Freelance Holography

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Holography

The Holographic Principle is a fundamental idea in quantum gravity that states that the information in a region of space is fully encoded on its boundary. ['t Hooft (1993), Susskind (1995)]

The key motivation is the **Bekenstein-Hawking entropy** which scales with area, not volume. [Bekenstein (1973), Hawking (1975)]

This motivates 't Hooft and Susskind to propose that gravitational degrees of freedom should be encoded on a lower-dimensional surface. ['t Hooft (1993), Susskind (1995)]

A concrete example of holography is the **AdS/CFT Correspondence** [Maldacena (1997), Gubser-Klebanov-Polyakov (1998), Witten (1998)]

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AdS/CFT

GKPW dictionary: [Gubser-Klebanov-Polyakov (1998), Witten (1998)]

$$\mathcal{Z}_{\text{bdry}}[\mathcal{J}] = \mathcal{Z}_{\text{bulk}}[\mathcal{J}]$$

Boundary partition function:

$$\mathcal{Z}_{\text{bdry}}[\mathcal{J}] = \int D\phi\, e^{-\hat{S}_{\text{bdry}}}\,, \qquad \hat{S}_{\text{bdry}} := S_{\text{CFT}} + \int_{\Sigma} \sqrt{-\gamma}\, \mathcal{J}\, \mathcal{O}$$

$\phi(x)$: dynamical fields of the bdry CFT.

$\mathcal{O}(x)$: is a gauge-invariant local operator of scaling dimension Δ.

$\mathcal{J}(x)$: is the coupling of $\mathcal{O}(x)$ which has scaling dimension $d - \Delta$.

Σ: denotes the AdS bdry at $r = \infty$.

Bulk partition function:

$$\mathcal{Z}_{\text{bulk}}[\mathcal{J}] = \int DJ\, e^{-S_{\text{bulk}}} \qquad \text{with} \qquad J\big|_{\Sigma} = r_{\infty}^{d-\Delta}\mathcal{J}$$

$J(x, r)$: bulk fields.

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Gauge/Gravity Correspondence: A special AdS/CFT limit where bulk gravity is classical, and the boundary theory has a large number of degrees of freedom.
[Witten (1998), Gubser (1999)]

In the gauge/gravity level, partition functions are dominated by saddle points:

$$\hat{S}^{\mathrm{D}}_{\mathrm{bdry}}[\mathcal{J};\Sigma] = S^{\mathrm{D}}_{\mathrm{bulk}}[J;\mathcal{M}]$$

RHS: $S^{\mathrm{D}}_{\mathrm{bulk}}[J;\mathcal{M}]$ is the finite on-shell bulk action with Dirichlet b.c. on $\mathcal{M}$

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LHS: On-shell boundary action (CFT with a single-trace deformation)

$$\hat{S}^{\mathrm{D}}_{\mathrm{bdry}}[\mathcal{J};\Sigma] = S_{\mathrm{CFT}}[\phi^*] + \int_\Sigma \sqrt{-\gamma}\,\mathcal{J}\,\mathcal{O}[\phi^*]$$

where ϕ^* is given by the solution of the saddle-point equation

$$\frac{\delta S_{\mathrm{CFT}}[\phi^*]}{\delta\phi} + \int_\Sigma \sqrt{-\gamma}\,\mathcal{J}\,\frac{\delta\mathcal{O}[\phi^*]}{\delta\phi} = 0 \quad\Longrightarrow\quad \phi^* = \phi^*[\mathcal{J}]$$

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Limitations of AdS/CFT

- AdS/CFT applies to asymptotically AdS spacetimes.

- Bulk fields obey Dirichlet boundary conditions:

$$\delta J(x, r_\infty) = 0 \qquad J(x, r_\infty) = r_\infty^{d-\Delta}\mathcal{J}(x)$$

- The dual theory resides on the asymptotic timelike boundary of AdS.

Freelance Holography:

- Freelance I: Relaxing boundary conditions in gauge/gravity correspondence.

- Freelance II: Moving the AdS boundary into the bulk to construct finite-cutoff holography.

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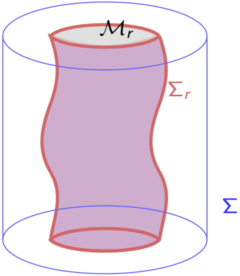


$\mathcal{M}$: A $(d+1)$-dimensional asymptotically AdS spacetime with metric $g_{\mu\nu}$.

Σ: denotes AdS boundary Located at $r=\infty$.

