Cosmic Inversion for Tensor Modes

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Motivations

- Physical information of early universe (inflation) can be extracted from primordial perturbation.

- Until now, there are no particle accelerator can be used to prove inflation.

⇒ Determination of the shape of primordial spectrum is one of the most important issues in cosmology.

Tools and Methods

- CMB can be used to study primordial perturbation because the primary sources of CMB anisotropies are primordial perturbations.

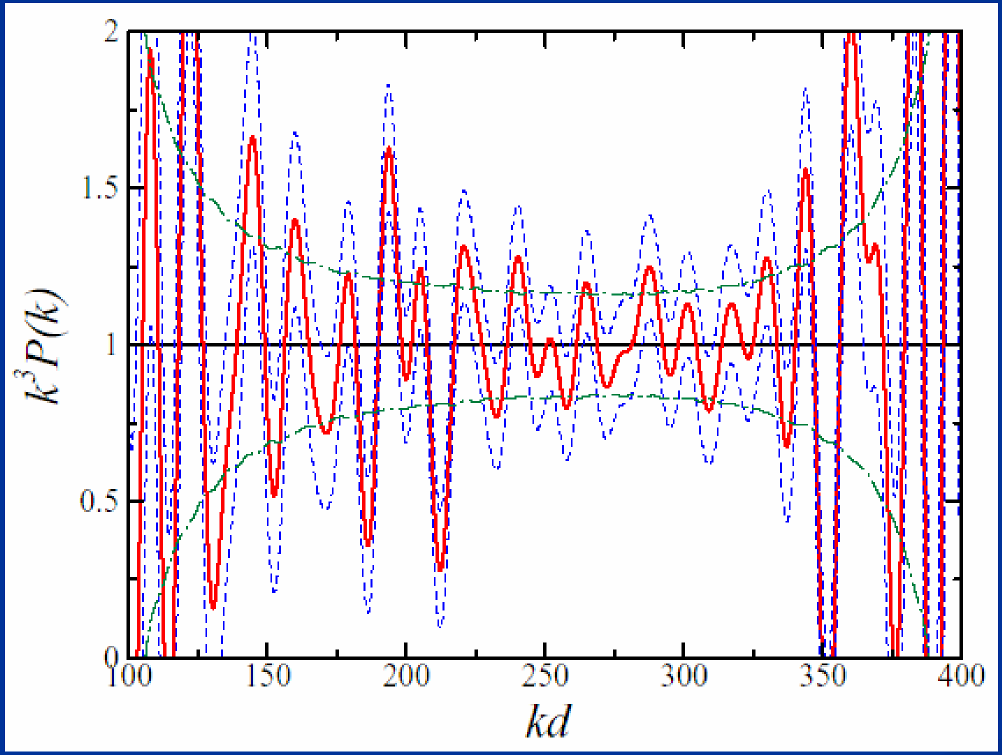
- In general, there two methods to study the shape of primordial perturbation:
  - "*Standard*" (with assumption on shape of primordial perturbations)
  - "*Inverse Method*" (no assumption on shape of primordial perturbations). Example: **Cosmic Inversion**

Cosmic Inversion

- 2002: Concept of Cosmic Inversion Method (Matsumiya et al)
- 2003: Cosmic Inversion Method for Density Perturbations from CMB Anisotropies (Matsumiya et al.)
- 2004: Reconstructing the Primordial Spectrum from WMAP Data by the Cosmic Inversion Method (Kogo et al.)
- 2004: Cosmic Inversion Method for Density Perturbations from CMB Anisotropies and its Polarization (Kogo et al.)
- 2005: Constraining Cosmological Parameters by the Cosmic Inversion Method (Kogo et al.)

