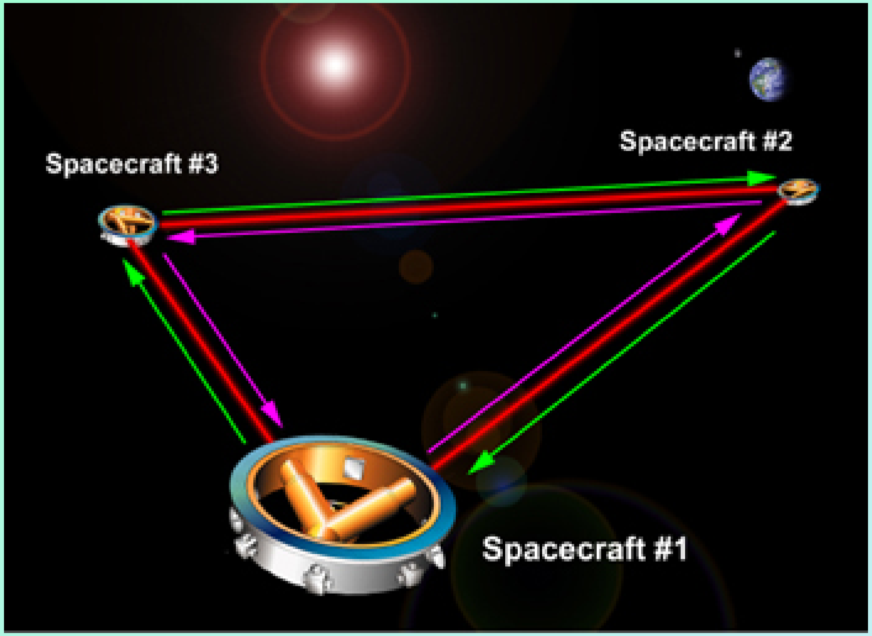
On the minimum flexing of arms of LISA

(Laser Interferometer Space Antenna)



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IISER, Pune, India.

ICSW-07, IPM, Tehran,Iran Jun4, 2007

Motivation

- **Einstein's General theory of relativity (GR) is the most beautiful & successful theory of gravity**. It has matched most of the tests of Gravitation remarkably well.

- Weak field, linearized GR predicts **Gravitational radiation**. Massless, propagating excitations of the gravitational field.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

$$\Box h_{\mu\nu} = 0$$

- **Strong, indirect evidence** of Gravitational waves (GW) deduced from measured slow down of Binary pulsars due to GW emission . ***(Hulse & Taylor won the 1993 Nobel prize for physics for this work.)***

- **Intense international efforts is underway for direct detection of GW.**

Terrestrial GW observatories

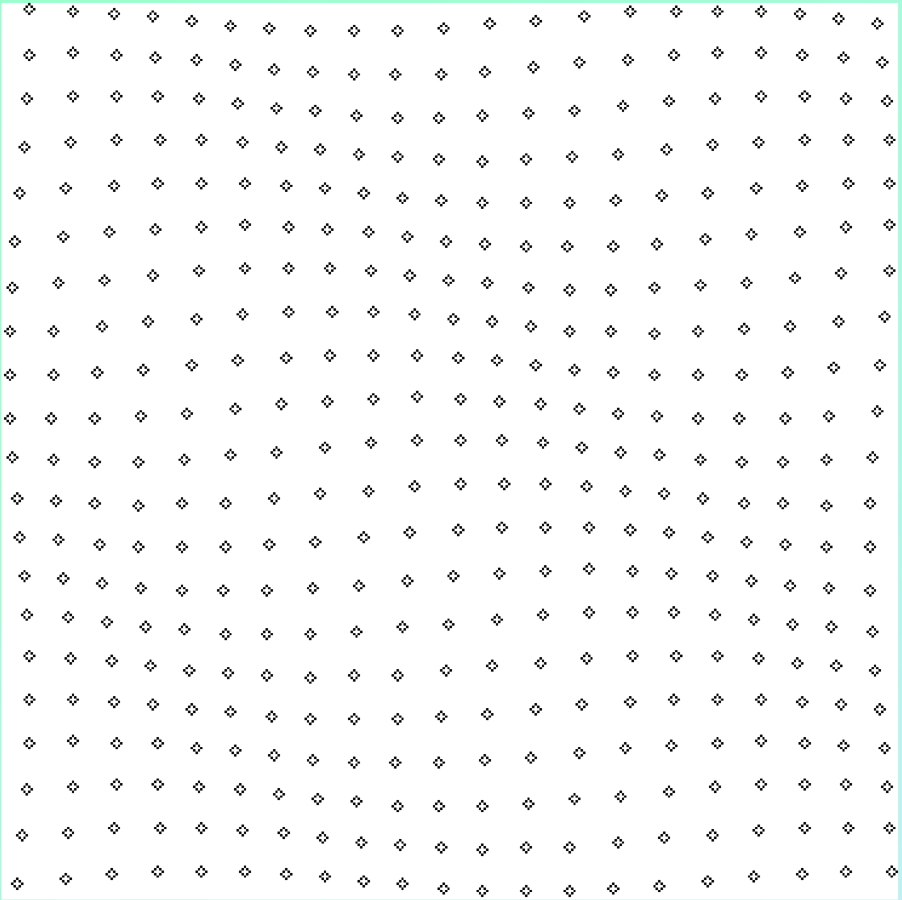
Weak interaction of Gravitational waves makes direct detection a major scientific challenge of this century.

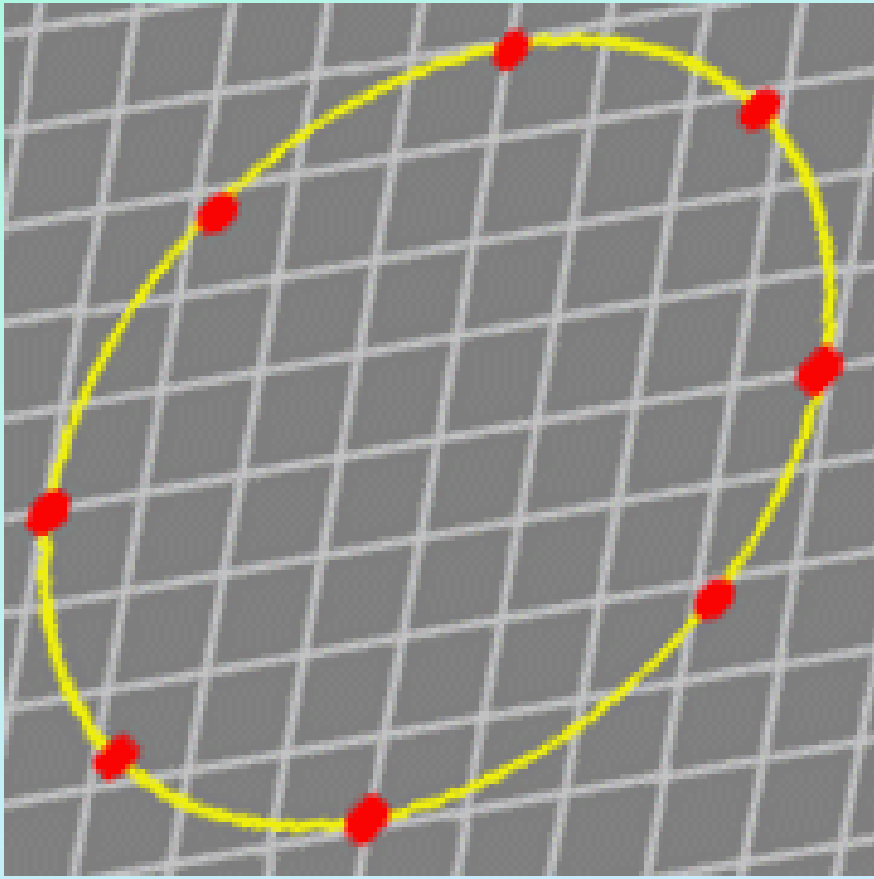
- *Number of Laser interferometers with Kilometer Scale arm length are already taking scientific data in hope of detecting GW signals from astrophysical sources.*

