Gravitational Energy Momentum Tensor at Null Infinity

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Matter and Gravitational EMT

Let us consider Gravity + Matter action

$$S[g_{\mu\nu}, \Phi] = S_{\text{gravity}} + S_{\text{matter}}$$

One can read the energy-momentum tensor of the matter as

$$T_{\mu\nu}^{\text{matter}} \sim \frac{1}{\sqrt{g}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}}$$

Bulk diffeomorphism invariance yields

$$\nabla_\mu T^{\mu\nu}_{\text{matter}} = 0$$

How about the energy-momentum tensor of gravity?

$$T_{\mu\nu}^{\text{gravity}} \sim \frac{1}{\sqrt{g}} \frac{\delta S_{\text{gravity}}}{\delta g^{\mu\nu}} = \text{eom} = 0$$

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Brown-York quasi-local EMT

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In addition to $g_{\mu\nu}$, we have the induced metric of the boundary $\gamma_{\mu\nu}$,

$$T^{\text{gravity}}_{\mu\nu} \sim \frac{1}{\sqrt{\gamma}} \frac{\delta S}{\delta \gamma^{\mu\nu}} \qquad \text{Brown-York EMT [Brown-York '93]}$$

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Boundaries: Asymptotic boundaries, Black hole horizon, etc

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Implications

- Gravitational energy, angular momentum, and radiation
- Holography
- Fluid/Gravity correspondence
  [S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani, 2008]

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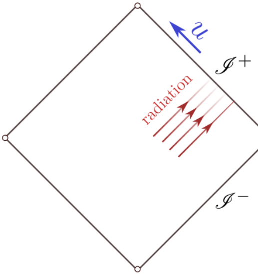
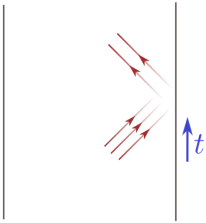
Null infinity v.s AdS timelike boundary



Minkowski

vs



Anti-de Sitter

Solution space

Theory: 3dim Einstein gravity with $\Lambda = 0$ $R_{\mu\nu} = 0$

Line element:

$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu = -V\,dv^2 + 2\eta\,dv\,dr + \mathcal{R}^2\,(d\phi + U\,dv)^2$$

Solution:

$$\mathcal{R} = \Omega + r\,\eta\,\lambda$$

$$U = \mathcal{U} + \frac{1}{\lambda\mathcal{R}}\,\frac{\eta'}{\eta} + \frac{8G\Upsilon - \Omega\,\Pi'}{2\lambda\mathcal{R}^2}$$

$$V = \frac{1}{\lambda^2}\Bigg[-8G\mathcal{M} - 2\,\mathrm{Sch}[\sigma;\phi] + \lambda\,\Omega\,\mathcal{D}_v\Pi + \left(\frac{\eta'}{\eta}\right)^2 + \frac{(8G\Upsilon - \Omega\,\Pi')^2}{4\mathcal{R}^2} - \frac{2\mathcal{R}}{\eta}\,\mathcal{D}_v(\eta\lambda) + \left(\frac{8G\Upsilon - \Omega\,\Pi'}{\mathcal{R}}\right)\frac{\eta'}{\eta}\Bigg]$$

where $\Pi := 2\ln|\eta\lambda\Omega^{-1}|$ and

$$\mathrm{Sch}[\sigma;\phi] := \frac{\sigma'''}{\sigma'} - \frac{3}{2}\left(\frac{\sigma''}{\sigma'}\right)^2, \qquad \sigma := \int^\phi \lambda\,d\phi$$

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Solution space

Integration functions: $\{\eta, \mathcal{U}, \Omega, \lambda, \mathcal{M}, \Upsilon\}$

Constraint equations

$$\mathcal{D}_v \mathcal{M} + \frac{1}{4G} \mathcal{U}''' = 0$$

$$\mathcal{D}_v \Upsilon - \lambda \left(\frac{\mathcal{M}}{\lambda^2} \right)' + \frac{1}{4G} (\lambda^{-1})''' = 0$$

we have defined

$$\mathcal{D}_v O_w := \partial_v O_w - \mathcal{U} \partial_\phi O_w - w O_w \partial_\phi \mathcal{U}$$

For example $\mathcal{M}, \Upsilon, \Omega, \Pi$ are of weight $2, 2, 1, 1$ respectively.

