Quantum Effects in Brane Cosmologies

David Jennings,
Department of Applied Mathematics and Theoretical Physics,
CMS, University of Cambridge, Cambridge, CB3 0WA, UK.

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Progressive Viewpoints

1. Earth is the special observer.

2. Inertial frames are special.

3. All observers are equal.

4. Comoving frames are special.

5. Brane is the special observer.

Brane Dynamics

In Randall-Sundrum-type models:

- *The universe is viewed as a 3-dimensional (mem)brane moving through a higher dimensional spacetime.*

- *Matter is confined to this hypersurface and General Relativity applies to the full spacetime.*

- *Gravity is localized around the brane.*

Symmetric Brane

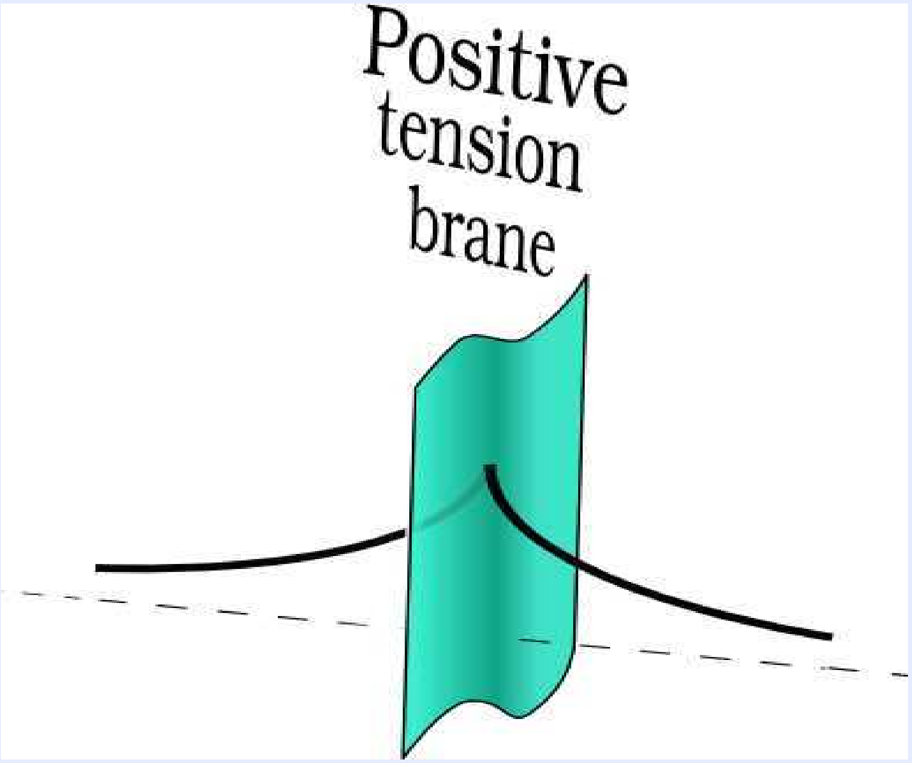


Figure 1: Single brane

Brane Dynamics

Motion of the brane universe is through a bulk spacetime.

