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Structured flat SFRW universe based on exact LTB junctions along the past light cone:

observational consequences

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Submitted to GRG

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outline

Our universe is not Homogeneous !

- FRW metric
- LTB metric
- Toy Model + Metric Singularities
- Choosing Appropriate Metric
- Modifies model >>> SFRW
- Observational Consequences
- Conclusion !

FRW Metric

Considering Homogeneity and Isotropy …

Scale Factor is only the function of t.

$$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]; \quad a = a(t)$$

Einstein Equation results in FRW equations:

$$\begin{cases} (\frac{\dot{a}}{a})^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \\ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \end{cases}$$

LTB Metric

Considering the Isotropy … **not Homogeneity** …

Scale Factor will be the function of t and r.

$$\begin{cases} ds^2 = -dt^2 + \dfrac{R'^2}{1+2E(r)} dr^2 + R^2(r,t) d\Omega^2; \quad a = a(r,t) \\ R(r,t) = r.a(r,t) \qquad E(r) > -1/2 \end{cases}$$

R: Physical distance

r: Co-moving distance

The overdot and prime denote partial differentiation with respect to t and r.

Biswas,Mansouri,Notari (astro-ph/0606703 v2)

LTB vs FRW

FRW

$$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]; \quad a = a(t)$$

$$\begin{cases} ds^2 = -dt^2 + \dfrac{R'^2}{1+2E(r)} dr^2 + R^2(r,t) d\Omega^2; \quad a = a(r,t) \\ R(r,t) = r.a(r,t) \qquad E(r) > -1/2 \end{cases}$$

LTB

$$ds^2 = -dt^2 + a^2\left[(1+\frac{a'r}{a})^2 \frac{dr^2}{1-k(r)r^2} + r^2 d\Omega^2\right]$$

$$k(r) = \frac{-2E(r)}{r^2}$$

LTB Metric and Einstein Eq.

Einstein Equation results in LTB equations:

$$\begin{cases} \frac{1}{2}\dot{R}^2(r,t) - \frac{GM(r)}{R(r,t)} = E(r) \\ 4\pi G\rho(r,t) = \frac{M'(r)}{R'R^2} \end{cases}$$

$$M(r) = \frac{4}{3}\pi r^3 \rho_c$$

Flat Universe

if $E(r) = 0$

$$R(r,t) = \left(\frac{9M(r)}{2}\right)^{1/3}\left[t - t_n(r)\right]^{2/3}$$

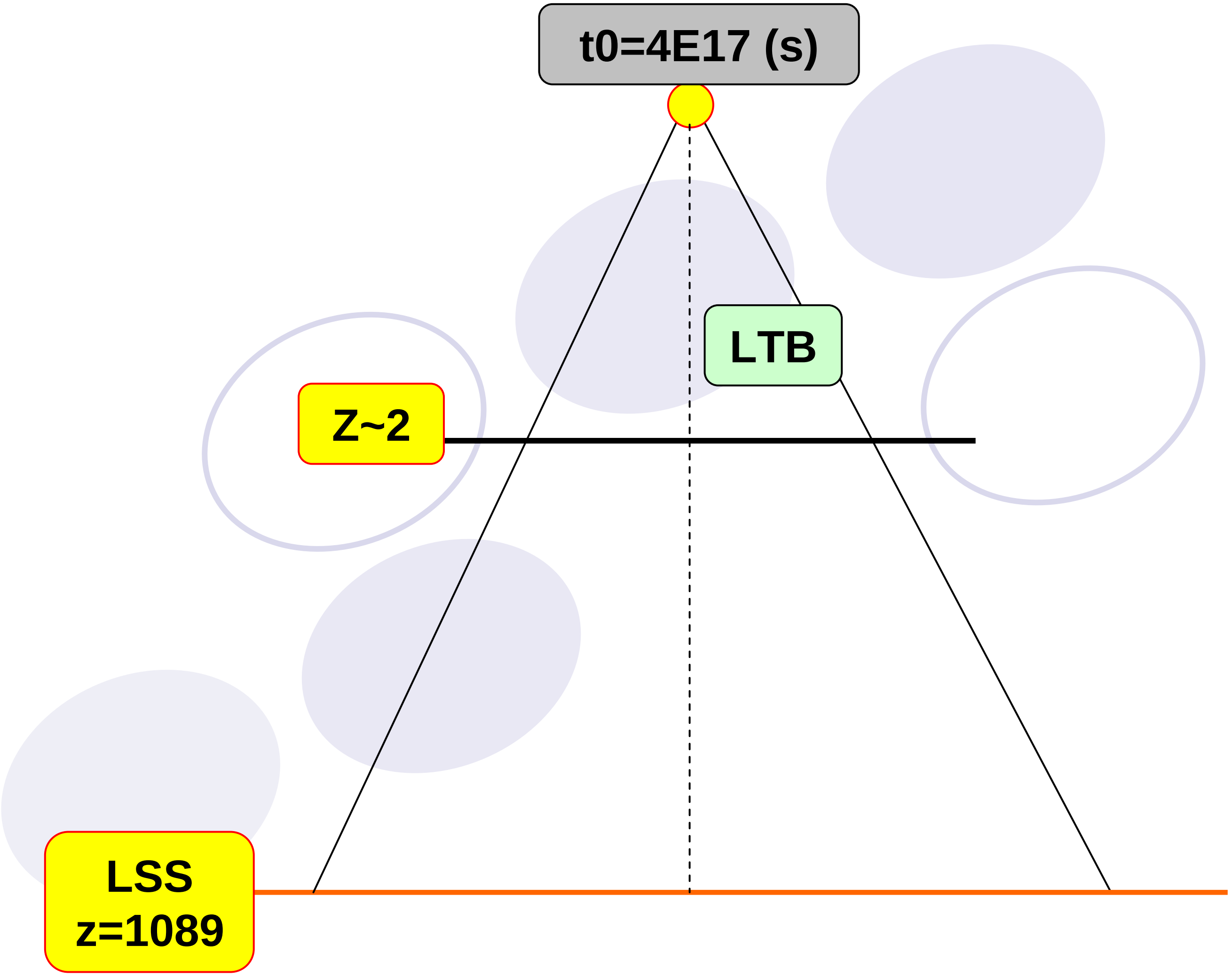
t_n(r) could be any arbitrary function of r.

LTB Metric for Flat Universes …

$$R(r,t) = \left(\frac{9M(r)}{2}\right)^{1/3}\left[t - t_n(r)\right]^{2/3}$$

$$\begin{cases} M(r) = 4\pi \displaystyle\int_0^{R(r,t)} \rho(r,t) R^2 dR \equiv \frac{4\pi}{3}\bar{\rho}(r,t) R^3 \\ M(r) \equiv \dfrac{4\pi}{3}\rho_c r^3 \end{cases}$$

Toy Model



Moving on light cone

$$R(r,t) = (6\pi\rho_c)^{1/3} r\left[t - t_n(r)\right]^{2/3}$$

Some proper relations between Redshift (z) and space-time coordinates. *(M. Célérier astro-ph/9907206)*

$$\begin{cases} \dfrac{dr}{dz} = \dfrac{1}{(1+z)\dot{R}'(r_z,t_z)} \\ \dfrac{dt}{dz} = -\dfrac{R'(r_z,t_z)}{(1+z)\dot{R}'(r_z,t_z)} \end{cases}$$

$$D_L(z) = (1+z)^2 R(z)$$

Luminosity Distance ...

Associated cosmic parameters

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$H(z) = \left[\frac{d}{dz}\frac{D_L(z)}{1+z}\right]^{-1}$$

Deceleration Parameter

$$q(z) = -1 + \left[\frac{1+z}{H(z)}\right]\frac{dH(z)}{dz}$$

Effective state parameter

$$\omega \equiv \frac{2}{3}\left[q(z) - 1/2\right] = \frac{2(1+z)}{3}\frac{d}{dz}\left[\ln\frac{H(z)}{(1+z)^{2/3}}\right]$$

jerk

$$j = \frac{\ddot{a}}{a}\left(\frac{\dot{a}}{a}\right)^3 = \frac{(a^2H^2)''}{2H^2}$$

Bang time and singularities

Vanishing of each of the metric functions and its derivatives may lead to different singularities:

$$R, R', \dot{R}, \ddot{R}, \dot{R}', R'' = 0$$

$$1 + E(r) = 0$$

The event horizon singularity

$$R''(t, r = 0) = 0$$

$$t_n'(0) = 0$$

Central weak singularity

Bang time and singularities

$$
\begin{aligned}
R &= 0 \to t = t_n \ , \\
\dot{R} &= 0 \to t = t_n \ , \\
R' &= 0 \to t = t_n \ , \qquad t = t_n + \frac{2}{3} r t_n' \ , \\
\dot{R}' &= 0 \to t = t_n \ , \qquad t = t_n - \frac{1}{3} r t_n' \ , \\
R'' &= 0 \to t = t_n \ , \qquad t = \frac{2 r t_n t_n'' + 5 t_n t_n' - \frac{2}{3} r t_n' - t_n}{1 - 5 t_n - 2 r t''} .
\end{aligned}
$$

