Accelerating black holes

Gravitational lensing and timelike geodesics

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Based on:
Ashoorioon, MBJP, and Mann, PRL 129, 031102 (2022); PRD 107, 044031 (2023)
Ashoorioon, MBJP, and Luongo, in preparation

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Some gauge theories allow the possibility of topological defects, such as cosmic strings. Vilenkin, *Phys.Rep.* 121, 263 (1985).

Cosmic string could break/fray to produce pair of accelerating black holes.

Hawking and Ross, *Phys.Rev.Lett.* 75, 3382, (1995).

Ashoorioon and MBJP, *Phys.Lett.B* 816, 136224, (2021).

Also, primordial black holes could be formed in the early Universe and get attached to cosmic strings.

Vilenkin, Levin, and Gruzinov, *JCAP* 2018, 008, (2018).

Black holes connected to cosmic string could evolve to supermassive black holes. Their velocity, however, should be small ($\lesssim 100\,\mathrm{km/s}$), so that they could be captured by galaxies. This, in turn, constrain tension of the cosmic string to $\mu \lesssim 10^{-19}\sqrt{M/M_{\odot}}$, where M is the mass of supermassive black hole connected to the cosmic string. Considering Newton second law $\mu = \alpha M$, for M87* with $M \simeq 6.5 \times 10^{9} M_{\odot}$, we find $\alpha \lesssim 10^{-25}\,\mathrm{m}^{-1}$.

Vilenkin, Levin, and Gruzinov, *JCAP* 2018, 008, (2018).

This acceleration is so small that a ray of light, passing the black hole M87* to the Earth, lies on the equatorial plane of the black hole in the whole way.
We will show that such *slowly* accelerating black holes have features which could be observed!

Spacetime around uniformly accelerated black holes is described by C metric. An interesting form to represent the C metric has been introduced as

$$ds^2 = \frac{1}{\alpha^2(x+y)^2}\left[-F(y)d\tau^2 + \frac{dy^2}{F(y)} + \frac{dx^2}{G(x)} + G(x)d\phi^2\right],$$

where

$$\begin{aligned} F(y) &= -(1-y^2)(1-2\alpha m y), \\ G(x) &= (1-x^2)(1+2\alpha m x). \end{aligned}$$

α is interpreted as the acceleration of the black hole.

Kinnersley and Walker, *Phys.Rev.D* 2, 1359 (1970).

Hong and Teo, *Class.Quant.Grav.* 20, 3269 (2003).

Using the transformations $x = \cos\theta$, $y = \frac{1}{\alpha r}$, and $\tau = \alpha t$, C metric can be written in the form

$$ds^2 = \frac{1}{(1+\alpha r\cos\theta)^2}\left[-Q(r)dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2 d\theta^2}{P(\theta)} + P(\theta)r^2\sin^2\theta d\phi^2\right],$$

where

$$Q(r) = (1-\alpha^2 r^2)\left(1-\frac{2m}{r}\right), \qquad P(\theta) = 1 + 2\alpha m\cos\theta.$$

We have Schwarzschild solution for $\alpha \to 0$.

Griffiths, Krtouš, and Podolský, *Class.Quant.Grav.* 23, 6745 (2006).

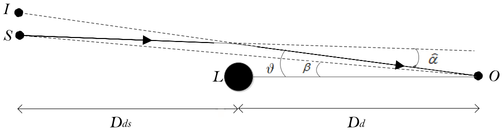
$$R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} = 48m^2\left(\tfrac{1}{r} + \alpha\cos\theta\right)^6 = 48m^2\alpha^6(x+y)^6,$$

The spacetime is flat for $x+y \to 0$, i.e. near conformal infinity. However, the curvature invariant does not vanish for $r \to \infty$ unless $\theta = \pi/2$. We would like to study the lensing effects on the equatorial plane of accelerating black holes, $\theta = \pi/2$. On this plane, the line element reduces to

$$ds^2 = -Qdt^2 + \frac{dr^2}{Q} + r^2 d\theta^2 + r^2 d\phi^2,$$

where $Q(r)$ is given by

$$Q(r) = \left(1 - \alpha^2 r^2\right)\left(1 - \tfrac{2m}{r}\right).$$



Due to the gravitational lensing by the black hole L, the observer O sees image I of the source S. The observer cannot see the source itself.

