PLAN of the LECTURES

- Motivation for going beyond FLRW
- Backreaction
  - PERTURBATIVE ESTIMATES
  - GENERAL CONSIDERATIONS
- Exact models (spherically symmetric)

MOTIVATION

- Usual homogeneous isotropic cosmology need an "adjustment": $\Lambda \sim (10^{-3}\,\mathrm{eV})^4$
- Are we 100% sure we are using the correct equations?
- (Even with Einstein gravity)
- After all the Universe has significant inhomogeneities:

- $\frac{\delta\rho}{\rho} \approx \mathcal{O}(1)$ on scales of $\mathcal{O}(10\ \mathrm{Mpc}/h)$
- there are structures observed on $\mathcal{O}(100\ \mathrm{Mpc}/h)$ scales (supercluster-void network)

[Sloan Great Wall ~ 400 Mpc/h]

· <u>So</u> consider cosmology with INHOMOGENEOUS DUST ($p=0$)

3 EFFECTS:

$$\boxed{\begin{array}{l} \text{GRAVITY IS NONLINEAR (BACKREACTION)} \\ \text{LIGHT PROPAGATION } (D_L - z) \\ \text{LOCAL EFFECTS} \end{array}}$$

<u>QUESTIONS</u> (1) Can we <u>reliably</u> quantify deviations from FLRW?

(2) <u>Moreover</u>, what if we live in a special region [LOCAL effect]?

USUAL HOMOGENEOUS DUST (EdS model)

$$T^{\mu\nu} = \rho u^\mu u^\nu \qquad u^\mu = (1,0,0,0) \qquad \rho(t)$$

FLAT FRW

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 = a^2(\tau)\left[-d\tau^2 + dx^i dx^j \delta_{ij}\right]$$

$$a \sim t^{2/3} \sim \tau^2$$

In general

$$ds^2 = -(1+\gamma_{00})\,dt^2 + 2\gamma_{0i}\,dt\,dx^i - a^2(t)\,\gamma_{ij}\,dx^i dx^j$$

$$u^\mu = u^\mu_{FLRW} + \delta u^\mu(x^\mu)$$

$$\rho = \rho_{FLRW} + \delta\rho\,(x^\mu)$$

<u>At linear level</u> (RELATIVISTIC PERTURBATION THEORY: <u>METRIC</u> AND <u>DENSITY</u> SMALL)

$$V^i = V_E^i + V_B^i \qquad \nabla \wedge v_E = 0 \qquad v_E = \vec{\nabla} V$$
$$\nabla \cdot v_B = 0$$

1. $\delta\rho$ and V are coupled
2. V_B^i (decays away)
3. GW's

<u>Non linear</u>: everything is mixed

IR. GAUGE $\gamma_{0i} = \gamma_{00} = 0$

A particle w. fixed coordinates follows a geodesic

ING GAUGE $u^{\mu} = (1,0,0,0)$

$p=0$ the fluid follow geodesics $\Rightarrow$ COMOVING AND SYNCHRONOUS

If $v_B=0$ one can do it globally)

[Phys. Rep. 231 : 1-105, 1993 Liddle, Lyth]
astro-ph/9303019

RTICLES OF DUST AT FIXED COORDINATES ALWAYS.
E METRIC STRETCHES between them $\bullet \xleftrightarrow{L} \bullet$
- changes)

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \gamma_{ij}(\tau, x^i)\, dx^i dx^j\right]$$

It becomes singular when 2 trajectories touch ("shell crossing") $L \to 0$

(One has to introduce other physical ingredients (like pressure) to resolve this problem.)

IDEALLY : Compute $D_L - z$ (LUMINOSITY DISTANCE - REDSHIFT)

within this metric, using a realistic matter distribution

MORE EASILY : Describe the average expansion inside a domain D

Fix a comoving domain D

Define averages of a quantity O as

$$\langle O \rangle_D \equiv \frac{\int_D d^3x \sqrt{\gamma}\, O}{\int_D d^3x \sqrt{\gamma}} \qquad \left(\begin{array}{l} \sqrt{\gamma} \equiv \sqrt{\det(\gamma_{ij})}, \\ \int_D d^3x \sqrt{\gamma} \equiv V_D \end{array} \right)$$

We would like to define an average scale factor a_D and see if it evolves differently from FLRW

Review of ZERO ORDER APPROXIMATION (FLRW)

Use two equations

0-0 E.E.

CONT. EQUATION

$$\begin{cases} G_{00}^{(0)} = k^2 T_{00} \\ \dot{\rho} = -\Theta \rho \end{cases}$$

$G_{00}^{(0)} = 3\left(\frac{\dot{a}}{a}\right)^2$

$T_{00} = \rho$

$\Theta = 3 \frac{\dot{a}}{a}$

How do I solve for ρ?

$$\begin{cases} \dot{\rho} = -\Theta \rho \\ \Theta = 3 \sqrt{\frac{k\rho}{3}} \end{cases}$$

On any domain D $\rho = \langle \rho \rangle_D$

$\langle \rho \rangle_D \sim t^{-2}$

$a \sim t^{2/3}$

FIRST ORDER

How does $\langle \rho \rangle_D$ scale? Like t^{-2}?

n other words if I define a_D by saying

$$\boxed{\langle \rho \rangle_D \sim a_D^{-3}}$$

Does a_D scale as $t^{2/3}$?

a) SHORT REVIEW OF FIRST ORDER IN SYNCHR. COMOVING GAUGE:

