Study the Nucleon Structure through Generalized Parton Distributions (GPDs)

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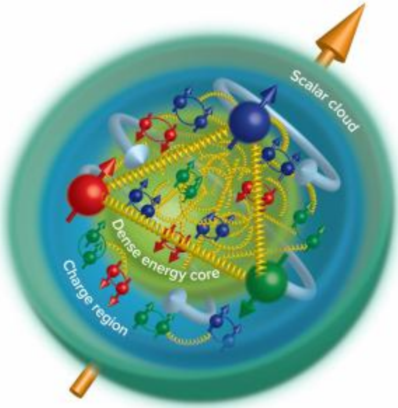
Outline

Introduction

Proton Structure

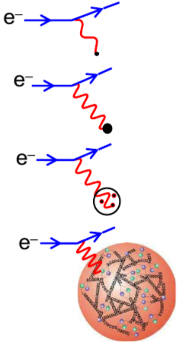


Probing the Structure of the Proton

Probing the Structure of the Proton

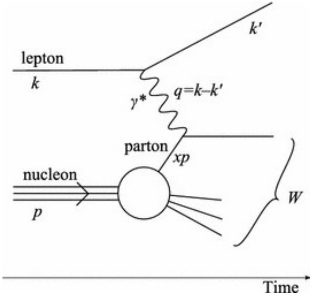
★ In $e^-p \to e^-p$ scattering the nature of the interaction of the virtual photon with the proton depends strongly on wavelength

- At **very low** electron energies $\lambda \gg r_p$: the scattering is equivalent to that from a "point-like" spin-less object
- At **low** electron energies $\lambda \sim r_p$: the scattering is equivalent to that from a extended charged object
- At **high** electron energies $\lambda < r_p$: the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks
- At **very high** electron energies $\lambda \ll r_p$: the proton appears to be a sea of quarks and gluons.



Prof. M.A. Thomson　　Michaelmas 2009　　153

Deep Inelastic Scattering (DIS)



$$W^2 = (p+q)^2 = M_p^2 + 2p \cdot q + q^2$$

If $W = M_p, \;\; \mu^2 = -q^2 \Rightarrow \quad x = \frac{\mu^2}{2p \cdot q} = 1 \Rightarrow$ elastic scattering

$W^2 \gg M_p^2 \Rightarrow$ Inelastic

$\mu^2 \gg M_p^2 \Rightarrow$ Deep

R. Devenish and A. Cooper-Sarkar, *Deep Inelastic Scattering* (Oxford, 2003).

Parton Distribution Functions (PDFs)

$$\frac{d\sigma}{dx d\mu^2} = \frac{2\pi\alpha^2}{x\mu^4}\left(Y_+ F_2(x,\mu^2) \pm Y_- x F_3(x,\mu^2) - y^2 F_L(x,\mu^2)\right) \quad (1)$$

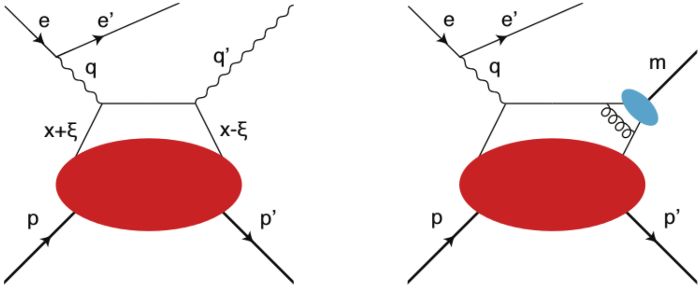
The factorization theorem states that the structure functions F_i can be written as the convolution of the nonperturbative parts and the hard coefficient functions which can be calculated in pQCD.

$$F_i(x,\mu^2) = \sum_{a=q,g} C_{i,a}(\alpha_s,\mu^2) \otimes f_a(x,\mu^2) \quad (2)$$

$xf_a(x,\mu^2) \equiv$ Parton Distribution Functions (PDFs)

