The Anisotropies of CMB in Krein Space Quantization

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QFT and the problems of gravity quantization

- **Gauge Quantum Field Theory in Minkowski Spacetime**

Two problems:

(a) Breaking of the covariance

(b) Appearance of the infrared divergence

It was shown (1978) that negative norm states or unphysical states are necessary for obtaining a causal and covariant Gauge Quantum Field Theory.

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The general properties of the indefinite metric Fock quantization were studied in 1979.

- Mintchev M., J. Phys. A: Math. Gen. **13**(1979) 1841, "Quantization in indefinite metric".

QFT and the problems of gravity quantization

It is inevitable to introduce conditions to the Fock space and Field operator for eliminating the N.F.S.

de-Sitter spacetime

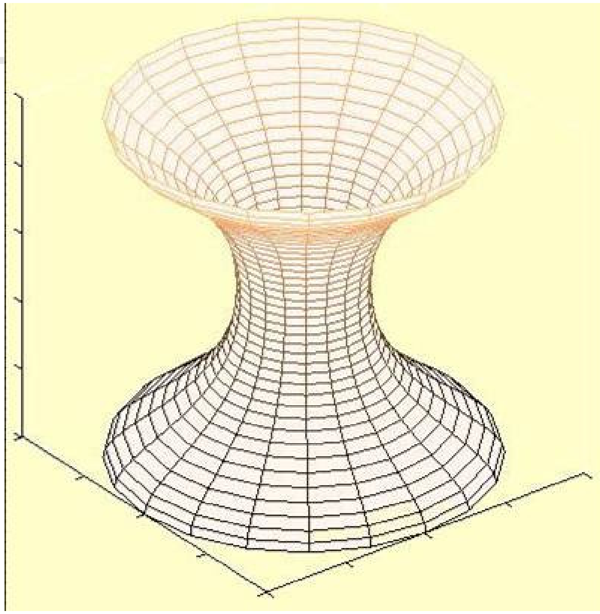
$$X_H = \{x \in R^5; x^2 = \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2}\}, \ \alpha, \beta = 0,1,2,3,4$$

$$\eta_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1,-1)$$

Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \ \Rightarrow \begin{cases} \Lambda = \dfrac{R^{(0)}}{4} = 3H^2 \\ R^{(0)}_{\mu\nu} = 3H^2 g^b_{\mu\nu} \end{cases}$$



Close coordinate:

$$X^0 = H^{-1} tg\rho$$

$$X^1 = (H \cos\rho)^{-1} \sin\alpha \sin\theta \cos\varphi$$

$$X^2 = (H \cos\rho)^{-1} \sin\alpha \sin\theta \sin\varphi \qquad \rho = \tan^{-1}(\sinh Ht)$$

$$X^3 = (H \cos\rho)^{-1} \sin\alpha \cos\theta$$

$$X^4 = (H \cos\rho)^{-1} \cos\alpha \qquad 0 \le \varphi \le 2\pi \ , 0 \le \theta \le \pi, 0 \le \alpha \le \pi, -\frac{\pi}{2} < \rho < \frac{\pi}{2}$$

de-Sitter spacetime

- **1930-1940** : Electron wave eq. in de Sitter spacetime (Dirac)
- **1940-1960** : Irreducible representations of de Sitter group
  (Newton – Tomas – Dexmier – Takahashi)
- **1960-1970** : Quantum field theory in de Sitter spacetime
- **1980** : Appearance of de Sitter metric in inflationary model
- **1986** : Minimally coupled scalar field in de Sitter spacetime

(a) **Breaking of the covariance.**
(b) **Appearance of the infrared divergence.**
- Allen B., Phys. Rev. D, **32** (1985) 3136, "Vacuum states in de Sitter space".

de-Sitter spacetime

- **Covariant Quantization and Negative Frequency States**:

- **Takook M.V.**, Phd thesis, Paris (1997)
- **De Bievre S.**, Renaud J., Phys. Rev. D., **57** (1998) 6230, “Massless Gupta-Bleuler vacuum on the (1+1)-dimensional de Sitter spacetime”.
- **Gazeau J. P.**, Renaud J., Takook M.V., Class. Quant. Grav. **17** (2000) 1415, gr-qc / 9904023, “Gupta-Bleuler quantization for minimally coupled scalar field in de Sitter spacetime.
- **Takook M.V.**, Mod. Phys. Lett. A, **16** (2001) 1691, gr-qc/0005020, “Covariant two-point function for minimally coupled scalar field in de Sitter spacetime.