[see astro-ph/9707278 for 1^{st} and 2^{ND} order
hep-ph/0409038]

$$\gamma_{ij} = (1 - 2\psi^{(1)})\,\delta_{ij} + D_{ij}\,\chi^{(1)} + \underbrace{\partial_i \chi_j^{(1)} + \partial_j \chi_i^{(1)}}_{\text{VECTORS}} + \underbrace{\chi_{ij}^{(1)}}_{\text{TENSOR}}$$

scalar

- Indices raised and lowered by δ^i_j
- $D_{ij} = \partial_i \partial_j - \frac{1}{3}\nabla^2 \delta_{ij}$
- $\partial^i \chi_i = 0$
- $\chi_{ij}^{(1)} = \chi_{ji}^{(1)}$, $\partial^i \chi_{ij}^{(1)} = 0$, $\chi^{(1)i}{}_i = 0$

SOLUTIONS

$$\psi^{(1)} = \overbrace{\frac{5}{3}\varphi(x)}^{\text{constant}} + \overbrace{\frac{\tau^2}{18}\nabla^2\varphi(x)}^{\text{growing mode}}$$

$$\chi^{(1)} = -\frac{1}{3}\tau^2\varphi(x)$$

$$\frac{\delta\rho^{(1)}}{\rho} \equiv \delta^{(1)} = \frac{\tau^2}{6}\nabla^2\varphi(x)$$

where $\varphi(x)$ is the potential (time-independent!) in the Newtonian / longitudinal gauge and we believe it comes directly from inflation (measured by CMB)

$$\boxed{\begin{cases} G_{00}^{(0)} + G_{00}^{(1)} = k^2 \rho & \rho = \rho^{(0)} + \rho^{(1)} \\ \dot{\rho} = -\theta \rho & \theta \equiv D_\mu u^\mu \end{cases}}$$

Average it

$$\boxed{\begin{cases} \langle G_{00}^{(0)} \rangle_D = k^2 \langle \rho \rangle_D - \langle G_{00}^{(1)} \rangle_D \\ \frac{d}{dt} \langle \rho \rangle_D = -\langle \theta \rangle_D \langle \rho \rangle_D \end{cases}} \quad (*) \quad \Longrightarrow \quad \boxed{\langle \theta \rangle_D \equiv 3 \frac{\dot{a}_D}{a_D}}$$

$$\theta = 3\frac{\dot{a}}{a} + \theta^{(1)} \qquad \text{in terms of } \langle \rho \rangle ?$$

get it from (*)

$$3\left(\frac{\dot{a}}{a}\right)^2 = k^2 \langle \rho \rangle_D - \langle G_{00}^{(1)} \rangle_D$$

$$\Longrightarrow \boxed{\begin{cases} \langle \theta \rangle_D = \sqrt{\frac{k^2 \langle \rho \rangle_D}{3}} + \langle \delta \theta \rangle_D \\ \frac{d}{dt} \langle \rho \rangle_D = -\langle \theta \rangle_D \langle \rho \rangle_D \end{cases}}$$

Without $\langle \delta\theta \rangle_D$ we would get $\langle \rho \rangle_D \sim t^{-2}$

$$\boxed{\frac{\langle \delta \theta \rangle_D}{3H} = \frac{\langle \theta^{(1)} \rangle_D}{3H} - \frac{\langle G_{00}^{(1)} \rangle_D}{6 a^2 H^2}}$$

SECOND ORDER (conceptually identical, more long)

$$\gamma_{ij} = \left(1 - 2\psi^{(1)} - \psi^{(2)}\right)\delta_{ij} + D_{ij}\left(\chi^{(1)} + \frac{1}{2}\chi^{(2)}\right) + \frac{1}{2}\left(\underbrace{\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)}}_{\text{2ND ORDER VECTOR}} + \underbrace{\chi_{ij}^{(2)}}_{\text{2ND ORDER TENSOR}}\right)$$

2ND ORDER SCALARS (QUADRATIC IN φ)

For example:

$$\psi^{(2)} = \underbrace{-\frac{50}{9}\varphi^2}_{\text{constant}} \underbrace{- \frac{5\tau^2}{54}\partial^k\varphi\,\partial_k\varphi}_{\substack{\text{2 gradients} \\ (\tau^2)}} + \underbrace{\frac{\tau^4}{252}\left((\nabla^2\varphi)^2 - \frac{10}{3}\partial^k\partial^i\varphi\,\partial_k\partial_i\varphi\right)}_{\substack{\text{4 gradients} \\ (\tau^4)}}$$

AS BEFORE:

$$\begin{cases} G_{00}^{(1)} + G_{00}^{(11)} + G_{00}^{(2)} = k^2\rho \\ \dot{\rho} = -\Theta\,\rho \end{cases}$$

AVERAGE IT (USE $\sqrt{\gamma}$!!)

$$\begin{cases} \frac{d}{dt}\langle\rho\rangle_D = -\langle\Theta\rangle_D\langle\rho\rangle_D \quad \longleftarrow \text{NON TRIVIAL!} \\ \langle\Theta\rangle_D = \sqrt{\frac{k^2\langle\rho\rangle_D}{3}} + \langle\delta\Theta\rangle_D \end{cases}$$

$$\frac{\langle \delta\Theta\rangle_D}{3H} = \frac{\langle \Theta^{(1)} + \Theta^{(11)} + \Theta^{(2)}\rangle}{3H} - \frac{\langle G_{00}^{(1)} + G_{00}^{(11)} + G_{00}^{(2)}\rangle}{6\,a^2H^2} - \frac{\langle G_{00}^{(1)}\rangle^2}{72\,a^4H^4}$$

Now, all the corrections can be expressed in terms of $\varphi(\vec{x})$

average with $\sqrt{\gamma}=1$

$$\frac{\langle\delta\Theta\rangle}{3H} = \frac{1}{a^2H^2}\Big[-\frac{5}{9}\langle\nabla^2\varphi\rangle_1 + \frac{5}{3}\Big(\langle\varphi\nabla^2\varphi\rangle_D + \frac{13}{18}\langle\partial_i\varphi\,\partial^i\varphi\rangle_D\Big)$$

$$+\frac{\tau^2}{27}\Big(\underbrace{\langle(\nabla^2\varphi)^2\rangle_D - \langle\partial^i\partial^j\varphi\,\partial_i\partial_j\varphi\rangle_D}_{\text{IT'S A TOTAL GRADIENT!}}\Big)\Big]$$

