In the name of God

Fractal Analysis of Cosmic Microwave Background Radiation

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Outline

1. CMB as a novel experiment in the recent cosmology
2. Determination of Statistical properties and classification of processes
3. CMB fluctuation and Self similar random process
4. Hurst exponent
   - Relation between H and other interesting quantities
5. Results
6. Summary

History of CMB

- Gamow, Alpher and Hermann 1948

- Penzias and Wilson 1964

OVRO 1	1.8′	0.03 deg^2	Ground 1989
FIRS	4◦	π	Balloon 1991
SP91	1.4◦	13.8 deg2	Ground 1992
OVRO 2	1.8′	0.1 deg2	Ground 1993
MAX 1	0.5◦		Balloon 1993
SK93	1.45◦		Ground 1993
Tenerife	5.6◦	350 deg2	Ground 1994
ARGO	1◦		Balloon 1994
MSAM	0.47◦	6 deg2	Balloon 1994
OVRO 2	1.8′	0.1 deg2	Ground 1994
WMAP	1.8′	Full sky	Satellite 2003
Planck		Full sky	Satellite 2007

Ground Based:

1: Tenerife
2: South Pole
3: Saskatoon
4: CAT
5: ATCA
6: Python
7: OVRO
8: SuZIE
9: Jodrell Bank
10: IAC Bartol
11: White Dish
12: Viper
13: COBRA
14: MAT
15: DASI
16: VSA
17: CBI
18: POLAR Brown/Wisc polarization

Balloon Borne :

1: FIRS
2: ARGO
3: MAX
4: BEAST
5: MSAM
6: BAM
9: ACE
10: QMAP
11: Boomerang
12: MAXIMA
13: Top Hat

Space based :

1: COBE
2: Planck
3: MAP

Our Goals

Investigation of

1- Statistical isotropy of CMB

2- Gaussianity of CMB

Gaussianity in CMB

- If the CMB is a Gaussian random field, many properties of this field are determined by the two-point correlation function as follows:

$$C(\hat{n}_1, \hat{n}_2) = \left\langle \Delta(\hat{n}_1)\Delta(\hat{n}_2) \right\rangle$$

$$\Delta(\hat{n}) = \frac{\delta T(\hat{n})}{\bar{T}}$$

- Non-Gaussianity is investigated with the higher order of correlation function:

$$C^m(\hat{n}_1, \hat{n}_2) = < [\Delta(\hat{n}_1)\,\Delta(\hat{n}_2)]^m >$$

$$\Delta(\hat{n}) = \frac{\delta T(\hat{n})}{\bar{T}}$$

Investigation of Gaussianity

- Test of Inflationary Models

- Using two point correlation function to extract many statistical properties of Universe

Statistical Isotropy
Power spectrum

$$C(\hat{n}_1, \hat{n}_2) = < [\Delta(\hat{n}_1)\, \Delta(\hat{n}_2)] >$$

$$\Delta(\hat{n}) = \frac{\delta T(\hat{n})}{\bar{T}}$$

$$C(\hat{n}_1, \hat{n}_2) = C(\hat{n}_1 \cdot \hat{n}_2) \equiv C(\gamma) \quad \cos(\gamma) = (\hat{n}_2 \cdot \hat{n}_1)$$

$$\Delta(\hat{n}) = \sum_{lm} a_{lm} Y_l^m(\hat{n})$$

$$C(\gamma) = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos(\gamma))$$

$$= \sum_{lm} C_l Y_l^m(\hat{n}_1) Y_l^{m*}(\hat{n}_2)$$

$$\left\langle a_{lm} a^*_{l'm'} \right\rangle = C_l \delta_{ll'} \delta_{mm'}$$

Classification of random processes

- ~~Direct computation and determination~~

Trend and unknown noise

- Indirect computation and determination

- Very often data are affected by non-stationarities such as trend and noise which have to be well distinguished from intrinsic fluctuations of system to find the correct scaling behavior of fluctuations
- Usually we don’t know the origin of noise
- Even, the scaling behavior of trend may be unknown

Fractal Analysis

1: Hurst’ rescaled range (R/S) analysis : By Hurst (1951)