- **Similarity between GQFT in Minkowski spacetime and QFT in de Sitter spacetime**

(a) Appearance of the infrared divergence

(b) Necessities of the N.F.S. for preservation of the covariance

Linear quantum gravity in de Sitter spacetime and infrared divergence

- **1986** : Graviton propagator (Iliopoulos – Tomaras – Antoniadis , Allen – Tyrun)

- **The appearance of infrared divergence**

- **Antoniadis I.**, Iliopoulos J., Tomaras T. N., Phys. Rev. Lett. **56** (1986) 1319, “Quantum instability of de Sitter space”.

- **Allen B.**, Tyrun M. Nucl. Phys. B. **292** (1987) 813, “An evaluation of the graviton propagator in de Sitter space”.

- **The infrared divergence is not physical**

- **Antoniadis I.**, Iliopoulos J., Tomaras T. N., Phys. B. **462** (1996) 437, “One-loop effective action around de Sitter space”.

- **Takook M.V.**, PhD thesis, Paris (1997).

Linear quantum gravity in de Sitter spacetime and infrared divergence

- **Takook M.V.**, Proceeding of the Wigsym6, 1999, Istanbul, gr-qc/0001052, “Covariant two-point function for linear gravity in de Sitter space”.

- **Garidi T.**, Gazeau J. P., Renaud J., Rohani S., Takook M.V., “Linear quantum gravity in de Sitter space”.

- **de Vega H. J.**, Ramirez J., Sanchez N., Phys. Rev. D. **60** (1999) 044007, astro-ph/9812465, “Generation of gravitational waves by generic sources in de Sitter spacetime”.

- **Hawking S. W.**, Hertog T., Turok N., hep-th/0003016, “Gravitational waves in open de Sitter space”.

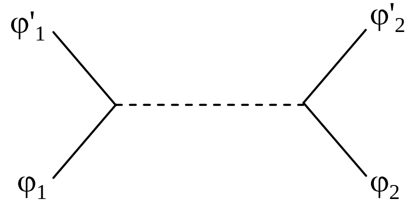
- **Higuchi A.**, Kouris S. S., gr-qc/0004079, “Large-distance behavior of the graviton two-point function in de Sitter spacetime”.

Linear quantum gravity in de Sitter spacetime and infrared divergence

$$G^{\mu\nu\rho'\sigma'} \propto \{\log(-H^2 x.x')\}$$

$$(x.x') \to \infty \quad G^{\mu\nu\rho'\sigma'}(x,x') = \Rightarrow \boxed{G^{\mu\nu\rho'\sigma'} \to \log arithmic - divergence}$$

Tree-level scattering amplitude:



$$M = \int T^{\mu\nu}(x) G^{F}_{\mu\nu\rho'\sigma'}(x,x') T^{\rho'\sigma'}(x') d^4x d^4x'$$

Field eq. in ambient space:

$$h_{\mu\nu}(X) = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x^\beta}{\partial X^\nu} K_{\alpha\beta}(x(X)) \qquad K_{\alpha\beta}(x) = D_{\alpha\beta}(x,\partial)\varphi_{Llm}(x)$$

$$\boxed{\Box\varphi = 0}$$

Two-point function:

$$W_{\alpha\beta\alpha'\beta'}(x,x) = \langle 0 | \; K_{\alpha\beta}(x) K_{\alpha'\beta'}(x') | 0 \rangle \qquad G^{\mu\nu\rho'\sigma'} = \frac{\partial X^\mu}{\partial x^\alpha}\frac{\partial X^\nu}{\partial x^\beta}\frac{\partial X^{\rho'}}{\partial x'^{\alpha'}}\frac{\partial X^{\sigma'}}{\partial x'^{\beta'}} W^{\alpha\beta\alpha'\beta'}$$

$$\boxed{W_{\alpha\beta\alpha'\beta'}(x,x') = \Delta_{\alpha\beta\alpha'\beta'}(x,x') W_{mc}(x,x')}$$