J. J. Ethier and E. R. Nocera, Ann. Rev. Nucl. Part. Sci. **70**, 43-76 (2020).

Hard Exclusive Processes

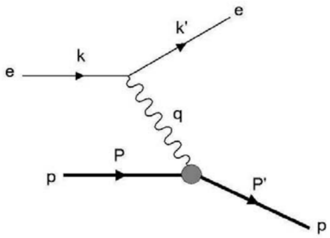


(a) Deeply virtual Compton scattering. (b) Deeply virtual meson production.

$$t = (p' - p)^2 = -Q^2 \qquad \xi \equiv \text{skewness}$$

K. Goeke *et al.*, Prog. Part. Nucl. Phys. **47**, 401-515 (2001).

Elastic Scattering from a Finite Size Proton



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon(1+\tau)} \tag{3}$$

$$\tau = \frac{Q^2}{4m^2}, \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

M. E. Christy *et al.*, Phys. Rev. Lett. **128**, 102002 (2022).

Elastic Scattering from a Finite Size Proton

The Sachs FFs G_E and G_M are usually expressed in terms of the Dirac and Pauli FFs of the nucleon, F_1 and F_2,

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,,$$
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m^2} F_2(Q^2)\,. \qquad (4)$$

The Dirac and Pauli FFs are related to the unpolarized valence GPDs H_v^q and E_v^q at zero skewness,

$$F_1^p(Q^2) = \sum_q e_q F_1^q(Q^2) = \sum_q e_q \int_0^1 dx\, H_v^q(x, \mu^2, Q^2, \xi = 0)\,,$$
$$F_2^p(Q^2) = \sum_q e_q F_2^q(Q^2) = \sum_q e_q \int_0^1 dx\, E_v^q(x, \mu^2, Q^2, \xi = 0)\,. \qquad (5)$$

M. Diehl and P. Kroll, Eur. Phys. J. C **73**, 2397 (2013).

Physical Processes Involving GPDs

Elastic scatterings: GPDs at $\xi = 0$

- Elastic lepton-nucleon scattering
- Elastic (anti)neutrino-nucleon scattering
- Wide-angle Compton scattering (WACS)

Hard exclusive processes: Whole GPDs

- Deeply virtual compton scattering (DVCS)
- Deeply virtual meson production (DVMP)
- Time-like Compton scattering (TCS)
- Heavy vector meson production (HVMP)
- Single diffractive hard exclusive processes (SDHEPs)
- Central exclusive production (CEP) processes at LHC

M. Diehl, "Generalized parton distributions," Phys. Rept. **388**, 41-277 (2003).

Other Quantities Related to GPDs

The axial form factor:

$$G_A(Q^2) = \int_0^1 dx \left[\widetilde{H}_v^u - \widetilde{H}_v^d\right] + 2\int_0^1 dx \left[\widetilde{H}^{\bar{u}} - \widetilde{H}^{\bar{d}}\right] \tag{6}$$

The charge and magnetic radii of the nucleon:

$$\langle r_E^2 \rangle = -6 \frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\Big|_{Q^2=0}, \qquad \langle r_M^2 \rangle = \frac{-6}{\mu}\frac{\mathrm{d}G_M}{\mathrm{d}Q^2}\Big|_{Q^2=0} \tag{7}$$

The gravitational form factors (GFFs):

$$\int_{-1}^1 dx\, x \sum_q H^q = M_2(Q^2) + \xi^2 D(Q^2) \tag{8}$$

The total angular momentum:

$$J^q(Q^2) = \frac{1}{2}\int_{-1}^1 dx\, x[H^q + E^q], \tag{9}$$

The Mechanical Properties of the Nucleon

If we define the Fourier transform of the GFF $D(t = -\boldsymbol{\Delta}^2)$ as

$$\widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3}\; e^{-i\boldsymbol{\Delta r}}\; D(-\boldsymbol{\Delta}^2)\,, \tag{10}$$

we can compute the **pressure** $p(r)$ and **shear forces** $s(r)$ as follows

$$p(r) = p(r) = \frac{1}{6m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\widetilde{D}(r)\right)\,, \tag{11}$$

$$s(r) = s(r) = -\frac{1}{4m}\; r\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\widetilde{D}(r)\right)\,. \tag{12}$$