LIGO

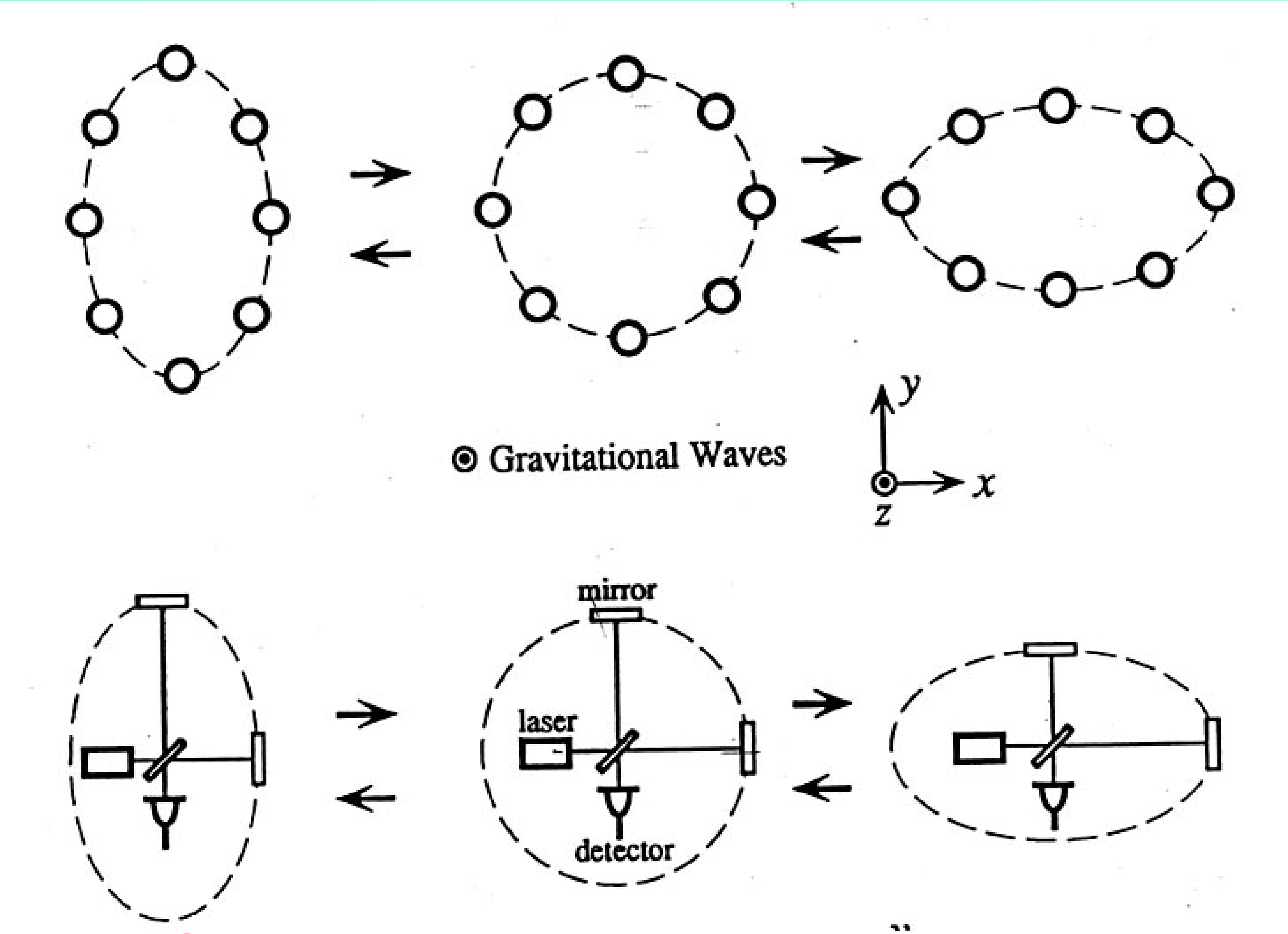
Laser Interferometer Gravitational-Wave Observatory

Effect of GW on test masses



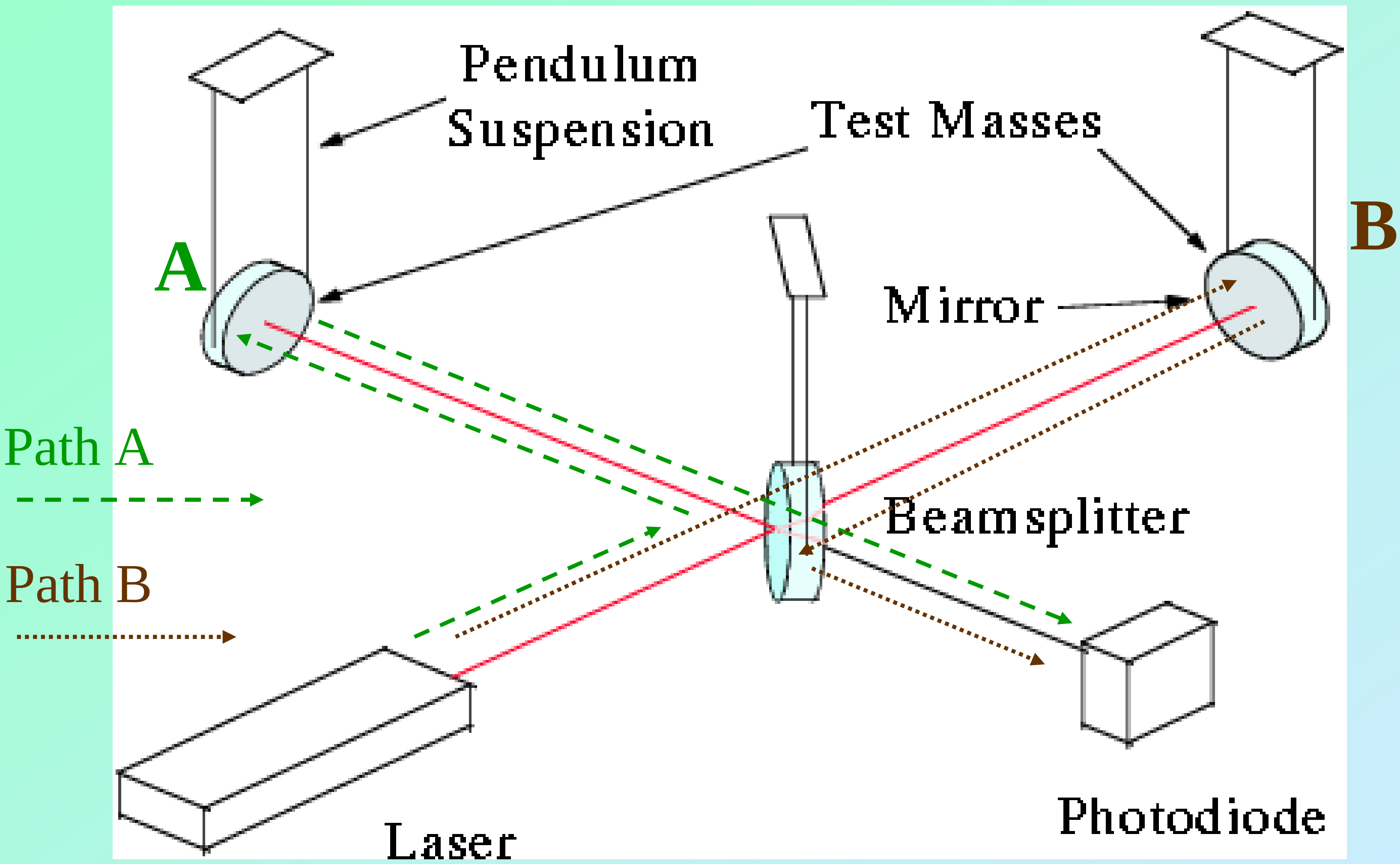


Effect of GW on a ring of test masses



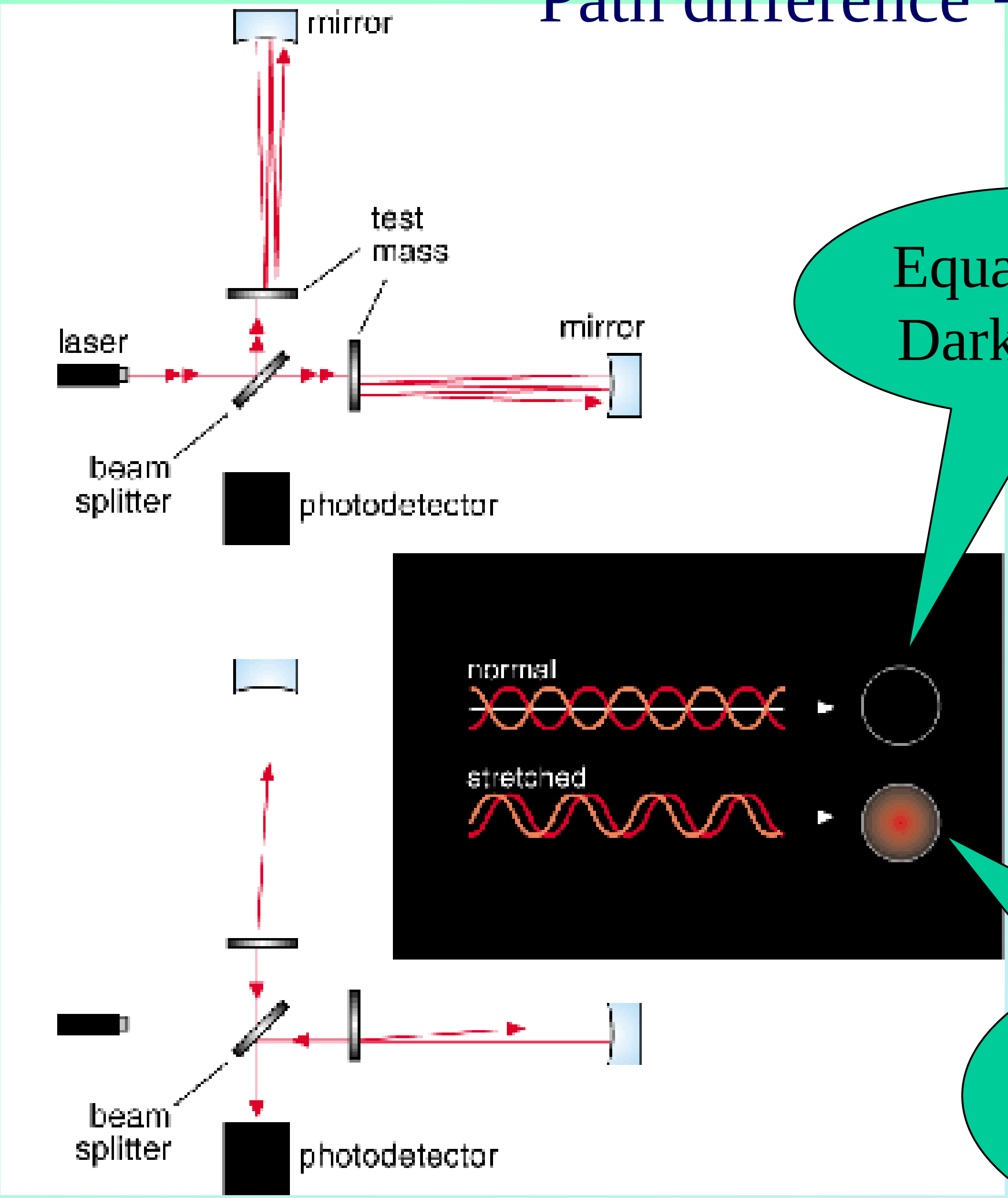
Interferometer mirrors as test masses

Detecting GW with Laser Interferometer



Difference in distance of Path A & B ➔ Interference of laser light at the detector (Photodiode)

Path difference ➔ phase difference



Equal arms:
Dark fringe

The effects of gravitational waves appear as a fluctuation in the phase differences between two orthogonal light paths of an interferometer.