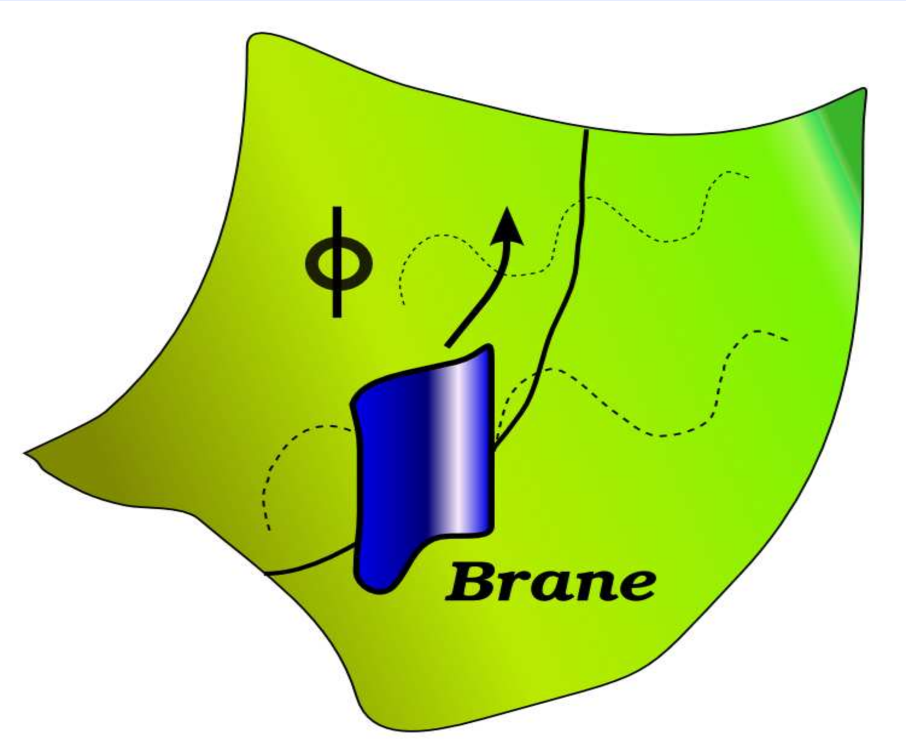


Figure 2: Brane and Bulk scalar field

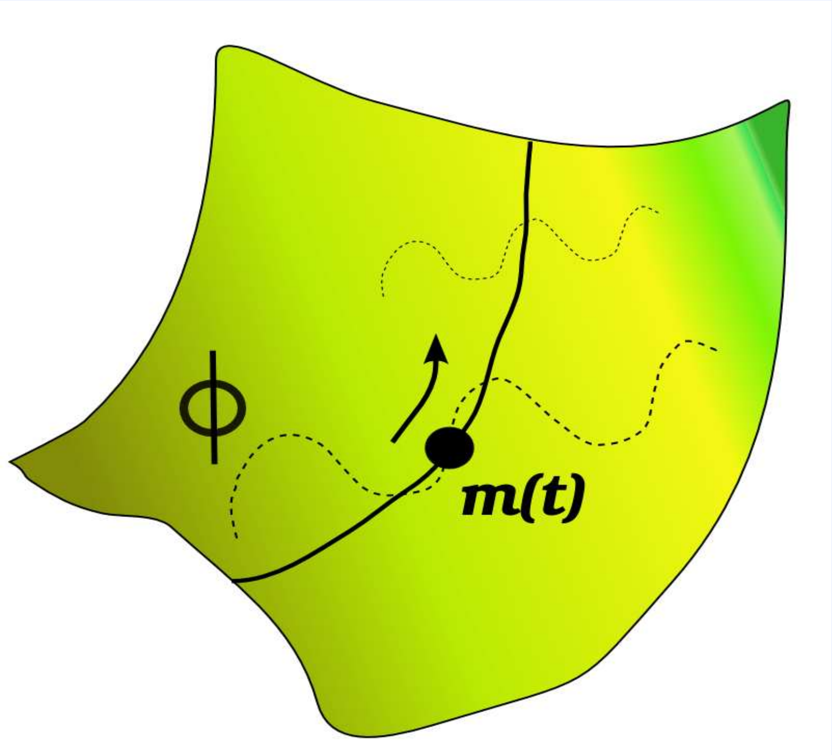
Unruh Effect

"Suddenly I saw the cold and Rook-delighting Heaven
That seemed as though ice burned, and was but the more ice."

W.B. Yeats

"An accelerated observer in Minkowski spacetime will measure the vacuum state to be a thermal bath of particles, with a temperature proportional to the acceleration of the observer."

Particle Detectors



Simplest case:

$$V = c^2 M(\tau)\phi(x(\tau)). \quad (1)$$

Excitations and the Wightman Function

Probability amplitude for an excitation is

$$P = c^2 \sum_E |< E|M|E_0 >|^2 \int dt dt' \mathcal{G}^+(x(t), x(t')) e^{-i(E-E_0)(t-t')}$$

with **Wightman function**

$$\mathcal{G}^+(x, x') =< 0|\phi(x)\phi(x')|0 > . \quad (2)$$

So what do we see in 4D Minkowski spacetime?

- For inertial detectors, $\dot{P} = 0$.
- For constantly accelerated detectors $\dot{P} \neq 0$

$$\text{Particle Detection Rate} \quad \propto \quad \frac{1}{e^{\frac{2\pi E}{a}} - 1}, \tag{3}$$

corresponding to a vacuum state temperature

$$T \quad = \quad \frac{a}{2\pi}. \tag{4}$$

The Unruh Brane

Work in D-dimensional Anti-de Sitter spacetime,

$$ds^2 = \frac{1}{k^2 z^2}(dt^2 - dx_1^2 - dx_2^2 - \cdots - dx_{D-2}^2 - dz^2) \tag{5}$$

Scalar field in the spacetime obeying

$$(\nabla^\mu \nabla_\mu + m^2 + \zeta R)\phi(x) = 0 \tag{6}$$

where ζ is its coupling to gravity.

We want

$$\mathcal{G}^+(x, x') = < 0|\phi(x)\phi(x')|0 > \tag{7}$$

Acceleration of the brane

Dynamics governed by Junction Condition

$$[K_{AB}]^+_- = -\kappa^2(T_{AB} - \frac{1}{3}h_{AB}T) \quad (8)$$

$K_{AB} =$ Extrinsic curvature of the brane.
Find: brane universe accelerates with

$$a = \frac{\kappa^2}{6}(2\rho + 3p - \sigma) \quad (9)$$

$\rho =$ matter density, $p =$ matter pressure, $\sigma =$ brane tension.