The nucleon's **mechanical and mass radii** are related to the GFF $D(t)$:

$$\langle r^2\rangle_{\text{mech}} = \frac{6D(0)}{\int_{-\infty}^{0} D(t)\,dt}\,, \qquad \langle r^2\rangle_{\text{mass}} = 6A'(0) - \frac{3D(0)}{2m^2}\,. \tag{13}$$

M. V. Polyakov *et al.*, Int. J. Mod. Phys. A **33**, no.26, 1830025 (2018).

Determination of GPDs

Various Approaches to Access GPDs

- GPDs or their moments can be calculated utilizing various theoretical models and techniques such as lattice QCD, chiral quark-soliton model, light-front Hamiltonian approach, nonrelativistic and light-cone constituent quark model, nonlocal chiral quark model, and so on.
- However, GPDs are nonperturbative objects like PDFs and can not be determined from the first principles of perturbative QCD.
- GPDs are usually determined through the QCD analysis of the experimental data.

M. Diehl and P. Kroll, Eur. Phys. J. C **73**, 2397 (2013).
K. Kumericki, S. Liuti and H. Moutarde, Eur. Phys. J. A **52**, 157 (2016).
H. Hashamipour, M. Goharipour *et al.*, Phys. Rev. D **107**, 096005 (2023).

Previous Studies

- H. Khanpour, M. Goharipour and V. Guzey, Eur. Phys. J. C **78**, no.1, 7 (2018).
- H. Hashamipour, M. Goharipour and S. S. Gousheh, Phys. Rev. D **100**, no.1, 016001 (2019).
- H. Hashamipour, M. Goharipour and S. S. Gousheh, Phys. Rev. D **102**, no.9, 096014 (2020).
- H. Hashamipour, M. Goharipour, K. Azizi and S. V. Goloskokov, Phys. Rev. D **105**, no.5, 054002 (2022).
- H. Hashamipour, M. Goharipour, K. Azizi and S. V. Goloskokov, Phys. Rev. D **107**, no.9, 096005 (2023).
- F. Irani, M. Goharipour, H. Hashamipour and K. Azizi, Phys. Rev. D **108**, no.7, 074018 (2023).

MMGPDs Collaboration

Modern Multipurpose GPDs (MMGPDs) Collaboration:

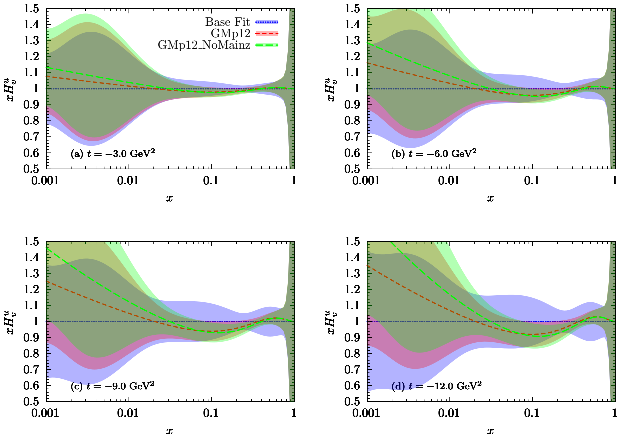
- **Kazem Azizi** (Faculty Member, University of Tehran)
- **Muhammad Goharipour** (Senior PostDoc, IPM)
- **Hadi Hashamipour** (PostDoc, INFN)
- **Fatemeh Irani** (PostDoc, University of Tehran)

Students:

- **Anoshiravan Moradi** (PhD, University of Tehran)
- **Apranik Fatehi** (MSc, University of Tehran)
- **Mohammad Hossein Amiri** (MSc, University of Tehran)
- **Behnam Falahi** (BSc, University of Tehran)

