Low curvature modifications of Gravity.

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Work done in collaboration with Ignacio Navarro.

•gr-qc/0506096 (Phys. Lett. B 622, 2005)
•gr-qc/0511045 (JCAP 0603:008, 2006)
•gr-qc/0512109 (JCAP 0609:006, 2006)
•gr-qc/0611127 (JCAP 0702:022, 2007)

The models:

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(R + F(R, P, Q)\right) \ ,$$

with $P = R_{\mu\nu} R^{\mu\nu}$ and $Q = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$.

with:

- $R \gg F$ for $R^2, Q, P \gg \mu^4$
- $F \to \infty$ when $R_{\alpha\beta\gamma\delta} \to 0$

Prime motivation: alternatives for Dark Energy

- $$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G_N} \left[R - \frac{\mu^2}{R} \right] ,$$

(Capozziello et al astro-ph/0303041, Carroll et al astro-ph/0306438)

- $$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G_N} \left[R - \frac{\mu^{4n+2}}{(aR^2 + bP + cQ)^n} \right] ,$$

(Carroll et al astro-ph/0410031, fit to the SN data: Mena et al astro-ph/0510453)

With crossover scale: $\mu \sim H_0$

Problem?

- Solar System and lab experiments rule out any long range gravitationally coupled fifth force.

- So the challenge is to modify gravity only at large (cosmic) distances, while keeping it unaltered at short (Solar System) distances.

f(R) gravity

=scalar-tensor gravity

$$\frac{M_p^2}{2}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu})$$

↓

$$\frac{M_p^2}{2}\int d^4x\sqrt{-g}\left(f'(X)(R-X)+f(X)\right) + S_m$$

↓

$$f'(X) = e^{\kappa\phi} \quad (\kappa^{-1} = \sqrt{3/2}M_p)$$

$$\hat{g}_{\mu\nu} = g_{\mu\nu}e^{\kappa\phi} = g_{\mu\nu}f'(X)$$

$$\int d^4x\sqrt{-\hat{g}}\left(\frac{M_p^2}{2}\hat{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) + S_m(\hat{g}_{\mu\nu}e^{-\kappa\phi})$$

$$\left(V(\phi) \equiv \frac{M_p^2}{2}e^{-\kappa\phi}\left(X[\phi] - e^{-\kappa\phi}f(X[\phi])\right)\right)$$

weak field expansion on a certain background solution.

$$\left.\begin{aligned} \phi &= \phi_0 + \tilde{\phi} \\ \hat{g}_{\mu\nu} &= e^{\kappa\phi_0}(\eta_{\mu\nu} + h_{\mu\nu}) \\ T^{\mu\nu} &= T_0^{\mu\nu} + \tilde{T}^{\mu\nu} \end{aligned}\right\} \rightarrow \begin{aligned} \mathcal{L}^{(2)} = \frac{M_p^2 e^{\kappa\phi_0}}{2}\mathcal{L}^{(2)}_{spin2} \\ - e^{\kappa\phi_0}\left(\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + e^{\kappa\phi_0}\frac{V''(\phi_0)}{2}\tilde{\phi}^2\right) \\ - \frac{\tilde{\phi}}{\sqrt{6}M_p}\tilde{T} + \tilde{T}_{\mu\nu}\frac{h^{\mu\nu}}{2} \end{aligned}$$

$$m_s^2(\phi_0) = e^{\kappa\phi_0}V''(\phi_0) = \frac{1}{3}\left(\frac{f'(R_0)}{f''(R_0)} - R_0\right)$$

$$M_p^{eff^2} = e^{\kappa\phi_0}M_p^2 = f'(R_0)M_p^2$$

Gravitational chameleon: both mass and effective Newton’s constant depend on the backgroundcurvature. (Violation of strong equivalence principle)