Choice of the bang time

$$t_n = \frac{\alpha}{p(r) + 1 + q(\frac{1}{r})},$$

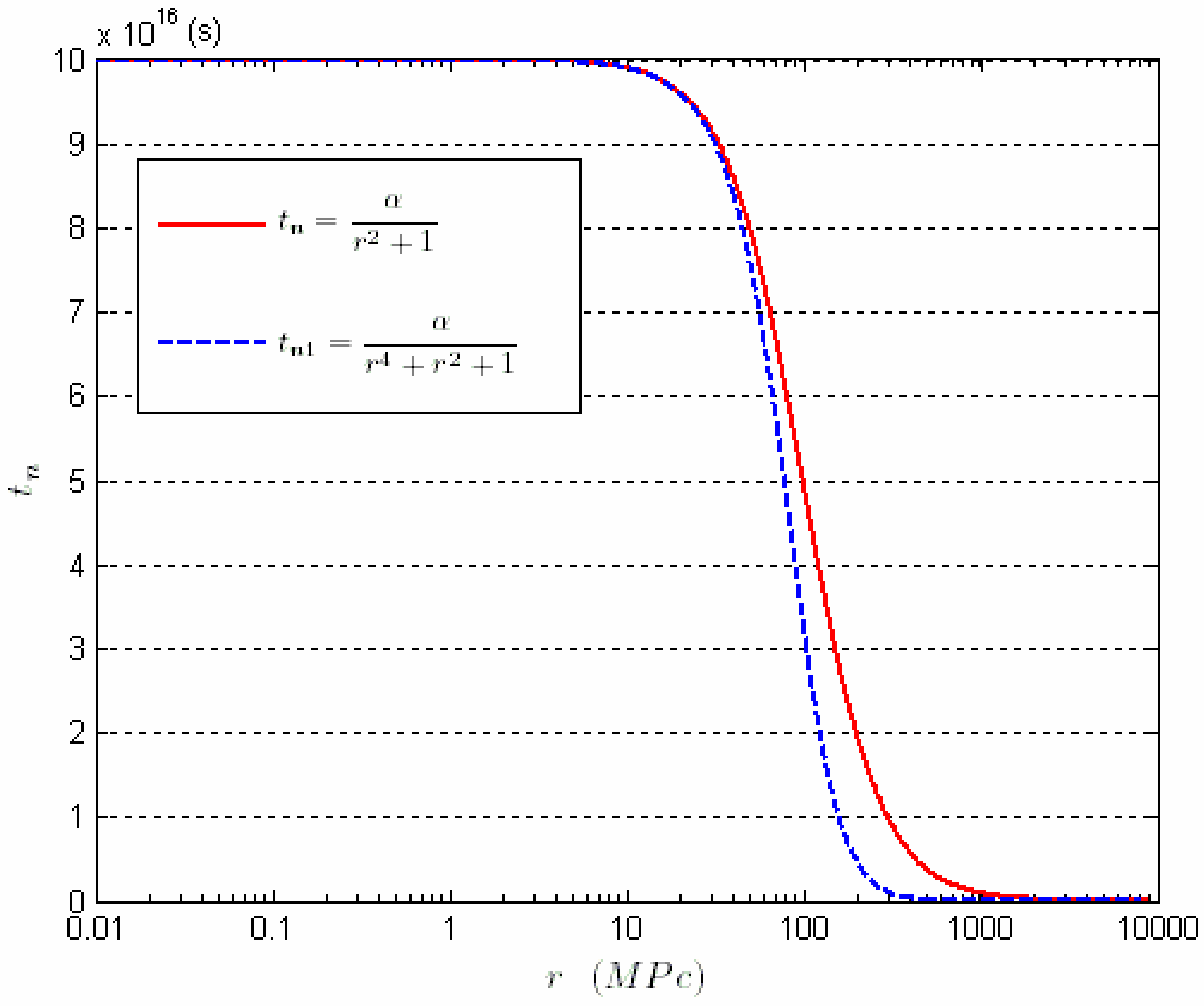
$$r \equiv \frac{r}{nL}$$

$$\begin{cases} n \sim 1 \quad inhomogeneity \quad scale \\ L \sim 100 \quad MPc \\ \alpha \sim 1.0E17 \quad s \end{cases}$$