The equations governing the geodesics can be obtained using the Lagrangian

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} = \frac{1}{2}\left(-Q\dot{t}^2 + \frac{\dot{r}^2}{Q} + r^2 \dot{\phi}^2\right).$$

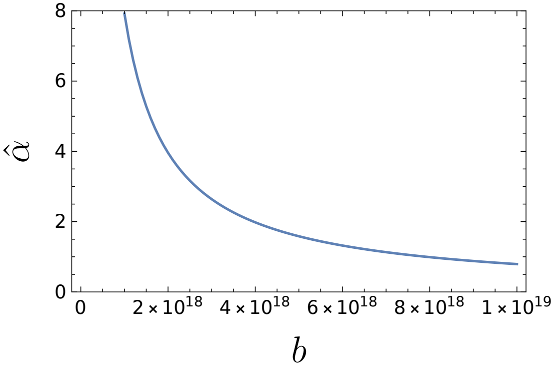
On the equatorial plane $\dot{\theta} = 0$. The constants of motion are

$$E = -\frac{\partial \mathcal{L}}{\partial \dot{t}} = Q\dot{t}, \qquad L_z = -\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = -r^2 \dot{\phi}.$$

At the point of closest approach to the black hole, $r = b$, we have $\frac{dr}{d\phi} = 0$ and $\frac{dr}{dt} = 0$. The deflection angle would then be found as

$$\hat{\alpha}(b) = 2\int_b^{\infty} \frac{dr}{r\sqrt{\left(\frac{r}{b}\right)^2 Q_b - Q}} - \pi.$$

Weinberg, (1972).



The deflection angle $\hat{\alpha}$ is in *arcseconds* and the impact parameter b is in *meters*. We have set $M_{\mathrm{M87}^*} = 9.6 \times 10^{12}\,\mathrm{m}$ and $\alpha = 10^{-25}\,\mathrm{m}^{-1}$.

The time delay is the difference between the time it takes for the light to travel the physical path from the source to the observer and the time it takes to travel the path from the source to the observer when there is no black hole. It can be found by the integral

$$\tau(b) = \left[\int_b^{r_s} dr + \int_b^{D_d} dr\right] \frac{1}{Q\sqrt{1-\left(\frac{b}{r}\right)^2 \frac{Q}{Q_b}}} - D_s \sec\beta.$$

The lens equation is

$$\tan\beta = \tan\vartheta - \mathcal{D}\left[\tan\vartheta + \tan(\hat{\alpha} - \vartheta)\right],$$

where $\mathcal{D} = D_{ds}/D_s$.

Virbhadra and Ellis, *Phys.Rev.D* 62, 084003 (2000).

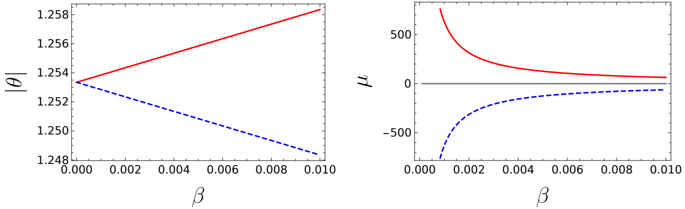
Also the image magnification is

$$\mu = \left(\frac{\sin\beta}{\sin\vartheta}\frac{d\beta}{d\vartheta}\right)^{-1}.$$

MBJP and Mann, *Phys.Rev.D* 99, 024035 (2019).

Image positions θ and magnifications μ of primary and secondary images due to lensing by M87*. These results are the same in the non-accelerating and accelerating cases. (a) p and s refer to primary and secondary images, respectively. (b) All angles are in *arcseconds*. (c) We have used $M_{\mathrm{M87^*}} = 9.6 \times 10^{12}\,\mathrm{m}$, $D_d = 5.2 \times 10^{23}\,\mathrm{m}$, $\mathcal{D} = 0.5$, and $\alpha = 10^{-25}\,\mathrm{m}^{-1}$.

β	θ_p	μ_p	θ_s	μ_s
0	1.25334	$\times$	−1.25334	$\times$
0.1	1.30436	6.79185	−1.20436	−5.78173
0.5	1.52806	1.82812	−1.02805	−0.826954
1	1.84943	1.26694	−0.849483	−0.267388
2	2.60342	1.05575	−0.603448	−0.0567785
3	3.45471	1.01705	−0.454745	−0.0176322
4	4.36033	1.00676	−0.360286	−0.00687431



(Left) Angular position of primary images θ_p (red plot) and the absolute value of angular position of secondary images $|\theta_s|$ (dashed blue plot). (Right) Magnification of primary (red plot) and secondary (dashed blue plot) images. Angles are in *arcseconds*. These results are nearly the same for non-accelerating and slowly accelerating case.