As expected, the scalar is very light on today's cosmic background

$$f(R) = R \pm \frac{\mu^{2+2n}}{R^n}$$

(with $\mu \sim H_0$)

$$\longrightarrow \quad m_s^2 \sim \pm \frac{R_0^{n+2}}{\mu^{2+2n}} \quad \left(\sim \pm H_0^2 (1+z)^{3n+6} \right)$$

for instance, solution corresponding to a spherically symmetric probe mass on the background:

$$ds^2 = -\left(1 - \frac{2G_N^{eff} M}{r} - \frac{2G_N^{eff} M}{3r} e^{-m_s r}\right) dt^2 + \left(1 + \frac{2G_N^{eff} M}{r} - \frac{2G_N^{eff} M}{3r} e^{-m_s r}\right) d\mathbf{x}^2$$

Cassini-satellite (Bertotti 2003): $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$

↓

f(R) actions that can generate late time cosmic acceleration are ruled out by Solar System experiments (Chiba 2003)

That is if we can trust the weak field expansion!

Weak field conditions

- ordinary gravity:

$$\boxed{h_{\mu\nu} \ll 1} \longrightarrow \begin{cases} \dfrac{G_N M}{r} \ll 1 \\ E \ll \Lambda_s = M_p \end{cases}$$

- Additional ingredient: expansion of the potential. ***typically***:

$$\boxed{\tilde{\phi} \ll |\phi_0| \sim M_p \left(\frac{\mu^2}{R_0}\right)^{n+1}} \longrightarrow \begin{cases} \dfrac{G_N M}{r} \ll \left(\dfrac{\mu^2}{R_0}\right)^{n+1} \\ E \ll \Lambda_s = M_p \left(\dfrac{\mu^2}{R_0}\right)^{n+1} \end{cases}$$

On a *realistic* cosmic background: $\Lambda_s \sim \dfrac{M_p}{(1+z)^{3n+3}}$

$\longrightarrow$ We can use the weak field expansion on today's cosmic background, so assuming adiabatic evolution, these theories are indeed ruled out.

Peculiar property: $R \approx R_0 \sim H_0^2$ even if $\rho \gg \rho_0 \sim M_p^2 H_0^2$

F(R,Q-4P) gravity:

Again one extra scalar with its mass depending on the background: $m_s^2 \sim \mathcal{R}_0 (\frac{\mathcal{R}_0}{\mu^2})^{2n+1} \sim H_0^2$ (on today's cosmic background)

$$\longrightarrow \quad ds^2 = -\left(1 - \frac{2G_N^{eff} M}{r} - \frac{2G_N^{eff} M}{3r} e^{-m_s r}\right) dt^2 + \left(1 + \frac{2G_N^{eff} M}{r} - \frac{2G_N^{eff} M}{3r} e^{-m_s r}\right) d\mathbf{x}^2$$

So the theory would be ruled out if the linearization was applicable...

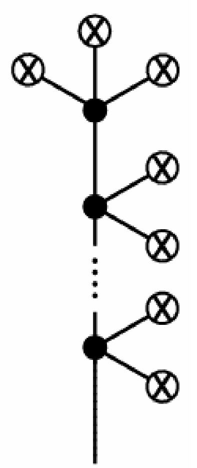
But it's not!

Higher order corrections

- **Intuitively**: the perturbative expansion breaks down when the curvature of the perturbation $h_{\mu\nu}$ becomes larger than the curvature of the background $g^{(0)}_{\mu\nu}$ ($\mathcal{R}_0$). (This is not what happens in F(R)!)

- **More quantitatively:**
  fourth order vertex: $\frac{(\partial^2 h^c \partial^2 h^c)^2}{\Lambda_s^8}$ with $\Lambda_s \sim (M_p \mu^3)^{1/4}$

  higher order corrections to the classical potential:



tree level tad pole diagram for the potential induced by the mass source $\int d^4x J\phi = \int d^4x M\delta^3(x)\phi$.

$$\rightarrow \frac{G_N M}{r}\left(\frac{r_V}{r}\right)^{8m}$$

with $r_V = (G_N M/\mu^3)^{1/4}$

(This distance is huge: ~10 kpc for the Sun, ~1 Mpc for the Milky Way)

The theory at short distances.