$\boxed{\text{Evaluate the terms}}$

Use $\varphi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\,\varphi_{\vec{k}}\,e^{i\vec{k}\cdot\vec{x}}$

Statistical averages: (Gaussian variable)

$$\begin{cases} \overline{\varphi_{\vec{k}}} = 0 \\ \overline{\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}} = (2\pi)^3\,\delta^3(\vec{k}_1+\vec{k}_2)\,P_\varphi(k_1) \\ \overline{\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\varphi_{k_4}} = \overline{\varphi_{k_1}\varphi_{k_2}}\;\overline{\varphi_{k_3}\varphi_{k_4}} + \overline{\varphi_{k_1}\varphi_{k_4}}\;\overline{\varphi_{k_2}\varphi_{k_3}} + \overline{\varphi_{k_1}\varphi_{k_3}}\;\overline{\varphi_{k_2}\varphi_{k_4}} \end{cases}$$

Why use statistical properties?

Statistical mean $\overline{(\ldots)}$ different from volume average $\langle \ldots \rangle_D$

Given a quantity $\mathcal{O}_D \equiv \langle \mathcal{O} \rangle_D$

we can estimate what is its most probable value (if we had many domains D)

$$\overline{\mathcal{O}_D}$$

And we can estimate its variance

$$\overline{\mathcal{O}_D^2} - \overline{\mathcal{O}_D}^2 \equiv \mathrm{Var}(\mathcal{O}_D)$$

TWO PROPERTIES

$\boxed{A}$ $\boxed{\begin{array}{l} \text{If } D \longrightarrow \infty \text{ volume} \\ \text{Then } \mathrm{Var} \longrightarrow 0 \end{array}}$

(statistical mean = ∞ volume average)

$\boxed{B}$ $\boxed{\begin{array}{l} \text{If } \mathcal{O}_D^{*} = \partial^i(\ldots\ldots) \\ \text{Then } \overline{\mathcal{O}_D} = 0 \end{array}}$

<u>First order term</u> Large variance on small D
Zero on average

<u>Second order term (k^2)</u> Non zero average of $O(10^{-5}$

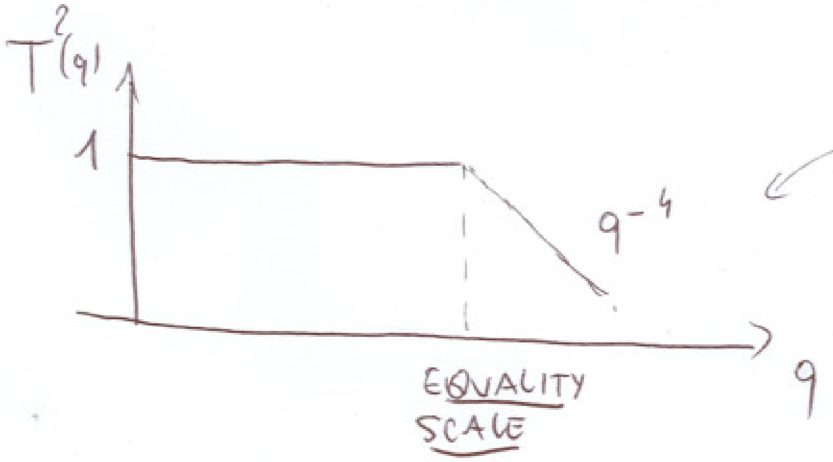
<u>Second order terms (k^4)</u> Zero on average
(total gradients)

<u>ALCULATIONS</u> (hep/ph 0409038)

<u>Use</u> the following

$$P_\varphi = \cancel{\ldots} \ \frac{9\pi^2}{2} A^2 \frac{T^2(k)}{k^3} \qquad A = 1.9\,10^{-5} \ (\text{CMB})$$

$$T^2(q) = \begin{cases} 1 & \text{small } q \\ q^{-4}\log^{-2} q & \text{large } q \end{cases} \qquad q \simeq \frac{k}{Mpc^{-1}}$$



STAGNATION OF PERTURBATIONS IN THE RADIATION ERA

$$\langle \ldots \rangle = \frac{1}{V(R)} \int_{V(R)} (\ldots)\, d^3x$$

SPHERICAL DOMAINS

Employ (for convenience) Gaussian window function $e^{-r^2/2R^2}$

Get

$$W(kR) = \frac{1}{V} \int d^3x\, e^{-r^2/2R^2} e^{ikx} = e^{-k^2R^2/2}$$

EXAMPLE

$$\tau^2 \overline{\langle \varphi \nabla^2 \varphi \rangle} = \frac{\tau^2}{V} \int e^{-r^2/2R^2} d^3x\, \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \left(\frac{k_1^2 + k_2^2}{2} \right) \overline{\varphi_{k_1} \varphi_{k_2}}\, e^{i(k_1+k_2)x}$$

$$= \tau^2 \int \frac{d^3k}{(2\pi)^3} k^2 P(k)\, W(0)^{=1}$$

$$\simeq \frac{a}{a_0} \underbrace{A^2}_{(10^{-5})^2} \underbrace{\left(\frac{h\,\mathrm{Mpc}^{-1}}{H_0} \right)^2}_{10^{+5}} \underbrace{\int_0^\infty dq\, q\, T^2(q)}_{O(1)}$$

$$\simeq 10^{-5} \frac{a}{a_0}$$

For a variance the $W(kR)$ does <u>not</u> disappear

$$\frac{\mathrm{Var}[\delta\theta]}{3H}\Bigg|_{\text{linear}} = \int \frac{dk}{k}\,\Delta^2(k)\,W^2(kR)$$

Only $\lambda > R$ contribute to this

So the second order effect is 10^{-5}

(agrees with ~~previous~~ other analyses Hui & Seljak '95
Räsänen '04
Siegel & Fry '05

Is that the end of the story?