Conformally Coupled Scalar field

The Wightman function for a *massless, conformally coupled* scalar field

$$\mathcal{G}^+(x,x') = \frac{k^{D-2}\Gamma(\frac{D}{2}-1)}{2(2\pi)^{\frac{D}{2}}}\left(\frac{1}{(v-1)^{\frac{D}{2}-1}} - \frac{1}{(v+1)^{\frac{D}{2}-1}}\right), \quad (10)$$

with

$$v(x,x') = \frac{z^2 + z'^2 + (\mathbf{x}-\mathbf{x}')^2 - (t-t'-i\epsilon)^2}{2zz'}. \quad (11)$$

So what happens in AdS Spacetime?

We find that

$$\text{Particle Detection Rate} = \begin{cases} 0 & a < k \\ 0 & a = k \\ \frac{F}{e^{2\pi\frac{E}{\omega}} - (-1)^D} & a > k \end{cases}$$

with

$$\begin{aligned} F &= \frac{\Gamma(\frac{D}{2}-1)}{(4\pi)^{\frac{D}{2}}}(\omega^{D-3} f_{D-3}(\frac{E}{\omega}) - k^{D-2} g(E,a,k)) \\ \omega^2 &= a^2 - k^2. \end{aligned} \tag{12}$$

Thus

$$T = \frac{\sqrt{a^2-k^2}}{2\pi}\Theta(a-k) \tag{13}$$

The Late-Time Universe

- Recall that

$$a = \frac{\kappa^2}{6}(2\rho + 3p - \sigma) \tag{14}$$

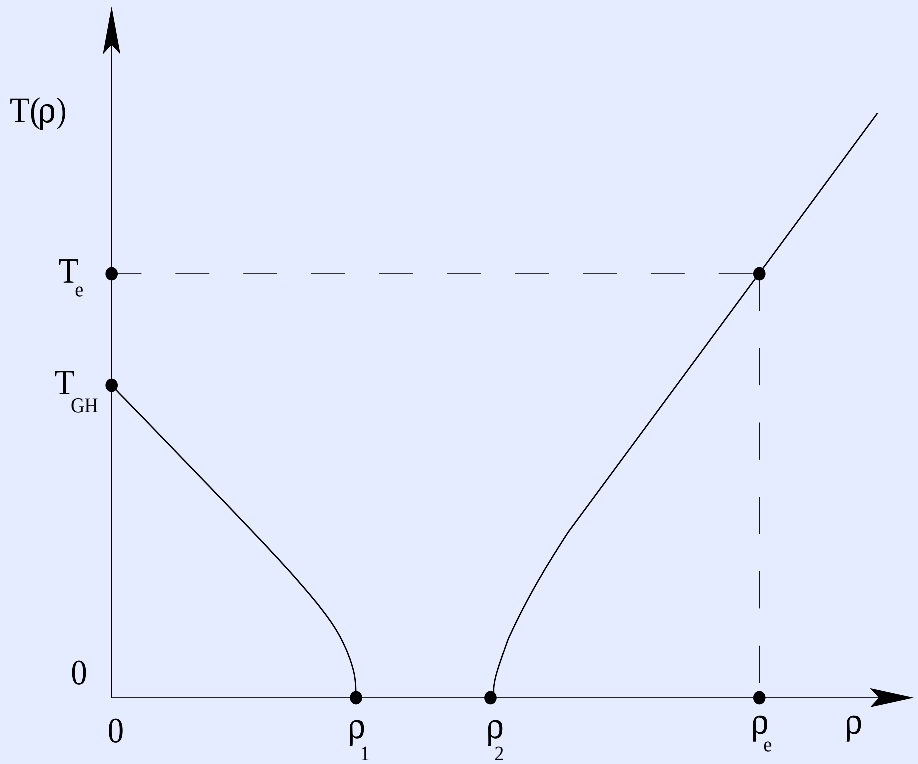
- Unruh temperature of the bulk is

$$T = \frac{\sqrt{a^2 - k^2}}{2\pi} \tag{15}$$

For the simple brane model

$$T_{\text{bulk}} = \frac{\kappa^2\sqrt{\rho(\rho - \sigma) + \frac{3\Lambda_4}{\kappa^4}}}{6\pi} \tag{16}$$

Temperature of Bulk



At $\rho = 0$, the **bulk Anti-dS** is at the **dS** temperature

$$T_{\text{AdS bulk}} = \frac{1}{2\pi}\sqrt{\frac{\Lambda_4}{3}} = T_{dS}. \qquad (17)$$

Conclusions

1. The motion of branes is non-inertial in general.

2. View brane as bulk particle detector.

3. An Unruh effect occurs that excites matter fields.

4. Affects early universe expansion.

5. Allows late-time stability of the universe.

6. Probes the dimensionality of spacetime and the bulk curvature.

7. Gibbons-Hawking temperature obtained as an embedded Unruh effect.

Special Announcement:

Happy Birthday Kate!