Modified gravity & Observational tests

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Topics :

- Introduction
- New Modified gravity model
- Field equations and special solution
- Equivalence of modified gravity with scalar field tensor theories
- Palatini formalism and dynamics of universe
- Cosmological observations and constraints
- Summary

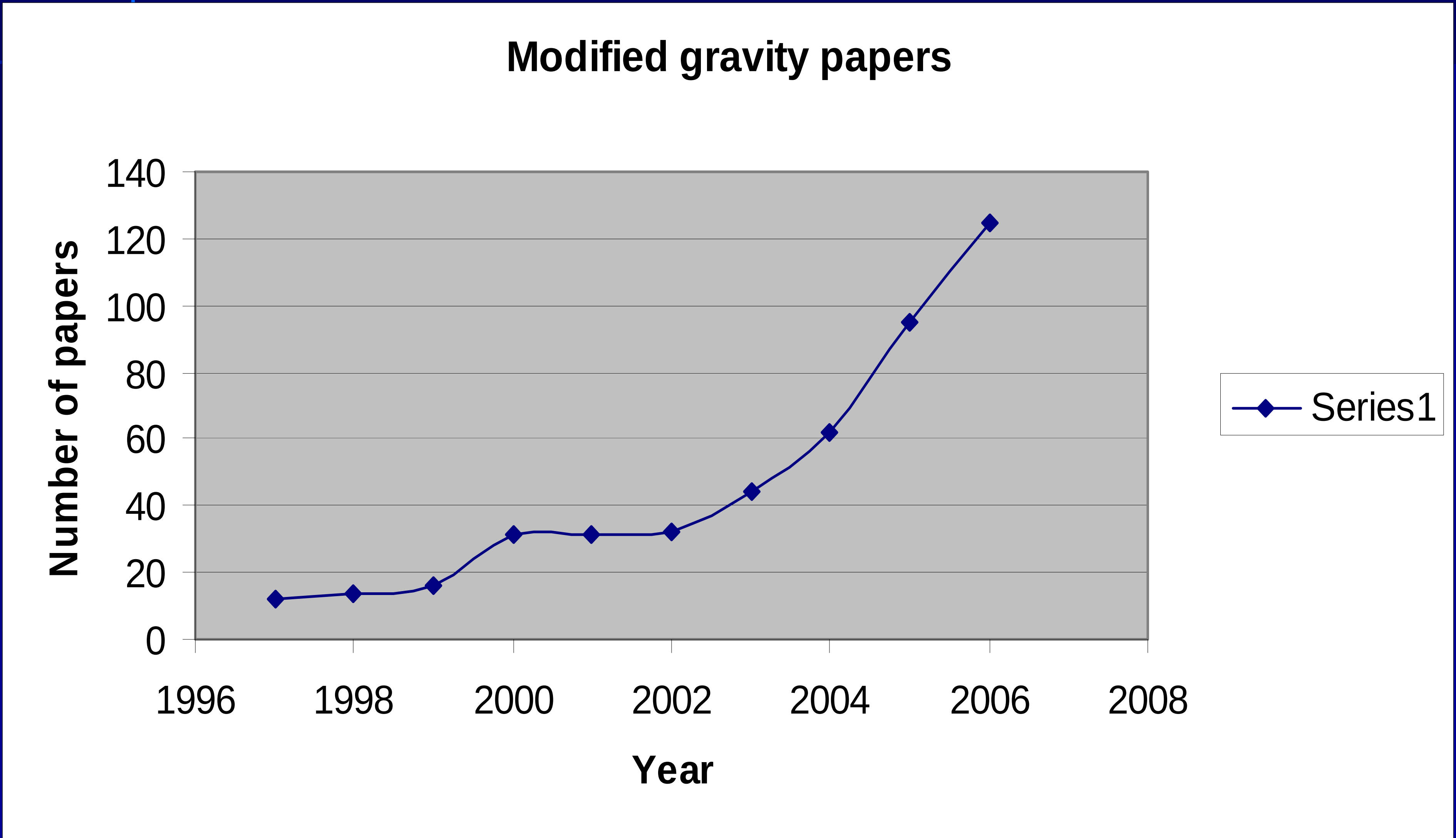
DARK ENERGY AND MODELS!

arXiv:hep-th/0603057

Dynamics of dark energy

Edmund J. Copeland,[1] M. Sami,[2,3] and Shinji Tsujikawa[4]

Increasing popularity of Modified gravity in last decade



Generalized form of the action

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\,d^4x + S_{matter}$$

If we choose the f(R)=R we will find the familiar Einstein-Hilbert action.