10^{-5} is <u>not</u> what one expects from 2ND order!

And the term grows in the future...

What about higher orders?

Do they decrease w.r.t. second order?

$$\begin{cases} 2^{\text{ND}}\text{ order} & K^2\varphi^2 & (K^4\varphi^2 \text{ total gradient}) \\ 4^{\text{TH}}\text{ order} & K^{8-2}\varphi^4 & (K^8\varphi^4 \text{ total gradient}) \\ n^{\text{TH}}\text{ order} & K^{2n-2}\varphi^n & (K^{2n}\varphi^n \text{ total gradient}) \end{cases}$$

$$\overline{\langle k^2 \varphi^2 \rangle}_D = \frac{a}{a_0} A^2 \left(\frac{h\,\mathrm{Mpc}^{-1}}{H_0} \right)^2 \int_0^\infty dq\, q\, T^2(q)$$

$$\overline{k^6 \varphi^4} = \left(\frac{a}{a_0} \right)^2 A^4 \left(\frac{h\,\mathrm{Mpc}^{-1}}{H_0} \right)^6 \int_0^{\infty \leftarrow \text{UV cutoff}} dq\, q^5\, T^2(q)$$

In general I can define an expansion parameter

$$\boxed{\epsilon = A \frac{a}{a_0} \left(\frac{h\,\mathrm{Mpc}^{-1}}{H_0} \right)^2}$$

$$\overline{k^{2n-2} \varphi^n} = A\, \epsilon^{n-1} \simeq 10^{-5}\, \epsilon^{n-1}$$

$\epsilon \sim O(1)$ at $z \approx 0$

ϵ grows in the future

<u>OBS</u>: The series is well behaved at $z \gtrsim 1$

<u>OBS</u> It is <u>not</u> due to UV cutoff.

(astro-ph/0503715)

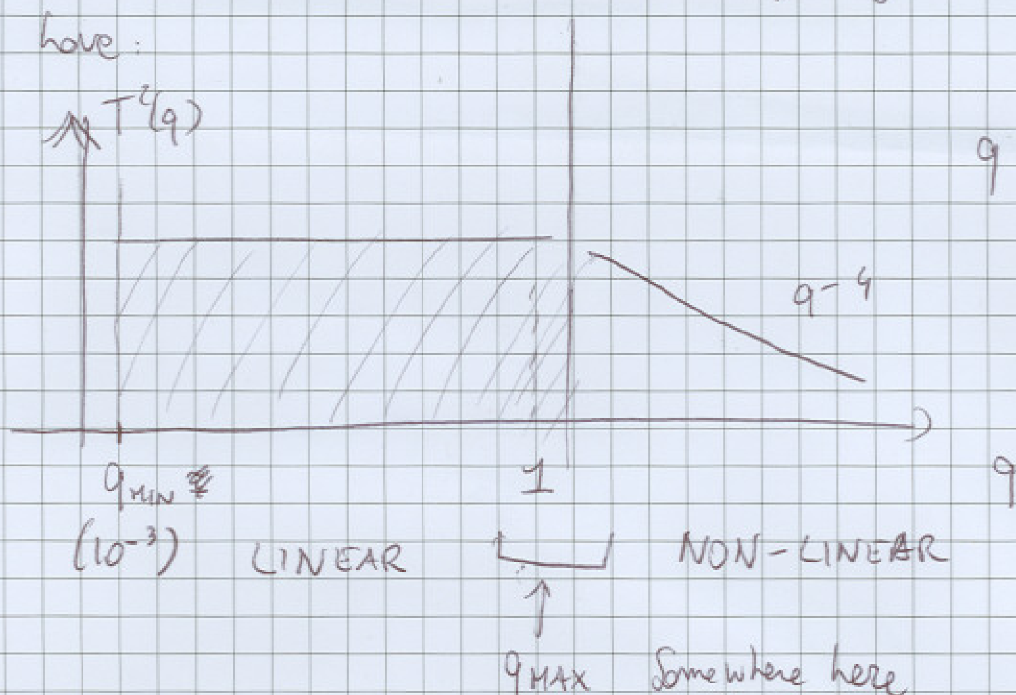
<u>OBS</u>: Is it a <u>physical effect</u> or just an <u>inadequate description</u> (relativistic perturbation theory)?

REMARKS

- We have used relativistic perturbation theory, in which $\gamma_{ij}, \delta << 1$
- But our universe has $\delta \gtrsim 1$ on scales $\lesssim \mathcal{O}(10\,Mpc)$
- So we may use relativistic pert. theory if we smooth out these scales.

  In other words when we evaluate integrals we use a cutoff $K_{MAX} \approx \frac{h}{10\,Mpc}$

- When we evaluate the Transfer function we have:



$$q \equiv \frac{K}{\Gamma\, h\, Mpc^{-1}}$$

Γ is given by: (in standard FLRW)

$$\Gamma = \Omega_0 h e^{-(\Omega_B - \sqrt{2}h\,\Omega_B/\Omega_0)}$$

- The integral is not very sensitive to the cutoff K_{MAX}

$$\frac{\overline{\langle \delta\Theta \rangle_D}}{3H} \simeq \underbrace{\frac{a}{a_0}}_{\frac{1.9\,10^{-5}}{\Omega_M(a)}} A^2 \underbrace{\left(\frac{h\,\mathrm{Mpc}^{-1}}{H_0}\right)^2}_{3000^2} \Gamma^2 \underbrace{\int_{q_{MIN}}^{q_{MAX}} dq\, q\, T^2(q)}_{2\cdot 10^{-2}}$$

$$\simeq \cancel{} \cdot A \cdot \varepsilon \simeq 2\cdot 10^{-5}\, \varepsilon$$

Where $\varepsilon \equiv \frac{A}{1+z}\left(\frac{h\,\mathrm{Mpc}^{-1}\,\Gamma}{H_0}\right)^2 \cdot \mathrm{Int}$

$$\simeq \frac{4\,\Gamma^2}{\Omega_M(1+z)}$$

Depending on the values of the parameters (h, Ω_B, Ω_M, ...) the expansion parameter ε can become of $\mathcal{O}(1)$ today.