Unequal arm:
Signal in PD

Challenge of Direct Detection

Gravitational waves are very weak!

Gravitational wave is measured in terms of strain, *h*

(change in length/original length)

$$2\frac{\delta L}{L} = h$$

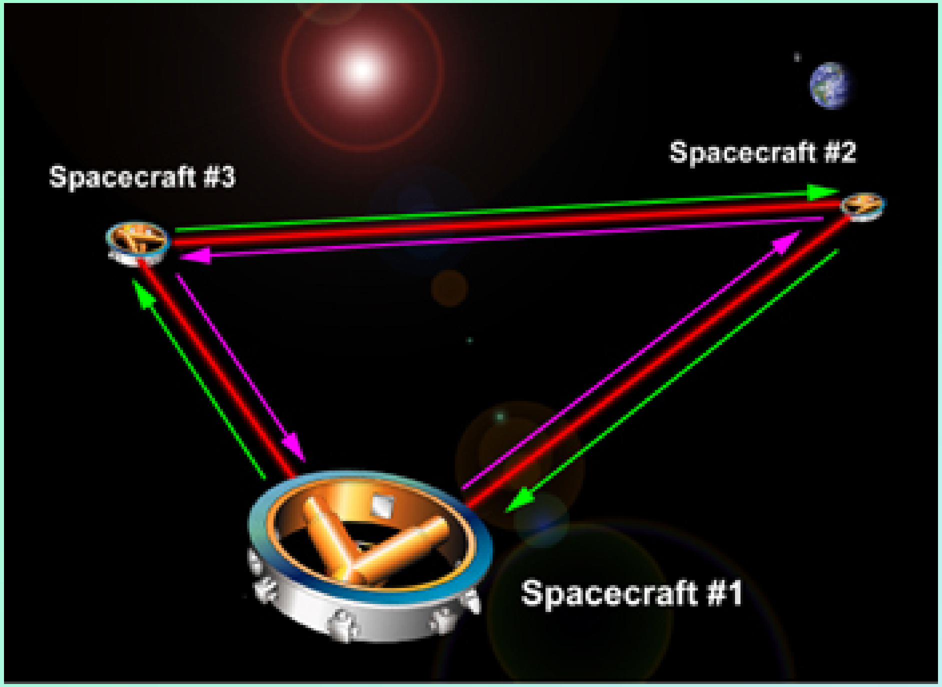
Expected amplitude of GW signals

$$h \approx 10^{-20} - 10^{-23}$$

Measure changes of

one part in thousand-billion-billion!

GW OBSERVATORY IN SPACE



LISA : Laser Interferometer Space Antenna

(An ambitious world-wide project)

NASA, ESA joint proposal for space based GW Observatory (expected launch 2015).

Why a GW Observatory in space ?

•Terrestrial GW observatories are limited to GW frequencies **above 10 Hz** due to **seismic noise**.

(10 Hz– 2000 Hz.)

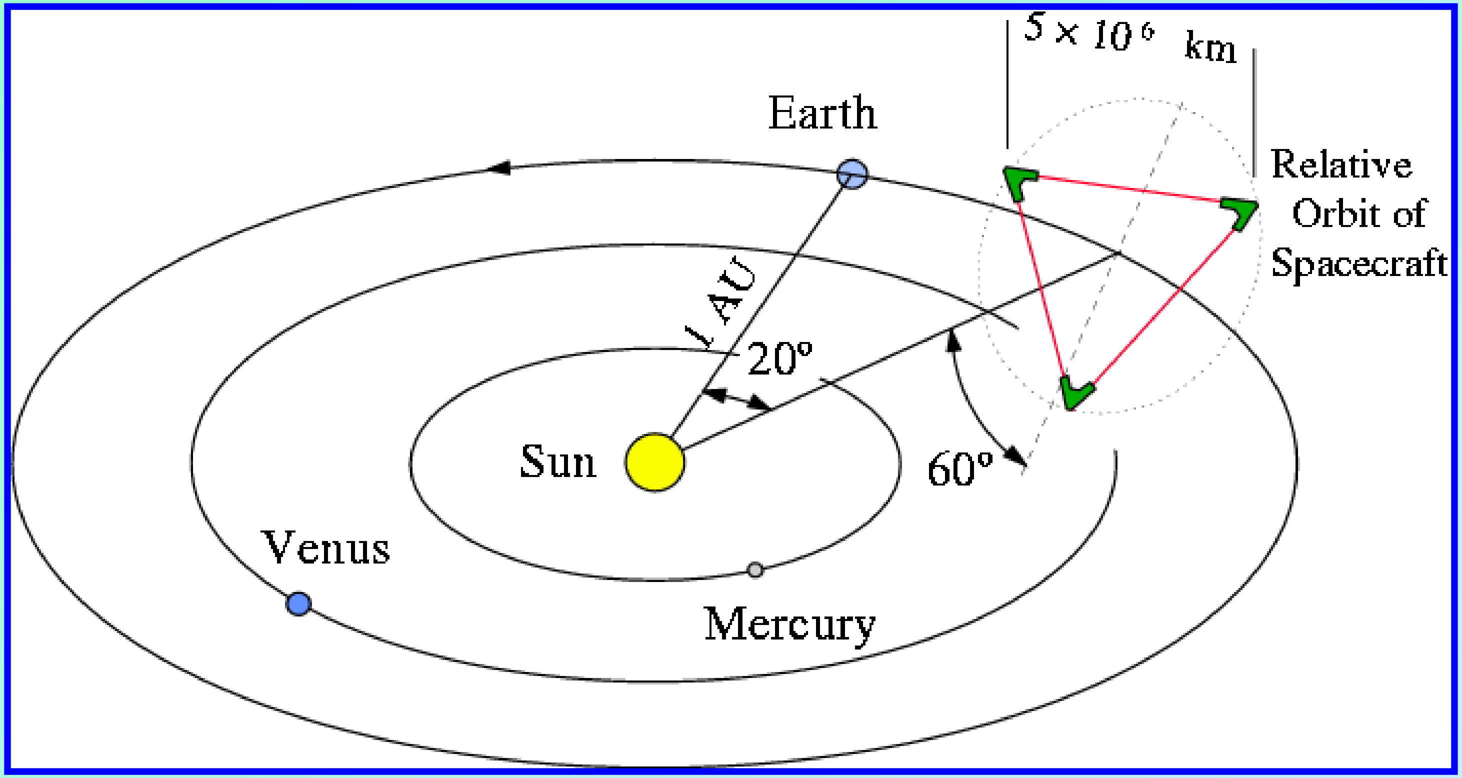
•Interesting sources abundant at sub-Hertz frequencies (milli-Hz to Hz range) are accessible.

•Easier to attain higher sensitivity with longer baselines.