Time delays τ and the differential time delay $t_d = \tau_s - \tau_p$ which is of observational importance. (a) β is in *arcseconds* and the (differential) time delays are in *seconds*. (b) Barred quantities refer to values of the case that the black hole is not accelerating and $\Delta t_d = \bar{t}_d - t_d$. (c) We have taken $\alpha = 10^{-25}\,\mathrm{m}^{-1}$.

β	τ_p	t_d	$\bar{\tau}_p$	Δt_d
0	3.13×10^{12}	0	1.63×10^{6}	0
0.1	3.13×10^{12}	20445	1.62×10^{6}	1.5×10^{-9}
0.5	3.13×10^{12}	102871	1.58×10^{6}	8.4×10^{-9}
1	3.13×10^{12}	209681	1.54×10^{6}	2.2×10^{-8}
2	3.13×10^{12}	448736	1.48×10^{6}	8.9×10^{-8}
3	3.13×10^{12}	737888	1.44×10^{6}	2.8×10^{-7}
4	3.13×10^{12}	1089214	1.41×10^{6}	7.0×10^{-7}

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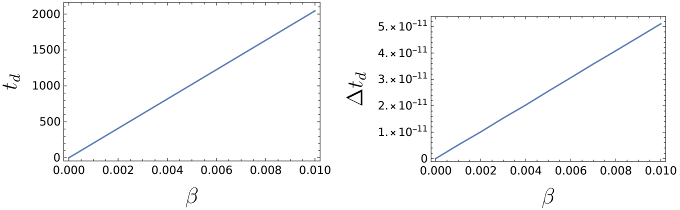


(Left)The differential time delay $t_d = \tau_s - \tau_p$ of secondary and primary images as a function of source angular position for accelerating black hole. (Right) The difference between the differential time delay in non-accelerating and slowly accelerating cases, $\Delta t_d = \bar{t}_d - t_d$.

Therefore, we conclude that, if in a future observation the images position match the prediction of general relativity, a possible deviation in the differential time delay can be due to the acceleration.

The tension of a cosmic string connected to supermassive black holes is $\mu \lesssim 10^{-19}\sqrt{M/M_{\odot}}$. For the black hole at the center of our Galaxy, with $M_{\mathrm{SgrA}^*} \simeq 4.23 \times 10^{6} M_{\odot}$, the tension of the cosmic string attached to it would be $\mu \lesssim 2.06 \times 10^{-16}$.

Johannsen, et al., *Phys.Rev.Lett* 116, 031101 (2016).

Therefore we find from the Newton second law, $\mu = \alpha M_{\mathrm{SgrA}^*}$, that the acceleration is $\alpha \lesssim 3.29 \times 10^{-26}\mathrm{m}^{-1}$.

The orbital period of S2 star around Sgr A* is about 16 years.

Boehle, et al., *The Astrophysical Journal* 830, 17 (2016).

Hees, et al., *Phys.Rev.Lett* 118, 211101 (2017).

We take the acceleration of Sgr A* to be $\alpha = 10^{-26}\mathrm{m}^{-1}$. Assuming the initial velocity of the black hole to be zero, the displacement of the black hole during the period of S2 is $2.29 \times 10^{8}\mathrm{m}$. This is much smaller than the Schwarzschild radius of the black hole which is of the order of $10^{10}\mathrm{m}$. Therefore we can assume that, if S2 starts its period near the equatorial plane of the black hole, it will remain near this plane during its orbit around the slowly accelerating black hole.

MBJP, *Phys.Rev.D* 106, 044037 (2022).