Cosmic Inversion Method

- No assumption on shape of primordial perturbations
- Applied on density perturbations



- Resolution:

  $\Delta k \approx 3.7 \times 10^{-4}$ Mpc^{-1} ($\Delta l \approx 5$)
- Predict some possible deviation from *scale-invariant* around

  $kd \approx 200$ ($k \approx 1.5 \times 10^{-2}$ Mpc^{-1})

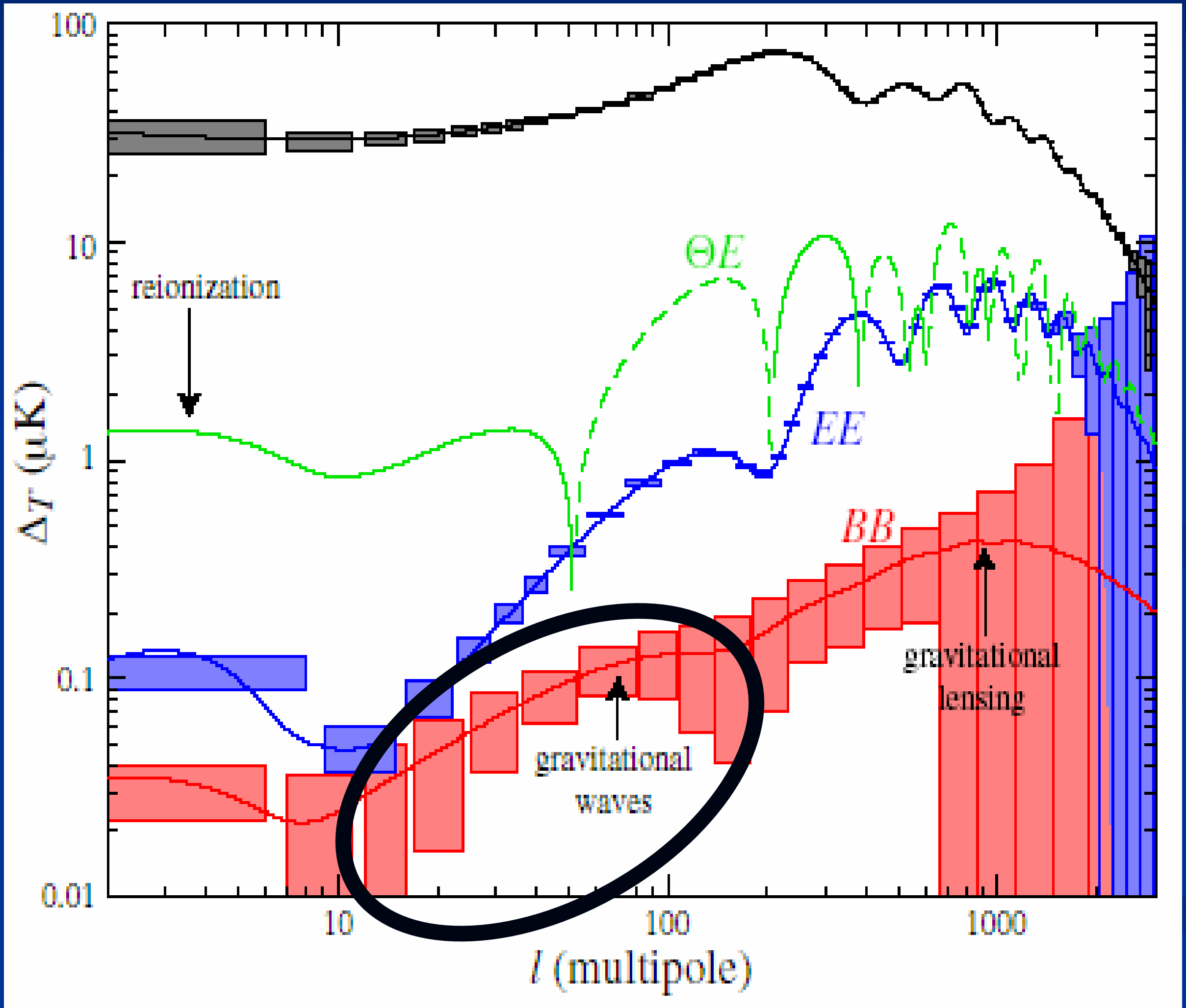
  and

  $kd \approx 350$ ($k \approx 2.6 \times 10^{-2}$ Mpc^{-1})

Kogo *et al.*, 2004

How if this method applied to tensor perturbations?