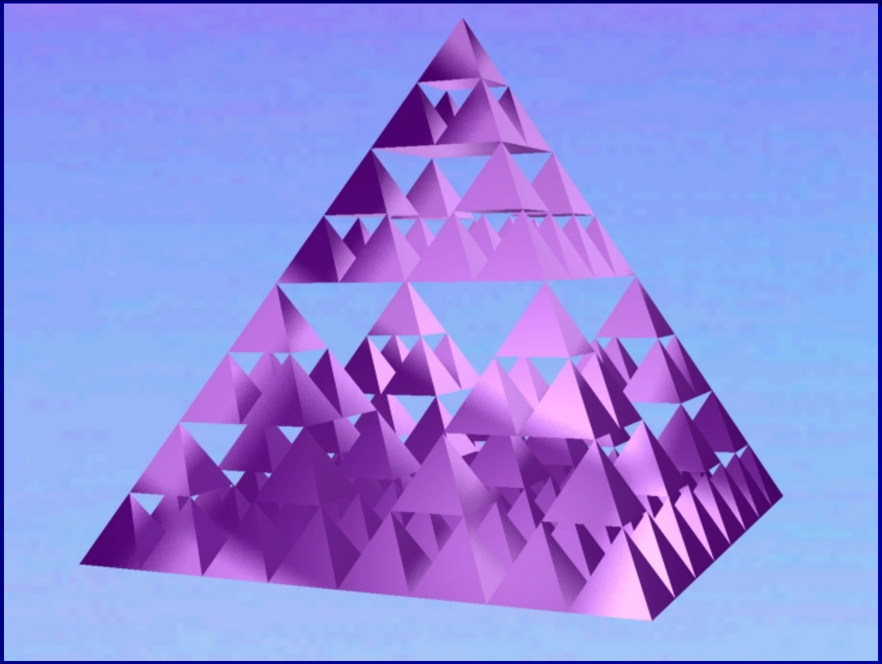
2: Scaled windowed variance analysis (SWV) : By Mandelbort (1985)

3: Dispersional analysis (Disp) : By Bassingthwaighte (1988)

4: Detrended fluctuation analysis (DFA) : By Peng (1994)

CMB and Self-similar processes

An object can be self-similar if it is formed by parts that are similar to the whole



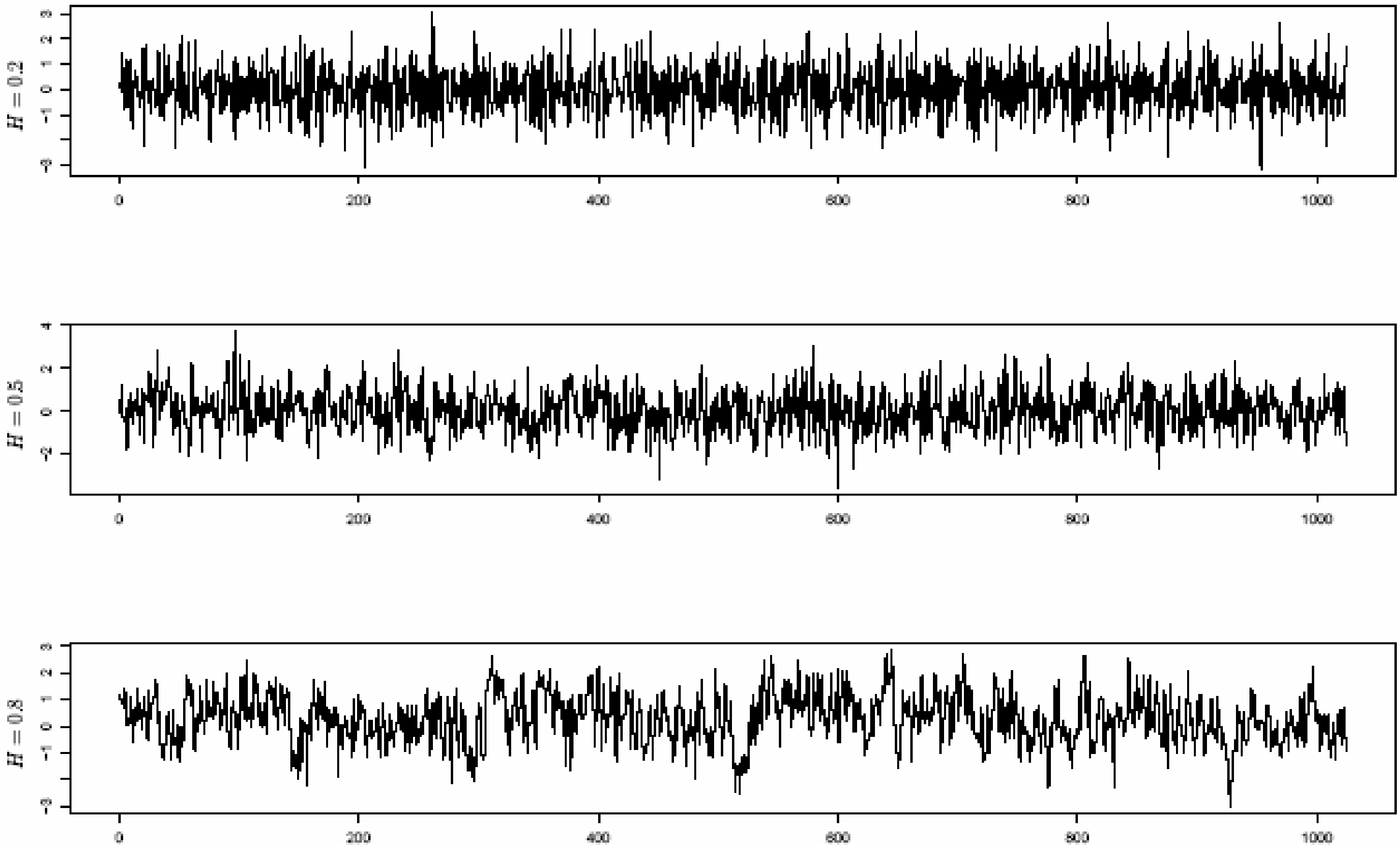
$$y(n) = x_1 + x_2 + x_3 + .. + x_n \square n^H x$$