LISA : Laser Interferometer Space Antenna

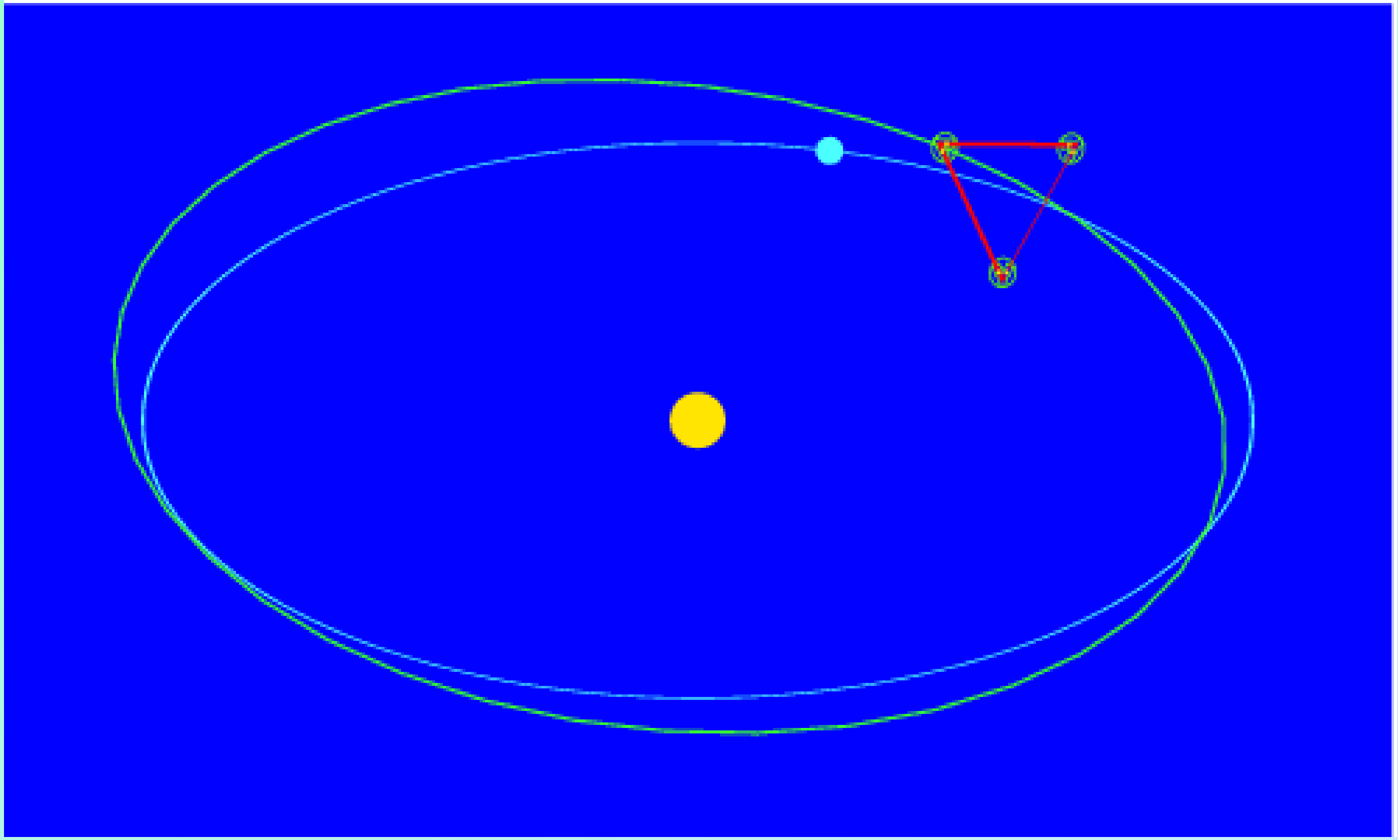
A NASA, ESA joint proposal for space based GW Observatory (launch 2015).

Frequency range: 10^{-4} Hz - 1 Hz



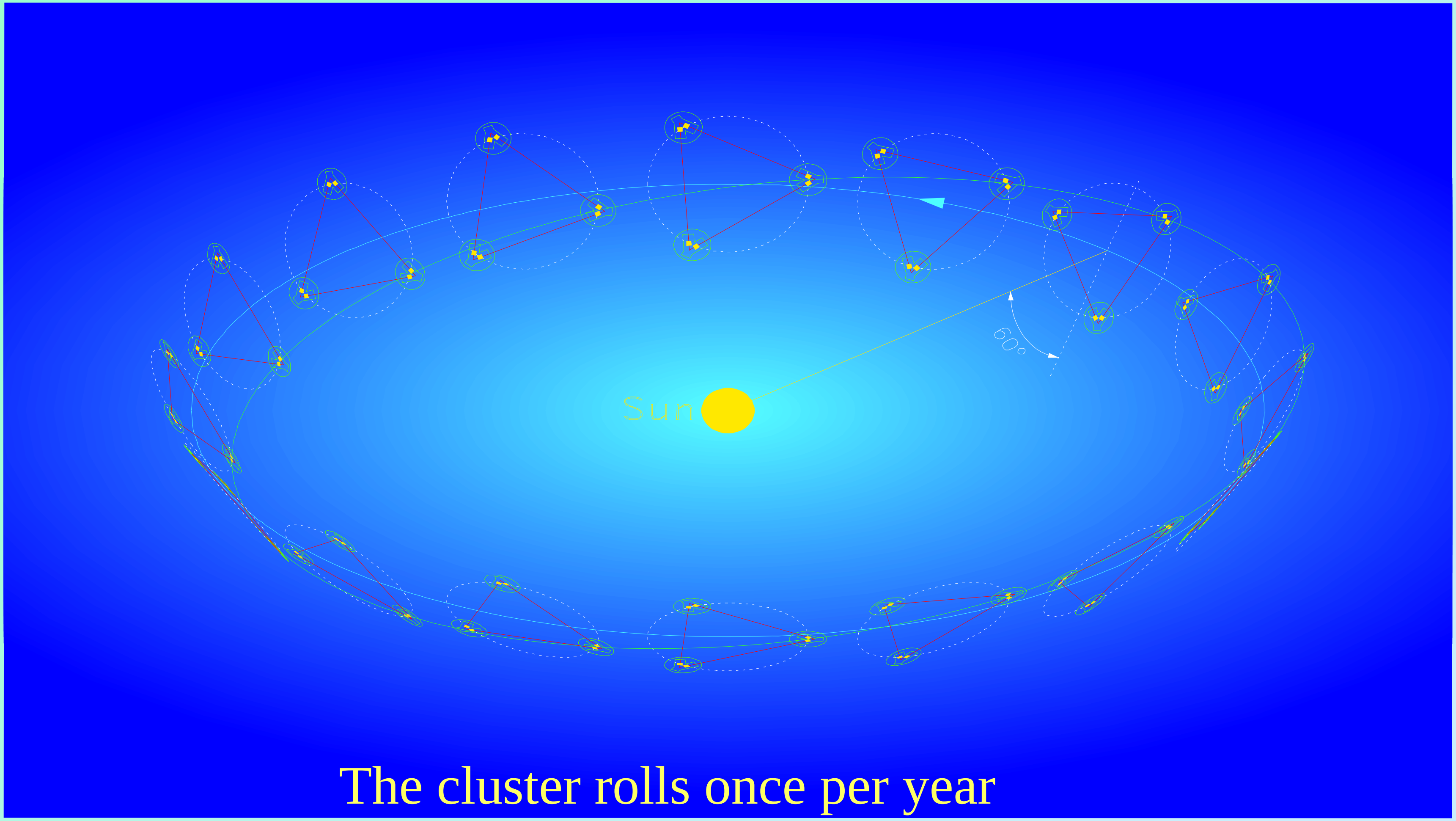
A configuration of three `freely falling' spacecrafts in earth-like orbit linked by optical laser beams working as an interferometer in space

The Orbit of LISA



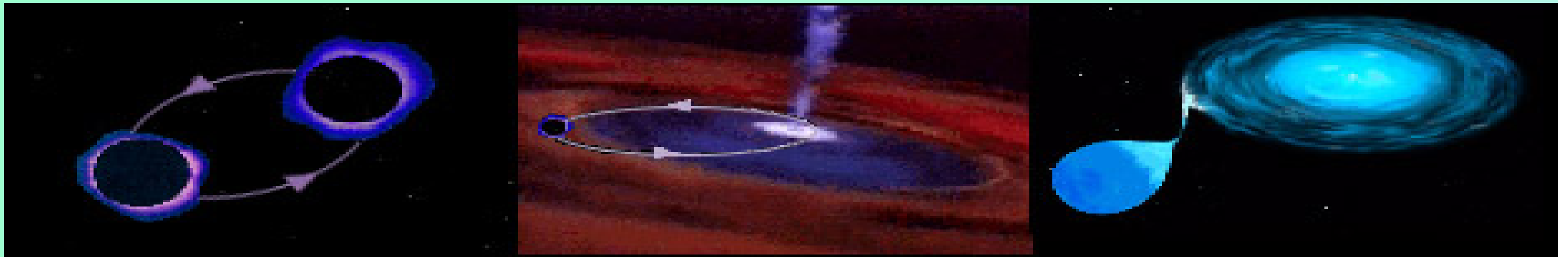
The spacecraft are freely falling in the Sun’s field .

The Orbit of LISA



The spacecraft are freely falling in the Sun’s field .