Massless minimally coupled scalar field

$$L = \frac{1}{2}[-g(x)]^{1/2}\{g^{\mu\nu}(x)\nabla_\mu\varphi(x)\nabla_\nu\varphi(x) - [m^2 + \zeta R(x)]\varphi^2(x)\}$$

$$\delta S = 0 \Rightarrow \quad \boxed{[\ \partial^2_H + (m^2 + \zeta R)]\varphi(x) = 0} \qquad \partial^2_H = \frac{1}{\sqrt{-g}}\partial_\nu\sqrt{-g}\,g^{\nu\mu}\partial_\mu \qquad g = \det g_{\mu\nu}$$

field solution: $\boxed{\varphi(x) = \chi(\rho)D(\Omega)}$ $\qquad D(\Omega) = Y_{Llm}(\Omega)$

$$\chi(\rho) = A_L(\cos\rho)^{\frac{3}{2}}[P^\lambda_{L+\frac{1}{2}}(\sin\rho) - \frac{2i}{\pi}Q^\lambda_{L+\frac{1}{2}}(\sin\rho)] \qquad A_L = H\frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(L-\lambda+\frac{3}{2})}{\Gamma(L+\lambda+\frac{3}{2})}\right)^{\frac{1}{2}}$$

$$\lambda = \sqrt{\frac{9}{4} - k} \qquad \frac{9}{4} \geq k \geq 0$$

$$\lambda = i\sqrt{k - \frac{9}{4}} \qquad \frac{9}{4} \leq k \qquad k = \frac{m^2}{H^2} + 12\zeta.$$

Massless minimally coupled scalar field

$$<\phi,\psi>=\frac{i}{H^2}\int_{\rho=0}\varphi^*(\rho,\Omega)\overleftrightarrow{\partial}_\rho\psi(\rho,\Omega)d\Omega$$

$$<\varphi^\lambda_{L'l'm'},(\varphi^\lambda_{Llm})^*>=0$$

$$<\varphi^\lambda_{L'l'm'},\varphi^\lambda_{Llm}>=\delta_{LL'}\delta_{ll'}\delta_{mm'}$$

Massless minimally coupled: m=ζ=0 ⇒ □φ= 0

$$A_0=H\frac{\sqrt{\pi}}{2}\frac{\Gamma(0)}{\Gamma(3)}=\infty$$

$$M_{\mu\nu}=-i(X_\alpha\partial_\beta-X_\beta\partial_\alpha)$$

$$(M_{03}+iM_{04})\varphi_{1,0,0}=-i\frac{4}{\sqrt{6}}\varphi_{2,1,0}+\varphi_{2,0,0}+\frac{3H}{4\pi\sqrt{6}}$$

Zero mode:

$$\varphi_{000}=\psi_g+\frac{\psi_s}{2}$$

$$\psi_g=\frac{H}{2\pi}\qquad \psi_s=-i\frac{H}{2\pi}[\rho+\frac{1}{2}\sin 2\rho]\qquad \rho=tg^{-1}(\sinh Ht)$$

Massless minimally coupled scalar field

$$(M_{03}+iM_{04})\varphi_0=(M_{03}+iM_{04})\psi_s=-i\frac{\sqrt{6}}{4}\varphi_{1,0,0}-i\frac{\sqrt{6}}{4}\varphi^*_{1,0,0}-\frac{\sqrt{6}}{4}\varphi_{1,1,0}-\frac{\sqrt{6}}{4}\varphi^*_{1,0,0}$$

Field operator: $\varphi(x)=\frac{1}{\sqrt{2}}(\varphi_p(x)+\varphi_n(x))$ $\quad a_k|0\rangle=b_k|0\rangle=0$

$$\varphi_p(x)=\sum_k a_k\varphi_k(x)+H.C\ ,\ \varphi_n(x)=\sum_k b_k\varphi^*_k(x)+H.C$$

$$k\in K'=K+\{0\}$$

$$a_k^+|0\rangle=|\text{physical state}\rangle,\quad b_k^+|0\rangle=|\text{unphysical state}\rangle$$

$$W^p_{mc}=\frac{H^2}{8\pi^2}\left[\frac{1}{1-z}-\ln(1-z)+f_{\alpha\beta}(\eta,\eta')+\text{const}\right]$$

$$W_{mc}(x,x')=\frac{H^2 i}{8\pi}\varepsilon(x^0-x'^0)[\delta(1-z)+\theta(z-1)]$$

$$\varepsilon(x^0-x'^0)=\begin{cases}1 & x^0\ \phi\ x'^0\\ 0 & x^0=x'^0\\ -1 & x^0\ \pi\ x'^0\end{cases}$$

$$z(x,x')=-H^2x.x'=-H^2\eta_{\rho\sigma}x^\rho x^\sigma$$

$$\rho,\sigma=0,1,2,3,4$$

Covariant graviton field operator

$$K_{\alpha\beta}(x) = \sum_{\lambda,L} a^{\lambda}_{Llm} D^{\lambda}_{\alpha\beta}(x,\partial)\varphi_{Llm}(\rho,\Omega) + H.C$$