The MMGPDs Recent Results

Impact of GMp12 JLab Data



M. Goharipour *et al.* [MMGPDs], Phys. Rev. D **109**, no.7, 074042 (2024).

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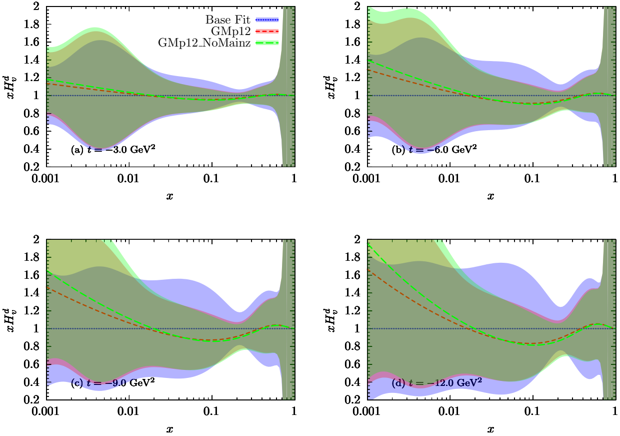

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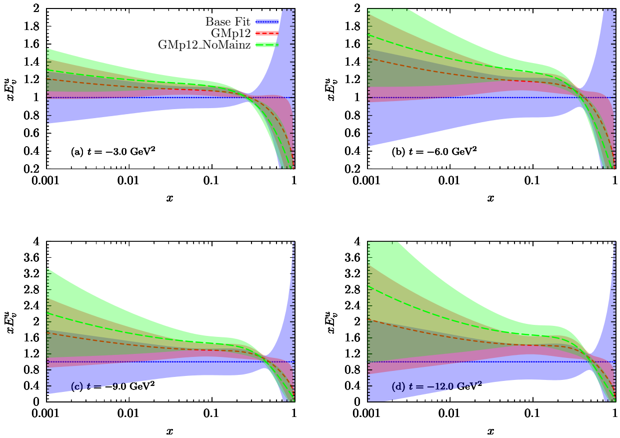

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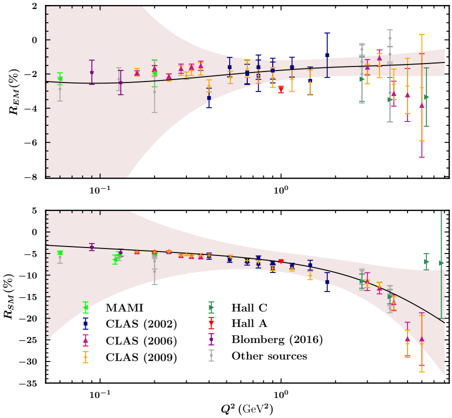
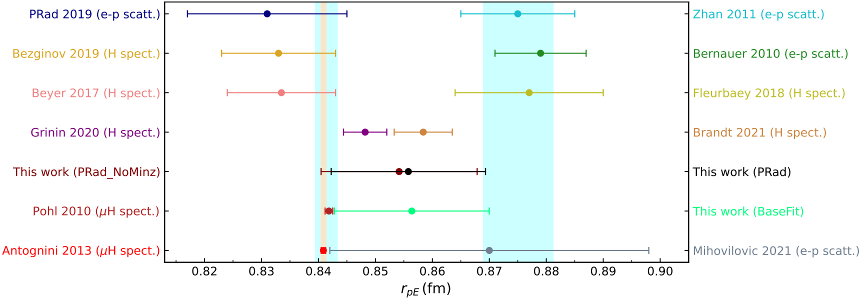


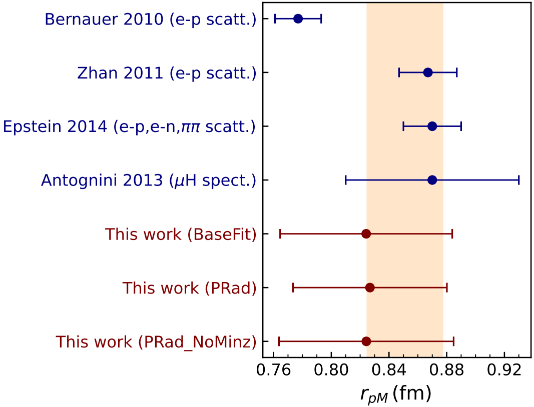
M. Goharipour *et al.* [MMGPDs], Phys. Rev. D **109**, no.7, 074042 (2024).