Σ_r: A family of codimension-one timelike hypersurfaces labeled by $r \in [0,\infty]$.

$\mathcal{M}_r$: The AdS region bounded by Σ_r.

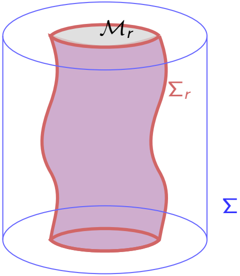
Σ_c: A cutoff surface at $r = r_c$.

$\mathcal{M}_c$: The AdS region with $r \leq r_c$, bounded by Σ_c.

Radial ADM decomposition:

$$ds^2 = N^2\, dr^2 + h_{ab}(dx^a + U^a\, dr)(dx^b + U^b\, dr)$$

Conformal induced metric: $\gamma_{ab} = r^{-2}\, h_{ab}$ with determinants $\sqrt{-h} = r^d \sqrt{-\gamma}$.



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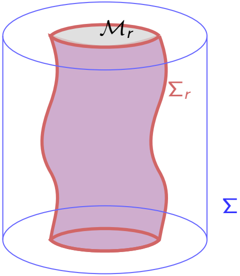

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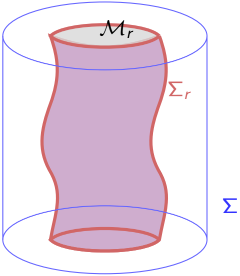

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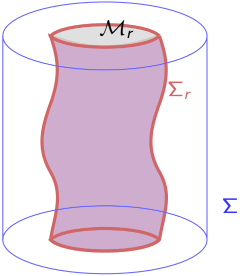

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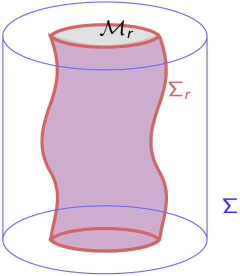

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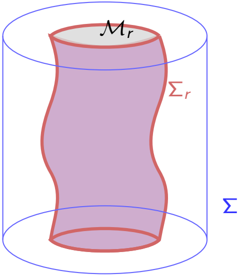

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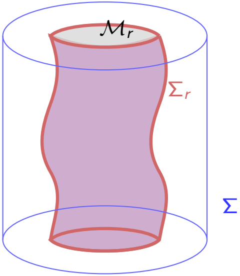

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Covariant Phase Space Formalism: [Lee, Wald (1990), Iyer, Wald (1994), Wald, Zoupas (2000)]

The action in the region $\mathcal{M}_r$ is given by

$$S^{\mathrm{W}}_{\mathrm{bulk}}[\mathcal{M}_r] = \int_{\mathcal{M}_r} \mathcal{L}^{\mathrm{W}}_{\mathrm{bulk}}[J] \qquad \mathcal{L}^{\mathrm{W}}_{\mathrm{bulk}} = \mathcal{L}^{\mathrm{D}}_{\mathrm{bulk}} + \partial_\mu W^\mu$$

where J represents dynamical fields.

Varying the action gives

$$\delta S^{\mathrm{W}}_{\mathrm{bulk}}[\mathcal{M}_r] = \int_{\mathcal{M}_r} \left(\mathrm{E}\,\delta J + \partial_\mu \Theta^\mu_{\mathrm{W}}\right)$$

where $\mathrm{E} = 0$ defines the bulk equations of motion.

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where $\mathrm{E} = 0$ defines the bulk equations of motion.

Covariant phase space (CPS) freedom:

$$\Theta^\mu_{\mathrm{W}} = \Theta^\mu_{\mathrm{D}} + \delta W^\mu + \partial_\nu Y^{\mu\nu}$$

W-Freedom as a Canonical Transformation

Role of W-freedom: modifying the bulk boundary conditions.

The Dirichlet symplectic potential has the following form

$$\Theta_{\mathrm{D}}(\Sigma_r) = \int_{\Sigma_r} \Theta^r = \int_{\Sigma_r} O\,\delta J$$

where O is the canonical conjugate momentum to J.

This symplectic potential is compatible with the boundary condition $\delta J\big|_{\Sigma_r} = 0$.