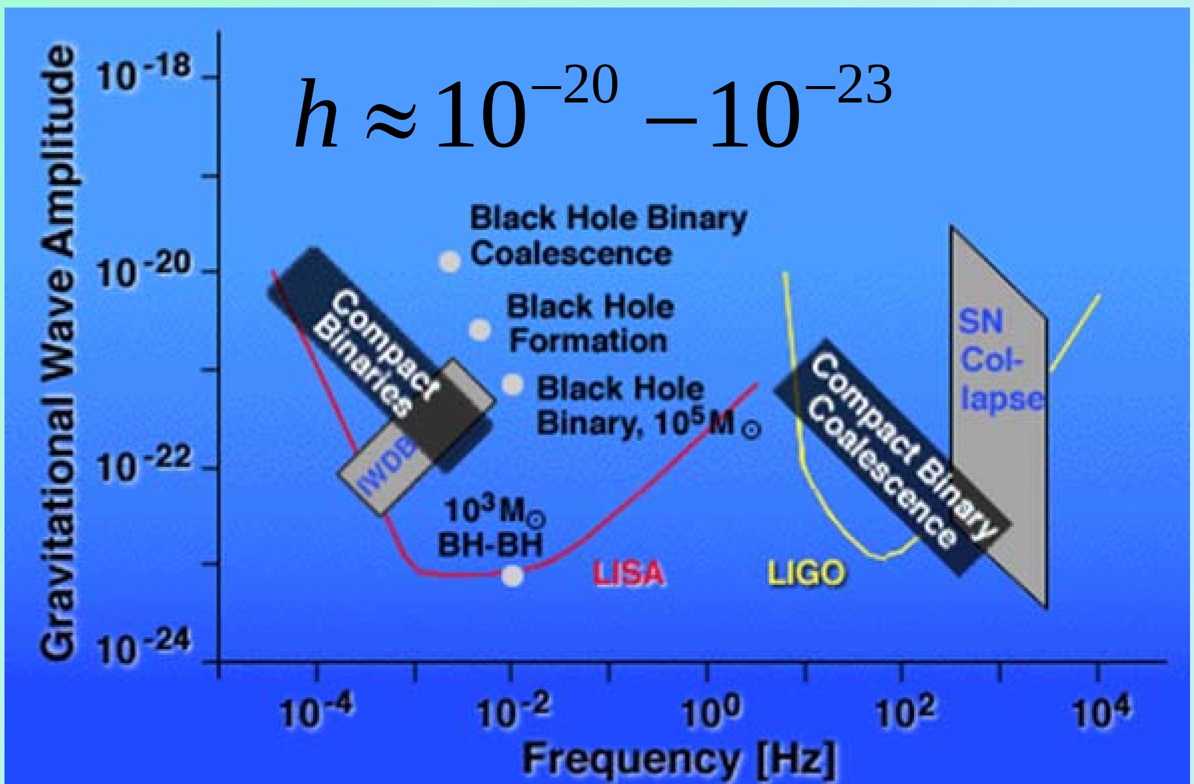
GW Source for LISA



Coalescence of massive black holes during collisions between galaxies, perhaps in formation of massive black holes, probing the central engines powering quasars.

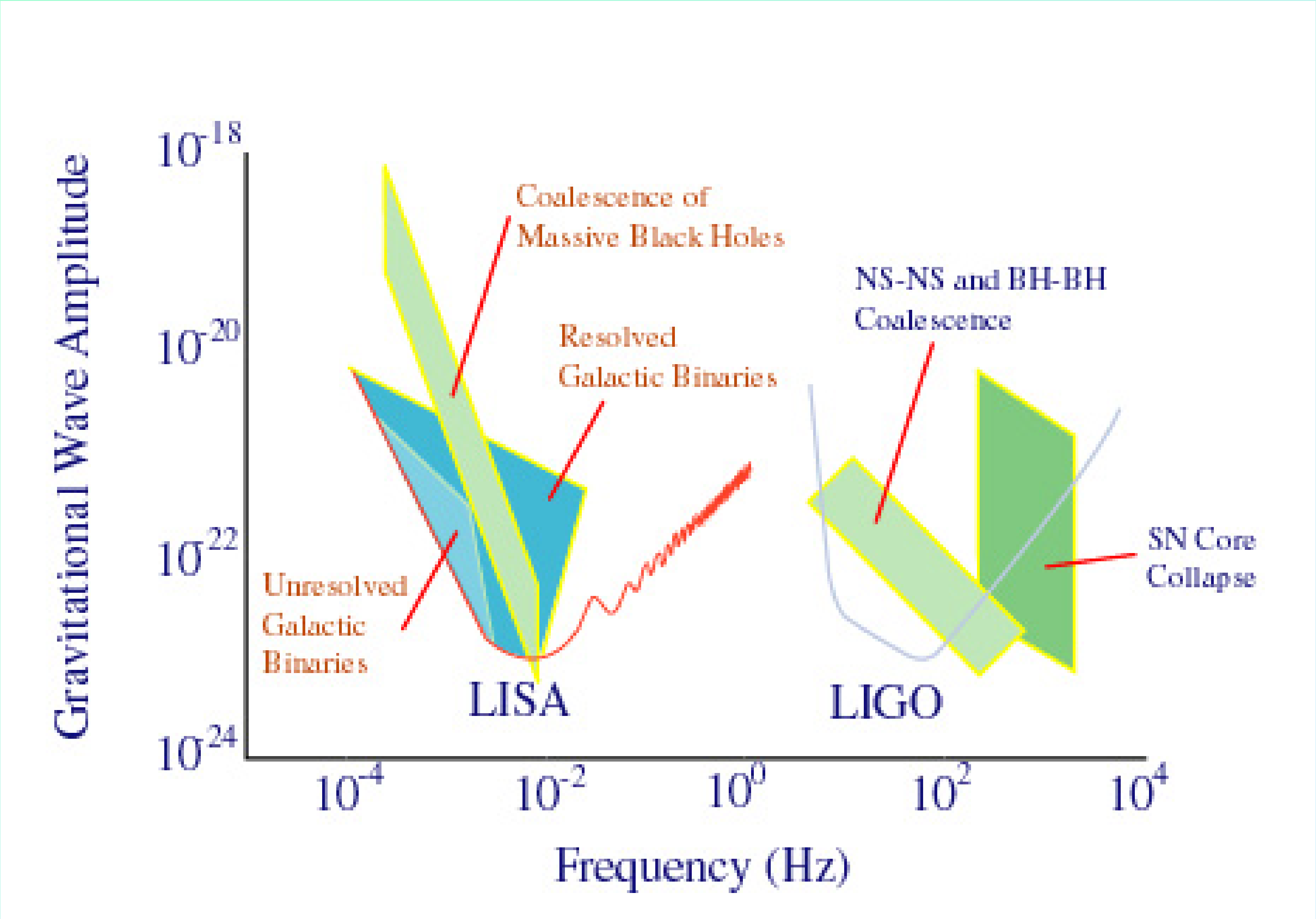
Black holes orbiting massive black holes, providing precision tests of gravitational theory in the high-field limit.

Hundreds of galactic binary star systems, many containing neutron stars or black holes, including several known binary systems.



$$h \approx 10^{-20} - 10^{-23}$$

•Terrestrial observatories and space observatories compliment each other



Technical Challenges of LISA

-- an extremely ambitious mission

1. Cancellation of Laser frequency noise.

- Laser frequency Fluctuations is orders of magnitude larger than the GW signal
- For terrestrial observatory, **equal arm length** of interferometer can be ensured ➔ **automatic cancellation** of laser frequency noise.
- For LISA, distances between spacecrafts do not remain constant.

Technical Challenges of LISA

-- an extremely ambitious mission

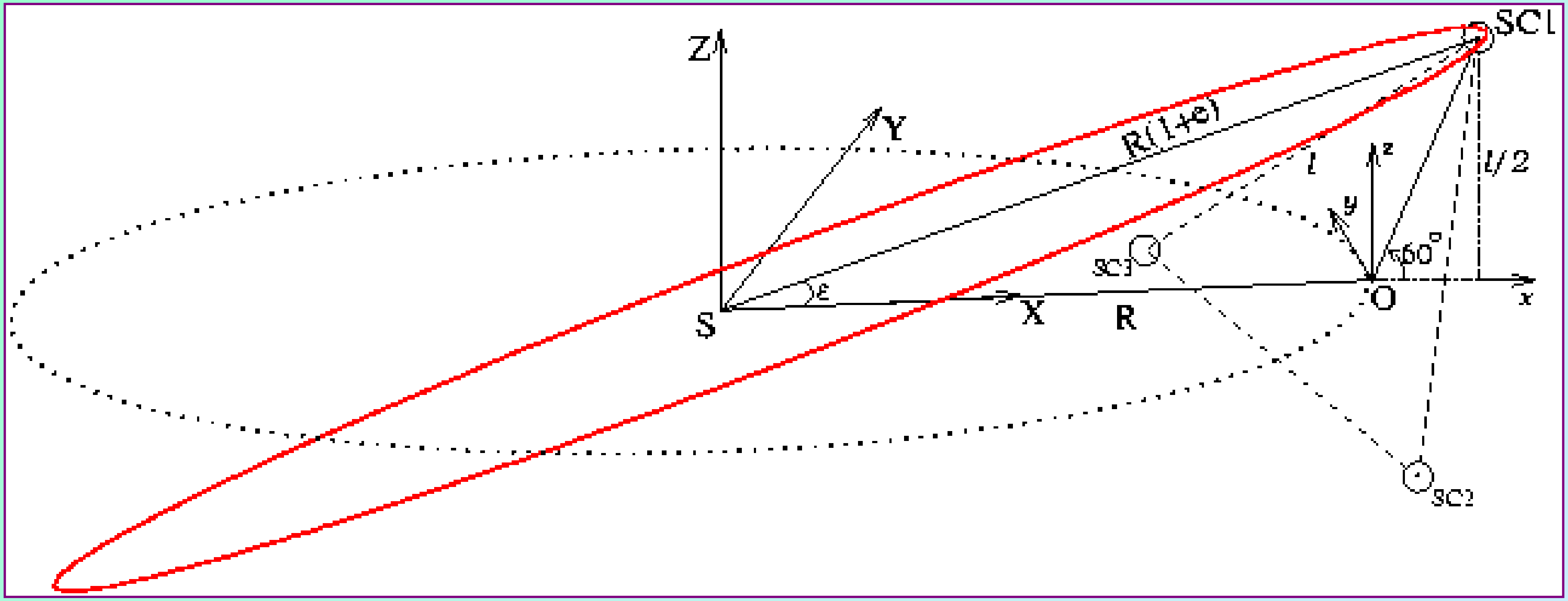
2. Stable flight formation & configuration

The spacecraft configuration needs to maintain **rigidity**

•Distances between spacecrafts should not change.