Plebański-Demiański metric:

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dtd\phi,$$

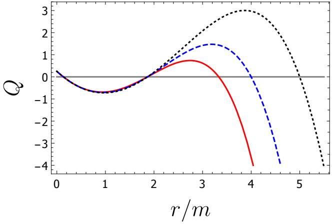
with

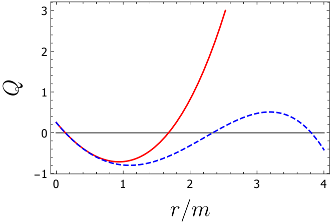
$$\begin{aligned}
g_{tt} &= -\frac{Q - a^2 P \sin^2(\theta)}{\rho^2\Omega^2}, \quad g_{rr} = \frac{\rho^2}{Q\Omega^2}, \quad g_{\theta\theta} = \frac{\rho^2}{P\Omega^2}, \\
g_{\phi\phi} &= -\frac{\sin^2(\theta)\left(a^2 Q \sin^2(\theta) - P\left(a^2 + r^2\right)^2\right)}{\rho^2\Omega^2}, \\
g_{t\phi} &= -\frac{a \sin^2(\theta)\left(P\left(a^2 + r^2\right) - Q\right)}{\rho^2\Omega^2},
\end{aligned}$$

where

$$\begin{aligned}
\Omega &= 1 - \alpha r \cos(\theta), \quad \rho^2 = a^2 \cos^2(\theta) + r^2, \\
P &= \cos^2(\theta)\left(\alpha^2 a^2 + \frac{a^2\Lambda}{3}\right) - 2\alpha m \cos(\theta) + 1, \\
Q &= \left(1 - \alpha^2 r^2\right)\left(a^2 - 2mr + r^2\right) - \frac{1}{3}\Lambda r^2\left(a^2 + r^2\right).
\end{aligned}$$

Griffiths and Podolskỳ, *IJMPD* 15, 335 (2006).





(Left) Horizons of rotating accelerating black holes in flat background, i.e. $\Lambda = 0$. We have $m\alpha = 0.3$ for the solid red plot, $m\alpha = 0.25$ for the dashed blue plot, and $m\alpha = 0.2$ for the dotted black plot. (Right) Horizons of a rotating accelerating black hole in anti de Sitter background with $m^2\Lambda = -0.1$ (solid red plot) and de Sitter background with $m^2\Lambda = 0.1$ (dashed blue plot). Here we have fixed $m\alpha = 10^{-3}$. We have taken $a/m = 0.5$ in both panels. Acceleration horizon disappear for $\Lambda < -3\alpha^2$.

Lagrangian (in equatorial plane):

$$2\mathcal{L} = g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2 + 2g_{t\phi}\dot{t}\dot{\phi} = -\mu^2.$$

Constants of motion:

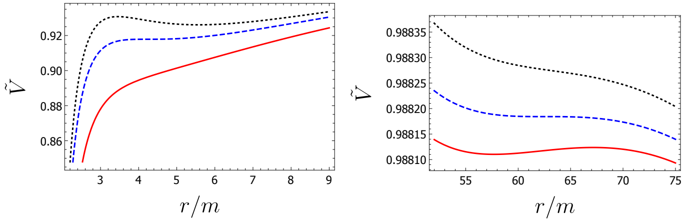
$$\begin{aligned} E &= p_t = -\frac{\partial\mathcal{L}}{\partial\dot{t}} = -\left(g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\right), \\ L &= p_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = g_{\phi\phi}\dot{\phi} + g_{t\phi}\dot{t}. \end{aligned}$$

We find

$$g_{rr}\dot{r}^2 = \frac{g_{\phi\phi}\tilde{E}^2+2g_{t\phi}\tilde{L}\tilde{E}+g_{tt}\tilde{L}^2}{g_{t\phi}^2-g_{\phi\phi}g_{tt}} - 1.$$

The potential is

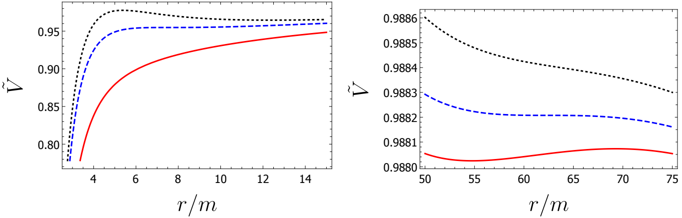
$$\tilde{V} = \frac{\left(g_{t\phi}^2-g_{tt}g_{\phi\phi}\right)\left(\frac{2\sqrt{-g_{\phi\phi}-\tilde{L}^2}}{\sqrt{g_{tt}g_{\phi\phi}-g_{t\phi}^2}} - \frac{2g_{t\phi}\tilde{L}}{g_{t\phi}^2-g_{tt}g_{\phi\phi}}\right)}{2g_{\phi\phi}}.$$