[see fig. 1 of astro-ph/0503715]

- HIGHER ORDERS : Corrections of the n^{TH} order go like $A\,\varepsilon^{n-1}$

IS IT A GAUGE ARTIFACT?
WHAT ABOUT NEWTONIAN GAUGE?

(Hui & Seljak '95
Fry & Siegel '05)

$$ds^2 = -(1+\varphi)\,dt^2 - (1-\varphi)\,dx^2$$ (WEAK FIELD)

- It gives a correction $\langle \varphi \nabla^2 \varphi \rangle \tau^2$

- φ is small but what about higher orders?

[astro-ph/9707278]

$$\varphi = \varphi^{(1)} + \varphi^{(2)}$$ (SECOND ORDER RELATIVISTIC PERTURBATION THEORY)

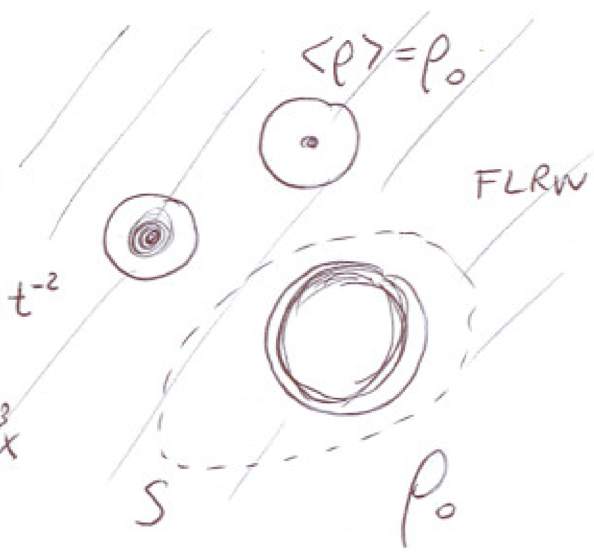
And $\varphi^{(2)}$ contains gradients!

$$\varphi^{(2)} = \underbrace{\frac{4}{3}\varphi^2}_{\mathcal{O}(10^{-10})} + \underbrace{\tau^2\left(\frac{1}{6}\partial_i\varphi\,\partial^i\varphi\right)}_{\mathcal{O}(10^{-5})} + \dots$$ today $(z \approx 0)$

Newtonian terms

Why are they total gradients?

By Gauss theorem this configuration has no effect $\langle \rho_D \rangle \propto \rho_{FLRW} \propto t^{-2}$



$$\int_V (\text{Newtonian term})\, d^3x = \int_V \partial(\Theta)\, d^3x$$

$$\int_S \Theta\, dS = 0$$

$\Theta = 0$ on the surface S

- The same result can be extended to Relativistic spherically symmetric swiss-cheese configuration?

? Birkhoff's theorem? $\longrightarrow$ no backreaction

$$\langle \rho \rangle = \rho_{FLRW} \propto a_{FLRW}^{-3} \propto t_{FLRW}^{-2}$$

- Nonetheless we can use Swiss-Cheese Lemaître - Tolman - Bondi (LTB) models matched to FLRW to study

$$\begin{cases} \text{LOCAL EFFECTS of LARGE STRUCTURES} \\ \text{LIGHT PROPAGATION (on } D_L - z) \end{cases}$$

MORE ON BACKREACTION!

gr-qc/9906015
astro-ph/0503715
astro-ph/0506534

1 IS ACCELERATION POSSIBLE?

- General approach (non-perturbative but only qualitative)

$$\begin{cases} ds^2 = -dt^2 + \gamma_{ij}\, dx^i dx^j \\ \Theta^i{}_j \equiv u^i{}_{;j} = \frac{1}{2}\gamma^{ik}\dot{\gamma}_{kj} \\ \quad \equiv \frac{\Theta}{3}\delta^i_j + \sigma^i{}_j \end{cases}$$

TRACE : $\boxed{\dot{J} = \Theta J}$ $\quad (J \equiv \sqrt{\det \gamma})$

0-0 eq.) $\quad \frac{1}{2}R + \frac{1}{3}\Theta^2 - \sigma^2 = 8\pi G\rho \qquad (\sigma^2 \equiv \frac{1}{2}\sigma^i{}_j\sigma^j{}_i)$

(R spatial Ricci scalar)

CONT. EQ.) $\quad \dot{\rho} = -\Theta\rho$

- So local expansion is $\frac{\Theta}{3}$

But Raychaudhuri equation (Trace i-j combined with 0-0)

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 + 4\pi G\rho = 0$$

gives for the deceleration parameter (Hirota & Seljak '05, Flanagan '05)

$$q_{\text{LOCAL}} \equiv -1 - 3\frac{\frac{d}{dt}\Theta}{\Theta^2} = \frac{6(\sigma^2 + 2\pi G\rho)}{\Theta^2} > 0$$

No local acceleration!