•We do detail analysis of orbits of LISA spacecrafts.

CW Frame and Equations



CW: Clohessy-Wiltshire eqns.

$$\ddot{x} - 2\Omega\dot{y} - 3\Omega^2 x = 0,$$
$$\ddot{y} + 2\Omega\dot{x} = 0,$$
$$\ddot{z} + \Omega^2 z = 0.$$

The general solution of CW equations

Reference orbit circular: Ω = constant

$$x(t) = \frac{\dot{x}_0}{\Omega}\sin\Omega t - (3x_0 + \frac{2\dot{y}_0}{\Omega})\cos\Omega t + 2(2x_0 + \frac{\dot{y}_0}{\Omega}),$$
$$y(t) = (6x_0 + \frac{4\dot{y}_0}{\Omega})\sin\Omega t + \frac{2\dot{x}_0}{\Omega}\cos\Omega t - 3(2\Omega x_0 + \dot{y}_0)t + (y_0 - \frac{2\dot{x}_0}{\Omega}),$$
$$z(t) = z_0\cos\Omega t + \frac{\dot{z}_0}{\Omega}\sin\Omega t.$$

Initial conditions: $x_o, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$

Offset terms: $2x_0 + \frac{\dot{y}_0}{\Omega}, \quad y_0 - \frac{2\dot{x}_0}{\Omega}$

Stability requirements

1. No offsets :

$$2x_0 + \frac{\dot{y}_0}{\Omega} = 0, \quad y_0 - \frac{2\dot{x}_0}{\Omega} = 0$$

2. Constant distance from the origin of the CW Frame

$$\Longrightarrow \quad \begin{aligned} x(t) &= \frac{1}{2}\rho_0 \cos(\Omega t - \phi_0), \\ y(t) &= -\rho_0 \sin(\Omega t - \phi_0), \\ z(t) &= \pm\frac{\sqrt{3}}{2}\rho_0 \cos(\Omega t - \phi_0), \end{aligned}$$

where

$$\rho_0 = \sqrt{4x_0^2 + y_0^2}, \quad \tan\phi_0 = \frac{y_0}{2x_0}$$

Orbits must lie in a plane making $\pm 60°$ angle with the ecliptic

Orbit equations

- First order in orbit eccentricity, e

Coordinates of spacecraft, *k*

$$\left\{ x_k(t), y_k(t), z_k(t) \right\}$$

$$x_k(t) = e \quad R \quad \cos\left[\Omega t - (k-1)\frac{2\pi}{3}\right]$$

$$y_k(t) = -2 \quad eR \quad \sin\left[\Omega t - (k-1)\frac{2\pi}{3}\right]$$

$$z_k(t) = \sqrt{3} \quad eR \quad \cos\left[\Omega t - (k-1)\frac{2\pi}{3}\right]$$

The distance between spacecraft

$$l_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2\right]^{\frac{1}{2}}$$

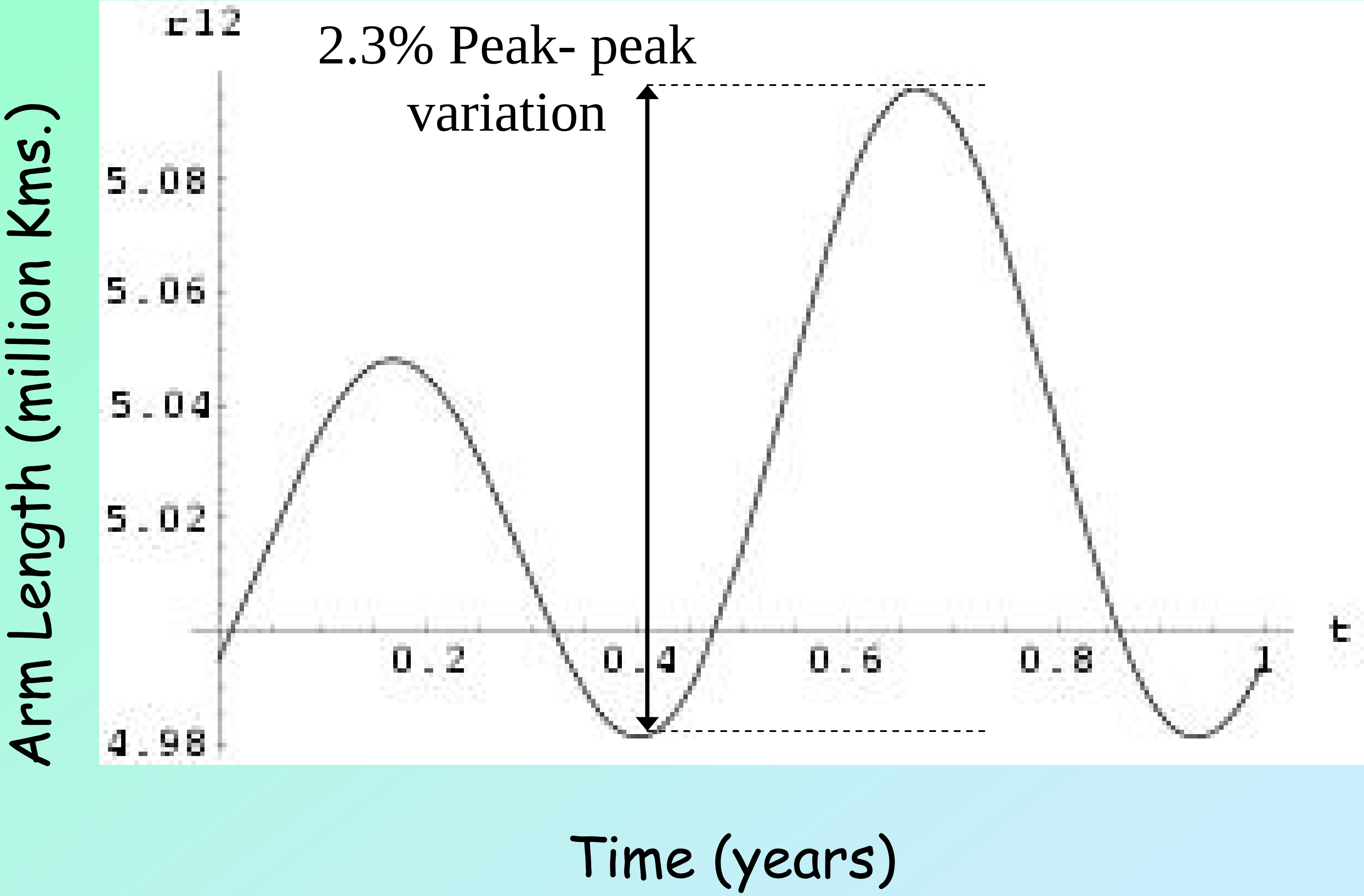
To the first order in eccentricity,
The distances between space craft
Remain constant.

But

To the second order in eccentricity,
The distances between space craft change

This is called arm flexing.

'Breathing modes' for 60± tilt



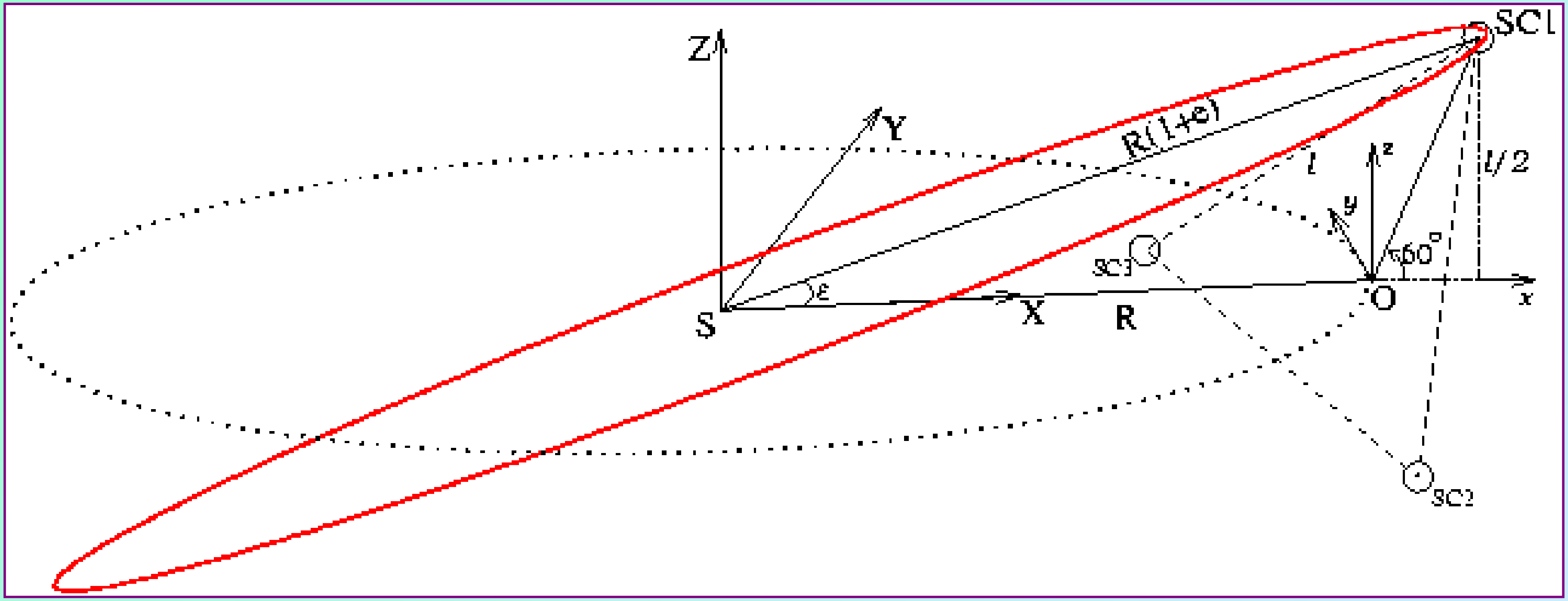
Tuning the tilt of LISA plane

We *readjust the tilt* of the plane of LISA triangle

from $60^{\pm} \rightarrow 60^{\pm}+\delta$

Then find the condition on δ .