Prediction of CMB Anisotropy and Polarization from Planck



Hu dan Dodelson, (2002)

Assumptions

- Tensor perturbations can be detected from CMB anisotropies
- Tensor perturbations are Gaussian and adiabatic
- Curvature of the universe is flat.

Integral Solution for Boltzmann Equation

$$\frac{\Theta_\ell^t(\eta_0,k)}{2\ell+1} = \int_0^{\eta_0} d\eta e^{-\tau}\left[\dot{\tau}\Pi_2^t - \dot{H}\right]\sqrt{\frac{3}{8}\frac{(\ell+2)}{(\ell-2)}\frac{j_\ell(k\Delta\eta)}{(k\Delta\eta)^2}}$$

$$\frac{E_\ell^t(\eta_0,k)}{2\ell+1} = -\frac{\sqrt{6}}{4}\int_0^{\eta_0} d\eta v\Pi_2^t\left[-j_l(k\Delta\eta) + j_l''(k\Delta\eta) + \frac{2j_l(k\Delta\eta)}{(k\Delta\eta)^2} + \frac{4j_l'(k\Delta\eta)}{k\Delta\eta}\right]$$

$$\frac{B_\ell^t(\eta_0,k)}{2\ell+1} = -\sqrt{\frac{3}{2}}\int_0^{\eta_0} d\eta v\Pi_2^t\left[2j_l'(k\Delta\eta) + \frac{4j_l(k\Delta\eta)}{k\Delta\eta}\right]$$

Rewrite Solutions for Polarization

$$\frac{E_\ell^t(\eta_0,k)}{2\ell+1} = -\sqrt{\frac{3}{2}}\int_0^{\eta_0} d\eta \left[\Pi_2^t\left(-\upsilon + \frac{6\upsilon}{(k\Delta\eta)^2} + \frac{4\dot{\upsilon}}{k^2\Delta\eta} + \frac{\ddot{\upsilon}}{k^2}\right)\right.$$

$$\left. + \dot{\Pi}_2^t\left(\frac{2\dot{\upsilon}}{k^2} + \frac{4\upsilon}{k^2\Delta\eta}\right) + \ddot{\Pi}_2^t\frac{\upsilon}{k^2}\right] j_l(k\Delta\eta)$$

$$\frac{B_\ell^t(\eta_0,k)}{2\ell+1} = -\sqrt{\frac{3}{2}}\int_0^{\eta_0} d\eta \left[\frac{\dot{\upsilon}\Pi_2^t + \dot{\Pi}_2^t\upsilon}{k} + 2\frac{\upsilon\Pi_2^t}{k\Delta\eta}\right] j_l(k\Delta\eta)$$

Definition

$$\sqrt{\frac{3}{8}}\int_{*start}^{*end} d\eta \left[\frac{v\Pi_2^t - e^{-\tau}}{k\Delta\eta}\right] \equiv x(k)h(0,\mathbf{k})$$

$$-\sqrt{\frac{3}{2}}\int_{*start}^{*end} d\eta \left[\Pi_2^t\left(-v+\frac{6v}{(k\Delta\eta)^2}+\frac{4\dot{v}}{k^2\Delta\eta}+\frac{\ddot{v}}{k^2}\right)+\dot{\Pi}_2^t\left(\frac{2\dot{v}}{k^2}+\frac{4v}{k^2\Delta\eta}\right)+\ddot{\Pi}_2^t\frac{v}{k^2}\right] \equiv y(k)h(0,\mathbf{k})$$