Introducing W-freedom leads to a canonical transformation:

$$\Theta_{\mathrm{W}}(\Sigma_r) = \int_{\Sigma_r} (O\,\delta J + \delta W^r[J, O]) = \int_{\Sigma_r} \tilde{O}\,\delta \tilde{J}$$

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Radial Evolution of Symplectic Potential

Start with the on-shell variation of the Lagrangian:

$$\delta \mathcal{L}^{\mathrm{W}}_{\mathrm{bulk}}\Big|_{\text{on-shell}} = \partial_\mu \Theta^\mu_{\mathrm{W}} = \partial_r \Theta^r_{\mathrm{W}} + \partial_a \Theta^a_{\mathrm{W}}$$

Now, integrate over Σ_r:

$$\frac{\mathrm{d}}{\mathrm{d}r}\Theta_{\mathrm{W}}(\Sigma_r) = \delta \int_{\Sigma_r} \mathcal{L}^{\mathrm{W}}_{\mathrm{bulk}}\Big|_{\text{on-shell}}$$

This equation governs the radial evolution of the symplectic potential.

Its integrated form is:

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Duality at a finite distance:

$$\hat{S}^{\rm D}_{\rm bdry}[\mathcal{J}_c; \Sigma_c] = S^{\rm D}_{\rm bulk}[J_c; \mathcal{M}_c]$$

RHS: $S^{\rm D}_{\rm bulk}[J_c; \mathcal{M}_c]$ is the on-shell bulk action in $\mathcal{M}_c$ with Dirichlet conditions:

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With the Dirichlet boundary condition

$$J_c \equiv J(r_c, x) = r_c^{\Delta - d}\mathcal{J}_c \quad \text{with} \quad \delta\mathcal{J}_c = 0$$

LHS: The boundary theory at the cutoff evolves via the deformation flow equation

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Proof:

We begin with the variation of the GKPW dictionary:

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At the saddle point, we obtain:

$$\underbrace{\delta S_{\text{CFT}} + \int_{\Sigma} \delta(\sqrt{-\gamma}\,\mathcal{O})\mathcal{J}}_{=0} + \int_{\Sigma} \sqrt{-\gamma}\,\mathcal{O}\,\delta\mathcal{J} = \delta S^{\text{D}}_{\text{bulk}}[J;\mathcal{M}]$$

Now, we use the following rescalings:

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$$\sqrt{-\gamma_c} = r_c^{-d} \sqrt{-h_c} \qquad \mathcal{J}_c = r_c^{d-\Delta} J_c \qquad \mathcal{O}_c = r_c^{\Delta} O_c$$

We arrive at:

$$\int_{\Sigma_c} \sqrt{-\gamma}\, \mathcal{O}\, \delta \mathcal{J} = \delta S^{\mathrm{D}}_{\mathrm{bulk}}[J_c; \mathcal{M}_c]$$

The previous equation was:

$$\int_{\Sigma} \sqrt{-h}\, O\, \delta J = \delta S^{\mathrm{D}}_{\mathrm{bulk}}[J; \mathcal{M}]$$

Now, recall the following relation:

$$\int_{\Sigma_{r_2}} \sqrt{-h}\, O\, \delta J - \int_{\Sigma_{r_1}} \sqrt{-h}\, O\, \delta J = \delta \int_{r_1}^{r_2} \mathrm{d}r \int_{\Sigma_r} \mathcal{L}^{\mathrm{D}}\Big|_{\text{on-shell}}$$

Taking $r_2 = \infty$ and $r_1 = r_c$, we obtain:

$$\begin{aligned}\int_{\Sigma_c} \sqrt{-h}\, O\, \delta J =& \delta S^{\mathrm{D}}_{\mathrm{bulk}}[J; \mathcal{M}] - \delta \int_{r_c}^{\infty} \mathrm{d}r \int_{\Sigma_r} \mathcal{L}^{\mathrm{D}}\Big|_{\text{on-shell}} \\ =& \delta \int_{0}^{r_c} \mathrm{d}r \int_{\Sigma_r} \mathcal{L}^{\mathrm{D}}\Big|_{\text{on-shell}} = \delta S^{\mathrm{D}}_{\mathrm{bulk}}[J_c; \mathcal{M}_c]\end{aligned}$$

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Now, consider a QFT on Σ_c with a single trace deformation:

$$\hat{S}_{\mathrm{bdry}}[\mathcal{J}_c; \Sigma_c] = S_{\mathrm{bdry}} + \int_{\Sigma_c} \sqrt{-\gamma}\, \mathcal{O}\, \mathcal{J}$$