- At short distances the curvature blows up, so we expect to recover GR.
- What about the extra scalar?

$$m_s^2 \sim \mathcal{R}_0 (\frac{\mathcal{R}_0}{\mu^2})^{2n+1}$$

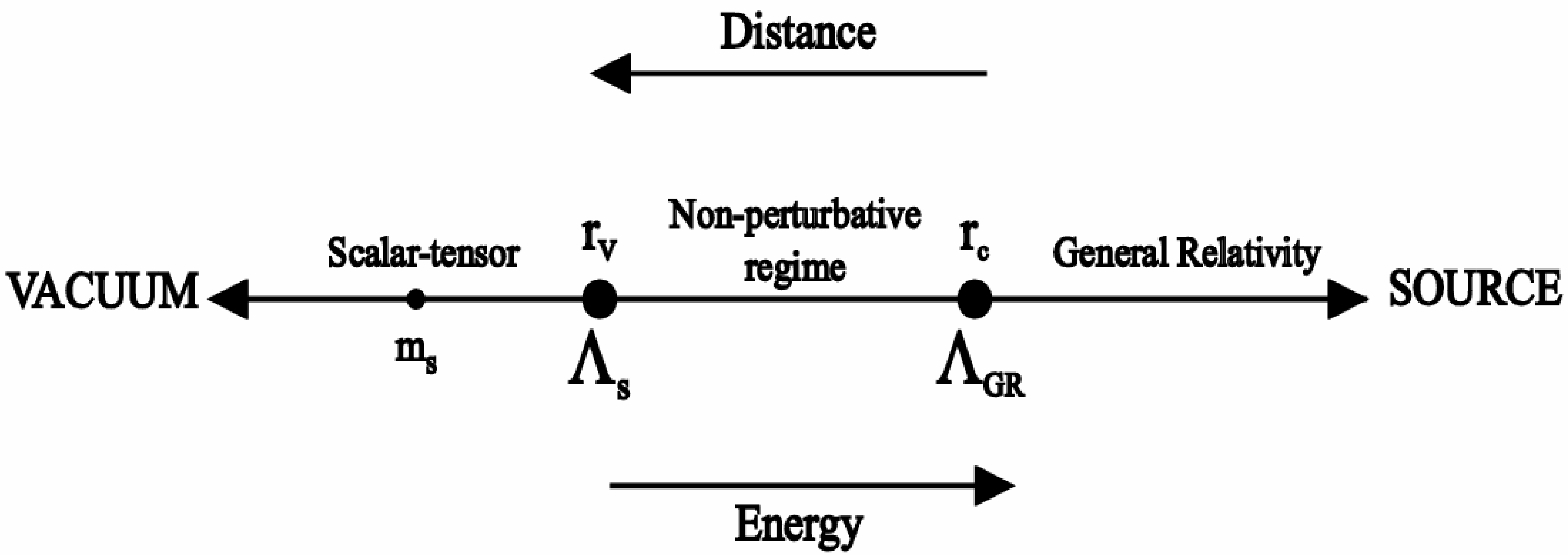
On Schwarzschild: $\mathcal{R}_0 \sim \frac{G_N M}{r^3} \quad \rightarrow \quad m_s(r) \times r > 1 \;\; \text{for} \;\; r < r_c$

With: $r_c = (\frac{(G_N M)^{n+1}}{\mu^{2n+1}})^{\frac{1}{3n+2}}$

Explicitly:

$$\begin{aligned} ds^2 \;\; \simeq \;\; & -\left[1 - \frac{2G_N M}{r}\left(1 - \alpha\left(\frac{r}{r_c}\right)^{6n+4} + \mathcal{O}\left((r/r_c)^{12n+8}\right)\right)\right] dt^2 \\ & + \;\; \left[1 - \frac{2G_N M}{r}\left(1 + \frac{\alpha(6n+3)}{2}\left(\frac{r}{r_c}\right)^{6n+4} + \mathcal{O}\left((r/r_c)^{12n+8}\right)\right)\right]^{-1} dr^2 + r^2 d\Omega_2^2 \end{aligned}$$

Schematically:



Some issues in the quantumworld

- Besides the cosmological constant problem, for ordinary GR+the Standard Model the quantumcorrections are under control up to the TeV scale.