$$-\sqrt{\frac{3}{2}}\int_{*start}^{*end} d\eta \left[\frac{\dot{v}\Pi_2^t + \dot{\Pi}_2^t v}{k} + 2\frac{v\Pi_2^t}{k\Delta\eta}\right] \equiv z(k)h(0,\mathbf{k})$$

$$\frac{\Theta_\ell^{t,app}(\eta_0,k)}{2\ell+1} = \sqrt{\frac{(\ell+2)!}{(\ell-2)!}}x(k)h(0,\mathbf{k})j_l(k\Delta\eta)$$

$$\frac{E_\ell^{t,app}(\eta_0,k)}{2\ell+1} = y(k)h(0,\mathbf{k})j_l(k\Delta\eta)$$

$$\frac{B_\ell^{t,app}(\eta_0,k)}{2\ell+1} = z(k)h(0,\mathbf{k})j_l(k\Delta\eta)$$

Angular Power Spectrum (Approximation)

$$C_\ell^{\theta\theta,app} = \int_0^\infty \frac{dk}{k} k^3 \frac{(\ell+2)!}{(\ell-2)!} x^2(k) P_t(\mathbf{k}) j_\ell^2 k\Delta\eta$$

$$C_\ell^{EE,app} = \int_0^\infty \frac{dk}{k} k^3 y^2(k) P_t(\mathbf{k}) j_\ell^2 k\Delta\eta$$

$$C_\ell^{BB,app} = \int_0^\infty \frac{dk}{k} k^3 z^2(k) P_t(\mathbf{k}) j_\ell^2 k\Delta\eta$$

$$C_\ell^{\theta E,app} = \int_0^\infty \frac{dk}{k} k^3 \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} x(k) y(k) P_t(\mathbf{k}) j_\ell^2 k\Delta\eta$$

Two Point Correlation Function

$$\tilde{C}^{\theta\theta}(\theta) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \frac{(\ell-2)!}{(\ell+2)!} C_\ell^{\theta\theta,app} P_\ell(\cos\theta)$$

$$\tilde{C}^{\theta E}(\theta) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \sqrt{\frac{(\ell-2)!}{(\ell+2)!}} C_\ell^{\theta E,app} P_\ell(\cos\theta)$$

Used Only in Cosmic Inversion

$$C^{EE}(\theta) \equiv \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} C_\ell^{EE,app} P_\ell(\cos\theta)$$

$$C^{BB}(\theta) \equiv \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} C_\ell^{BB,app} P_\ell(\cos\theta)$$

Two Point Correlation Function (Approximation)

$$\tilde{C}^{\theta\theta,app} = \frac{1}{2\pi^2}\int_0^\infty dk k^2 x^2(k) P_t(\mathbf{k}) \frac{\sin kr}{kr}$$

$$\tilde{C}^{\theta E,app} = \frac{1}{2\pi^2}\int_0^\infty dk k^2 x(k) y(k) P_t(\mathbf{k}) \frac{\sin kr}{kr}$$

$$C^{EE,app} = \frac{1}{2\pi^2}\int_0^\infty dk k^2 y^2(k) P_t(\mathbf{k}) \frac{\sin kr}{kr}$$

$$C^{BB,app} = \frac{1}{2\pi^2}\int_0^\infty dk k^2 z^2(k) P_t(\mathbf{k}) \frac{\sin kr}{kr}$$

$$r = 2d \sin\frac{\theta}{2}$$

General TPCF (Approximation)

$$C^{X\bar{X},app} = \frac{1}{2\pi^2}\int_0^\infty dk k^2 f_X(k) f_{\bar{X}}(k) P_t(\mathbf{k}) \frac{\sin kr}{kr}$$

Applied the Fourier Sine Formula to TPCF (Approximation):

$$P_t(\mathbf{k}) = \frac{4\pi}{f_X(k) f_{\bar{X}}(k) k}\int_0^\infty dr r C_t^{X\bar{X},app} \sin kr$$