Vacuum definition: $\mathbf{a^{\lambda}_{Llm}\,|0\rangle = 0}$ $\quad \forall\ \mathbf{0 \le l \le L} \quad \mathbf{-l \le m \le l}$

Covariant field operator:

$$K_{\alpha\beta}(x) = \sum_{\lambda,L} a^{\lambda}_{Llm} D^{\lambda}_{\alpha\beta}(x,\partial)\varphi_{Llm}(\rho,\Omega) + H.C + \sum_{\lambda,L} b^{\lambda}_{Llm} D^{\lambda}_{\alpha\beta}(x,\partial)\varphi^{*}_{Llm}(\rho,\Omega) + H.C$$
$$\forall\ 0 \le l \le L \qquad -l \le m \le l$$

Gupta-bleuler vacuum:

$$a^{\lambda}_{Llm}|0\rangle = 0 \qquad b^{\lambda}_{Llm}|0\rangle = 0$$

Generalization to QFT in Minkowski spacetime

The principles of QFT:

(1) Covariance
(2) Causality
(3) Existence of the Vacuum
(4) Positivity

The principles of QFT with the negative frequency states (N.F.S) :

(1) Covariance
(2) Causality
(3) Existence of the Vacuum

Krein QFT calculation

(a) Free scalar field:

$$(\Box + m^2)\phi(x) = 0$$

The two sets of solutions:

$$u_p = u_k(x) = \frac{e^{-ik.x}}{\sqrt{(2\pi)^3 2\omega_k}}$$ **Positive energy**

$$u_n = u_k^*(x) = \frac{e^{ik.x}}{\sqrt{(2\pi)^3 2\omega_k}}$$ **Negative energy**

$$k^0 = \omega_k = \omega_{-k} = \sqrt{k^2 + m^2} \geq 0$$

$$u_p(k,x) = u_n^*(k,x) \quad , \quad u_n(k,x) = u_p^*(k,x)$$

$$\phi_K(x) = \frac{1}{\sqrt{2}}\left[\phi_p(x) + \phi_n(x)\right]$$

$$\phi_p(x) = \int d^3\mathbf{k}\left[a(\mathbf{k})u_p(k,x) + a^+(\mathbf{k})u_p^*(k,x)\right]$$

$$\phi_n(x) = \int d^3\mathbf{k}\left[b(\mathbf{k})u_n(k,x) + b^+(\mathbf{k})u_n^*(k,x)\right]$$

The nonzero commutation relations:

$$\left[a(\mathbf{k}), a^+(\mathbf{k}')\right] = \delta^3(\mathbf{k} - \mathbf{k}') \quad , \quad \left[b(\mathbf{k}), b^+(\mathbf{k}')\right] = -\delta^3(\mathbf{k} - \mathbf{k}')$$

Krein QFT calculation

The vacuum state :

$$a^+(\mathbf{k})|0>=|1_k> \quad =|Physical-state\rangle \quad ;$$

$$b^+(\mathbf{k})|0>=|\bar{1}_k> \quad =|Unphysical-state\rangle \quad ;$$

$$a(\mathbf{k})|0>=0 \quad \forall\mathbf{k}$$

$$b(\mathbf{k})|0>=0 \quad \forall\mathbf{k}$$

$$a(\mathbf{k})|unphysical-state>=0 \ , \ b(\mathbf{k})|physical-state>=0 \quad \forall\mathbf{k}$$

The norm in Krein space:

$$<0|0>=1 \qquad <1_k|1_{k'}>=\delta^3(\mathbf{k}-\mathbf{k}') \qquad <\bar{1}_k|\bar{1}_{k'}>=-\delta^3(\mathbf{k}-\mathbf{k}')$$