$$y(a) = a^H x$$

Hurst exponent by using MF-DFA

- Hurst exponent H = h (q =2)
- 0 < H < 1 Stationary processes
- 0 < H < 0.5 Anti-correlated
- 0.5 < H < 1.0 long range correlated
- 1.0 < h (q=2) non-stationary processes
- H = h (q=2) - 1

fGn



Generalized Hurst Exponent

$$\tau(q) = qh(q) - 1$$

Classical multifractal scaling

$$D_f = 2 - H$$

Fractal dimension

$$D_q = \frac{\tau(q)}{q-1}$$

Generalized multifractal dimension

Generalized Hurst Exponent

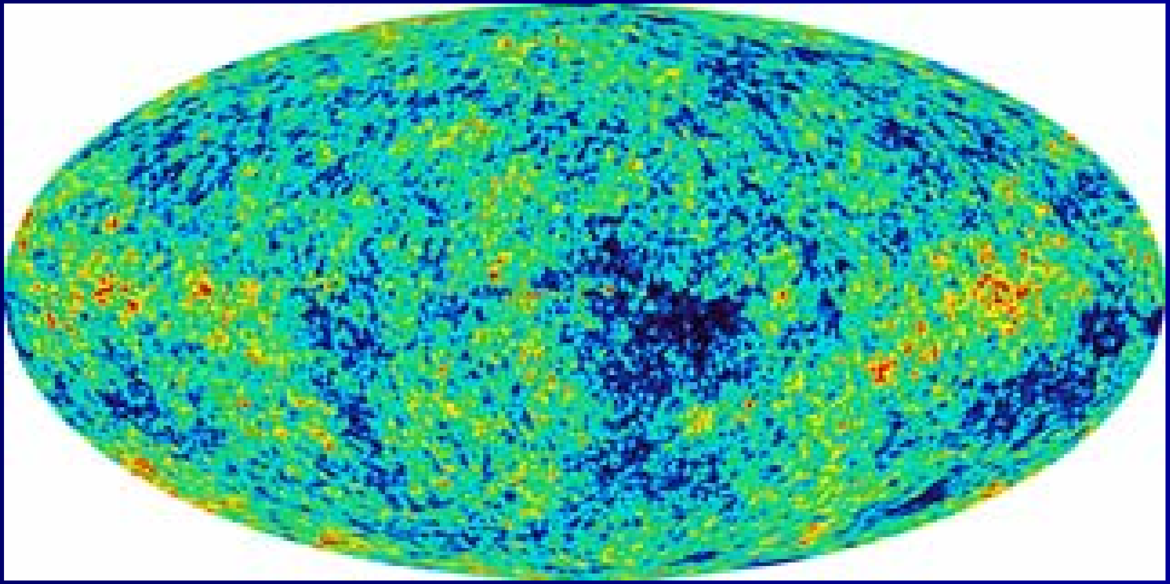
$$C\left(\Theta\right) = \left\langle T(\hat{n})T(\hat{n}')\right\rangle \propto \Theta^{-\Delta} \qquad \cos(\Theta) = \hat{n}\cdot\hat{n}'$$