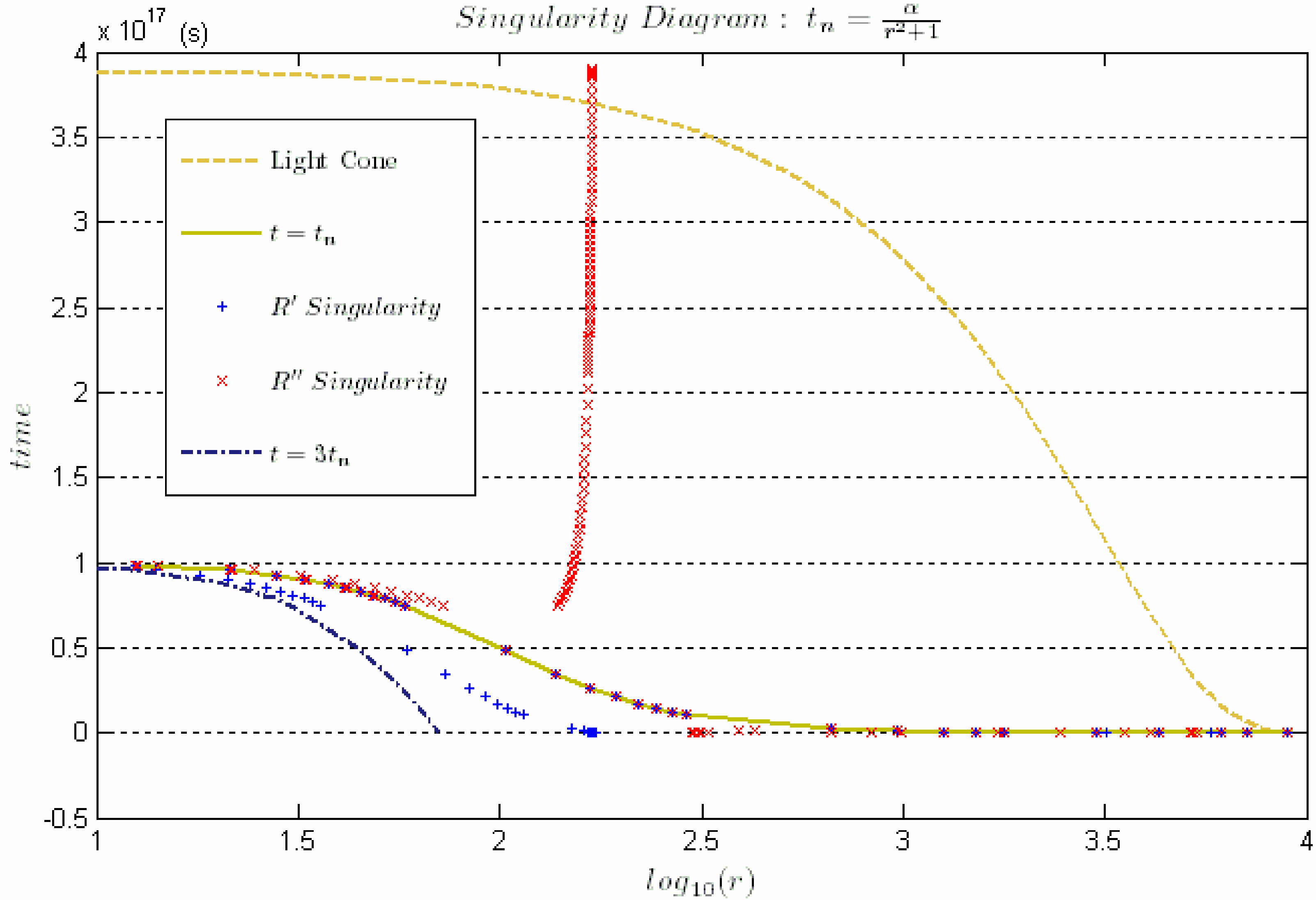
$$t_n = \frac{\alpha}{r^2 + 1}$$

$$t_{n1} = \frac{\alpha}{r^4 + r^2 + 1}.$$

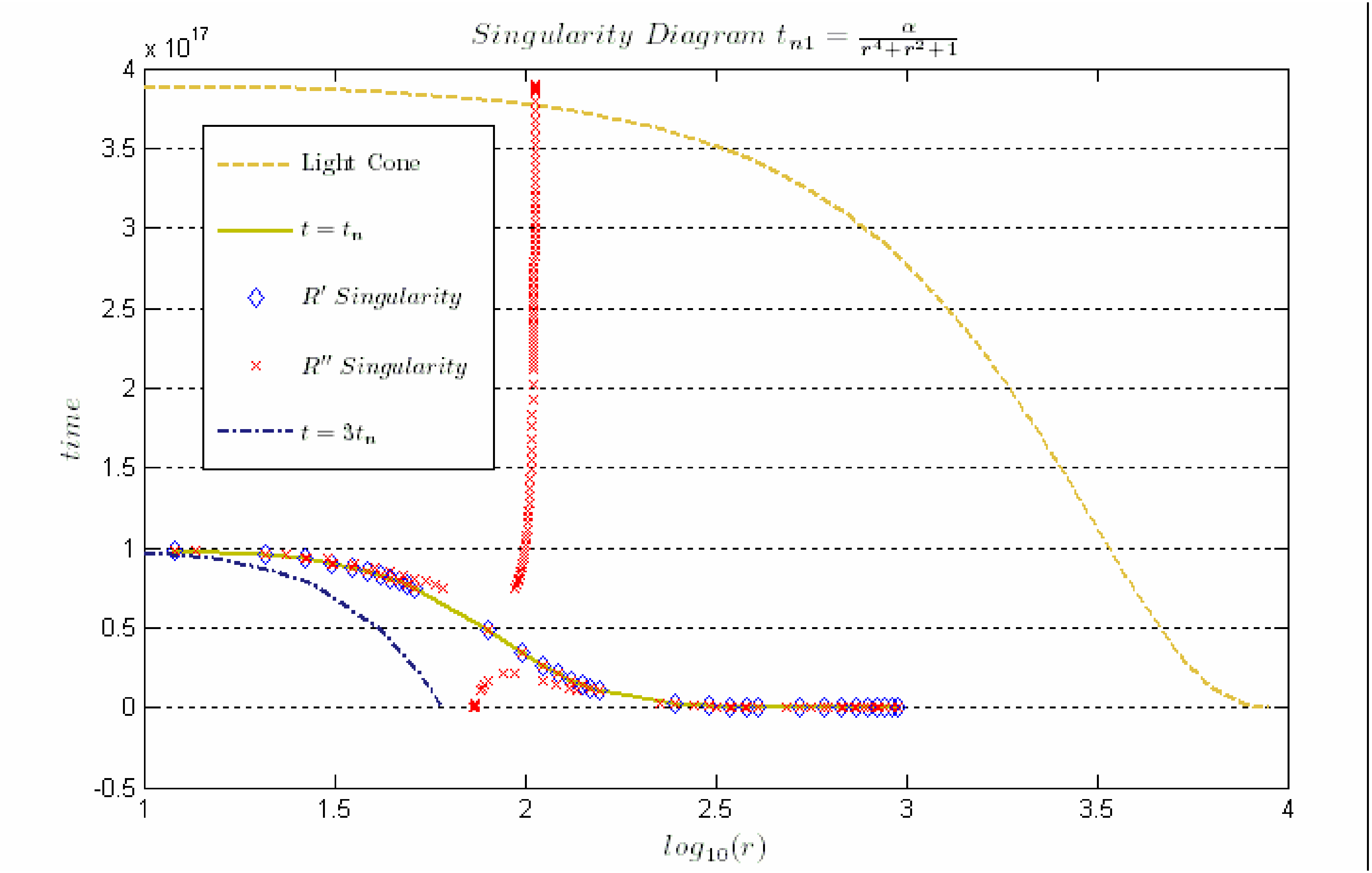
Choice of the bang time



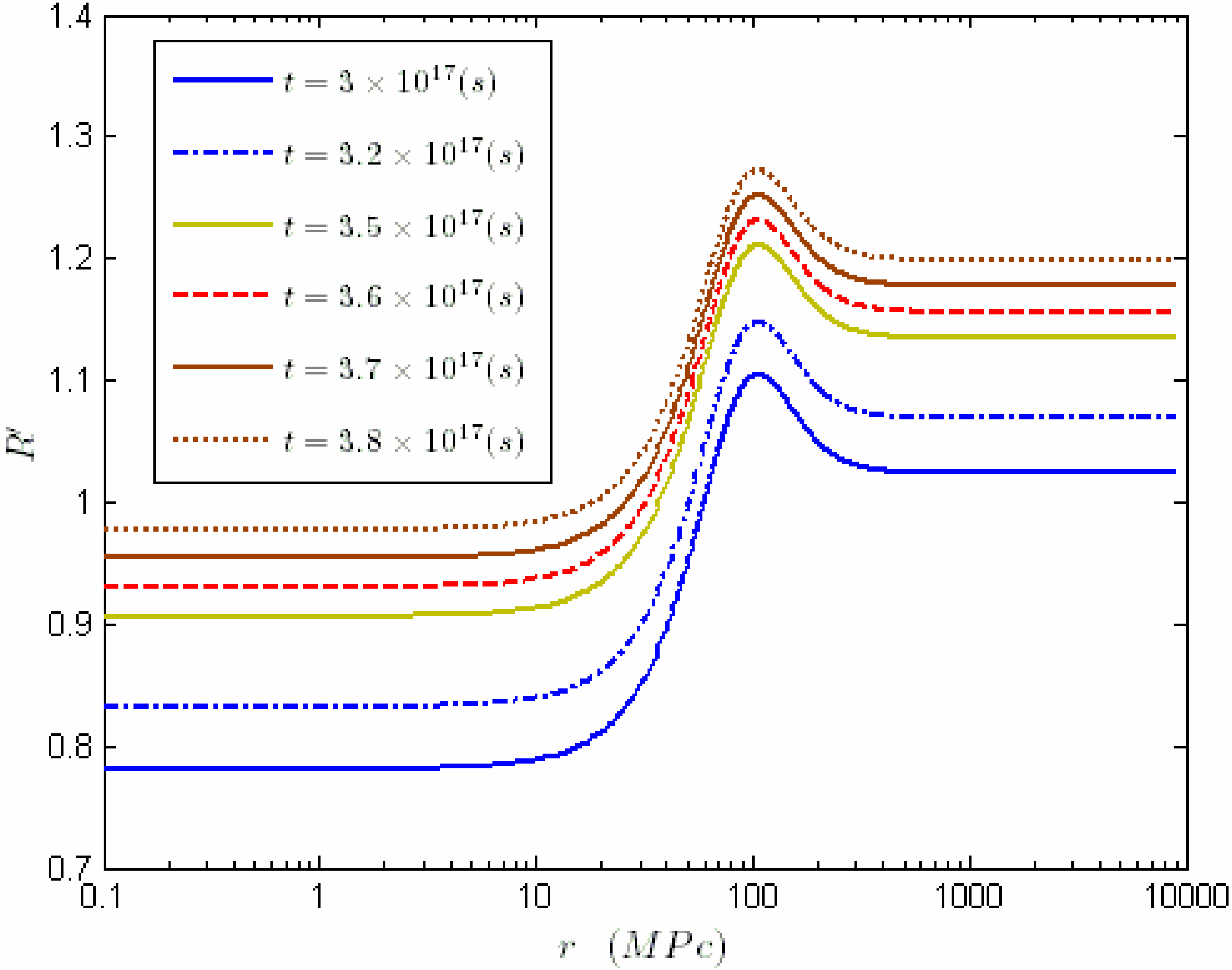
Singularity Diagram : t_n



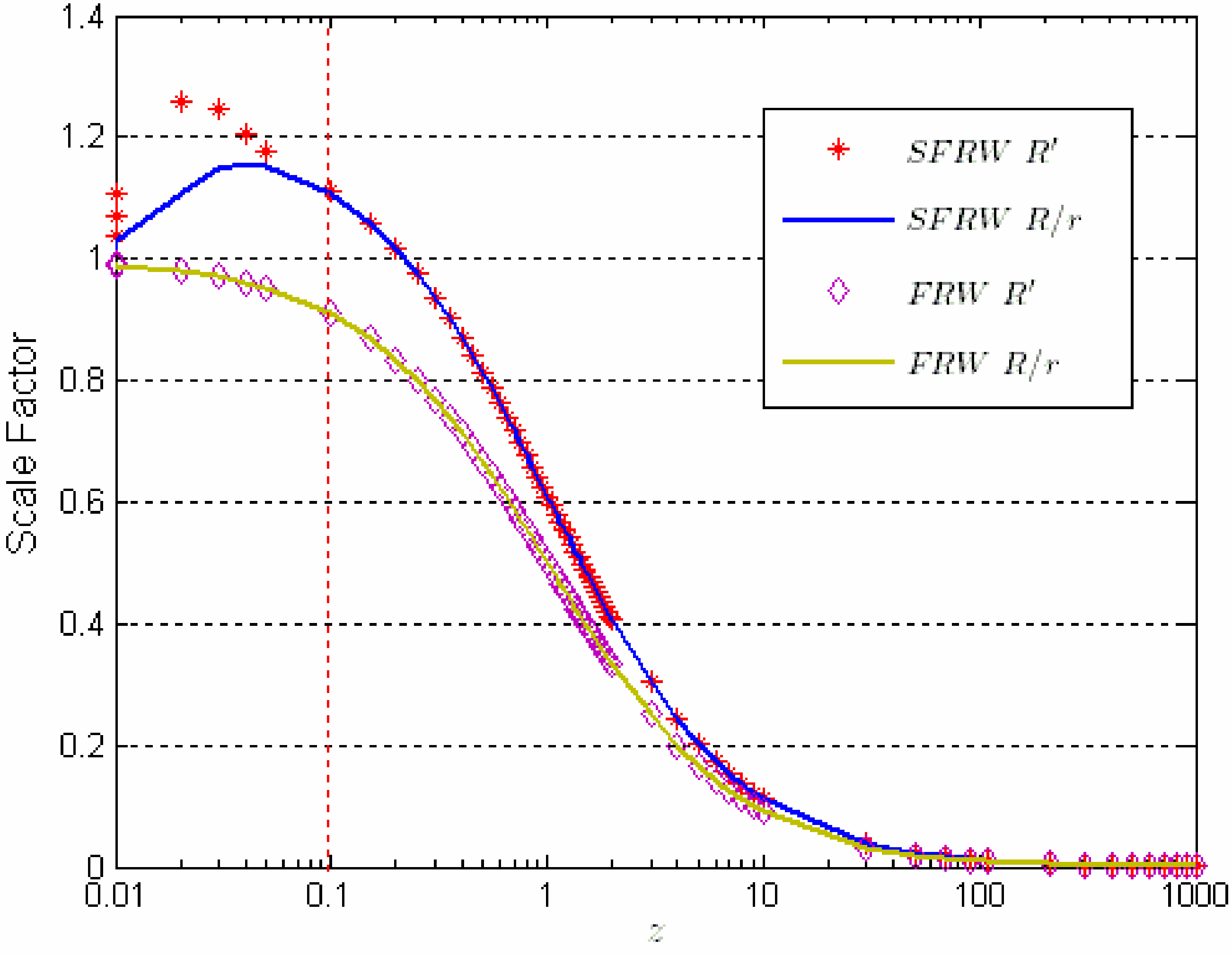
Singularity Diagram : t_{n1}



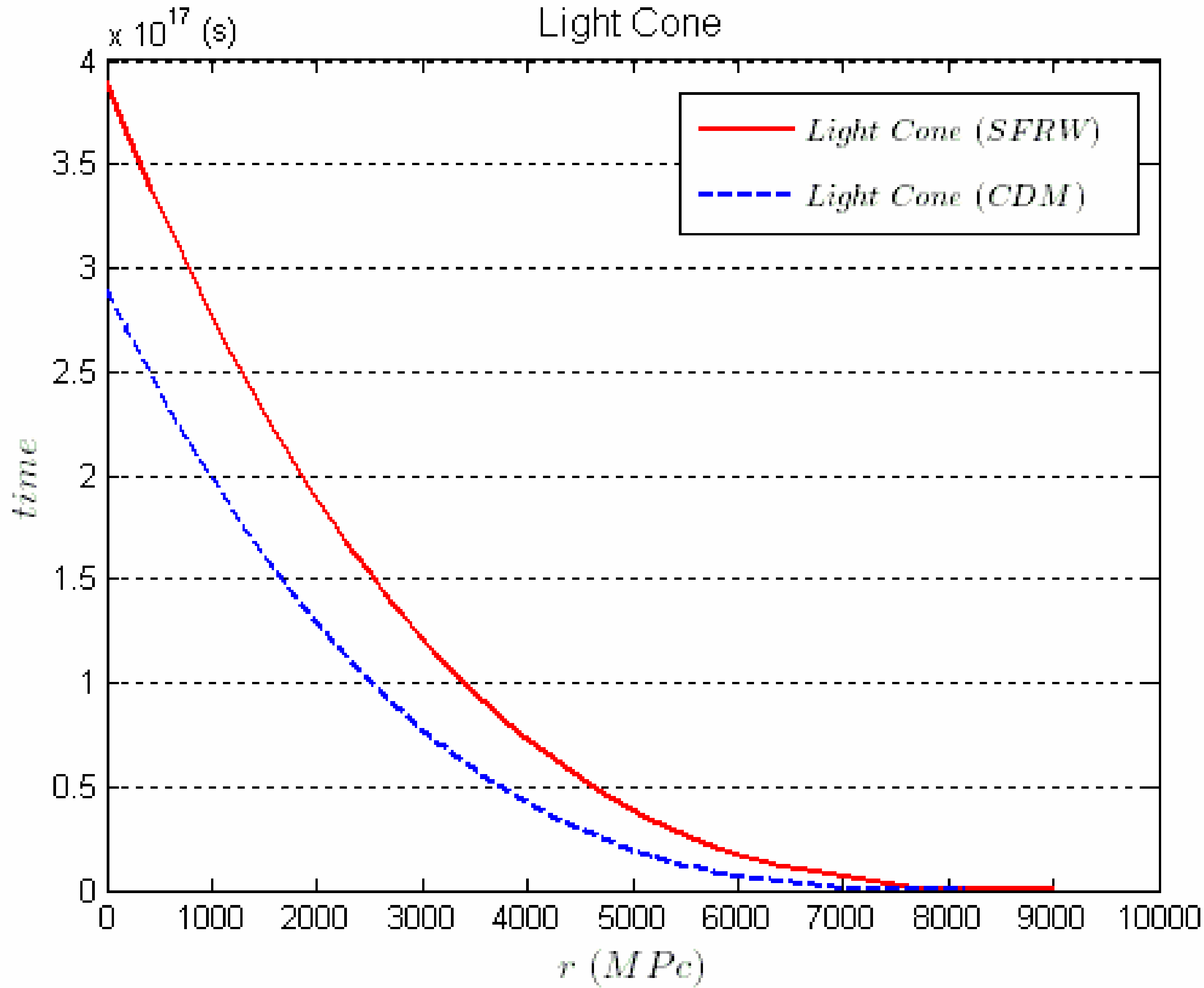
Cosmic Expansion : Singularities



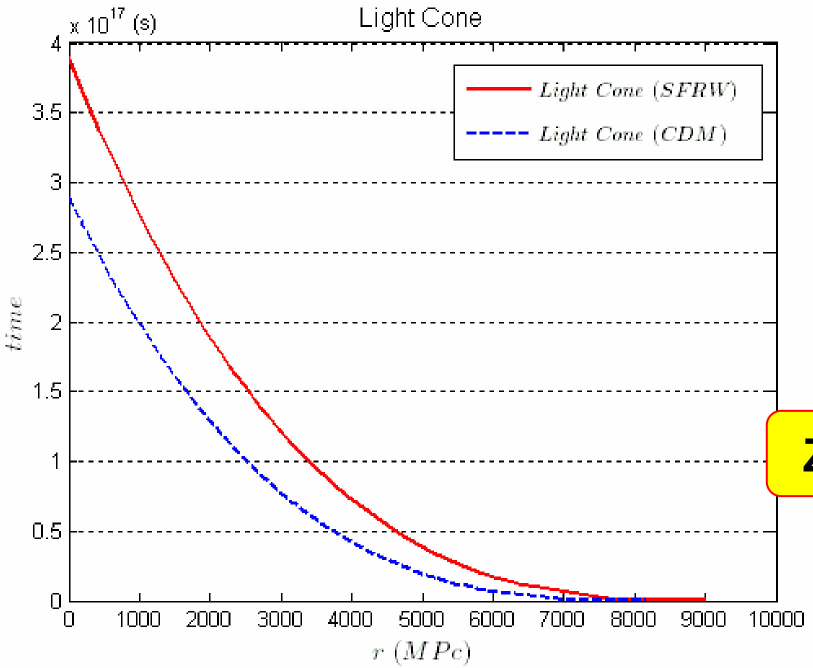
Scale Factor

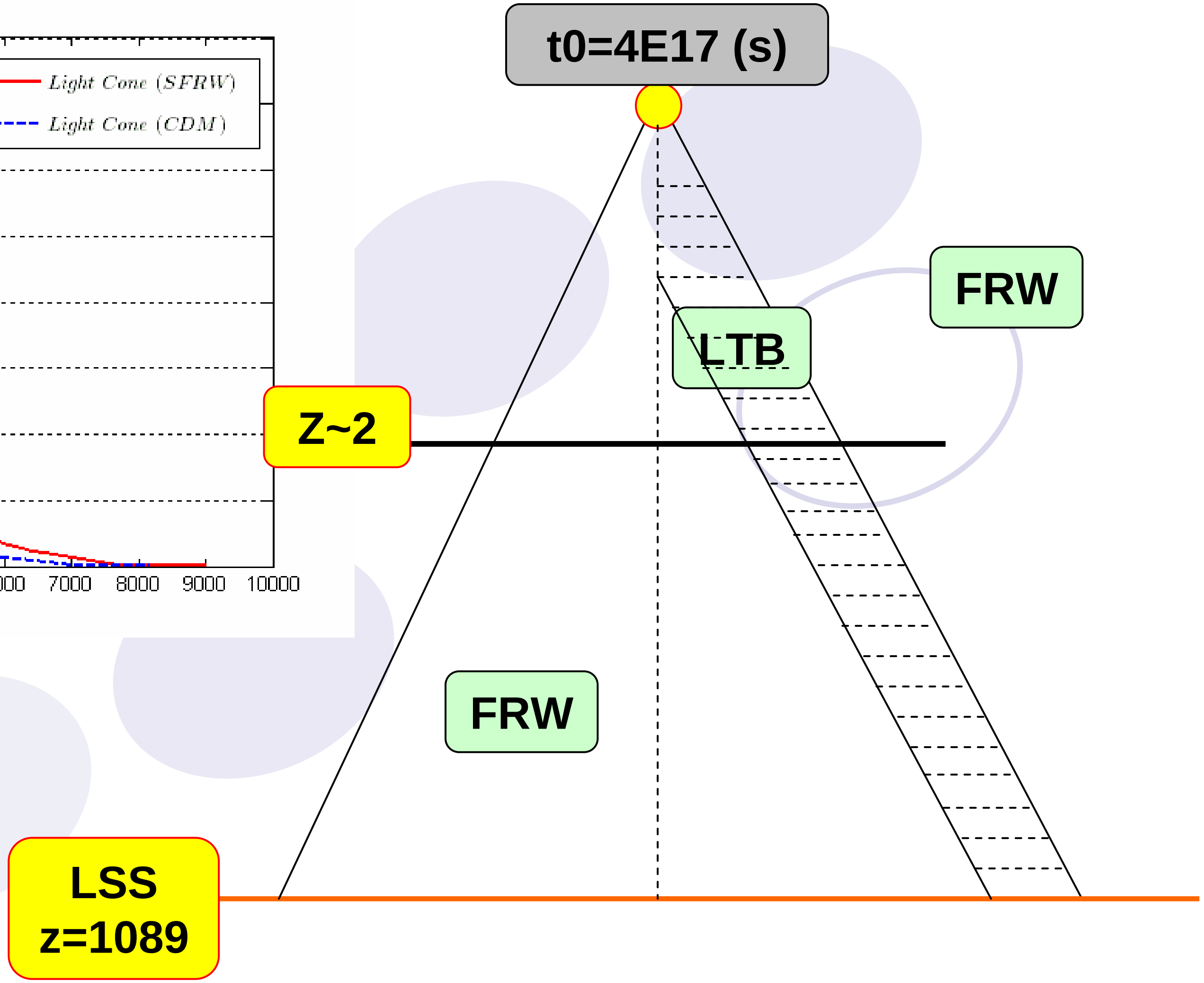