The vacuum energy:

$$H=\int d^3\mathbf{k}\left[a^+(\mathbf{k})a(\mathbf{k})+b^+(\mathbf{k})b(\mathbf{k})+a^+(\mathbf{k})b^+(\mathbf{k})+a(\mathbf{k})b(\mathbf{k})\right]k^0$$

$$<0|a^+a|0>=<0|b^+b|0>=<0|a^+b^+|0>=<0|ab|0>=0$$

$$<0|H|\ 0>=0$$

Krein QFT calculation

The two point function: $$iG_T(x,x') = <0|T\phi(x)\phi(x')|0>$$

$$G_T(x,x') = \mathrm{Re}\, G_F^P(x,x') = -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}$$

$$2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu) \geq 0$$

The Feynman propagator:

$$G_F^P(x,x') = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik.(x-x')}}{k^2 - m^2 + i\varepsilon} = -\frac{1}{8\pi}\delta(\sigma) + \frac{m^2}{8\pi}\theta(\sigma)\left[\frac{J_1(\sqrt{2m^2\sigma}) - iN_1(\sqrt{2m^2\sigma})}{\sqrt{2m^2\sigma}}\right] - \frac{im^2}{4\pi^2}\theta(-\sigma_0)\frac{K_1(\sqrt{2m^2(-\sigma_0)})}{\sqrt{2m^2(-\sigma_0)}}$$

The divergence appears in the imaginary part of this eq. and the real part is convergent :

$$\lim_{\sigma_0\to 0} \mathrm{Re}\, G_F^P(x,x') = \frac{m^2}{16\pi} - \frac{1}{8\pi}\delta(\sigma_0) \quad , \quad \lim_{\sigma_0\to\infty} \mathrm{Re}\, G_F^P(x,x') = 0$$

$$\lim_{z\to 0}\frac{J_1(z)}{z} = \frac{1}{2} \quad , \quad \lim_{z\to 0}\frac{N_1(z)}{z} = -\frac{2}{\pi}\frac{1}{z^2} \quad , \quad \lim_{z\to 0}\frac{K_1(z)}{Z} = \frac{1}{z^2}$$

$$J_1(z) = \frac{z}{2}\sum_{s=0}^{\infty}\frac{(-1)^s}{s!(s+1)!}\left[\frac{z}{2}\right]^{2s}$$

$$N_1(z) = 2J_1(z)\log\frac{z}{2} - \frac{2}{z}$$

$$K_1(z) = -\frac{\pi}{2}\left[J_1(iz) + iN_1(iz)\right]$$

Krein QFT calculation

Considering quantum metric fluctuations:

$$< G_T(x,x') >= \frac{-1}{8\pi}\sqrt{\frac{\pi}{2\langle \sigma_1{}^2 \rangle}}\exp(-\frac{\sigma_0{}^2}{2\langle \sigma_1{}^2 \rangle}) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}$$

For $2\sigma_0 = 0$: $h_{\mu\nu}$, $\langle \sigma_1{}^2 \rangle \neq 0$

$$< G_T(0) >= \frac{-1}{8\pi}\sqrt{\frac{\pi}{2\langle \sigma_1{}^2 \rangle}} + \frac{m^2}{8\pi}$$

- Rouhani S., Takook M. V., “A naturally renormalized quantum field-theory”,gr-qc/0607027.
- Ford H. L., “Quantum field theory in curved spacetime”, gr-qc/9707062.

Krein QFT calculation

(b) Free massless vector field:

$$\Box A_{\mu} - (1-\Lambda)\partial_{\mu}(\partial_{\nu}A^{\nu}) = 0$$

$$A_{K_{\mu}}(x) = \frac{1}{\sqrt{2}}\int d^{3}\mathbf{k}\sum_{\lambda=0}^{3}\varepsilon_{\mu}(\mathbf{k},\lambda)\left[(a_{\lambda}(\mathbf{k}) + b_{\lambda}^{+}(\mathbf{k}))u_{p}(k,x) + (a_{\lambda}^{+}(\mathbf{k}) + b_{\lambda}(\mathbf{k}))u_{n}(k,x)\right]$$

$$\left[a_{\lambda}(\mathbf{k}), a_{\lambda'}{}^{+}(\mathbf{k}')\right] = -\eta^{\lambda\lambda'}\delta^{3}(\mathbf{k}-\mathbf{k}') \ , \ \left[b_{\lambda}(\mathbf{k}), b_{\lambda'}{}^{+}(\mathbf{k}')\right] = +\eta^{\lambda\lambda'}\delta^{3}(\mathbf{k}-\mathbf{k}')$$

$$\left|one-physical-state\right\rangle \equiv |1_{\mathbf{k},\lambda}^{a}> \propto a_{\lambda}^{+}(\mathbf{k})|0> \ , \ \lambda = 1,2$$

$$\left|one-unphysical-state\right\rangle \equiv |\overline{1}_{\mathbf{k},\lambda}^{a}> \propto a_{\lambda}^{+}(\mathbf{k})|0> \ , \ \lambda = 0,3$$

$$\left|one-unphysical-state\right\rangle \equiv |\overline{1}_{\mathbf{k},\lambda}^{b}> \propto b_{\lambda}^{+}(\mathbf{k})|0> \ , \ \lambda = 0,1,2,3$$

Four of the above states have negative norms, and two of them are unphysical positive norm states.