Charge and Magnetic Radii of the Nucleons



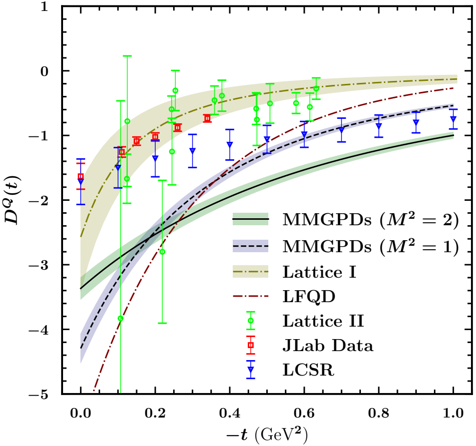
M. Goharipour *et al.* [MMGPDs], Phys. Lett. B **864**, 139423 (2025).

Charge and Magnetic Radii of the Nucleons



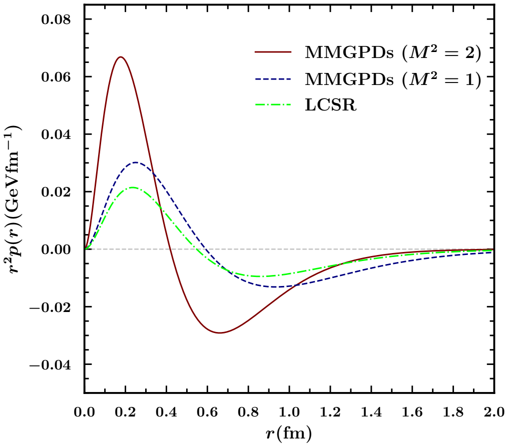
M. Goharipour *et al.* [MMGPDs], Phys. Lett. B **864**, 139423 (2025).

Gravitational Form Factor D



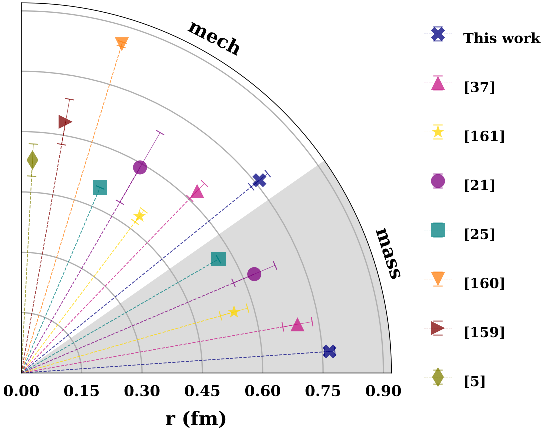
M. Goharipour *et al.* [MMGPDs], [arXiv:2501.16257 [hep-ph]].

Pressure Distribution



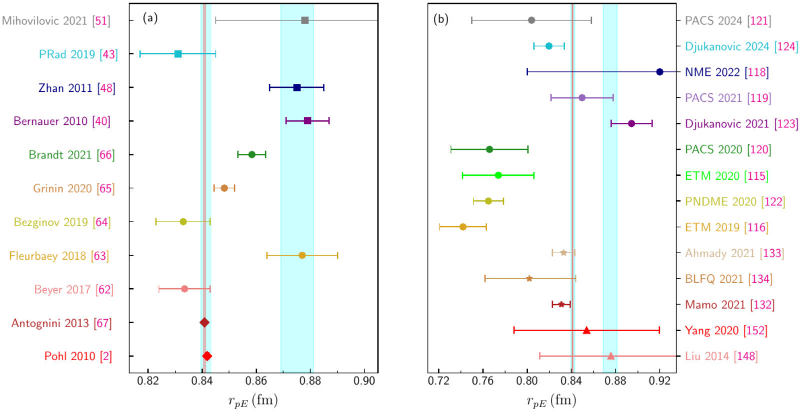
M. Goharipour *et al.* [MMGPDs], [arXiv:2501.16257 [hep-ph]].

