangle_{\tilde{s}_1,\tilde{s}_2} = \delta_{\ell_1,\tilde{\ell}_1}\delta_{\ell_2,\tilde{\ell}_2}\;\delta_{s_1,\tilde{s}_1}\delta_{s_2,\tilde{s}_2} \quad (47)$$

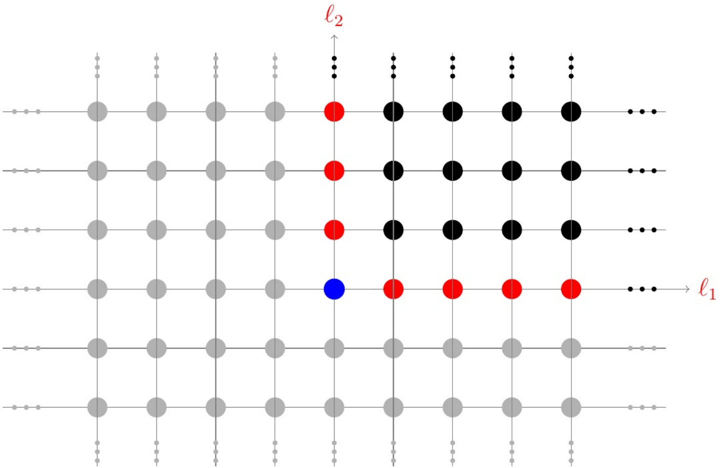
$\mathbf{L}_i$ constraint for Class 2P

Therefore, for a generic ℓ_1, ℓ_2, the Hilbert space is divided into 4 sectors, for 4 (s_1, s_2) choices.
If we work with one sector, say the one with $s_1 = s_2 = +1$, one readily sees that

$$ {}_{+,+}\langle \ell_1, \ell_2 | \mathbf{L}_i | \tilde{\ell}_1, \tilde{\ell}_2 \rangle_{+,+} = 0, \qquad \forall \ \ell_i, \tilde{\ell}_i. \tag{48} $$

So, we have solved for $\mathbf{L}_i$ constraints by modding out the Hilbert space by $\mathbb{Z}_2 \times \mathbb{Z}_2$. We remark that each $|\ell_1, \ell_2\rangle_{+,+}$ state for a given ℓ_i is infinitely degenerate.

$\mathbf{L}_i$ constraint for Class 2P



$\mathbf{K}_{\vec{n}}$ constraint for Class 2P

The $\mathbf{K}_{\vec{n}}$ constraint in fact restricts the type of monomials $|l_1, l_2\rangle$ that can be used in the construction of physical states.

We define 2 parameters[1] attributed to a given monomial that $\mathbf{K}_{\vec{n}}$ necessarily changes.

Requiring the monomials used in the construction of physical states to have fixed such parameters, we can see that the $\mathbf{K}_{\vec{n}}$ constraint will be automatically satisfied.

[1]We need 2 parameters, since $\mathbf{K}_{\vec{n}}$ has 2 degrees of freedom n_1 and n_2.

$\mathbf{K}_{\vec{n}}$ constraint for Class 2P

Given a monomial

$$|\alpha\rangle = \prod_{i,\vec{p}} (\mathbf{C}^{i\dagger}_{\vec{p}})^{\alpha^i_{\vec{p}}} |\Omega\rangle \tag{49}$$

we construct two numbers defined as

$$A = \sum_{\vec{p}} \epsilon^{ij} p_j \alpha^i_{\vec{p}} \tag{50}$$

and

$$B = \sum_{\vec{p}} \alpha^i_{\vec{p}} p_i \tag{51}$$

$\mathbf{K}_{\vec{n}}$ constraint for Class 2P

We can show that $\mathbf{K}_{\vec{n}}$ transforms this monomial to a sum of some monomials whose A and B either changes as

$$A \to A - n_1, \qquad B \to B - n_2 \tag{52}$$

or as

$$A \to A - n_2, \qquad B \to B - n_1. \tag{53}$$

Therefore if we require our physical states to have a fixed (A, B) structure we can guarantee that the $\mathbf{K}_{\vec{n}}$ constraint is satisfied.

$\mathbf{K}_{\vec{n}}$ constraint for Class 2P

Regarding completeness, suppose we have fixed our physical states to have $A = A_0$ and $B = B_0$.

Consider the spectrum is not complete and we can add another state with $A = A_0 + \delta$ and $B = B_0 + \lambda$ to the spectrum.

Then the action of $\mathbf{K}_{(\delta,\lambda)}$ on this states results states with $A = A_0$ and $B = B_0$ among other terms. Then since the mentioned part was already considered in the spectrum, the $\mathbf{K}_{\vec{n}}$ constraint will be violated. Therefore our chosen spectrum is complete.

Coclusions and Outlook

• Physicality is a constraint that should be satisfied for a set of states as a whole.

• There is a new set of states in the null string theory that has not been anticipated in the literature.

• Similar states form the interesting part of null brane theory where satisfying the $\mathbf{K}_{\vec{n}}$ constraint becomes a dilemma.

• We have proposed a solution for this dilemma by introducing some parameters atributed to a monomial that $\mathbf{K}_{\vec{n}}$ changes. By requiring these parameters to be fixed in the physical hilbert space, we solve the $\mathbf{K}_{\vec{n}}$ problem.

• There seems to be an important mathematical basis for this choice that has to do with the fact that $\mathbf{K}_{\vec{n}}$ is the generator of area preserving diffeomorphisms. This needs to be clarified.

Thank you.