Why a Modified gravity

- Provide gravitational alternative for Dark Energy and Dark Matter. Cosmic speed up is explained where sub-dominant term (like 1/R) may become essential at small curvatures.
- Maybe modified gravity can be used for unification of early-universe inflation and late time acceleration.
- Some actions of modified gravity can be predicted by other theories.
- Maybe be a candidate for unified explanation of Dark Matter and Dark Energy.

- No need for introducing exotic matter
- Naturally describe the transition of the deceleration to acceleration in universe history.
- There is no Coincidence problem.
- Useful in High energy physics.
- Brane Cosmology predict some new actions like DGP-model

What is a good Modified gravity action?

- Pass the solar system test
- Give the standard history of universe
- Be consistent with Cosmological observations
- Have no gravitational instabilities
- Explain something more than standard model

Field equations

- Variation of the action with respect to the metric results in the field equations:

$$f^{'}(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-(\nabla_{\mu}\nabla_{\nu}-g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha})f^{'}(R)=8\pi GT_{\mu\nu}$$

Differential equations of forth order results in complicated Friedman equations.

Most studied modified gravities

- Chiba $$f(R) = R + \alpha \ln R$$
- Caroll et al. $$f(R) = R - \frac{\mu^4}{R}$$
- Barrow $$f(R) = R^{1+\delta}$$

Each new modified gravity has free parameter, which must be constraint by observations.

New Modified gravity

Proposed: Shiekh Jabbari

Shant Baghram , Marzieh Fahrang ,Sohrab Rahvar

Phys.Rev. D75 (2007) 044024

$$f(R)=\sqrt{R^2-R_0^2}$$

R_0 is free parameter interpreted as intrinsic curvature of space-time

Trace of field equation for this f(R) model in metric formalism is:

$$\frac{R^2}{\sqrt{R^2 - R_0^2}} - 2\sqrt{R^2 - R_0^2} + \frac{R^2}{\left(R^2 - R_0^2\right)^{\frac{3}{2}}}\left[3\nabla_\alpha \nabla^\alpha R - \frac{9R}{R^2 - R_0^2}\nabla_\alpha R \nabla^\alpha R\right]$$

$$= kT$$

Special solution

- Maximally symmetric space-time, with

$$R = \sqrt{2}\,R_0$$

which can happen at the ultimate stage of the expansion of the universe with vanishing energy momentum tensor.

We find the Schwarz shield de-Sitter space , in which our free parameter play the role of cosmological constant.

$$ds^2 = -(1 - \frac{2GM}{c^2 r} + \frac{\sqrt{2}}{12} R_0 r^2) c^2 dt^2 + (1 - \frac{2GM}{c^2 r} + \frac{\sqrt{2}}{12} R_0 r^2)^{-1} dr^2$$
$$+ r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

Constraint On R_0 In solar system

- Solar System test

Perihelion of Mercury as a test for Modified gravity:

$$R_0 < 10^{-55} cm^{-2}$$

Comparing to the horizon size of universe

$$R_0^{\frac{1}{2}} \approx H_0$$

Cosmological solutions

- *Radiation dominant era*

$$P = \frac{1}{3}\rho$$

$$R = 6\dot{H} + 12H^2$$

- *FRW metric → Modified Friedman equations*

$$a(t) \propto t^{\frac{1}{2}} + \sigma t^{\frac{5}{2}}$$

$$\sigma << 1$$

- *Does not alter dynamics of early universe*

Cosmological solutions and the problem of Matter Domination era

- Matter Dominant era

Complicated equations ,most modified gravity models can not predict proper matter dominant era.

arXiv:gr-qc/0612180

Luca Amendola, **Radouane Gannouji**, **David Polarski**, **Shinji Tsujikawa**

Categories modified gravities in four groups

Dynamics of universe:

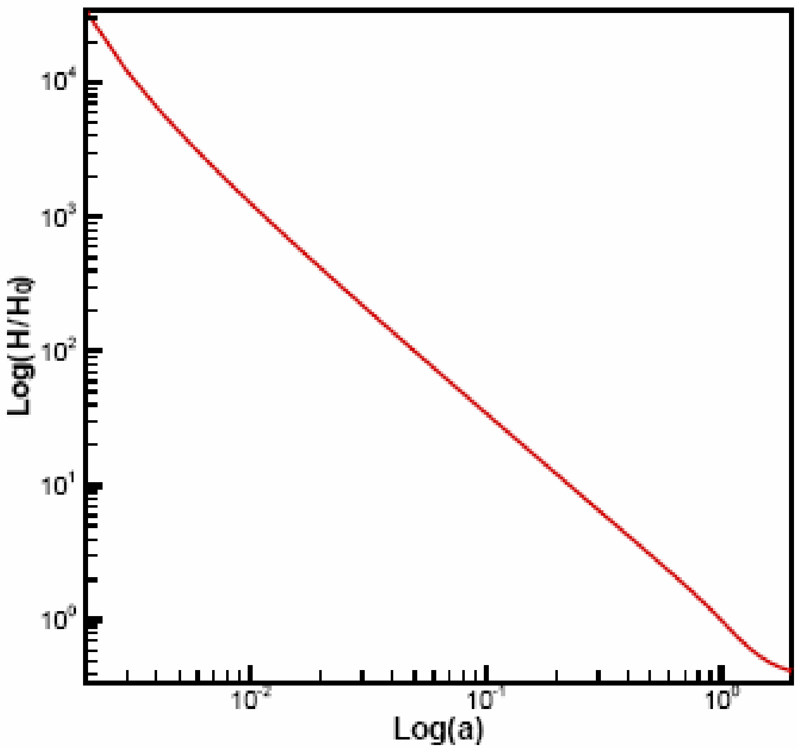


FIG. 2: Numerical solution of Equations (25) and (26). Here we omit the Ricci scalar between the two equations and obtain a direct relation between the Hubble parameter and scale factor in the logarithmic scale.

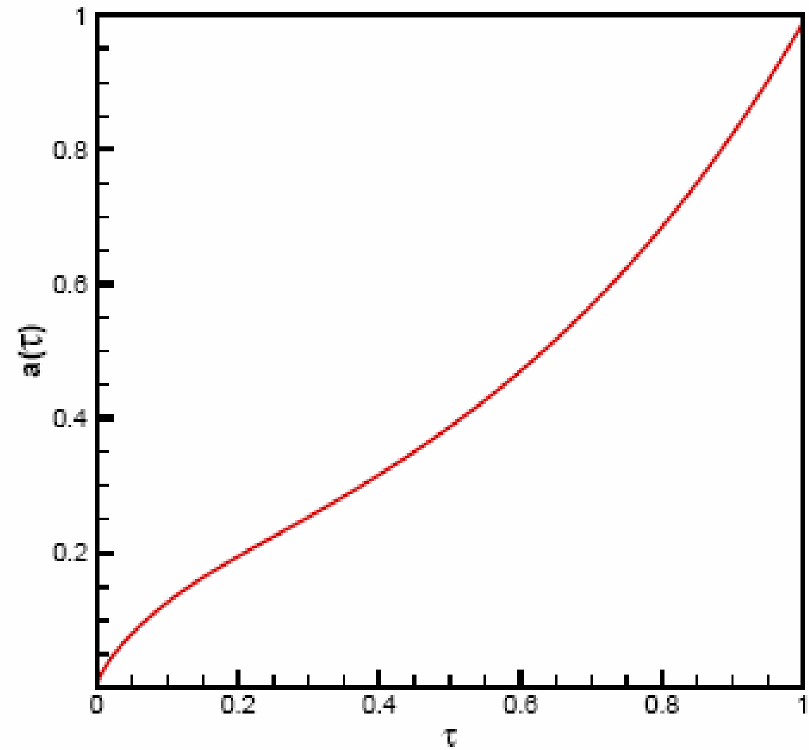


FIG. 3: Numerical solution of scale factor in terms of cosmic time, normalized to the $R_0^{-1/2}$.