Mechanical and Mass Radii



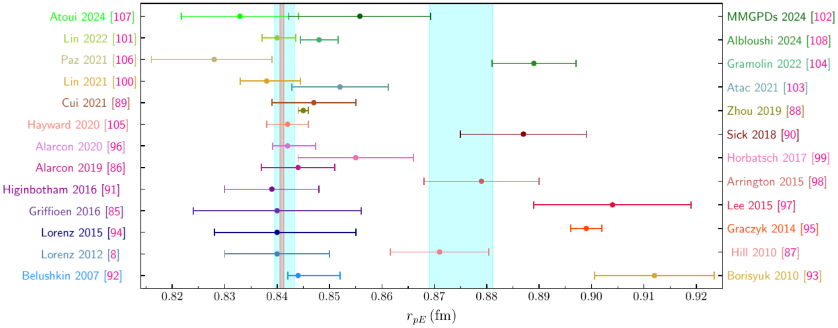
M. Goharipour *et al.* [MMGPDs], [arXiv:2501.16257 [hep-ph]].

Proton Charge Radius



M. Goharipour *et al.* [MMGPDs], [arXiv:2503.08847 [hep-ph]].

Proton Charge Radius



M. Goharipour *et al.* [MMGPDs], [arXiv:2503.08847 [hep-ph]].

The Nucleon radii

TABLE I. Summary of combined experimental, lattice QCD, theoretical, and phenomenological results for different nucleon radii. The last column provides the combination of all experimental, theoretical, and phenomenological results, offering a comprehensive overview of the nucleon's electromagnetic, weak, and mechanical properties. All values are given in femtometers (fm), except for the mean-square charge radius of the neutron $\langle r^2_{nE} \rangle$, which is given in fm^2.

Quantities	Experiment	Lattice QCD	Theory	Phenomenology	All
r_{pE}	0.857 ± 0.005	0.815 ± 0.016	0.827 ± 0.013	0.857 ± 0.002	0.847 ± 0.005
r_{pM}	0.846 ± 0.017	0.759 ± 0.040	0.786 ± 0.021	0.830 ± 0.011	0.821 ± 0.010
$\langle r^2_{nE} \rangle$	-0.115 ± 0.002	-0.075 ± 0.013	-0.076 ± 0.020	-0.110 ± 0.005	-0.100 ± 0.007
r_{nM}	0.890 ± 0.030	0.746 ± 0.078	0.786 ± 0.045	0.847 ± 0.019	0.843 ± 0.019
r_A	0.645 ± 0.101	0.573 ± 0.051	$0.533^{+0.071}_{-0.072}$	0.628 ± 0.093	0.604 ± 0.053
$r_{\rm mass}$	$---$	0.554 ± 0.029	0.724 ± 0.020	0.785 ± 0.014	0.755 ± 0.012
$r_{\rm mech}$	$---$	0.698 ± 0.035	0.736 ± 0.016	$0.753^{+0.021}_{-0.019}$	$0.744^{+0.013}_{-0.012}$

M. Goharipour *et al.* [MMGPDs], [arXiv:2503.08847 [hep-ph]].

Summary and Conclusions

- GPDs have the potential to study hadron structure in three dimensions (3D).
- GPDs are an essential ingredient of different types of elastic scattering and hard exclusive processes.
- GPDs can be determined through the global analysis of the experimental data just as for PDFs.
- The accurate determination of GPDs is required to make precise predictions for various processes in both low- and high-energy particle physics, and hence for testing the standard model and setting the stage for new physics discoveries.

The main mission of MMGPDs Collaboration is to make significant improvements in the determination of GPDs and their uncertainties. The extracted GPDs enable us to investigate various aspects of the hadron structure.


THANK
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