Key Equations of Cosmic Inversion for Tensor Modes

$$P_t(\mathbf{k}) = \frac{4\pi}{x^2(k)k}\int_0^\infty dr\, r\tilde{C}_t^{\theta\theta,app}\sin kr$$

$$P_t(\mathbf{k}) = \frac{4\pi}{x(k)y(k)k}\int_0^\infty dr\, r\tilde{C}_t^{\theta E,app}\sin kr$$

$$P_t(\mathbf{k}) = \frac{4\pi}{y^2(k)k}\int_0^\infty dr\, rC_t^{EE,app}\sin kr$$

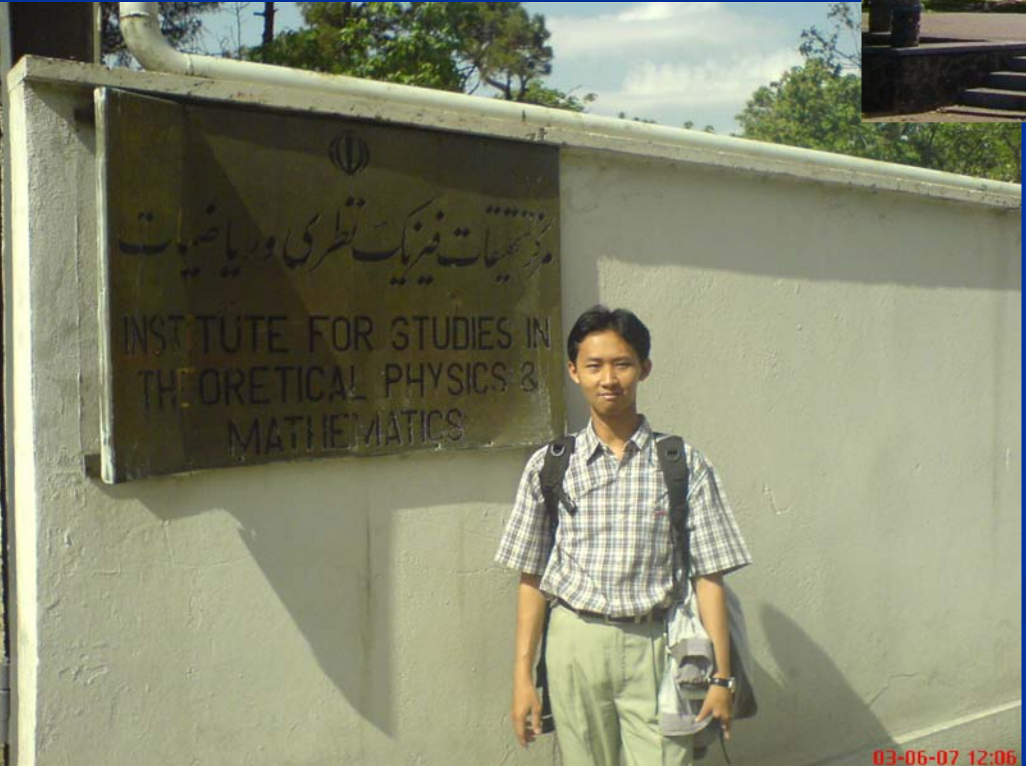
$$P_t(\mathbf{k}) = \frac{4\pi}{z^2(k)k}\int_0^\infty dr\, rC_t^{BB,app}\sin kr$$

Schematic of Method for Reconstructing Primordial Perturbations

$$\cdots \Rightarrow P^{(n)}(k) \rightarrow \frac{C_\ell^{X\bar{X},\,\mathrm{ex}(n)}}{C_\ell^{X\bar{X},\,\mathrm{app}(n)}} = b_\ell^{X\bar{X},(n)}$$

$$\rightarrow \frac{C_\ell^{X\bar{X},\,\mathrm{obs}(n)}}{b_\ell^{X\bar{X},(n)}} = C_\ell^{X\bar{X},\,\mathrm{app}(n+1)} \rightarrow P^{(n+1)}(k) \Rightarrow \cdots$$

Thank You



Terima Kasih