<u>However</u> if I define the expansion on a finite volume

$$\langle\theta\rangle_D = \frac{1}{V_D}\int d^3x\,\sqrt{\gamma}\,\theta \qquad \left(V_D = \int\sqrt{\gamma}\,d^3x\right)$$

$$q_D \equiv -1-3\,\frac{\frac{d}{dt}\langle\theta\rangle_D}{\langle\theta\rangle_D^2}$$

Then $$q_D = \frac{6\left(\langle\sigma^2\rangle + 2\pi G\langle\rho\rangle\right)}{\langle\theta\rangle^2} - \frac{1}{2}\,\frac{\langle\theta^2\rangle-\langle\theta\rangle^2}{\langle\theta\rangle^2} - \frac{3}{2}\left(\frac{\langle\theta\sigma\rangle-\langle\theta\rangle\langle\sigma\rangle}{\langle\theta\rangle^2}\right)$$

$$\left(\text{SINCE } \frac{d}{dt}\langle\theta\rangle_D \neq \left\langle\frac{d\theta}{dt}\right\rangle_D\right)$$

$q<0$
ACCELERATION IS POSSIBLE!

Effective Buchert equations ('99)

$$a_D \equiv \left(\frac{V_D(t)}{V_{D_0}}\right)^{1/3}$$

Use $\dot{J} = \theta J$ to prove

$$\begin{cases} \dfrac{\dot{V}_D}{V_D} = \langle\theta\rangle_D = 3\,\dfrac{\dot{a}_D}{a_D} \\ \langle\rho\rangle_D^{\cdot} = -\langle\theta\rangle_D\,\langle\rho\rangle_D \end{cases}$$

$$\begin{cases} \left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{eff} & (*) \\ \frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3}\left(\rho_{eff} + 3 p_{eff}\right) & (**) \\ \langle\rho\rangle_D^{\cdot} = -3\frac{\dot{a}_D}{a_D}\langle\rho\rangle_D & (***) \end{cases}$$

~~Consistent of~~

Where the effective ρ_{eff} and p_{eff} are

$$\begin{cases} \rho_{eff} = \langle\rho\rangle_D - \frac{Q_D}{16\pi G} - \frac{\langle R\rangle_D}{16\pi G} \\ p_{eff} = -\frac{Q_D}{16\pi G} + \frac{\langle R\rangle_D}{48\pi G} \end{cases}$$

$$\boxed{Q_D = \frac{2}{3}\left(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\right) - 2\langle\sigma^2\rangle_D}$$

Consistency of (*) (**) and (***) give

$$\left(a_D^6 Q_D\right)^{\cdot} + a_D^4\left(a_D^2\langle R\rangle_D\right)^{\cdot} = 0$$

Acceleration <u>IF</u> $\boxed{Q_D > 4\pi G\langle\rho\rangle_D}$

. particular solution $Q_D = -\frac{1}{3}\langle R\rangle_D = \text{const}$

mimicks Λ

- But we do not know how to calculate Q_D non perturbatively (magnitude and time dependence)!

- It's purely relativistic

- Go <u>beyond Pert. theory</u> and <u>beyond Spherical Symmetry</u>

$\boxed{\text{LTB}}$ <u>MODELS</u> (EXACT, RELATIVISTIC SPHERICALLY SYMMETRIC) CAN BE GLUED TO FLRW

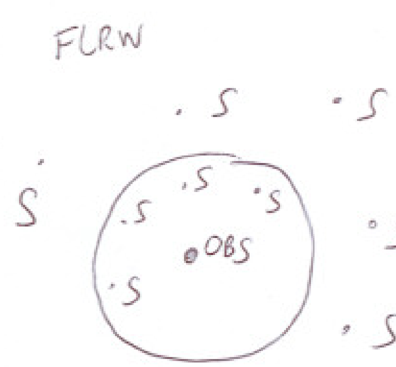
Why are they useful?

0 <u>Check</u>: backreaction is not there? (

1 [SWISS-CHEESE]

<u>LIGHT PROPAGATION</u>

2 <u>LOCAL special position</u> of OBSERVER



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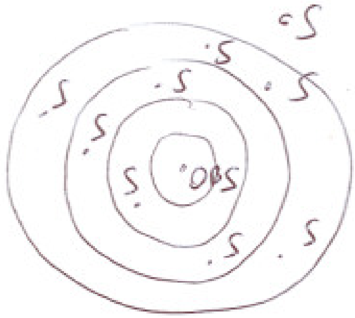


ONION - LIKE

REMIND • $\mathcal{O}(1)$ fluctuations in $\frac{\delta\rho}{\rho}$ at $\mathcal{O}(10\ \mathrm{Mpc}/h)$

• STRUCTURES observed up to $\mathcal{O}(100\ \mathrm{Mpc}/h)$

Sloan Great Wall $\sim 400\ \mathrm{Mpc}/h$

• HORIZON 3000 Mpc/h

LTB Metrics

$$ds^2 = -dt^2 + S^2(r,t)\,dr^2 + R^2(r,t)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

COMOVING COORD., proper time

DUST. Einstein equations:

Arbitrary functions
$M(r)$, $E(r)$

$$\begin{cases} S^2(r,t) = \dfrac{R'^2(r,t)}{1+2E(r)} \\ \dfrac{1}{2}\dot{R}^2(r,t) = \dfrac{G\,M(r)}{R(r,t)} + E(r) \\ 4\pi\rho(r,t) = \dfrac{M'(r)}{R'(r,t)\,R^2(r,t)} \end{cases}$$

In other words

$$\begin{cases} ds^2 = -dt^2 + \dfrac{R'^2(r,t)}{1+2E(r)}\, dr^2 + R^2(r,t)\, d\Omega \\[2ex] M(r) = 4\pi \displaystyle\int_0^r \rho(\tilde{r},t)\, R^2(\tilde{r},t)\, R'(\tilde{r},t)\, d\tilde{r} \\[2ex] \dfrac{1}{2}\dfrac{\dot{R}^2}{R^2} = \dfrac{G M(r)}{R^3} + \dfrac{E(r)}{R^2} \end{cases}$$