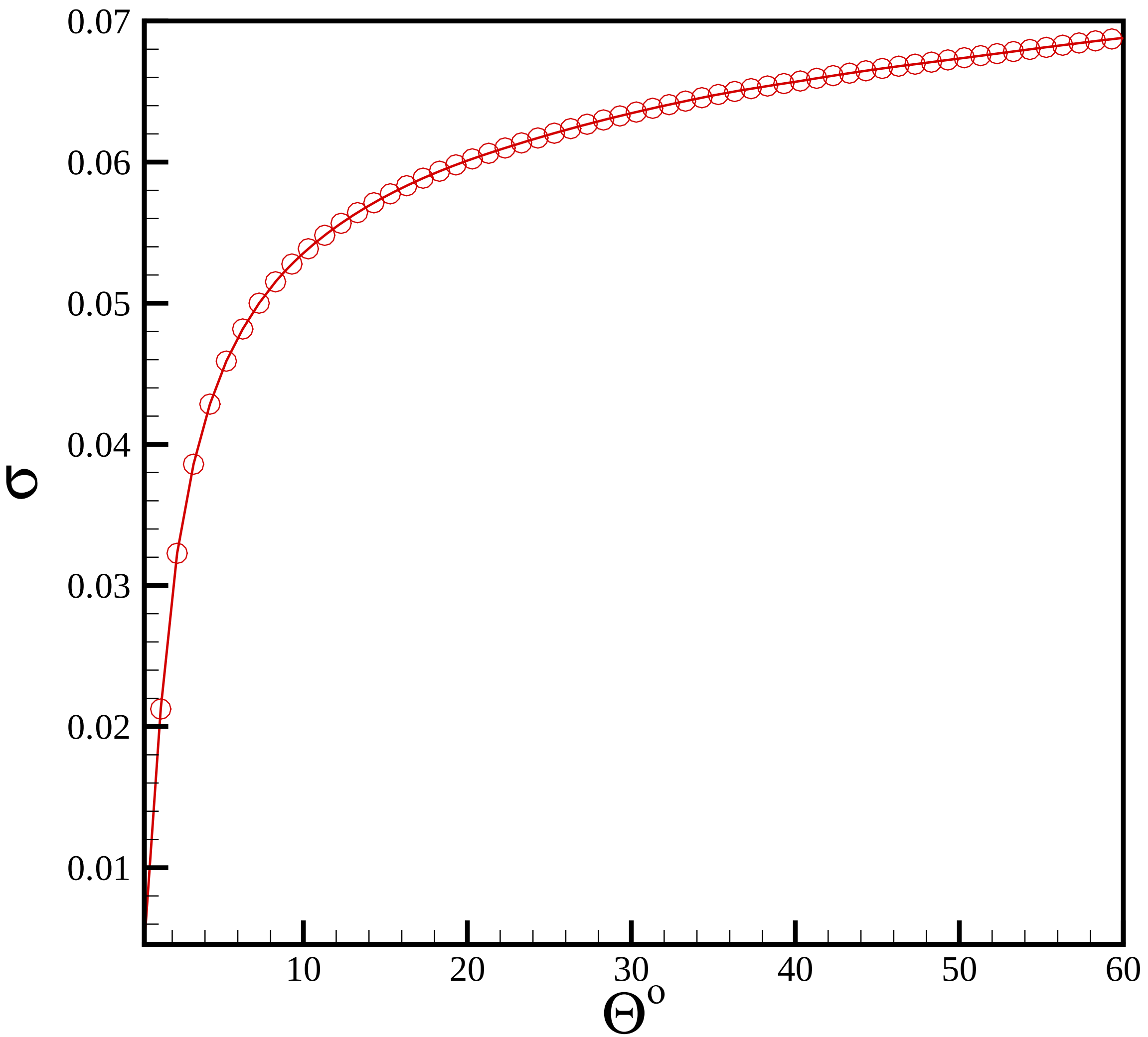
Δ=2-2H

$$s(\omega) \approx \omega^{-\beta}$$

β=2H-1

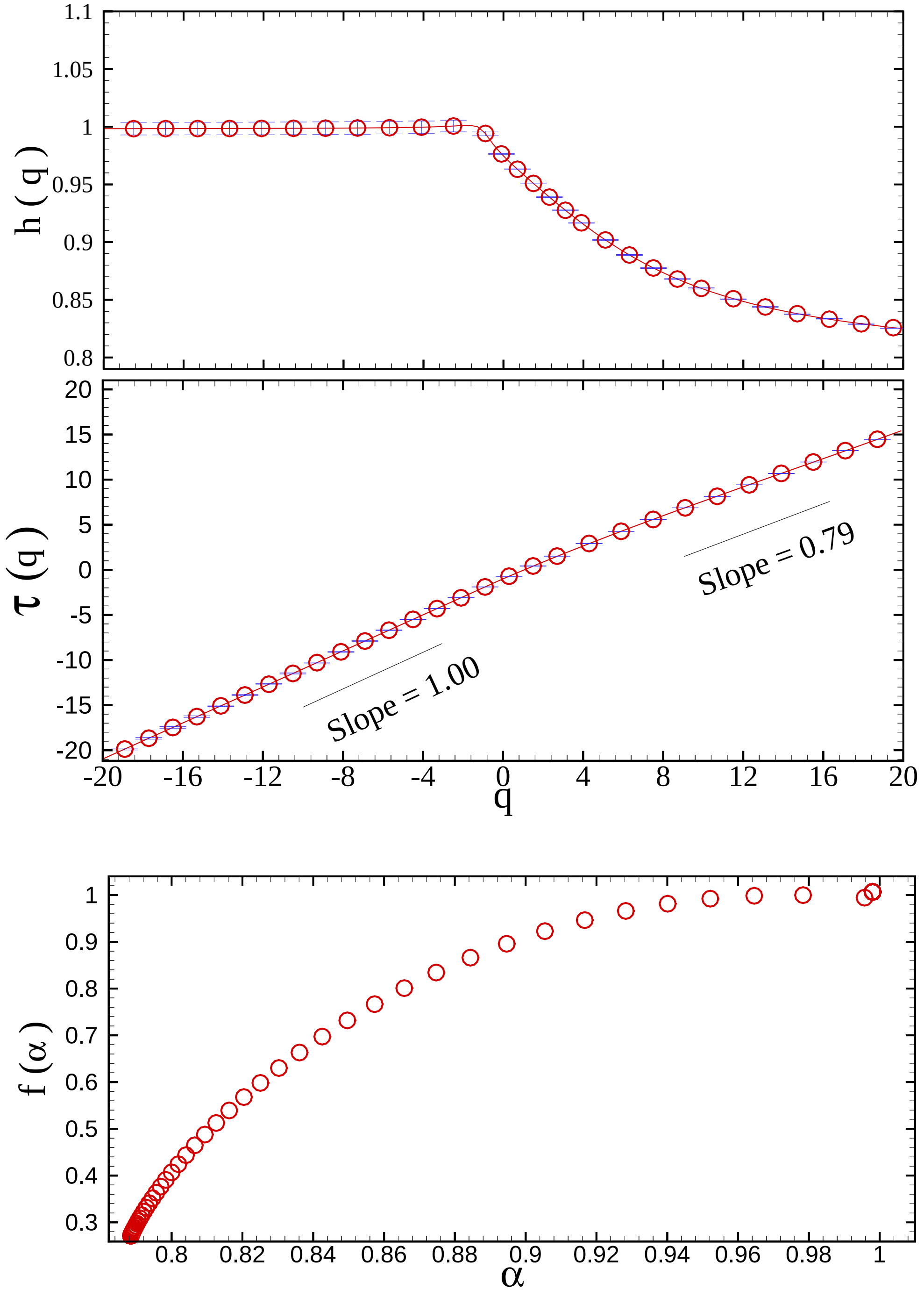
Results




0.07
0.06
0.05
0.04
0.03
0.02
0.01
σ
10
20
30
40
50
60
Θ°

Original Data

$$H = 0.94 \pm 0.01$$



1- CMB is a multifractal process with long range correlation

2- The value of Hurst exponent guarantees the statistical isotropy in the WMAP data