The corresponding equation of motion (at the saddle point) is:

$$\frac{\delta \hat{S}_{\mathrm{bdry}}[\mathcal{J}_c; \Sigma_c]}{\delta\phi} = 0 \quad \Longrightarrow \quad \delta S_{\mathrm{bdry}} + \int_{\Sigma_c} \delta(\sqrt{-\gamma}\, \mathcal{O})\, \mathcal{J} = 0$$

Using the saddle point equation, we obtain:

$$\underbrace{\delta S_{\mathrm{bdry}} + \int_{\Sigma_c} \delta(\sqrt{-\gamma}\, \mathcal{O})\mathcal{J}}_{=0} + \int_{\Sigma_c} \sqrt{-\gamma}\, \mathcal{O}\, \delta\mathcal{J} = \delta S^{\mathrm{D}}_{\mathrm{bulk}}[J_c; \mathcal{M}_c]$$

This equation is the variation of the following equation:

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The only remaining question is the form of the boundary action $\hat{S}_{\text{bdry}}[\mathcal{J}_c; \Sigma_c]$.

To address the above question, we compare the following two equations

$$\hat{S}^{\text{D}}_{\text{bdry}}[\mathcal{J}; \Sigma] = S^{\text{D}}_{\text{bulk}}[J; \mathcal{M}]$$

$$\hat{S}_{\text{bdry}}[\mathcal{J}_c; \Sigma_c] = S^{\text{D}}_{\text{bulk}}[J_c; \mathcal{M}_c]$$

From the subtraction of these equations, we find

$$\begin{aligned}\hat{S}_{\text{bdry}}[\mathcal{J}; \Sigma] - \hat{S}_{\text{bdry}}[\mathcal{J}_c; \Sigma_c] &= S^{\text{D}}_{\text{bulk}}[J; \mathcal{M}] - S^{\text{D}}_{\text{bulk}}[J_c; \mathcal{M}_c] \\ &= \int_{r_c}^{\infty} \mathrm{d}r \int_{\Sigma_r} \mathcal{L}^{\text{D}}_{\text{bulk}}\Big|_{\text{on-shell}}\end{aligned}$$

The differential form of the above equation is as follows

$$\frac{\mathrm{d}}{\mathrm{d}r}\hat{S}^{\text{D}}_{\text{bdry}}[\mathcal{J}(r); \Sigma_r] = \mathcal{S}_{\text{D}\to\text{D}}(r) \qquad \mathcal{S}_{\text{D}\to\text{D}}(r) := \int_{\Sigma_r} \mathcal{L}^{\text{D}}_{\text{bulk}}\Big|_{\text{on-shell}}$$

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Finite cutoff duality with arbitrary boundary conditions

The duality is given by:

$$\hat{S}^{\mathrm{W}}_{\mathrm{bdry}}[\tilde{\mathcal{J}}_c; \Sigma_c] = S^{\mathrm{W}}_{\mathrm{bulk}}[\tilde{J}_c; \mathcal{M}_c]$$

RHS: is the bulk on-shell action on $\mathcal{M}_c$ compatible with W boundary conditions, $\delta \tilde{J}_c = 0$.

LHS: To determine the boundary theory, we proceed in two steps:

1. Radial Flow: We begin by deforming the theory from radius r_∞ to r_c, while maintaining the asymptotic boundary condition, say, W′.

2. Boundary Condition Change: Next, we apply a deformation to transition from boundary condition W′ to W.

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Example: Einstein's Gravity

Pure Einstein-Hilbert action:

$$S^{\mathrm{D}}_{\mathrm{bulk}} = \int_{\mathcal{M}_r} \mathcal{L}^{\mathrm{D}}_{\mathrm{bulk}} = \frac{1}{2}\int_{\mathcal{M}_r} \sqrt{-g}\left(R + \frac{d(d-1)}{\ell^2}\right) + \int_{\Sigma_r}\left(\sqrt{-h}\,K + \mathcal{L}_{\mathrm{ct}}\right)$$

The counterterms are:

$$d = 2: \quad \mathcal{L}_{\mathrm{ct}} = -\frac{1}{\ell}\sqrt{-h} \qquad\qquad d = 3: \quad \mathcal{L}_{\mathrm{ct}} = -\frac{2}{\ell}\sqrt{-h}\left(1 + \frac{\ell^2}{4}\hat{R}\right)$$

Renormalized Brown-York energy-momentum tensor (rBY-EMT):

$$\mathcal{T}^{ab} = \mathring{\mathcal{T}}^{ab} + \mathcal{T}^{ab}_{\mathrm{ct}}$$