Geometry of LISA



$$\alpha = \frac{l}{2R} = \frac{1}{60}, \qquad e = \frac{\alpha}{\sqrt{3}} + \frac{\alpha^2}{2} - \alpha\delta, \qquad \varepsilon = \alpha - \frac{\alpha^2}{\sqrt{3}} + \frac{\alpha\delta}{\sqrt{3}}$$

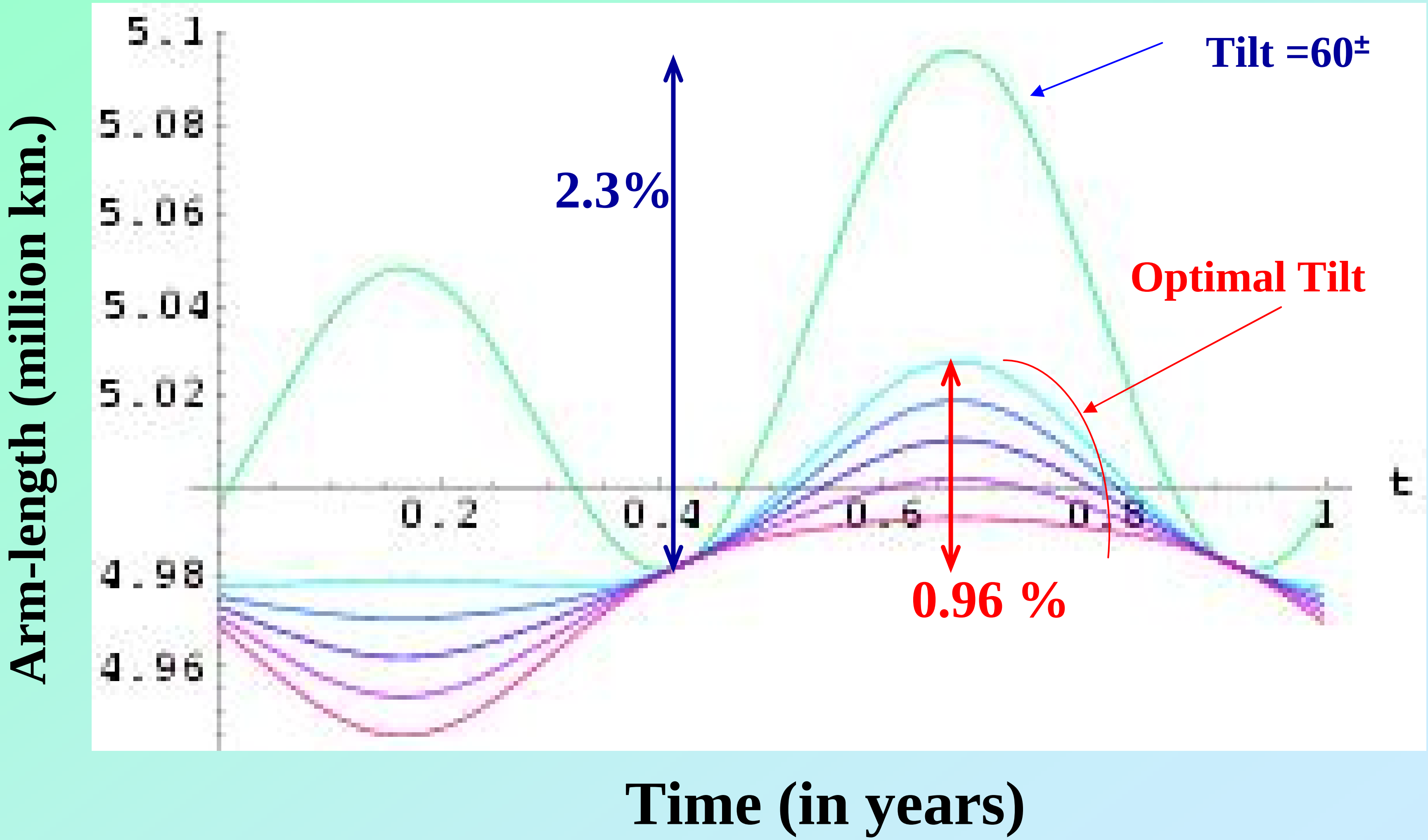
$$l_{12}(t,\delta) = l + \Delta l_{12}(t,\delta)$$

$$\Delta l_{12}(t,\delta) = \frac{\alpha^2 R}{16\sqrt{3}}[48(\frac{3}{8} - \delta_1) - 15\cos\theta + 48(\frac{5}{8} - \delta_1)\cos 2\theta - \cos 3\theta)]$$

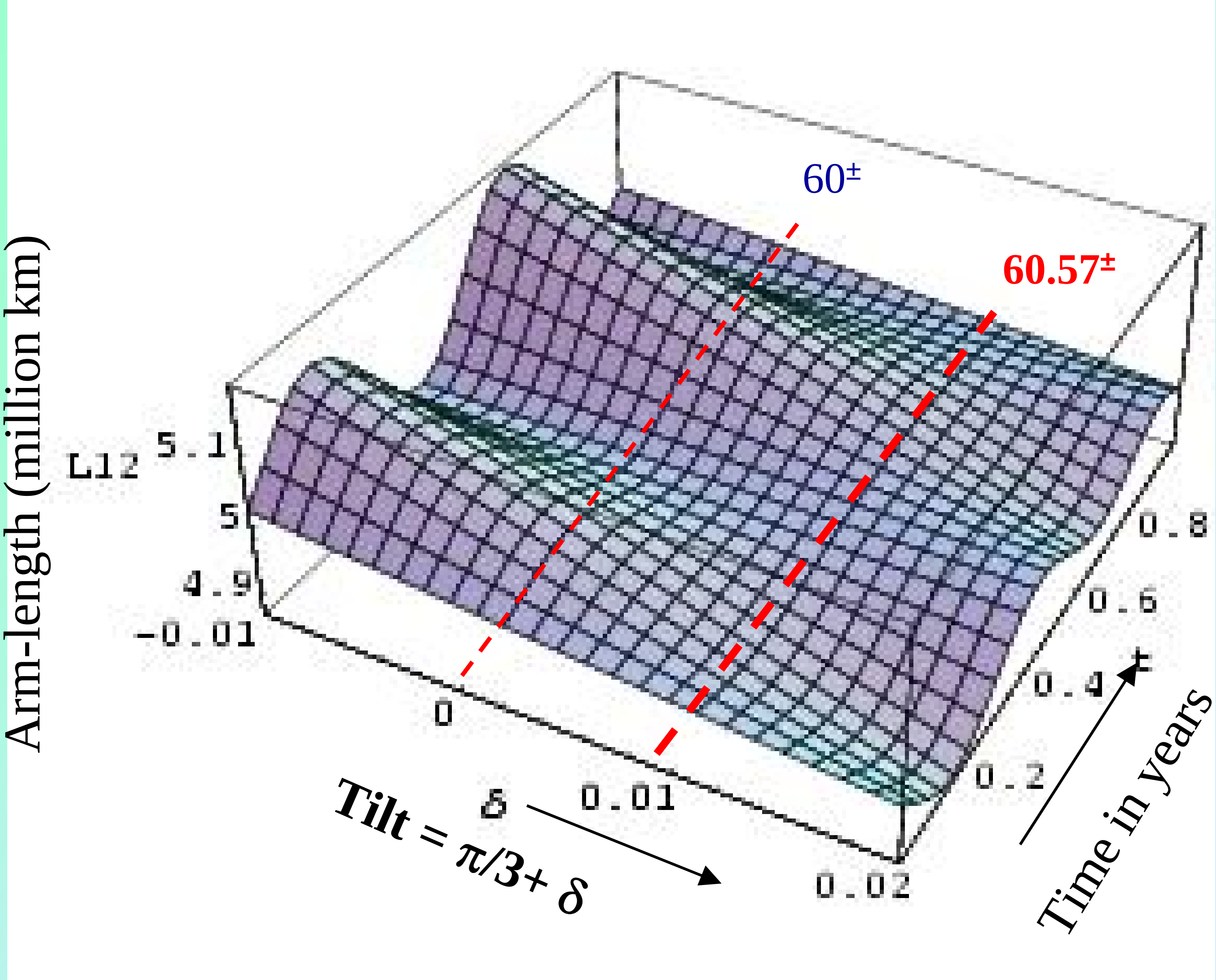
$$\delta = \alpha\delta_1 \quad \textit{and} \quad \theta = 2\pi t - \frac{\pi}{3}$$

$$\Delta l_{12}(t,\delta = 0) = \frac{\sqrt{3}}{32}\alpha l(6 - 5\cos\theta + 10\cos 2\theta - \frac{1}{3}\cos 3\theta)$$