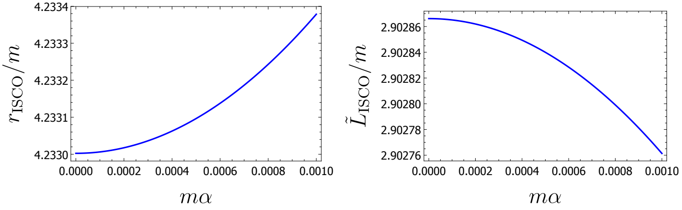
(Left) The effective potential for co-rotating particles in the region near the black hole horizon with $\tilde{L}/m = 2.7$ for solid red plot, $\tilde{L}/m = \tilde{L}_{\rm ISCO}/m \simeq 2.90276$ for dashed blue plot, and $\tilde{L}/m = 3$ for dotted black plot. (Right) The effective potential for co-rotating particles in the region far from the black hole with $\tilde{L}/m = 7$ for solid red plot, $\tilde{L}/m = \tilde{L}_{\rm OSCO} \simeq 7.03792$ for dashed blue plot, and $\tilde{L}/m = 7.09$ for dotted black plot. We have taken $a/m = 0.5$, $\theta = \pi/2$, $m\alpha = 10^{-3}$, and $m^2\Lambda = 10^{-52}$ in both panels.

Timelike geodesics

Stable circular orbits



(Left) The effective potential for counter-rotating particles in the region near the black hole horizon with $\tilde{L}/m = -3$ for solid red plot, $\tilde{L}/m = \tilde{L}_{\rm ISCO}/m \simeq -3.88349$ for dashed blue plot, and $\tilde{L}/m = -4.2$ for dotted black plot. (Right) The effective potential for counter-rotating particles in the region far from the black hole with $\tilde{L}/m = -7$ for solid red plot, $\tilde{L}/m = \tilde{L}_{\rm OSCO} \simeq -7.08774$ for dashed blue plot, and $\tilde{L}/m = -7.2$ for dotted black plot. We have taken $a/m = 0.5$, $\theta = \pi/2$, $m\alpha = 10^{-3}$, and $m^2\Lambda = 10^{-52}$ in both panels.



(Left) The radius of ISCO as a function of acceleration parameter for the case of co-rotating particles. (Right) The angular momentum of the particle in ISCO as a function of acceleration parameter. We have taken $a/m = 0.5$, $\theta = \pi/2$, and $m^2\Lambda = 10^{-52}$ in both panels.

The precession can be found as

$$\begin{aligned}\Delta\phi &= 2\left[\phi(r_a) - \phi(r_p)\right] - 2(1+2\alpha m)^{-1}\pi \\ &= 2 \int_{r_p}^{r_a} \left(\frac{g_{rr}(g_{t\phi}\tilde{E} + g_{tt}\tilde{L})^2}{(g_{t\phi}^2 - g_{tt}g_{\phi\phi})\delta}\right)^{1/2} - 2(1+2\alpha m)^{-1}\pi,\end{aligned}$$

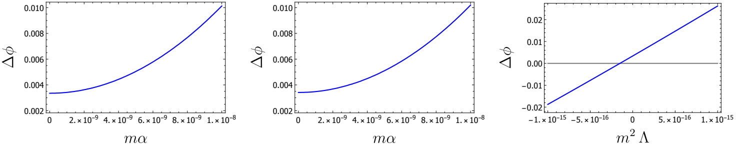
where

$$\delta = g_{\phi\phi}\tilde{E}^2 + 2g_{t\phi}\tilde{E}\tilde{L} + g_{tt}\tilde{L}^2 + g_{tt}g_{\phi\phi} - g_{t\phi}^2.$$

The semi-major axis and eccentricity of S2 are $b = 1.543 \times 10^{14}\mathrm{m}$ and $e = 0.88$. Eisenhauer, et al., *The Astrophys. J. Lett.* **597**, L121 (2003).

Timelike geodesics

Precession of perihelion



(Left) The precession (in radians) of co-rotating particles. (Center) The precession of counter-rotating particles. We have taken $a = 0.5$, $\theta = \pi/2$, and $m^2\Lambda = 10^{-52}$ in top row. We have used the semi-major axis and eccentricity of the orbit of S2 around Sgr A*. (Right) We have taken $a = \alpha = 0$.

Thank you