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$$d=2: \quad \mathcal{T}^{ab}_{\mathrm{ct}} = \frac{1}{\ell}h^{ab} \qquad\qquad d=3: \quad \mathcal{T}^{ab}_{\mathrm{ct}} = \frac{2}{\ell}h^{ab} - \ell\hat{G}^{ab}$$

Deformation in Einstein's Gravity

Deformation action for Dirichlet to Dirichlet transition:

$$d = 2: \quad \mathcal{S}_{\mathrm{D}\to\mathrm{D}} = -\int_{\Sigma_c} \sqrt{-h}\, N \left[\mathcal{O}_{\mathcal{T}\bar{\mathcal{T}}} + \frac{\mathcal{T}}{\ell}\right],$$

$$d = 3: \quad \mathcal{S}_{\mathrm{D}\to\mathrm{D}} = -\int_{\Sigma_c} \sqrt{-h}\, N \Big[\mathcal{O}_{\mathcal{T}\bar{\mathcal{T}}} + \frac{\mathcal{T}}{\ell} + \ell\, \hat{R}^{ab}\, \mathcal{T}_{ab} - \frac{\ell}{4} \hat{R}\, \mathcal{T}\Big]$$

where the $\mathcal{O}_{\mathcal{T}\bar{\mathcal{T}}}$ term is the $T\bar{T}$ operator: [Taylor (2018), Hartman, Kruthoff, Shaghoulian, and Tajdini (2018)]

$$\mathcal{O}_{\mathcal{T}\bar{\mathcal{T}}} := \mathcal{T}^{ab}\, \mathcal{T}_{ab} - \frac{\mathcal{T}^2}{d-1}$$

Deformation action for Neumann to Neumann transition:

$$d = 2: \quad \mathcal{S}_{\mathrm{N}\to\mathrm{N}} = \int_{\Sigma_r} N\sqrt{-h} \left[-\frac{1}{2}\mathcal{O}_{\mathcal{T}\bar{\mathcal{T}}} - \frac{\mathcal{T}}{\ell}\right]$$

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Simple Argument for $T\bar{T}$ as Radial Deformation

A simple argument for the emergence of $T\bar{T}$:

Consider the variation of the Einstein-Hilbert action:

$$\delta_\xi S^{D}_{\text{bulk}}[\mathcal{M}_r]\Big|_{\text{on-shell}} = -\frac{1}{2}\int_{\Sigma_r}\sqrt{-h}\,\mathcal{T}^{ab}\,\delta_\xi h_{ab}$$

Radial evolution of the bulk on-shell action arises by choosing $\xi = \partial_r$:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r}S^{D}_{\text{bulk}}[\mathcal{M}_r]\Big|_{\text{on-shell}} &= -\frac{1}{2}\int_{\Sigma_r}\sqrt{-h}\,\mathcal{T}^{ab}\,\partial_r h_{ab} \\ &= -\int_{\Sigma_r}\sqrt{-h}\,N\,\mathcal{T}^{ab}\,K_{ab} \\ &= -\int_{\Sigma_r}\sqrt{-h}\,N\left(\mathcal{T}^{ab}\mathring{\mathcal{T}}_{ab} - \frac{\mathring{\mathcal{T}}\mathcal{T}}{d-1}\right)\end{aligned}$$

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Two Equivalent Interpretations

There are two interpretations for $T\bar{T}$-like deformations:

1. Generator of radial deformation. [McGough, Mezei, and Verlinde (2016)]
2. Modification of asymptotic boundary conditions. [Guica and Monten (2019)]

The following key equation shows the equivalence of these two interpretations:

$$\int_{\Sigma_c} \tilde{O}_c \, \delta \tilde{J}_c = \int_{\Sigma} \tilde{O} \, \delta \tilde{J} - \delta \int_{r_c}^{\infty} \mathrm{d}r \, \mathcal{S}_{\mathrm{W} \to \mathrm{W}}(r)$$

For the second interpretation, we have:

$$\tilde{J}_c = \tilde{J}_c[\tilde{J}, \tilde{O}] \quad \text{and} \quad \tilde{O}_c = \tilde{O}_c[\tilde{J}, \tilde{O}]$$

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Summary and Key Takeaways

- We extended the gauge/gravity correspondence in two key directions:
    1. Considering arbitrary boundary conditions.
    2. Moving the AdS boundary to finite distances.
- Radial deformation of the fluid/gravity correspondence.
- Shifting the AdS boundary to the black hole horizon and beyond [Ali, Almheiri, and Lin (2025)].
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