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Boundary symmetries

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$$\xi = \textcircled{T}\,\lambda\,\partial_v + \left[\textcircled{Y} - \mathcal{U}\,\lambda\,T + \frac{(T\,\lambda)'}{\lambda\,\mathcal{R}}\right]\partial_\phi + \xi^r\partial_r$$

with

$$\begin{aligned}\xi^r =& \frac{1}{\eta\,\lambda}\left[\textcircled{Z} - T\,\lambda\,\mathcal{D}_v\Omega - (\Omega Y)' - \frac{1}{\eta}\left(\frac{\eta\,(T\lambda)'}{\lambda}\right)'\right]\\ &- \frac{r}{2}\left(\textcircled{W} + T\,\lambda\,\mathcal{D}_v\Pi - 2\,e^{\Pi/2}Z + Y\,\Pi'\right)\\ &- \frac{8G\Upsilon - \Omega\,\Pi'}{2\,\eta\,\lambda^2\,\mathcal{R}}\,(T\lambda)'\end{aligned}$$

Symplectic potential and ambiguities

Symplectic potential [Lee-Wald '90]:

$$\Theta^\mu[g;\delta g] := \underbrace{\frac{\sqrt{-g}}{8\pi G}\nabla^{[\alpha}\left(g^{\mu]\beta}\delta g_{\alpha\beta}\right)}_{\text{Lee-Wald symplectic pot}} + \underbrace{\nabla_\nu Y^{\mu\nu}[g;\delta g]}_{Y\text{-freedom}} + \underbrace{\delta L^\mu_{\mathcal{B}}[g]}_{\text{boundary Lagrangian}}$$

We choose Y-freedom s.t to remove r-dependence:

$$Y^{\mu\nu}[\delta g; g] = \frac{1}{8\pi G}\left(2\delta\sqrt{-g}\, n^{[\mu} l^{\nu]} + 3\sqrt{-g}\,\delta n^{[\mu} l^{\nu]}\right)$$

where l^μ and n^ν are two null vector fields

$$l^\mu\partial_\mu = \partial_v + \frac{V}{2\eta}\partial_r - U\partial_\phi\,, \qquad n^\mu\partial_\mu = -\frac{1}{\eta}\partial_r\,, \qquad n\cdot l = -1$$

We fix the boundary Lagrangian later by requiring a well-defined action principle.

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On-shell symplectic potential

(partially) on-shell symplectic potential:

$$\boldsymbol{\Theta} := \int \Theta^\mu \, \mathrm{d}^2 x_\mu \,, \qquad \boldsymbol{\Theta} = \boldsymbol{\Theta}_{\mathcal{H}} + \boldsymbol{\Theta}_{\mathcal{C}} + (\textit{total variation term})$$

hydrodynamic and corner symplectic potentials

$$\boldsymbol{\Theta}_{\mathcal{H}} := -\frac{1}{2\pi} \int \mathrm{d}^2 x \left[\mathcal{M} \, \delta(\lambda^{-1}) + \Upsilon \, \delta \mathcal{U} \right]$$

$$\boldsymbol{\Theta}_{\mathcal{C}} := \frac{1}{16\pi G} \int \mathrm{d}^2 x \, \partial_v \left(\Omega \, \delta \Pi \right)$$

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Surface charge

Surface charge variation

Surface charge variation:

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} \mathrm{d}\phi \,(W\,\delta\Omega + Z\,\delta\Pi) + \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \,(T\,\delta\mathcal{M} + Y\,\delta\Upsilon)$$

The charge algebra is the direct sum of the $\mathfrak{Heisenberg}$ and the $\mathfrak{bms}_3$ algebras. The former is spanned by Ω and Π while the latter is spanned by $\mathcal{M}$ and Υ.