- For F(R,Q-4P) gravity we find that in order to have light extra fields at large distances and not violate the Solar System constraints, one needs a low cut-off on the Gravity side. (see also the DGP model).

- Addressing the UV-stability of the low-energy effective action therefore becomes way more difficult, not to mention that one does not solve the cosmological problem anyway.

Conclusions

- Both F(R) and F(R,Q-4P) models are characterized by an extra scalar degree of freedom, that behaves as a gravitational chameleon: $m_s^2 \sim \pm \frac{R_0^{n+2}}{\mu^{2+2n}}$
- In order to bypass the Solar System constraints we need a low cut-off on today's cosmic background so that when we approach a source, the curvature blows up and the extra scalar decouples.
- This does not happen for F(R) gravity, the cut-off is $\sim M_p$, and the curvature stays locked to its background value.
- This happens for F(R,Q-4P) gravity. But there are other issues: quantumstability, lorentzviolation (De Felice, Hindmarsh and Trodden, 2006)

Non-perturbative recovery of GR for **large backgroundcurvatures** through the Chameleon mechanism.

- Chameleon mechanism for scalar-tensor theories when the thin-shell condition holds. (Khoury , Weltman 2003)

$$\frac{\Delta r}{r_{\odot}} \simeq \frac{(\phi_0 - \phi_s)}{6\beta\Phi_N M_p} \ll 1 \longrightarrow \text{Field outside the source:} \quad \tilde{\phi} \simeq -\frac{3\Delta r}{r_{\odot}} \frac{\beta M}{4\pi M_p} \frac{e^{-m_s r}}{r}$$

- Applying this to f(R) we find:

$$\frac{\Delta r}{r_{\odot}} \sim \frac{(\frac{\mu^2}{R_0})^{n+1}}{\frac{G_N M}{r_{\odot}}} \ll 1 \longrightarrow \text{Field outside the source:} \quad \tilde{\phi} \sim M_p r_{\odot} (\frac{\mu^2}{R_0})^{n+1} \frac{e^{-m_s r}}{r}$$

Resolving some confusion.

- $\mu \to 0$ limit? (Faraoni, 2006) $\longrightarrow$ See previous slide

- Large curvature in our galaxy? (Cembranos, 2005; Shao et al 2006, Zhang, 2007)
  $\longrightarrow$ Peculiar property: $R \approx R_0 \sim H_0^2$ even for $\rho \gg \rho_0 \sim M_p^2 H_0^2$

- Fine tuned actions? (Dick, 2003; Nojiri, Odintsov 2003)

$$f(R) = R - \frac{\mu^4}{R} + \alpha\frac{\mu^6}{R^2} \longrightarrow m_s \geq 10^{-3} eV \quad \text{if} \quad R_0 = 3\alpha\mu^2(1 \pm 10^{-60})$$

But: $V(\phi) \approx \mu^2 M_p^2 + \mu^2 M_p^{1/2}(\phi - \phi_t)^{3/2} \longrightarrow \Lambda_s \sim M_p \left(\frac{H_0}{m_s}\right)^4$

so linearization is not applicable!