Nonminimally coupled scalar tensor gravity

- Conformal transformation

Suitable conformal transformation on metric

NLG Lagrangian from JORDAN frame

To EINSTEIN frame ,scalar tensor gravity.

$$S = \frac{1}{2k}\int f(R)\sqrt{-g}\,d^4x \longrightarrow$$

$$S = \frac{1}{2k}\int d^4x\sqrt{-g}\,[R - \frac{3}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)]$$

NLG→STG

$$S = \frac{1}{2k}\int d^4x\sqrt{-g}\,(f'(A)(R-A)+f(A))$$

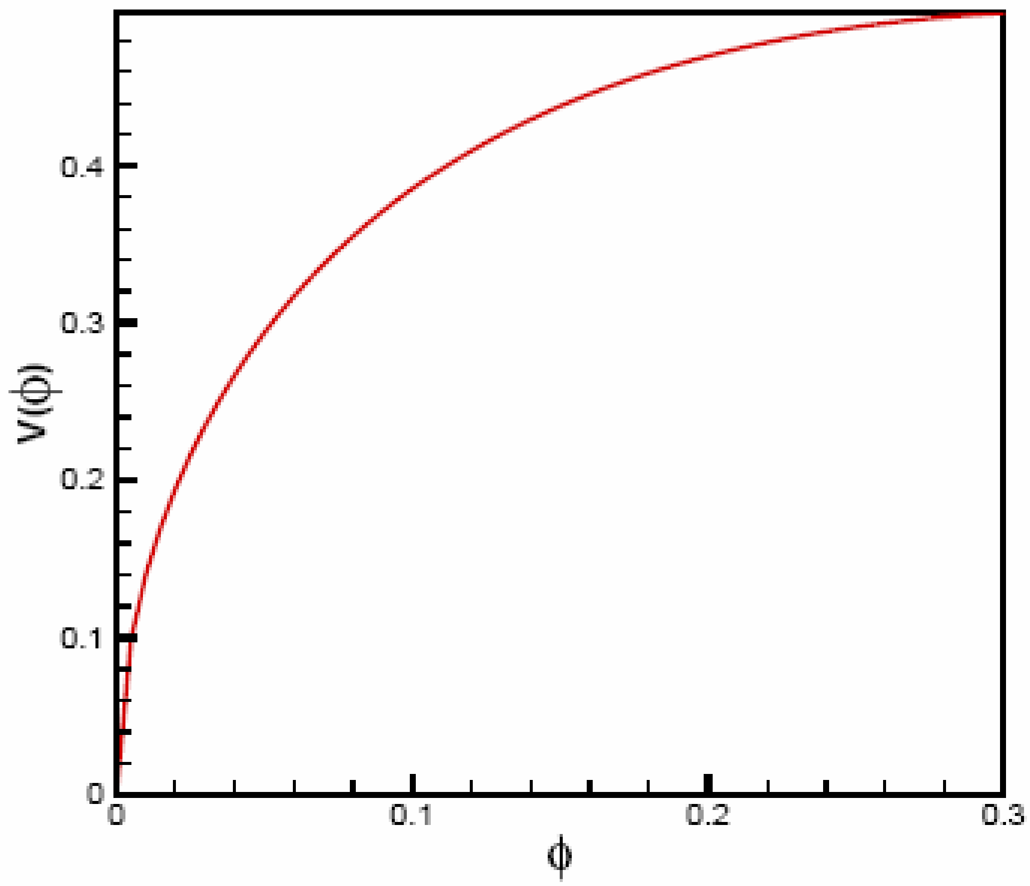
$$g_{\mu\nu} \longrightarrow \exp(\varphi)g_{\mu\nu}$$

$$\varphi = -\ln f'(A)$$

$$S = \frac{1}{2k}\int d^4x\sqrt{-g}\,[R-\frac{3}{2}(\frac{f''(A)}{f'(A)})g^{\mu\nu}\partial_\mu A\partial^\nu A$$

$$-\frac{A}{f'(A)}+\frac{f(A)}{f'(A)^2}]$$

Corresponding scalar potential



New Approach PALATINI FORMALISM

Connections $\Gamma^{\alpha}_{\mu\nu}$ are considered independent variables.