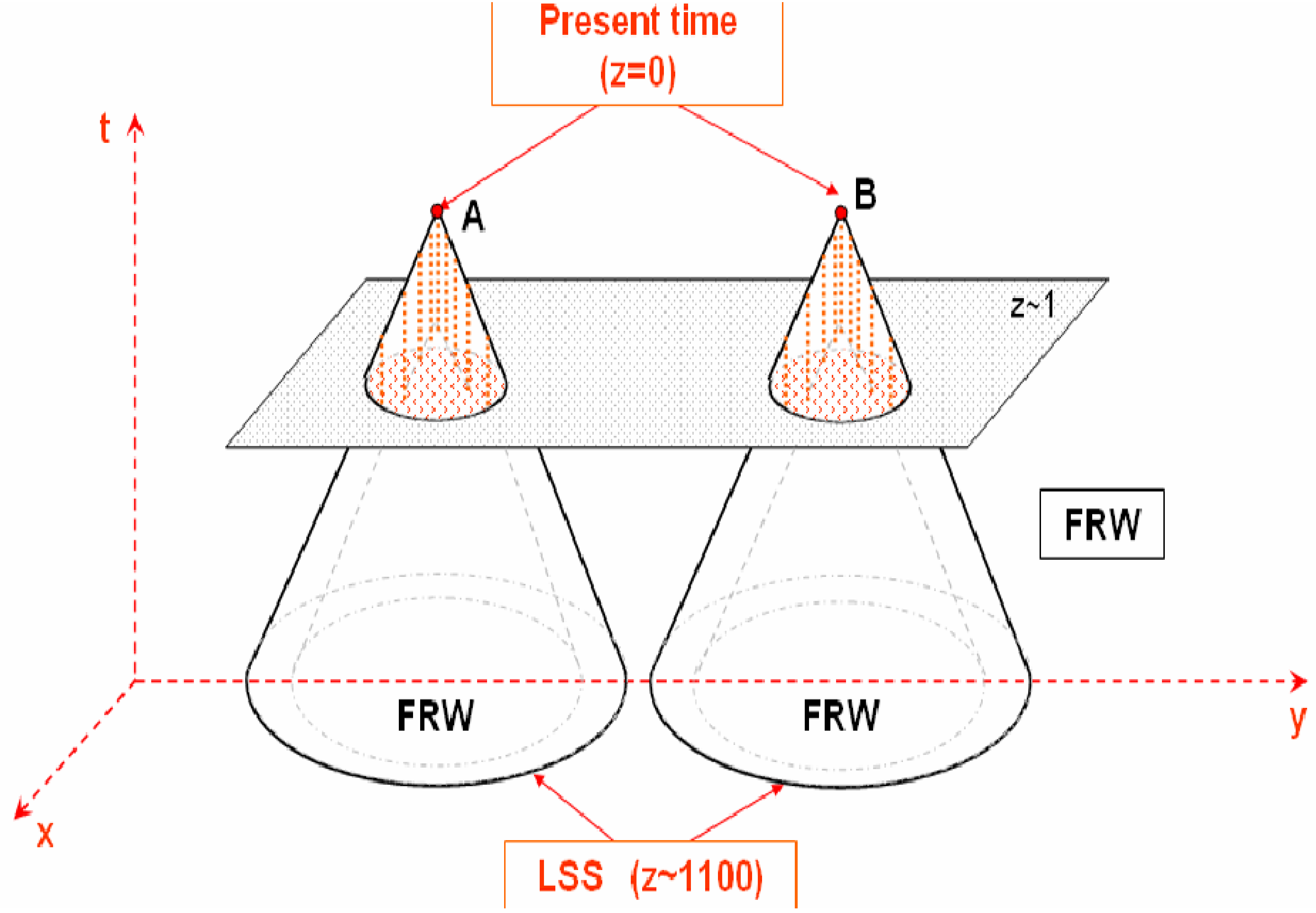
Light Cone


Light Cone
x 10^17 (s)
time
r (MPc)
Light Cone (SFRW)
Light Cone (CDM)
0
0.5
1
1.5
2
2.5
3
3.5
4
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

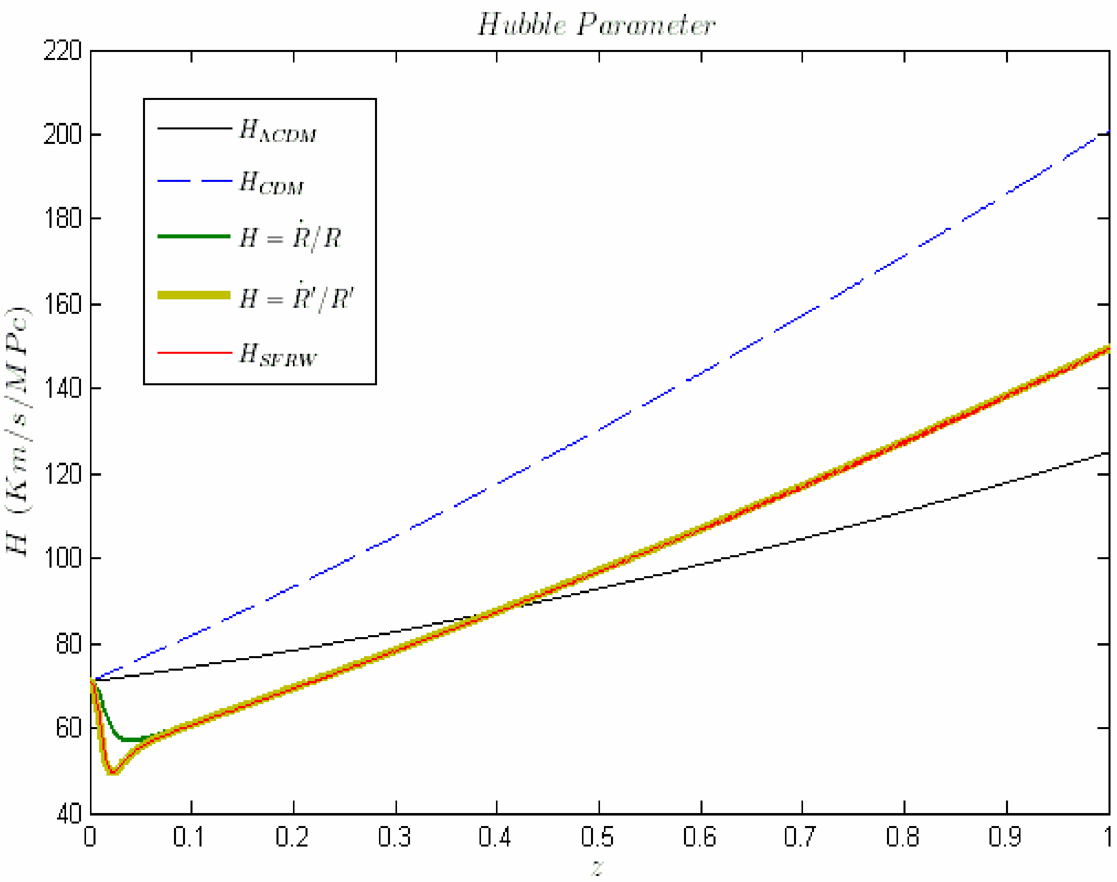
SFRW Model


Light Cone
x 10^17 (s)
time
r (MPc)
Light Cone (SFRW)
Light Cone (CDM)
0
0.5
1
1.5
2
2.5
3
3.5
4
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000


t0=4E17 (s)
FRW
LTB
Z~2
FRW
LSS
z=1089


Present time
(z=0)
t
A
B
z~1
FRW
FRW
FRW
y
x
LSS (z~1100)