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Geometry of null infinity

Conformal induced metric:

$$\mathrm{d}s^2|_{\mathcal{I}} = \lim_{r\to\infty} \frac{\mathrm{d}s^2}{\mathcal{R}^2}$$

$$\gamma_{ab} = k_a k_b \,, \qquad k_a\,\mathrm{d}x^a := \mathrm{d}\phi + \mathcal{U}\,\mathrm{d}v$$

Kernel of induced metric:

$$\gamma_{ab} l^b = 0 \,, \qquad l^a \partial_a := \lambda(\partial_v - \mathcal{U}\partial_\phi)$$

Dual of the kernel:

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Projection:

$$P^a{}_b := \delta^a_b + n_b l^a \,, \qquad P^a{}_b l^b = P^a{}_b n_a = 0$$

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$$h^{ac}\gamma_{cb} = P^a{}_b \,, \qquad h^{ab} = k^a k^b \,, \qquad k^a\partial_a := \partial_\phi$$

Null-volume form:

$$\gamma = \lambda^{-2}$$

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Null boundary connection

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$$\Gamma^c_{ab} = \frac{1}{2} h^{cd} \left(\partial_a \gamma_{bd} + \partial_b \gamma_{ad} - \partial_d \gamma_{ab}\right) + h^{cd} K_{da} n_b + l^c S_{ab}$$

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EMT from symplectic potential

Canonical structure of symplectic potential:

$$\Theta_{\mathcal{H}} = \frac{1}{2\pi}\int \mathrm{d}x^2 \sqrt{\gamma}\, T^{ab}\delta\gamma_{ab}$$ non-degenerate case

$$\Theta_{\mathcal{H}} = \frac{1}{2\pi}\int \mathrm{d}^2x\sqrt{\gamma}\left(\mathrm{T}^{ab}\delta\gamma_{ab} + P^a\delta n_a\right)$$ degenerate case

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$$\Theta_{\mathcal{H}} = -\frac{1}{2\pi}\int \mathrm{d}^2x\left[\mathcal{M}\,\delta(\lambda^{-1}) + \Upsilon\,\delta\mathcal{U}\right]$$

We recognize:

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Equation of state [J. de Boer et.al. 2022]:

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Conserved EMT

We can add a total variation term (boundary Lagrangian) to the symp. pot.:

Boundary action

$$S_{\mathcal{B}} := \int \mathrm{d}^2x\, L_{\mathcal{B}} = -\int \mathrm{d}v \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi\sigma'} \left(\mathcal{M} + \frac{1}{4G}\,\mathrm{Sch}[\sigma;\phi]\right)$$

where $\mathrm{Sch}[\sigma;\phi] := \frac{\sigma'''}{\sigma'} - \frac{3}{2}\left(\frac{\sigma''}{\sigma'}\right)^2$.

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Properties of null EMT

- Null EMT = Stress tensor + Null current.

- The (2,0)-type stress tensor is not symmetric. [J. de Boer et.al. 2022]

- It satisfies the following Equation of state

$$\mathrm{T} + P^a n_a = 0 \quad \rightarrow \quad \text{Energy+Pressure=0}$$

- Comparision with AdS case [M. Henningson, K. Skenderis '98]:

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Symplectic form

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Finally, the symplectic form is:

$$\boldsymbol{\Omega} = \frac{1}{2\pi}\int \mathrm{d}^2x \left[\delta(\sqrt{\gamma}\,\mathsf{T}^{ab}) \wedge \delta\gamma_{ab} + \delta(\sqrt{\gamma}P^a) \wedge \delta n_a\right] + \frac{1}{16\pi G}\int \mathrm{d}^2x\,\partial_v\left(\delta\Omega \wedge \delta\Pi\right)$$

Summary

- We constructed a conserved gravitational EMT (stress tensor + null current) at null infinity in 3dim asymptotically flat spacetimes.

- By demanding a well-defined action principle and the existence of a conserved EMT, we found a Schwarzian action as the boundary action.

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  - $\lim_{r\to\infty}$ (Brown-York EMT in bulk)
  - $\lim_{\ell\to\infty}$ (Brown-York EMT at AdS boundary)
- Implications of the emergence of the Schwarzian action at null infinity:
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