DFA $H = 0.94 \pm 0.01 < 1.0$

SWV $H = 0.95 \pm 0.02 < 1.0$

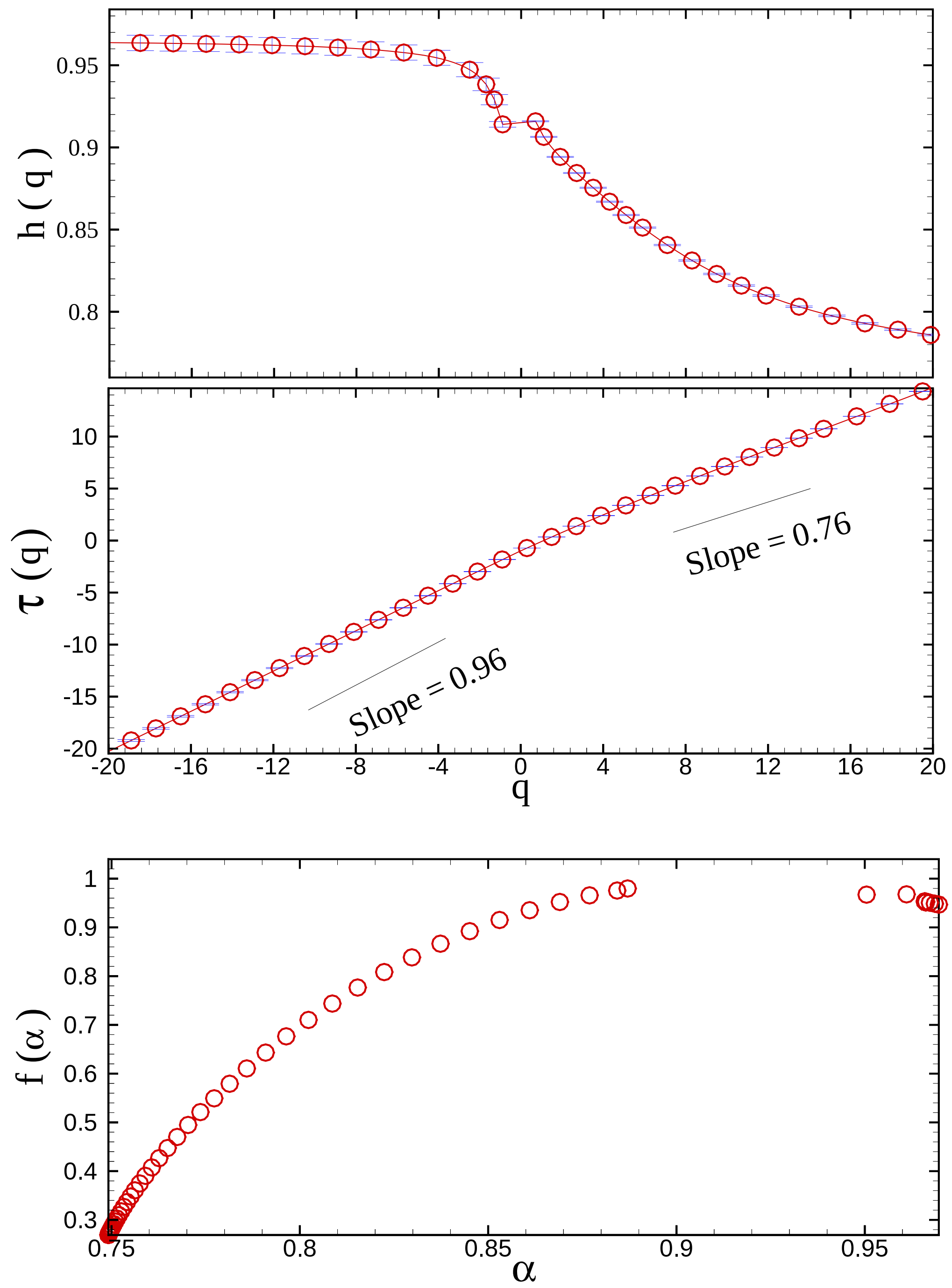
What is the origin of Multifractality in the CMB?

In general two different types of multifractality in certain data can be distinguished:

- Multifractality due to a fatness of PDF
- Multifractality due to different correlations in small and large scales