Hubble Parameter



$$H_R = \frac{\dot{R}}{R}$$

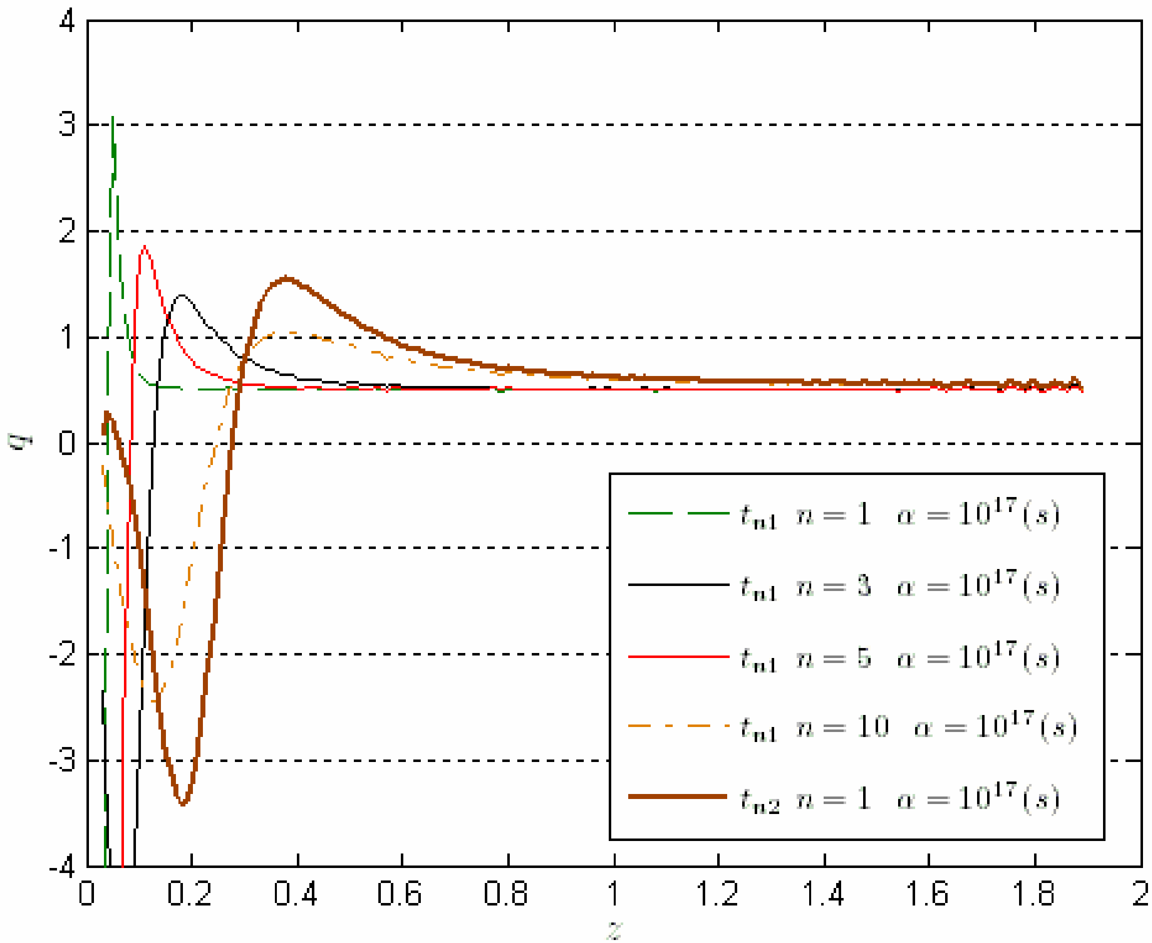
$$H_{R'} = \frac{\dot{R}'}{R'}$$

? $\Rightarrow \quad H_R = H_{R'}$

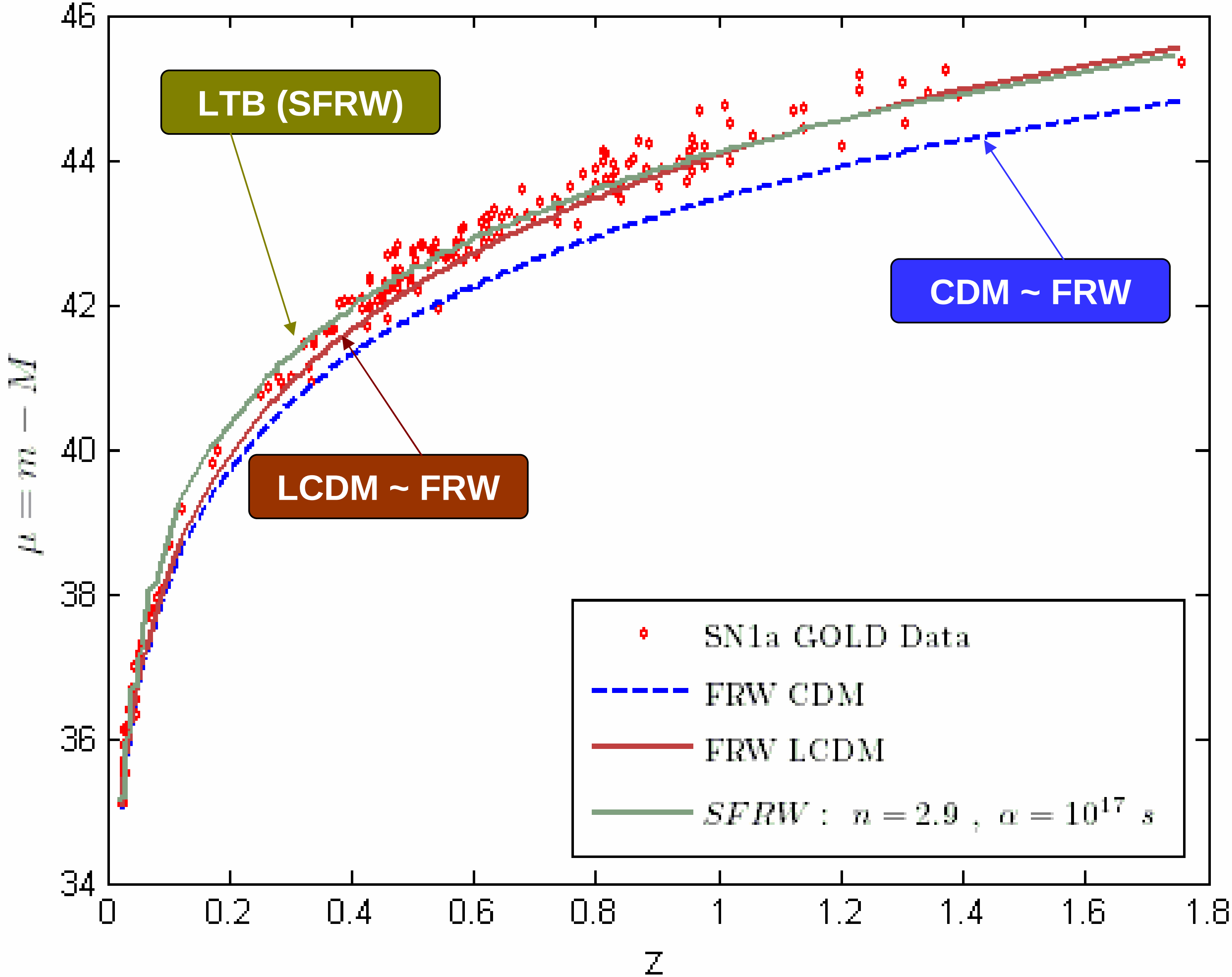
$$H(z) = \left[\frac{d}{dz}\frac{D_L(z)}{1+z}\right]^{-1}$$

deceleration Parameter: t_{n1}

using $t_{n1} = \frac{\alpha}{r^4+r^2+1}$ and $t_n = \frac{\alpha}{Ar^4+Br^2+1}$ for $A = 0.0001,\ B = 0.001.$



Using SN1a data as an observational Constraint …(182 data points – Gold Sample)