Variation by metric :

$$f'(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} = kT_{\mu\nu}$$

Variation by connections :

$$\nabla_{\lambda}(\sqrt{-g}\, f'(R) g_{\mu\nu}) = 0$$

Conformal transformations:

$$h_{\mu\nu} = f'(R) g_{\mu\nu}$$

Jordan frame –Einstein frame

Advantage: Second order differential equations

- We choose FRW metric for universe contained with perfect fluid and using the trace of field equation in Palatini formalism we obtained generalized Friedman equation as:

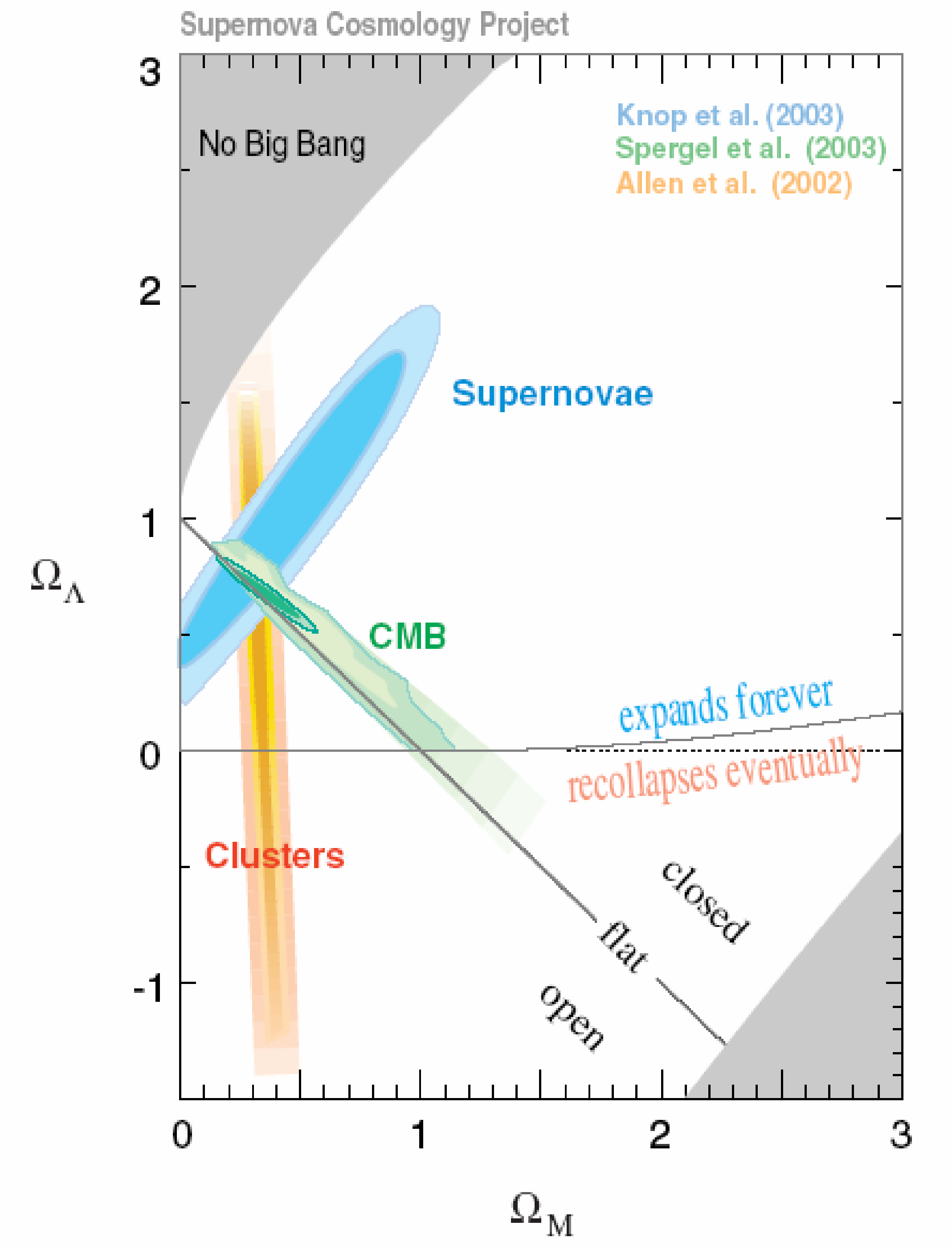
$$RF - 2f = kT$$

$$T = -\rho + 3p$$

$$(H + \frac{1}{2}\frac{\dot{F}}{F})^2 = \frac{1}{6}\frac{\kappa(\rho + 3p)}{F} + \frac{1}{6}\frac{F}{F'}$$