(If $M(r)$ is a growing function,
I can always redefine the radial coordinate such that:

$$M(r) = \frac{4\pi}{3} M_0^4 r^3 \quad)$$

- FLRW limit $\quad E(r) = \begin{cases} E_0 r^2 & \text{CLOSED} \\ 0 & \text{FLAT} \\ -E_0 r^2 & \text{OPEN} \end{cases}$

- GENERAL Solutions

$E=0)$
$$R(r,t) = \left[\frac{9\, G M(r)}{2}\right]^{1/3} \left[t - t_B(r)\right]^{2/3}$$

$E>0)$
$$\begin{cases} R = \dfrac{G M(r)}{2E(r)} (\cosh u - 1) \\[2ex] t - t_B(r) = \dfrac{G M(r)}{2E(r)} (\sinh u - u) \end{cases}$$

$E<0)$ $$R = \frac{GM(r)}{-2E(r)}(1-\cos u)$$

$$t - t_B(r) = \frac{GM(r)}{[-2E(r)]^{3/2}}(u - \sin u)$$

OSS As $t \to +\infty$ the bang time function is irrelevant, ~~i.e. the universe becomes more and more homogeneous~~

i.e. it is important only at early time (decaying mode).

So, we have the choice on one function $E(r)$

$$k(r) \equiv \frac{E(r)}{\tilde{M}^2 r^2}$$ [astro-ph/0606703 astro-ph/0702555]

$$\left(\tilde{M} \equiv \frac{M_0^2}{m_{PL}}\right)$$

Two possibilities

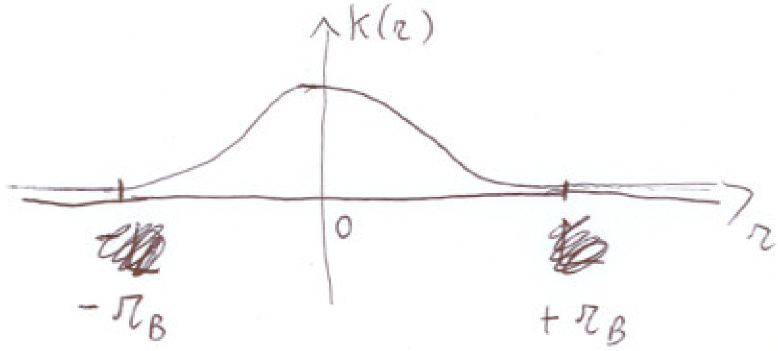
) $k(r)$ describes an "Onion" Universe for $0 \le r \le \infty$

$k(r)$ non trivial inside a sphere and then match to FLRW $(k = \text{const})$

OBS: In general $k'(0) = 0$ to avoid cusps and singularities in the density or in the metric.

OBS We choose for simplicity $k(r) > 0$

② Swiss-cheese - Match to FLRW [gr-qc/0307023]



$$\begin{cases} k'(r_B) = 0 \\ k(r_B) = k_{FLRW} = \frac{4\pi}{3}\Omega_k \end{cases}$$

$$R(r,t) = \begin{cases} \text{non trivial} & \text{for } r \leq r_B \\ r\, a(t) & \text{for } r \geq r_B \quad (\text{FLAT FRW}) \end{cases}$$

$$\boxed{\begin{array}{l} R(r,t),\ R'(r,t) \quad \text{CONTINUOUS} \\ M(r) = M_0^4 \frac{4\pi}{3} r^3 \end{array}}$$

$\Rightarrow \rho(r_B) = \rho_{FLRW}$ CONTINUOUS SINCE $\rho = \dfrac{M'}{4\pi R^2 R'}$

Now one can easily show that (if $E \ll 1$!!)

$\langle \rho \rangle_{r_B} = \rho_{FLRW} \propto a_{FLRW}^{-3} \propto t^{-2}$ $\boxed{\text{NO BACKREACTION}}$

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$$\langle \rho \rangle_{r_B} = \frac{\int d^3x\, \rho(x) \sqrt{\gamma}}{\int d^3x \sqrt{\gamma}} = \frac{\int dr\, \cancel{d\theta d\varphi}\, \rho(r)\, R^2 R'}{\int dr\, \cancel{d\theta d\varphi}\, R^2 R'} =$$

$$= \frac{\int dr\, M'(r)}{4\pi \int dR\, R^2} = \frac{M(r_B)}{4\pi R^3/3} =$$

$$= \frac{M_0^4\, \cancel{\frac{4\pi r_B^3}{3}}}{\cancel{\frac{4\pi r_B^3}{3}}\, a^3_{FLRW}} = \frac{M_0^4}{a^3_{FLRW}}$$

By continuity $M_0^4 = \rho_{0,FLRW}$ (valid also for generic $M(r)$)

The ~~very~~ fact that we can match to FLRW is ~~exactly~~ because there is no backreaction?

(Otherwise $a(t)$ outside would not evolve as FLRW $a(t) \sim t^{2/3}$)

This is ~~analogous~~ due to the fact that there is spherical symmetry. (for $E \ll 1$)

It is the same thing that would happen if I wanted to study GW with a spherically symmetric model (they are zero by construction).

Void surrounded by dense structure _ Example:

$$K(r) = K_0 \left[\left(\frac{r}{L}\right)^2 - 1 \right]^2$$

What is the corresponding $\rho(r,t)$?