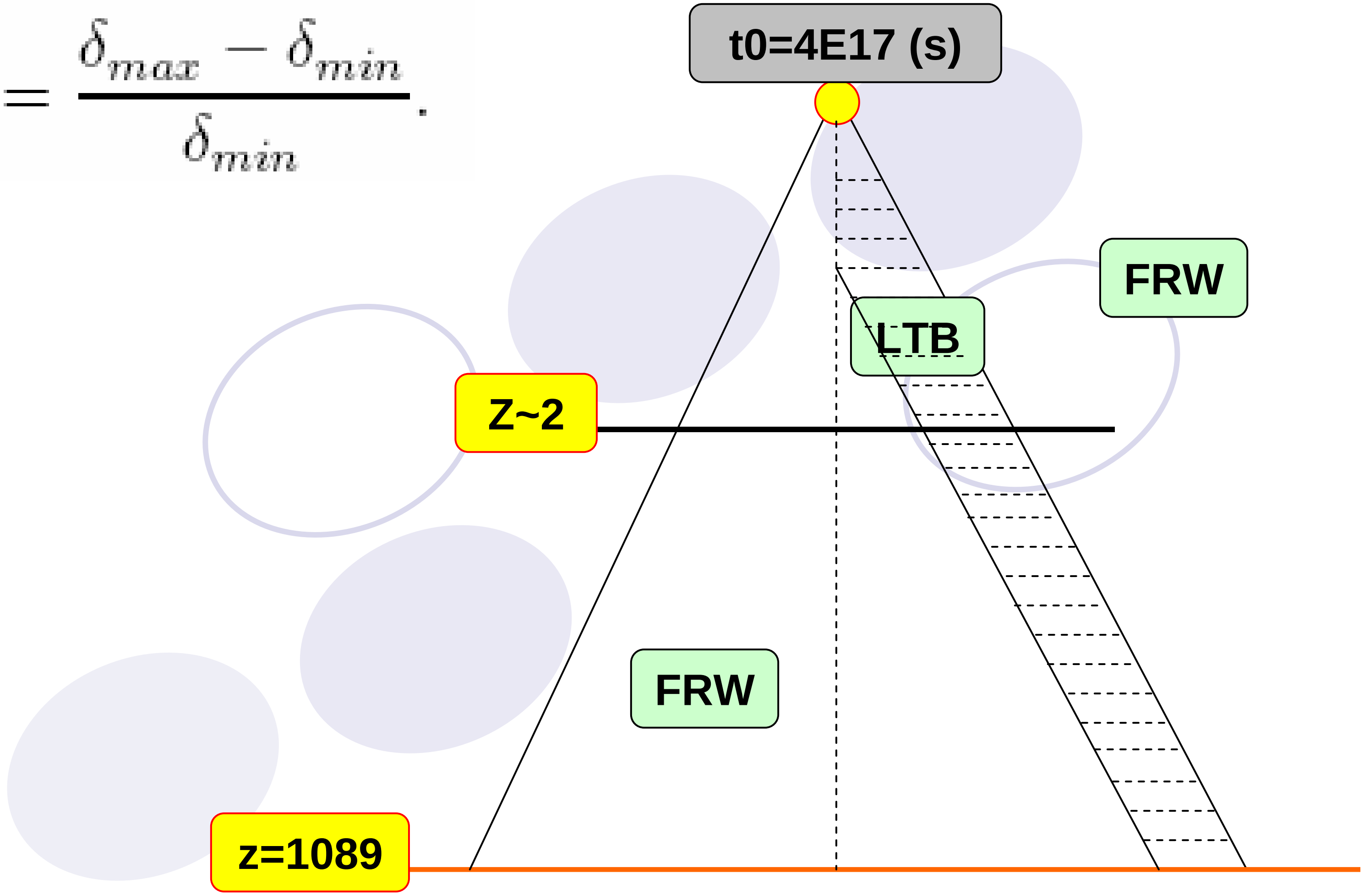
Luminosity Distance Analysis

$t_n = \frac{\alpha}{p(r)}$	χ^2 (for 182 SNe Ia)
$p(r) = r^4 + 1$	255
$p(r) = r^4 + r^2 + 1$	265
$p(r) = r^2 + 1$	226
$p(r) = r^2 + 1 + r^{-1}$	234
$p(r) = r^2 + r + 1 + r^{-1} + r^{-2}$	239
$p(r) = r^2 + 1 + r^{-1} + r^{-2}$	251
$p(r) = r + 1$	209
$p(r) = r + 1 + r^{-1}$	220
$p(r) = r + 1 + r^{-1} + r^{-2}$	226
$p(r) = 0.0001r^4 + 0.001r^2 + 1$	286
CDM	1700
$\Lambda CDM \quad (\Omega_\Lambda = 0.7)$	393
$n = 1,\ \alpha = 10^{17}s,\ t_0 = 3.9 \times 10^{17}s$	

Bang time	$\alpha\ (10^{17}s)$	n	χ^2 (for 182 SNe Ia)
$t_n = \alpha(r^2 + 1)^{-1}$	1.1	2.2	188
$t_n = \alpha(r^4 + 1)^{-1}$	1.0	2.2	212
$t_{n1} = \alpha(r^4 + r^2 + 1)^{-1}$	1.0	2.9	198

Density Contrast

$$\delta = \frac{\delta_{max} - \delta_{min}}{\delta_{min}}.$$



Density Contrast

$$t_{n1} = \frac{\alpha}{r^4+r^2+1}$$

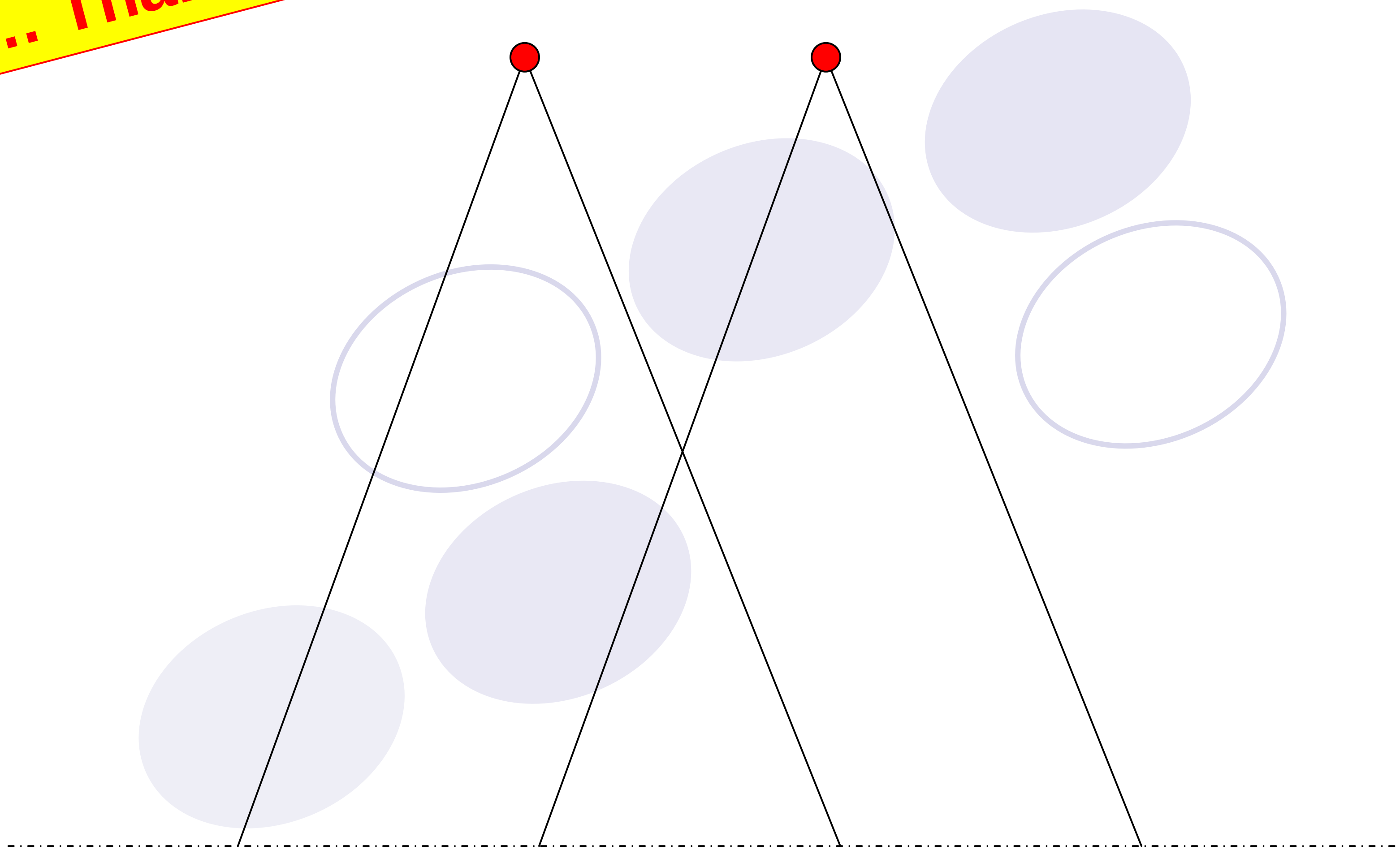
$\times 10^{17} s$	Inhomogeneity Factor	
$\alpha < 0.5$		*Not valid*
$\alpha = 1$	$n \sim 1$	$\delta(z \approx 1100) \sim 10^{-7}$
$\alpha = 1$	$2 < n < 3$	*Valid for both conditions*
$\alpha = 1.5$	$1 < n < 4$	*Valid for both conditions*
$\alpha = 2$	$3 < n < 4$	*Valid for both conditions*
$3 < \alpha < 5$	$4 < n < 5$	*Valid for both conditions*

The above results for t_{n1}, and those of the luminosity distance ($\alpha = 10^{17}\ s$), shows that the inhomogeneity factor n should be chosen in the range $1 < n < 3$. Note that the density contrast should not be greater than the desired order of magnitudes. In the opposite case, one can compensate the lack of contrast by adding some density perturbations as it is done in FRW models.

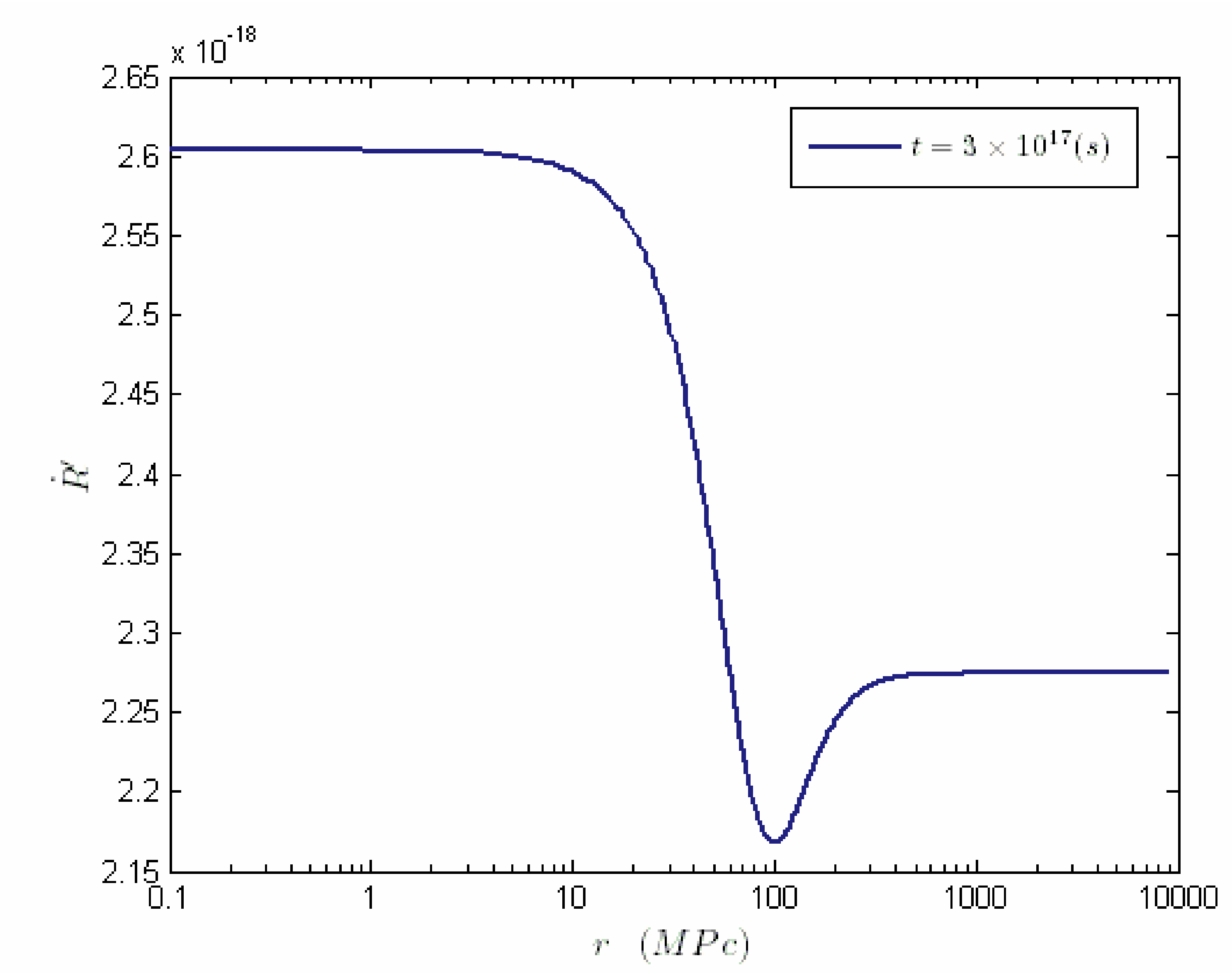
Conclusion !