Sadegh Movahed , Shant Baghram, Sohrab Rahvar

arXiv:0705.0889v1 [asrto-ph]



Observational constraints from background evolution

- SNIa data
- Location of Baryonic acoustic oscillation peak
- CMB shift parameter

We need to know the dynamic of universe

$$H=H(a)$$

- Also with the use of continuity equation we can find the time derivative of Ricci Scalar which help us to determine Hubble parameter and scalar factor as a function of Ricci scalar

$$\dot{R} = 3H \frac{(1-3p^{'}(\rho))(\rho+3P)}{RF^{'}-F}$$

$$H^{2} = \frac{1}{6(1-3\omega)F} \frac{3(1+\omega)f-(1+3\omega)RF}{[1+\frac{3}{2}(1+\omega)\frac{F(RF-2f)}{F-RF^{'}}]^{2}}$$

$$a = \left[\frac{1}{\kappa\rho_{0}(1-3\omega)}(2f-RF)\right]^{\frac{-1}{3(1+\omega)}}$$

- We redefine the action as:

$$f(R) = H_0^2 \sqrt{X^2 - X_0^2}$$

$$X \equiv \frac{R}{H_o^2}; X_0 \equiv \frac{R_0}{H_0^2}$$

Define the dimensionless functions as:

$$f(R) = H_0^2 F(X)$$

$$f'(R) = F'(X)$$

$$f''(R) = \frac{F''(X)}{H_0^2}$$

$$H(X) = \frac{H(R)}{H_0}$$

SNIa Constraint

- Apparent luminosity of SNIa is dependent on luminosity distance and in consequence to the dynamic of universe.

$$D_l(z) = (1+z)\int_0^z \frac{dz}{H(z)}$$

$$D_l(z) = \frac{1}{3}(\Omega_m H_0^2)^{\frac{1}{2}}(Rf' - 2f)^{\frac{1}{3}}\int_{R_0}^{R} \frac{Rf'' - f'}{(Rf' - 2f)^{\frac{2}{3}}}\frac{dR}{H(R)}$$