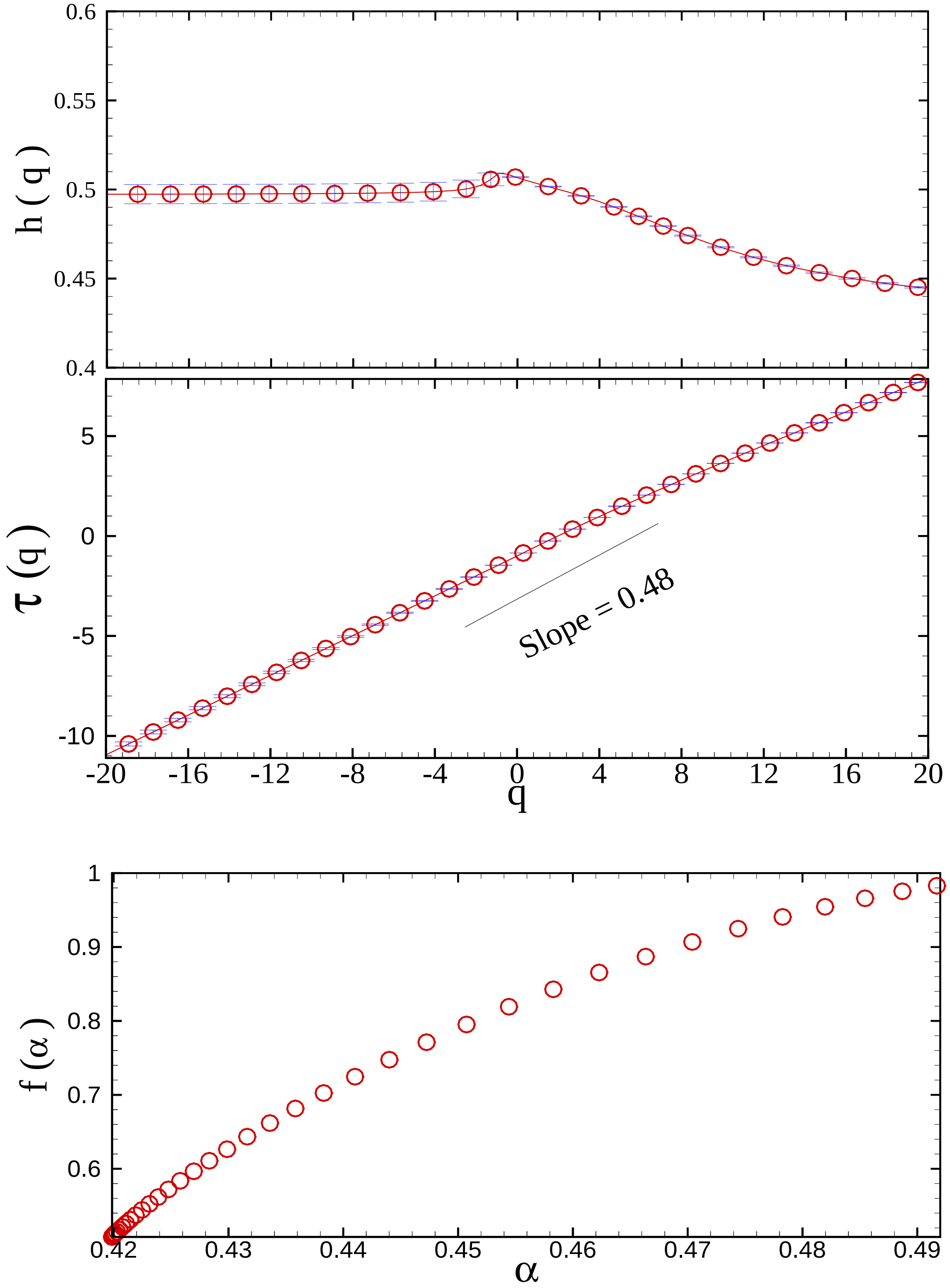
Surrogate Data

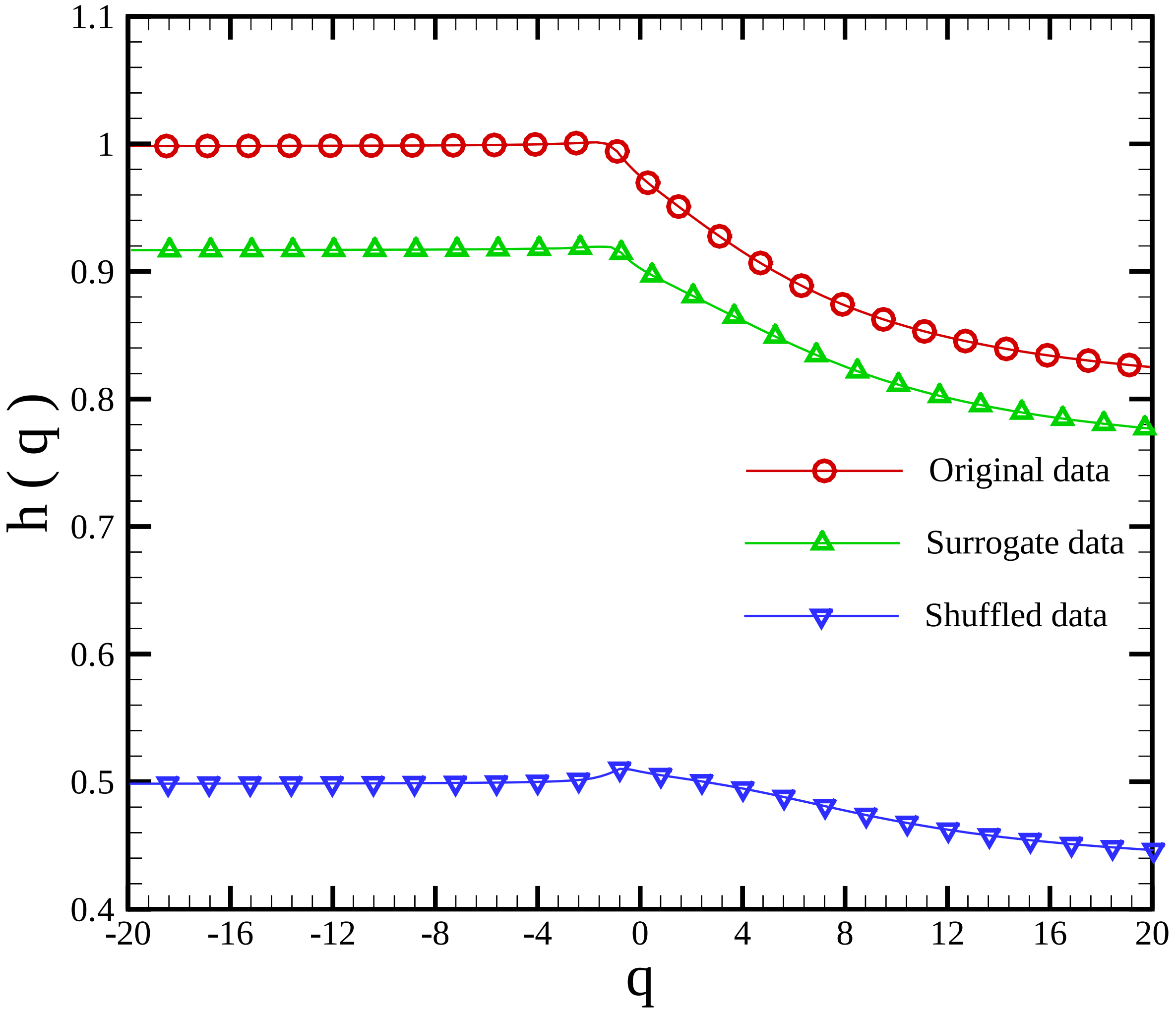
$$H = 0.88 \pm 0.01$$



Shuffled Data

$$H = 0.50 \pm 0.001$$




1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
h (q)
-20
-16
-12
-8
-4
0
4
8
12
16
20
q
Original data
Surrogate data
Shuffled data

Singularity Spectrum of CMB

$$\Delta\alpha = \alpha(q_{\min}) - \alpha(q_{\max})$$

$$\Delta\alpha_{Original} = 0.21$$

$$\Delta\alpha_{Surrogate} = 0.18$$

$$\Delta\alpha_{Shuffled} = 0.07$$

$$\chi^2 = \sum_i \frac{\left[h_O(q_i) - h_{Sur}(q_i)\right]^2}{\sigma_O{}^2(q_i) + \sigma_{Sur}{}^2(q_i)} = 37.27$$

$$\chi^2 = \sum_i \frac{\left[h_O(q_i) - h_{Shu}(q_i)\right]^2}{\sigma_O{}^2(q_i) + \sigma_{Shu}{}^2(q_i)} = 3015.96$$

1- Multifractality in the CMB is almost due to the long range correlation

2- CMB fluctuation has a Gaussian distribution

Summary

- According to the value of Hurst exponent of CMB data, is homogenous process, i.e. there is no evidence for violation of SI.
- Comparing the MF-DFA results of the original temperature fluctuation to those for shuffled and surrogate series, displayed that, its multifractality nature is due to long-range correlations.
- The similarity between MF-DFA results of original and surrogate data demonstrated that, CMB is consistent with a Gaussian distribution

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Thank you for attention