F(R,P,Q) gravity: one can generically expect 8 degrees of freedom (Hindawi et al hep-th/9509147):

- □ 2 polarizations of the massless spin 2 graviton
- □ 1 massive scalar
- □ 5 polarizations of a massive spin 2 ghost

For perturbations on DeSitter space:

(Minkowski will typically not be a solution.)

$$S^{(2)} \cong \int d^4x \sqrt{-g} \frac{1}{16\pi G_N} \left[-\Lambda + \delta R + \frac{1}{6m_0^2} R^2 - \frac{1}{2m_2^2} C^{\mu\nu\lambda\sigma} C_{\mu\nu\lambda\sigma} \right]$$

$$m_2^{-2} \equiv -\langle f_P + 4 f_Q \rangle_0$$

$m_g \sim m_2$,so $f(R, Q-4P)$ gravity has no ghosts and only one extra scalar degree of freedom, besides the massless spin 2 graviton. **(On deSitter space!)**

Explicitly:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad g^{(0)}_{\mu\nu} = diag(-1, e^{2Ht}, e^{2Ht}, e^{2Ht})$$

$$h_{00} = \phi \ , h_{0i} = w_i \ \ , h_{ij} = \chi_{ij} + \delta_{ij}\tau$$

$$< \chi_{ij}\chi_{ij} >_0 \ = \ \frac{1}{C_1 M_p^2(\omega^2 - \tilde{k}^2)} , \qquad < \tau\tau >_0 \ = \ \frac{2}{3}\frac{1}{C_1 M_p^2(\omega^2 - \tilde{k}^2 - m_s^2)}$$

$$< w_i w_i >_0 \ = \ \frac{1}{C_1 M_p^2 \tilde{k}^2} , \qquad < \tilde{\phi}\tilde{\phi} >_0 \ = \ \frac{6m_s^2}{C_1 M_p^2 \tilde{k}^4} .$$

- Again, we see the chameleon behavior:

$$M_p^{eff\,2} = C_1 M_p^2 = (1 + (\ldots)(\frac{\mu^2}{\mathcal{R}_0})^{2n+1})$$

$$m_s^2 \sim \mathcal{R}_0(\frac{\mathcal{R}_0}{\mu^2})^{2n+1}$$

Even more explicit:

$$S^{(2)} = \frac{1}{16\pi G_N}\int d^4x\,\left(\chi_{ij}\hat{O}_\chi\chi_{ij} + w_i\hat{O}_w w_i + \phi\hat{O}_1\tau + \tau\hat{O}_2\tau + \phi\hat{O}_3\phi\right),$$

$$\begin{aligned}
\hat{O}_1 &= -\frac{C_2}{16}\frac{\tilde{\nabla}^2}{H^2}\partial_0^2 + \frac{C_2}{24}\frac{\tilde{\nabla}^4}{H^2} + \frac{3C_2}{16H}\partial_0^3 - \frac{3C_2}{16H}\tilde{\nabla}^2\partial_0 - \frac{9}{32}C_2\partial_0^2 + \left(C_1 + \frac{51}{64}C_2\right)\tilde{\nabla}^2 \\
&\quad -H\left(3C_1 + \frac{75}{64}C_2\right)\partial_0 + 3H^2\left(\frac{3}{2}C_1 + \frac{75}{128}C_2\right), \\
\hat{O}_2 &= -\frac{3C_2}{32H^2}\partial_0^4 + \frac{C_2}{8H^2}\partial_0\tilde{\nabla}^2\partial_0 - \frac{C_2}{24}\frac{\tilde{\nabla}^4}{H^2} + \left(\frac{3}{2}C_1 + \frac{51}{64}C_2\right)\partial_0^2 - \left(\frac{1}{2}C_1 + \frac{3}{32}C_2\right)\tilde{\nabla}^2 \\
&\quad -H^2\frac{9}{4}\left(\frac{3}{2}C_1 + \frac{75}{128}C_2\right), \\
\hat{O}_3 &= -\frac{C_2}{96}\frac{\tilde{\nabla}^4}{H^2} + \frac{3C_2}{32}\partial_0^2 - \frac{7C_2}{32}\tilde{\nabla}^2 - H^2\left(\frac{3}{2}C_1 + \frac{75}{128}C_2\right).
\end{aligned}$$

- **An illustration of an apparent higher derivative theory, that has NO ghosts.**