$$\mu = 5\log D_L(z; X_0) + \bar{M}$$

- By least square fitting and using the likelihood statical analyzes we find

$$X_0 = 6.207^{+0.230}_{-0.147}$$

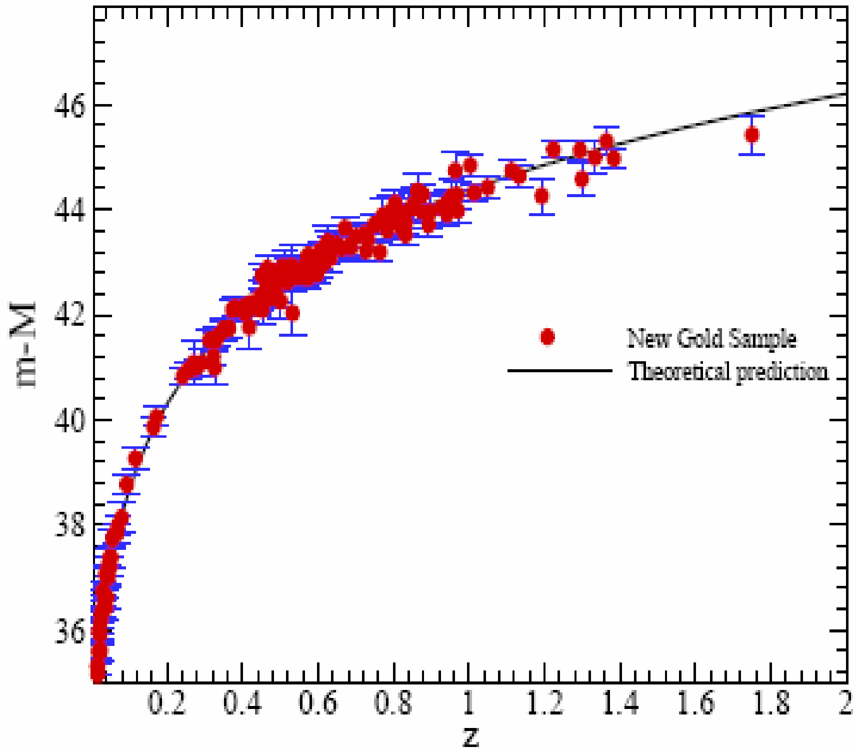


FIG. 1. Distance modulus of the SNIa new Gold sample in terms of redshift. Solid line shows the best fit values with the corresponding parameters of $h = 0.63$, $\Omega_m = 0.276^{+0.376}_{-0.240}$, $X_0 = 6.207^{+0.230}_{-0.147}$ in 1σ level of confidence with $\chi^2_{min}/N_{d.o.f} = 0.912$ for $f(R)$ model.

CMBR Shift parameter

- Instead of peak location of power spectrum of CMB ,we can use model independent Shift parameter as:

$$R \propto \frac{l_1^{flat}}{l_1}$$

l_1^{flat} Corresponds to flat pure CDM model and its easy to show:

CMBR SHIFT PARAMETER

$$R = \sqrt{\Omega_m H_0^2} \int_0^{z_{dec}} \frac{dz}{H(z)}$$

$$R = \frac{1}{3^{\frac{4}{3}}} (\Omega_m H_0^2)^{\frac{1}{6}} \int_{R_0}^{R_{dec}} \frac{Rf'' - f'}{(Rf' - 2f)^{\frac{2}{3}}} \frac{dR}{H(R)}$$

Baryonic Oscillation

Correlation function of CMB → peak

Which is the expanding spherical wave of baryonic perturbation with comoving scale of 150Mpc.

A dimensionless parameter can be defined as:

Baryon Oscillation

- Angular diameter distance at survey

$$A = \sqrt{\Omega_m} E(z_1)^{\frac{-1}{3}} [\frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)}]^{\frac{2}{3}}$$

$$E(z) = \frac{H(z)}{H_0}$$

Constraint by large scale structure

- Obtaining the growth index $f = \frac{d \ln \delta}{d \ln a}$ of structures and compare it with the 2dF Galaxy red shift survey will put constraint on the free parameters of model

- Perturbation on sub horizon scales(linear)