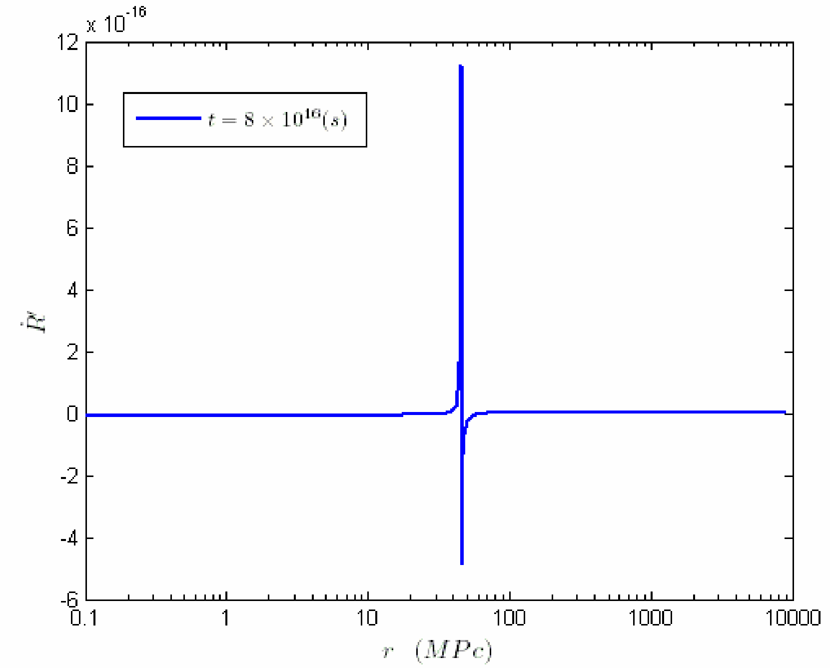
- Using LTB metric the effect of inhomogeneities is considerable
- At least, some portion of D.E. could be interpreted as inhomogeneities effect
- Every cosmological model including inhomogeneities should be understood by looking at past ligh cone
- …

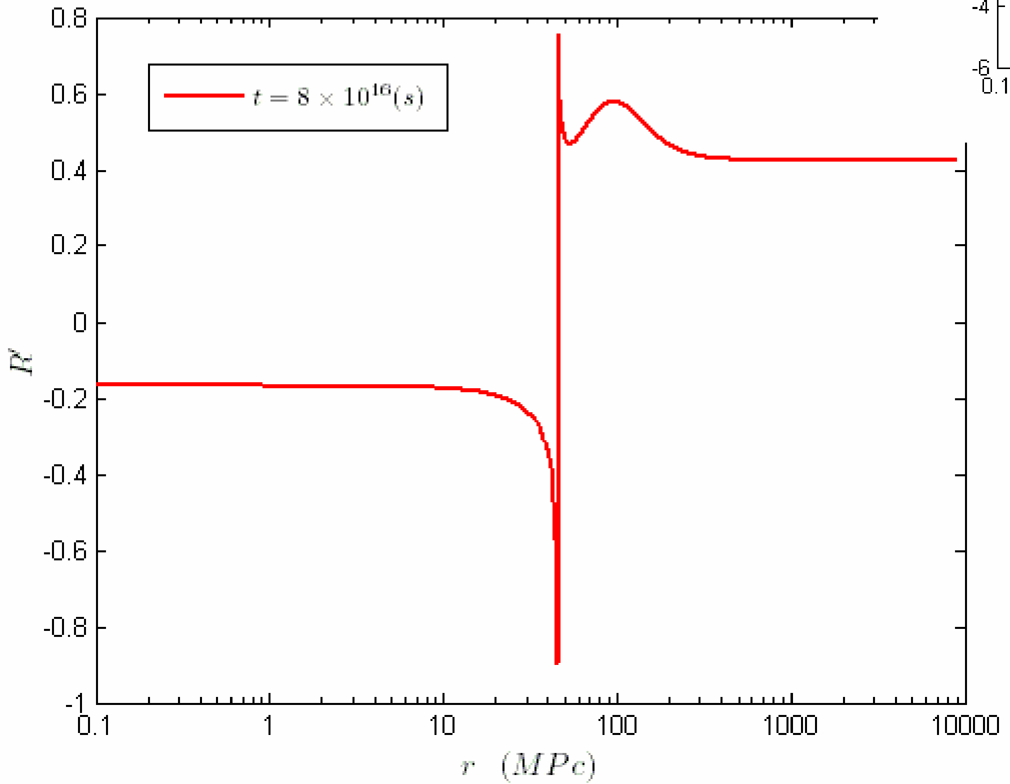
... Thank you ...

