Dynamics of DGP Cosmology with a Bulk Field

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Plan of the Talk

- Introduction
- Effective Einstein Equation
- Toy Model
- DGP Freidman equations
- Late time cosmology
- Conclusions

Introduction

Our universe is 3-dimensional surface(brane) embedded in a higher dimensional space-time (Bulk) has generated lot of interest in cosmology.

Pioneering works were done by Randall-Sundrum, ADD, DGP.
Matter confined to brane and gravity propagates in all dimensions.

Modified cosmological evolution, Friedmann equation on the brane

$H \alpha \rho$

DGP model: Gravity is modified at large scales.
Cross over scale between 4d and 5D gravity.
It explains Late time acceleration

The possibility of a Non-empty bulk. And effect of scalar field in the bulk on the cosmological evolution on the brane.

Sasaki has shown that inflation on brane without inflaton on the brane

(PRD 63 044015)

Effective Einstein Equations

The metric is given by

$$ds^2 = g_{AB}dx^A dx^B = dy^2 + g_{\mu\nu}dx^\mu dx^\nu$$

Five dimensional Einstein equations

$$R_{ab} - \frac{1}{2}g_{AB}R = k_5^2 T_{AB}$$

The Four dimensional equations are derived using Codacci equation as

$$^{(4)}G_{AB} = \frac{2k_5^2}{3}[T_{CD}\ q_A^B q_B^A + (T_{CD}n^C n^D - \frac{1}{4}T_C^C q_{AB}]$$
$$+Kk_{AB} - K_A^C K_{BC} - \frac{1}{2}q_{AB}(K^2 - K^{CD}K_{CD}) - E_{AB}$$

The Five dimensional energy momentum has form

$$T_{AB} = -\Lambda g_{AB} + S_{AB}\delta(y)$$

The energy momentum tensor on the brane

$$S_{AB} = -\sigma q_{AB} + \tau_{AB}$$

Where Λ and σ are the brane tension and cosmological constant respectively

- Assuming Z_2 Symmetric bulk space-time with respect to brane. This implies junction conditions

$$[K_{\mu\nu}] = -K_5^2(S_{\mu\nu} - \frac{1}{3}q_{\mu\nu}S)$$

Using the above junction conditions, the effective Einstein equation

$$G_{\mu\nu} = K_5^4 \Pi_{\mu\nu} - E_{\mu\nu}$$

Where

$$\Pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\rho}\tau_{v}^{\rho} + \frac{1}{12}\tau\tau_{\mu v} + \frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}\tau^2$$

Compare with result obtained by Shiromizu, Maeda,Sasaki

(PRD 62 024012)

$$^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} + 8\pi G_N \tau_{\mu\nu} + \kappa_5^4 \pi_{\mu\nu} - E_{\mu\nu}$$

Toy model with a bulk field

- Now we consider a bulk scalar field in the bulk

$$R_{AB} - \frac{1}{2} g_{AB} R = k_5^2 (\hat{T}_{AB} + S_{AB}\ \delta(y))$$

The bulk energy momentum tensor is

$$\hat{T}_{AB} = \phi_{,A}\phi_{,B} - g_{AB}(\frac{1}{2} g^{AB}\phi_{,C}\phi_{,D} + V(\phi))$$

The effective equations on the brane become

$$G_{\mu\nu} = \frac{K_5^2}{6} T_{\mu\nu} + K_5^4 \Pi_{\mu\nu} - E_{\mu\nu}$$

where

$$\hat{T}_{\mu\nu} = \left(4\varphi_{,\mu}\,\varphi_{,\nu} + \left(\frac{3}{2}(\varphi_{,\chi})^2 - \frac{5}{2}q^{\alpha\beta}\varphi_{,\alpha}\,\varphi_{,\beta} - 3V(\varphi)\right)q_{\mu\nu}\right)$$

Friedmann equation

The metric on the brane becomes

$$ds^2|_{y=0} = -dt^2 + S^2(t)\delta_{ij}dx_i dx_j$$

We consider a homogeneous and isotropic universe on the brane. The bulk scalar field satisfies the condition

$$\phi,y|_{y=0} = 0$$

The Friedmann equation on the brane becomes

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{k_5^2}{18}\rho_B + \frac{k_5^4}{36}\rho_b^2 - E_{00}$$

$$\rho_B = 3(\frac{1}{2}\dot{\phi}^2 + V(\phi))$$

is the bulk energy density,

$$H^2 + \frac{k}{S^2} = \frac{k_5^2}{18}\rho_B$$

DGP Friedmann equations

- Consider DGP Model with induced 4D Ricci scalar on the brane.The action is

$$S_{(5)} = -\frac{1}{2\kappa^2}\int d^5X\sqrt{-\tilde{g}}\tilde{R} + \int d^5X\sqrt{-\tilde{g}}\mathcal{L}_m - \frac{1}{2\mu^2}\int d^4x\sqrt{-g}R.$$