$$\ddot{\delta} + 2H(a)\dot{\delta} - 4\pi G\rho\delta = 0$$

- Numerical solution of equation

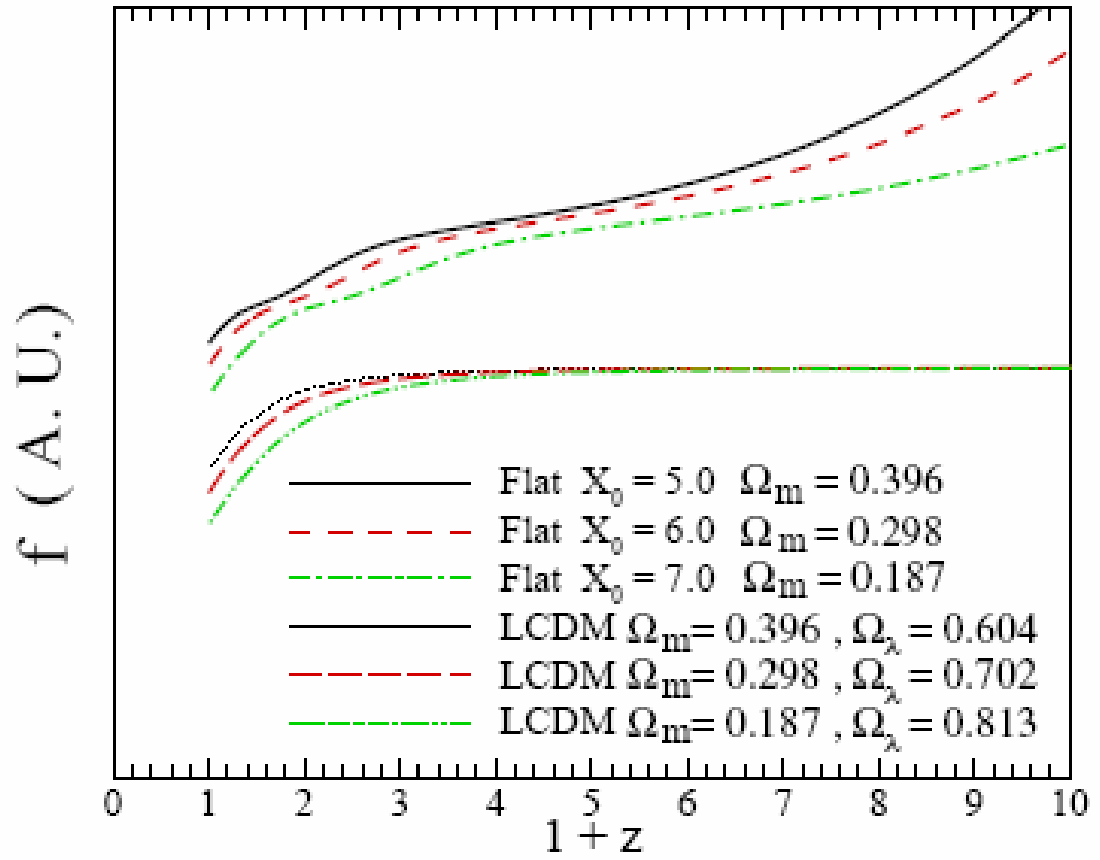


FIG. 4. Growth index versus redshift for different values of X_0. To make sense we plot f for ΛCDM with the same Ω_m.

$$\frac{df}{d\ln a} = \frac{3\Omega_m}{2aH(X)} - f^2 - f\left[2 + \frac{aH'(X)}{H(X)}\frac{dX}{da}\right]$$

$$X_0 = 6.129^{+0.167}_{-0.177}$$

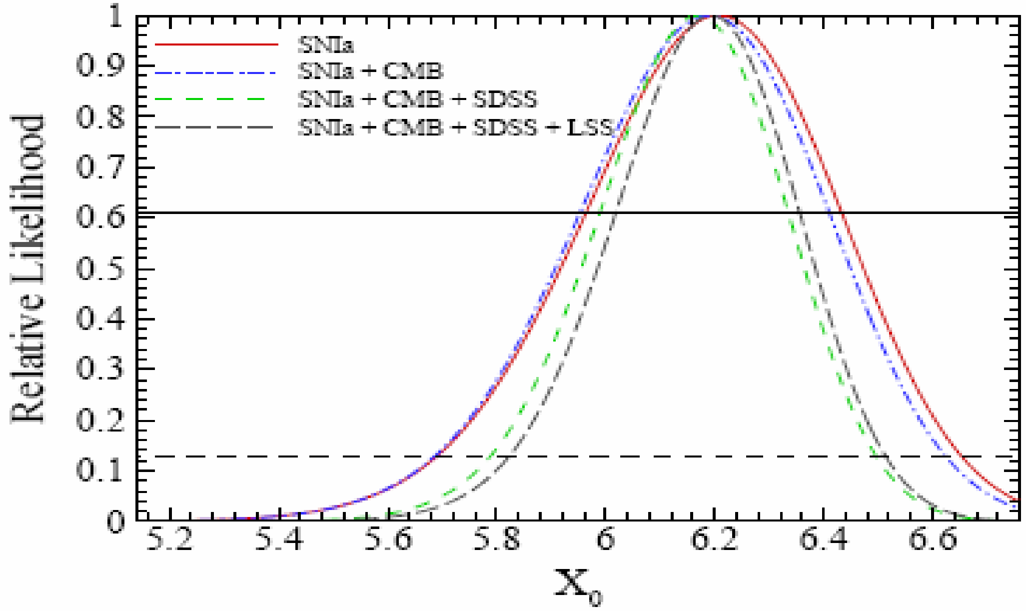


FIG. 5. Marginalized likelihood functions of $f(R)$ modified gravity model free parameter, X_0. The solid line corresponds to the likelihood function of fitting the model with SNIa data (new Gold sample), the dashdot line with the joint SNIa+CMB+SDSS data and dashed line corresponds to SNIa+CMB+SDSS+LSS. The intersections of the curves with the horizontal solid and dashed lines give the bounds with 1σ and 2σ level of confidence respectively.

Summary

- Cosmological observations are consistent with the solar system test for this model.
- Investigation should be continued for good modified gravities

Special thanks