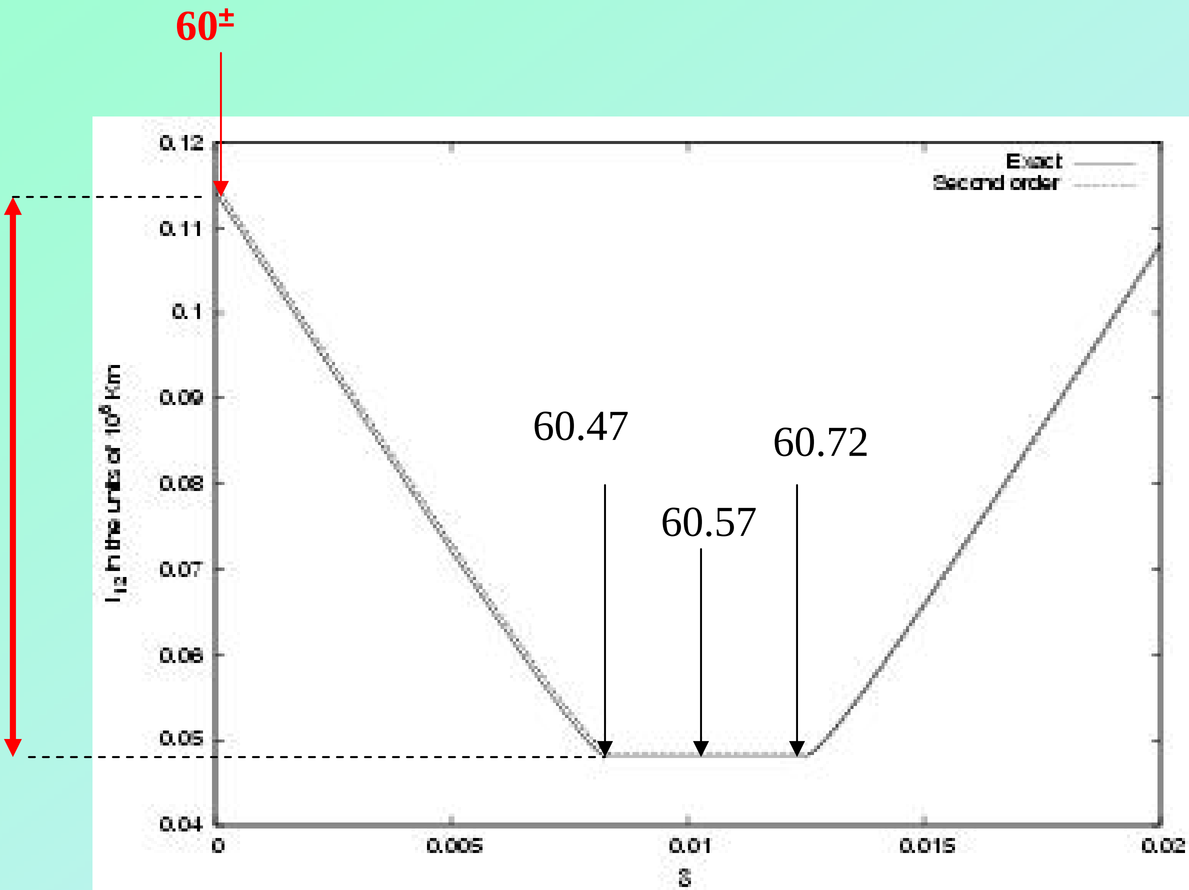
Optimal variation in arm-length vs. time



Arm-length vs. time and tilt



Maximum Variation of arm length for different tilts

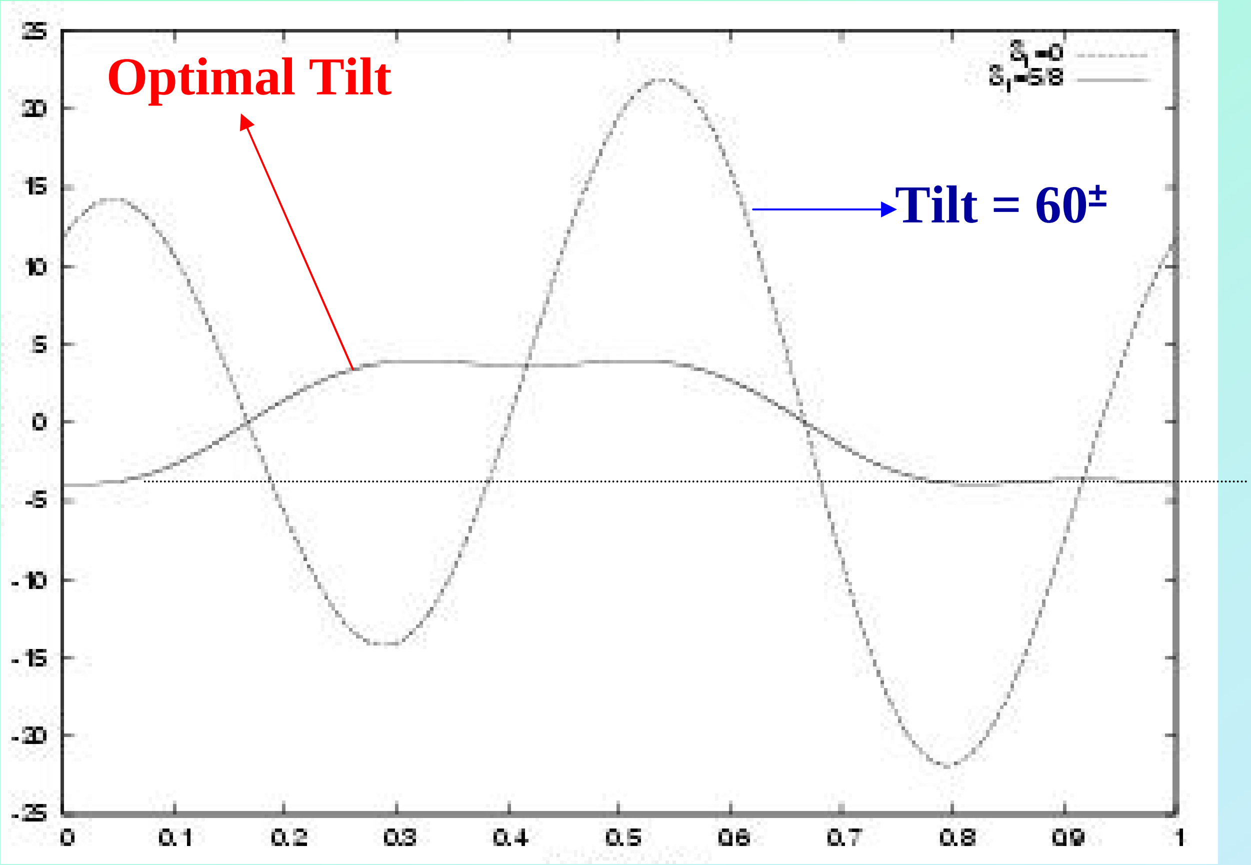


Reduction by factor 5.5

- Variation is minimized around **Tilt angle = 60±** **+ 0.57±**

Relative velocities (➔ Doppler shifts)

Rel. velocity of spacecrafts (m/s)



Time (in years)

Optimal tilt: Maximum separation velocity is 5.6 times smaller!!

Doppler shifts reduction is more crucial than arm length variation

Main results

- We show that there *exist* heliocentric orbits which can form stable flight configurations
  - The inter-spacecraft distances remain constant during flight to the first order in eccentricity
  - The configuration moves rigidly.
- The configuration can have *any* number of spacecraft and arbitrary shape
  - Need not be an equilateral triangle (LISA).

Reducing flexing of arms of LISA

(Class.Quantum Grav. , 23(2006)1763-1778)

At higher order in eccentricity :

- **LISA arms slowly change over the time scale of a year. This is called as flexing of arms or breathing modes.**

- **The maximum flexing amplitude is ~115000 km (2.3% variation) if the configuration is in a 60± Plane with the ecliptic.**

- **The maximum flexing amplitude can be *reduced further* to ~48000 km (0.96% variation) by *readjusting the tilt* of the plane of LISA triangle.**

Significance beyond LISA

- The stable constellation can consist of **arbitrary number N** of spacecraft and can have **arbitrary shape**.
  - For LISA with N=3, the equilateral triangular configuration is not the only one.
    - » N=4 square configuration has been proposed for detecting stochastic GW background.
  - **Big Bang Observer (NASA: Beyond Einstein program)** would like to have several LISA like, but smaller (0.5 million km. arm lengths) constellations flown simultaneously.

Thank you !!!

➢References

➢Stable flight formation of LISA

S.V. Dhurandhar, R. Nayak ,S. Koshti & J-Y Vinet

Class.Quantum Grav.22(2005)481-487

On the minimum flexing of LISA's arms

R. Nayak ,S. Koshti, S.V. Dhurandhar, & J-Y Vinet

Class.Quantum Grav.23(2006)1763-1778