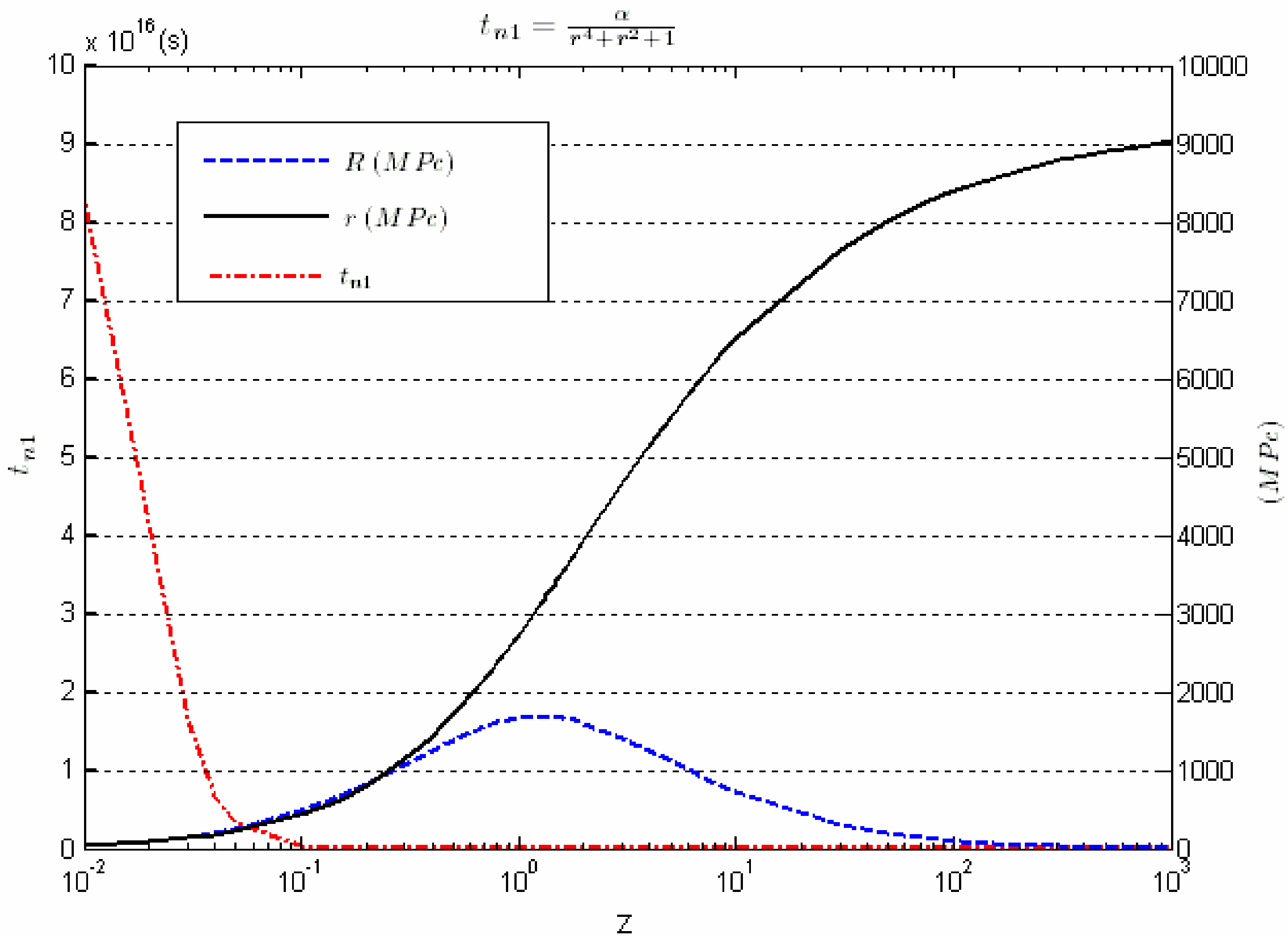


Cosmic Expansion : Singularities



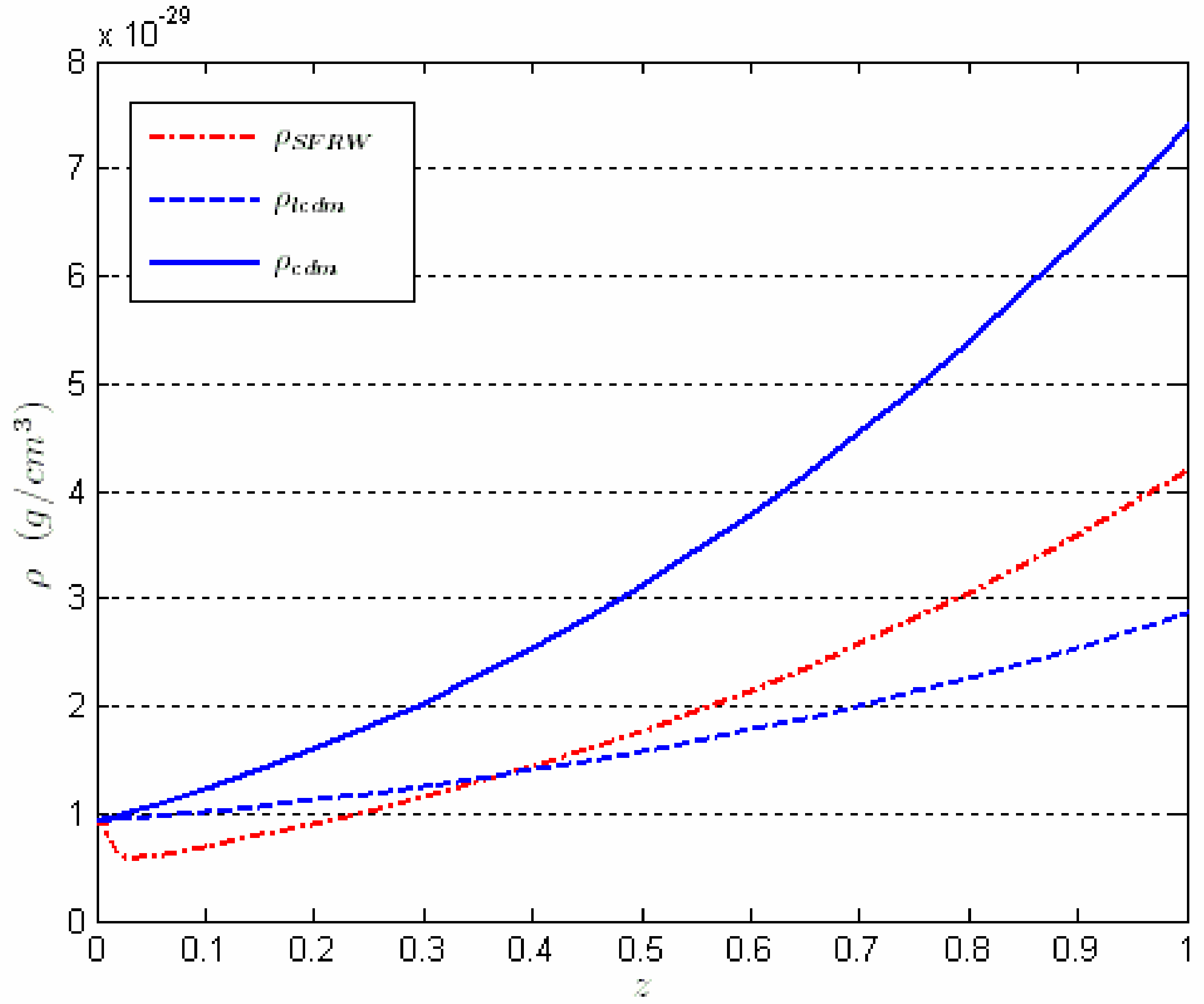

x 10^-16
12
10
8
6
4
2
0
-2
-4
-6
0.1
1
10
100
1000
10000
$\dot{R}'$
$r \quad (MPc)$
$t = 8 \times 10^{16}(s)$


0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.1
1
10
100
1000
10000
R'
$r \quad (MPc)$
$t = 8 \times 10^{16}(s)$

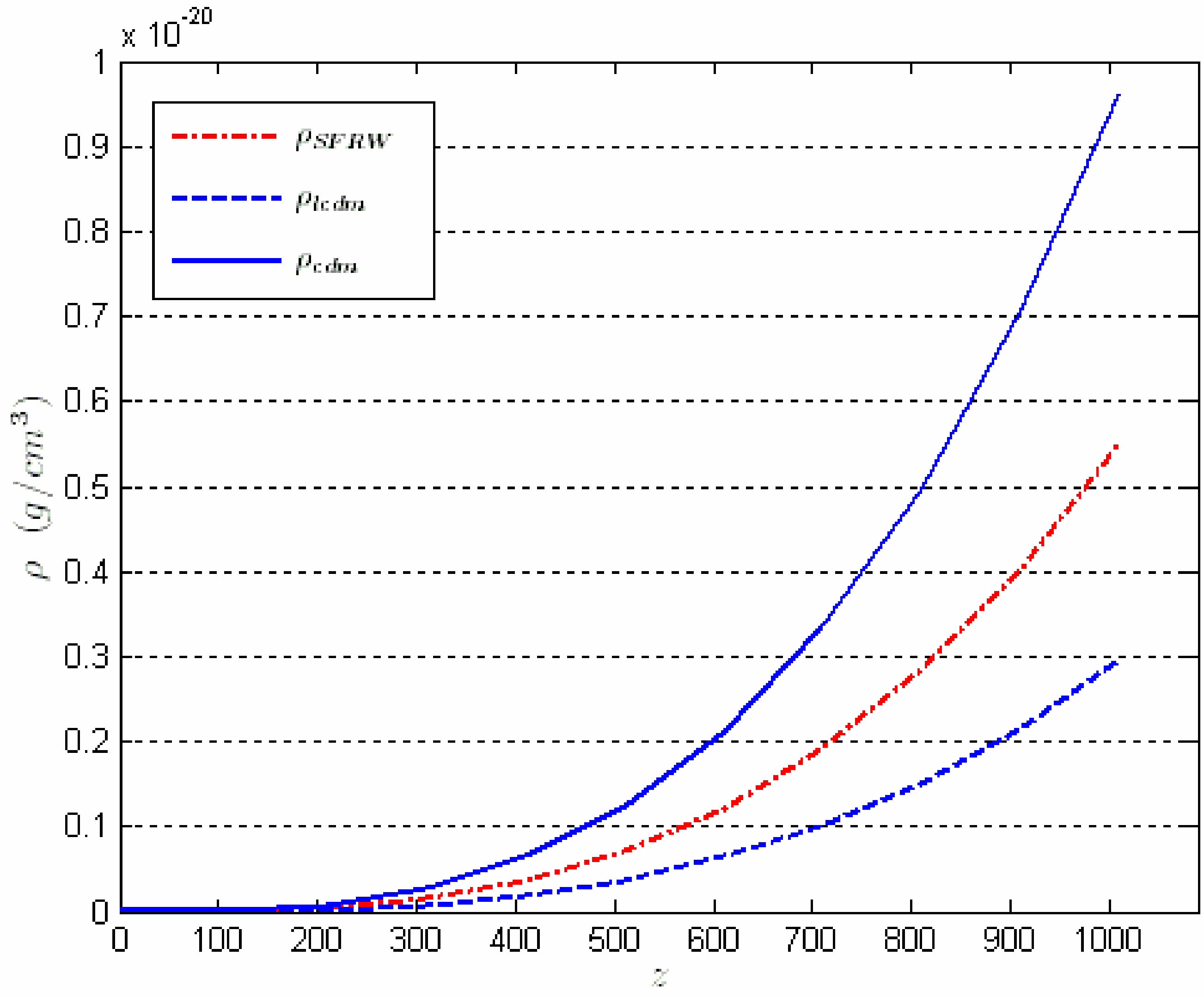


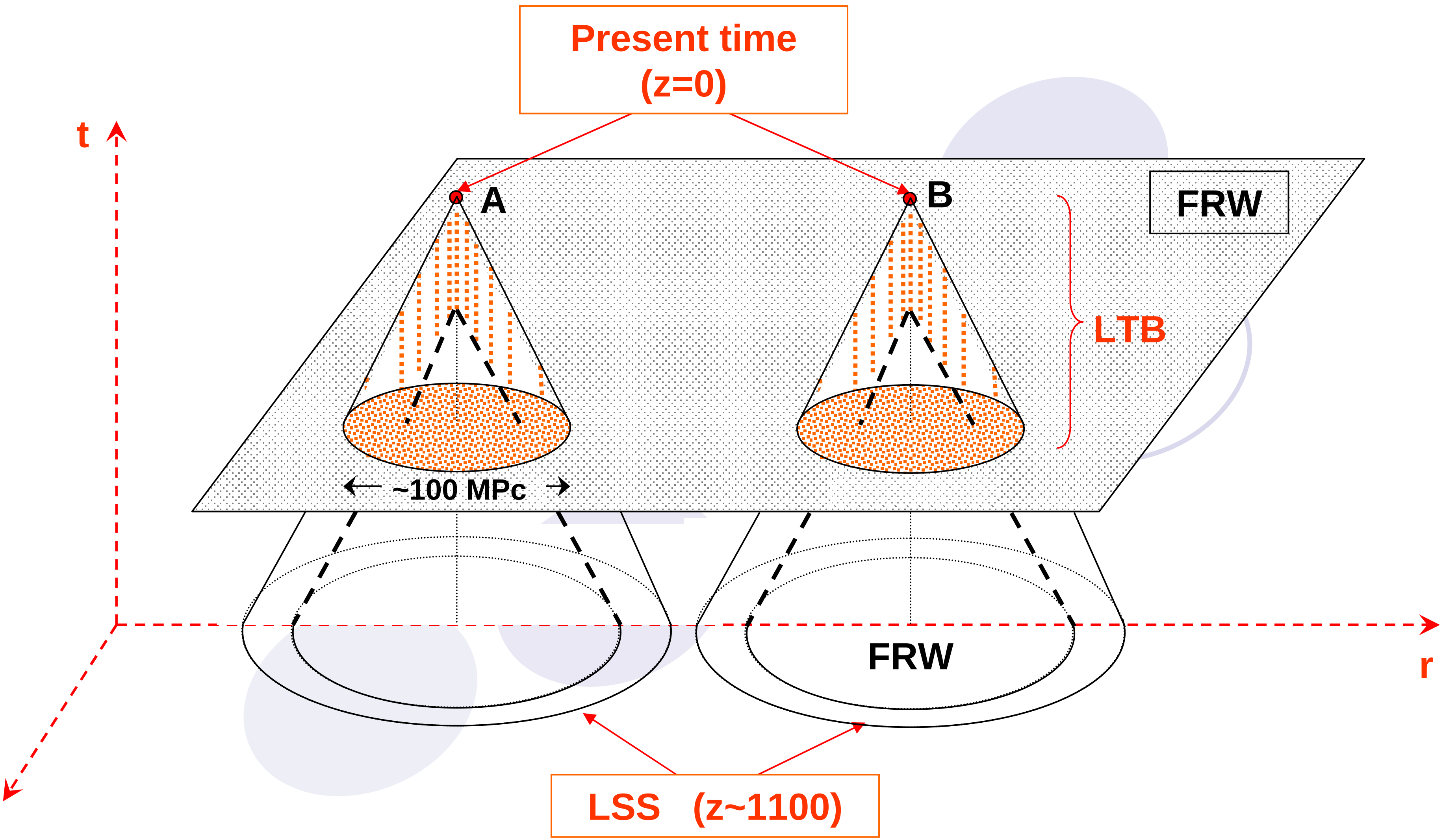
$t_{n1} = \frac{\alpha}{r^4+r^2+1}$
x 10^16 (s)
$R\ (MPc)$
$r\ (MPc)$
t_{n1}
t_{n1}
(MPc)
Z
10
9
8
7
6
5
4
3
2
1
0
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Density vs Z (redshift)



Density vs Z (redshift)




Present time
(z=0)
t
A
B
FRW
LTB
~100 MPc
FRW
r
LSS (z~1100)

deceleration Parameter: t_n

$$\left(t_n = \frac{\alpha}{r^2+1}\right)$$