- Numerically
- Analitically? One has to solve for $u(r,t)$

For small u the following holds

$$\boxed{u \simeq \gamma \tau \sqrt{k(r)} \ll 1}$$

$$\tau \equiv (\tilde{M} t)^{1/3}$$
$$\gamma = \left(\frac{9\sqrt{2}}{\pi}\right)^{1/3}$$

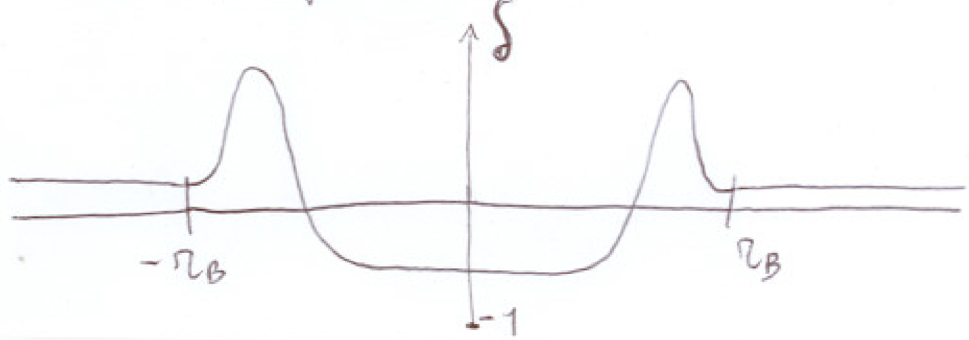
- Expand the $\cosh u$, $\sinh u$

$$R(r,t) \propto \tau^2 r \left[1 + \underset{\substack{\parallel \\ 1/20}}{R_2} \gamma^2 \tau^2 k(r)\right]$$

$$\rho(r,t) \simeq \underbrace{\frac{M_0^4}{6\pi(\tilde{M}t)^2}}_{\text{homogeneous part}} \frac{1}{\left[1 + R_2 \gamma^2 \tau^2 A(r)\right]}$$

$$A(r) = (r k)'$$

$$\begin{cases} \delta \propto \tau^2 A(r) & \text{at small } \delta \quad (\text{consistent with P.T.}) \\ \delta \propto \dfrac{1}{1 + R_2 \gamma^2 \tau^2 A(r)} & \text{at large } \delta \quad (\text{as in Zeldovich approx.}) \end{cases}$$



To have a "Onion" model we choose

$$k(r) = \frac{1}{r} K_0 \frac{L}{2\pi} \sin^2\left(\frac{\pi r}{L}\right)$$

- IT GOES TO ZERO for $r \to \infty$
  (we do not introduce curvature that can spoil CMB 1^{st} peak)

- It gives $\rho \simeq \rho_0(t) \dfrac{1}{1 + \underbrace{\varepsilon(t)}_{\substack{\propto \\ K_0 \cdot \tau^2}} \sin^2\left(\frac{\pi r}{L}\right)}$

Now we can compute propagation of photons in this metric to make contact with observations (D_L and z)

Three steps

1) derive $t(r)$ along a photon trajectory

2) derive $z(r)$ " " " "

3) derive $D_L(r)$ " " " "

① $t(r)$ can be solved using

$$ds = 0 \implies \frac{dt}{dr} = -\frac{R'}{\sqrt{1+2E(r)}}$$

NUMERICAL SOLUTION

ANALYTICAL (small u approx.)

$$t(r)^{1/3} = t_F^{1/3}\left[1 + \frac{\varepsilon}{2\pi}\left(\frac{L}{r_{HOR}}\right)\cos\left(\frac{2\pi r}{L}\right)\right]$$

(ONION)

② $$\frac{dz}{dr} = -\frac{(1+z)\,\dot{R}'}{\sqrt{1+2E}}$$ (Célérier astro-ph/0512103)

Consider two light rays

$t = T(r) + \tau(r)$ $\quad (\tau(r) \ll T(r))$

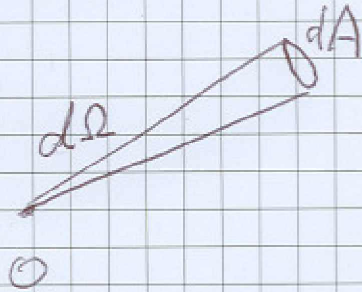
Solve for $\frac{d\tau}{dr}$ and relate τ to z $\quad \left(\tau(r) = \frac{\tau(0)}{1+z}\right)$

$$z \simeq \frac{2\beta\tilde{M}\,\delta r}{(\tilde{M}t_0)^{1/3}}\left[1 - \frac{\epsilon}{\pi}\frac{L}{\delta r}\cos\left(\frac{2\pi r}{L}\right)\right]$$ (for small z)

$\delta r = r - r_0$

③ D_L ?

Use $D_L = (1+z)^2 D_A$ (always true in GR)



D_A (angular distance) is defined as:

(Ellis et al.

$$D_A^2 \equiv \frac{dA}{d\Omega} = \frac{d\theta_S\, d\phi_S \sqrt{g_{\theta\theta}\, g_{\phi\phi}}}{d\theta_O\, d\phi_O}$$

where the observer coordinates satisfy:

1. THE TIME-LIKE COORDINATE IS THE PROPER TIME OF THE OBSERVER

2. THE FIRST SPATIAL COORDINATE IS A MONOTONIC PARAMETER ALONG THE PAST LIGHT CONE

3. THE REMAINING TWO COORDINATES LOOK LIKE

$$ds^2 = r^2 \left(d\bar{\theta}^2 + d\tilde{\phi}^2 \sin^2\bar{\theta} \right)$$

↑ AFFINE PARAMETER

If observer in the center

$$D_A = R\big|_{\text{SOURCE}}$$

Alternative (and more general) derivation

$$\frac{1}{\sqrt{A}} \frac{d^2(\sqrt{A})}{dv^2} = \mathcal{R} \qquad \mathcal{R} \equiv \frac{1}{2} R_{\mu\nu} k^\mu k^\nu$$

$$k^\mu = \frac{dx^\mu}{dv}$$

(Sachs '61)

$$\frac{dt}{dv} = -(1+z)$$

RESULTS : ⓪ Is there any backreaction? No (but: we assumed $\epsilon \ll 1$)

① Large corrections near observer

(can we ~~mimick~~ ~~SN data~~ fit SN data?)

② What kind of profile can fit the SN data?

③ In Swiss-cheese is there a net effect for a photon going through a patch?