- The cross over scale between 4D and 5D gravity is

$$r_c = \frac{k^2}{2\mu^2}$$

We consider a non-empty bulk, with a scalar field present in it. The 5D Einstein equation

$$R_{AB} - \frac{1}{2}g_{AB}R = k_5^2(\hat{T}_{AB} + S_{AB}\,\delta(y))$$

The 4D energy momentum tensor on brane with the induced term is

$$S_{AB} = \tau_{AB} - \sigma q_{AB} - \mu^{-2} G_{AB}$$

The equation for 4D Einstein tensor is obtained as

$$\left(1 + \frac{\sigma k_5^2}{6\mu^2}\right) G_{\mu\nu} = -\left(\frac{k^2\sigma}{2} + \frac{k^4\sigma^2}{12}\right) q_{\mu v} + \mu^2 \hat{T}_{\mu\nu} + \frac{\sigma k^4}{6}\tau_{\mu\nu} + \frac{K_5^4}{\mu^4} F_{\mu\nu} + K_5^4 \Pi_{\mu\nu} + \frac{K_5^4}{\mu^2} L_{\mu\nu} - E_{\mu\nu}$$

If we consider a pure DGP model, without brane tension and bulk cosmological constant

$$G_{\mu\nu} = \mu^2 \hat{T}_{\mu\nu} + \frac{K_5^4}{\mu^4} F_{\mu\nu} + K_5^4 \Pi_{\mu\nu} + \frac{K_5^4}{\mu^2} L_{\mu\nu} - E_{\mu\nu}$$

On the brane Robertson -Walker metric, The 00 component of above equation becomes

$$(H^2 + \frac{k}{S_0^2}) = \epsilon \frac{2\mu^2}{k_5^2} \sqrt{(H^2 + \frac{k}{S_0^2}) - \frac{\mu^2}{3}\rho_B - \frac{c}{S_0^4}} - \frac{\mu^2}{3}\rho_b$$

When no induced curvature term is included, result in agreement with others

$$(H^2 + \frac{k}{S_0^2}) = \frac{\mu^2}{3}\rho_B + \frac{k_5^4}{36}\rho_b^2$$

Recovery of Friedmann cosmology with a bulk field

$$H^2 + \frac{k}{S^2} = \frac{\mu^2}{3}\rho_B$$

Left hand side is sub dominant i.e

$$H^2 + \frac{k}{S^2} \ll \frac{k_5^2}{2\mu^2}$$

$$\rho_B = \frac{r_c}{3}(\frac{1}{2}\dot{\phi}^2 + V(\phi))$$

$$\Phi = \sqrt{\frac{r_c}{3}}\phi$$

Rescaled bulk scalar field on the brane

Late time cosmology

During late universe Hubble radius has two distinct behaviours depending on the value of ϵ

Є = ± 1 corresponds to possible embedding of brane into the bulks

The DGP Friedmann equation can be rewritten as

$$H^2 + \frac{k}{S^2} = \frac{1}{2r_c^2}\left[1 + \epsilon\sqrt{1 - \frac{4\mu^2}{3}\rho_B r_c^2}\right]$$

The above equation can be expanded under the condition

$$\mu^2 \rho_B \ll 1/r_c^2$$

Є = + 1

$$H^2 = \frac{1}{r_c^2}$$

Leads to accelerating late universe expansion

Є = -1

$$H^2 = 0$$

Universe is static

At first order

$$H^2 = \frac{1}{r_c^2} - \frac{\mu^2 \rho_B}{3}$$

$$H^2 = \frac{\mu^2 \rho_B}{3}$$

Conclusions

DGP Einstein equation with a bulk field is derived.

Recovering standard cosmology with a rescaled bulk field.

Late time acceleration in agreement with DGP model with a brane field.

Bulk field can explain cosmological evolution on the brane..

In the absence of induced in DGP model it coincides with other models

Thank You