The vacuum energy and momentum : $<0|\mathrm{H}|\ 0>=0$ $\quad <0|P|\ 0>=0$

The Anisotropies of CMB and the Spectrum of Density Perturbation

- Definition of the spectrum: $P_f = (\frac{Lk}{2\pi})^3 4\pi < |f_k|^2 >$

$\sigma_f^2 = \sum < |f_k|^2 > \quad \rightarrow \quad \sigma_f^2 \equiv < f^2(x) > = \int_0^\infty P_f(k) \frac{dk}{k}$

- Spectrum in Hilbert space:

de Sitter metric: $ds^2 = dt^2 - e^{2Ht} dx^2$

$$\Phi(x) = (2\pi)^{-\frac{3}{2}} \int d^3k [b(k) u_k(t) e^{iK.X} + b^+(k) u_k^*(t) e^{-iK.X}]$$

$$u_k^{''} + 3H u_k^{'} + \frac{k^2}{a^2} u_k = 0$$

for $\lambda << H^{-1}$ $\quad k >> aH \quad \rightarrow \quad u_k^{''} + \frac{k^2}{a^2} u_k = 0$

for $\lambda >> H^{-1}$ $\quad k << aH \quad \rightarrow \quad u_k^{''} + 3H u_k^{'} = 0$

with $u_k = \frac{\sigma_k}{a}$, $\quad \frac{k}{aH} = -k\tau \quad \sigma_k^{''} + (k^2 - \frac{a^{''}}{a}) \sigma_k = 0$

The Anisotropies of CMB

On subhorizon scales: $k^2 >> \frac{a''}{a} \quad \rightarrow \quad \sigma_k'' + k^2\sigma_k = 0 \quad \rightarrow \quad \sigma_k^{(p)} = \frac{e^{-ik\tau}}{\sqrt{2k}} \quad , \sigma_k^{(n)} = \frac{e^{+ik\tau}}{\sqrt{2k}}$

The solution of field equation: $u_k^{(p)} = \frac{e^{-ik\tau}}{a\sqrt{2k}} \quad , \quad u_k^{(n)} = \frac{e^{+ik\tau}}{a\sqrt{2k}}$

On superhorizon scales: $k^2 << \frac{a''}{a} \quad \rightarrow \quad \sigma_k'' - \frac{a''}{a}\sigma_k = 0 \quad \rightarrow \quad \sigma_k = B(k)a$

at $k = aH \quad \rightarrow \quad |B(k)| = \frac{1}{a\sqrt{2k}} = \frac{H}{\sqrt{2k^3}}$

The solution of field equation: $\sigma_k^{(p)} = \frac{e^{-ik\tau}}{\sqrt{2k}}(1+\frac{i}{k\tau})$ or $u_k^{(p)} = \frac{e^{-ik\tau}}{a\sqrt{2k}}(1+\frac{i}{k\tau})$

$$<\Phi^2> = \frac{1}{(2\pi)^3}\int |u_k^{(p)}| d^3k = \frac{1}{(2\pi)^3}\int \frac{d^3k}{k}(\frac{1}{2a^2} + \frac{H^2}{2k^2})$$

$$\Rightarrow \quad P_\Phi(k) = \frac{H^2}{4\pi^2}$$

Comparing with: $<\Phi^2> = \int_0^\infty P_\Phi(k)\frac{dk}{k}$

The Anisotropies of CMB in krein space

Spectrum in krein Space: $\phi_K(x)=\left[\phi_p(x)+\phi_n(x)\right]$

First perspective: The gravitational field appears as an internal phenomenon imposed on the structure of spacetime.

$$\Phi(x)=(2\pi)^{\frac{-3}{2}}\int d^3k\{\left[a(k)u_k^{(p)}(t)e^{+iK.x}+a^+(k)u_k^{*(p)}(t)e^{-iK.x}\right]$$
$$+\left[b(k)u_k^{(n)}(t)e^{-iK.x}+b^+(k)u_k^{*(n)}(t)e^{iK.x}\right]\}$$

$$u_k^{(p)}=\frac{e^{-ik\tau}}{a\sqrt{2k}}(1+\frac{i}{k\tau})$$

$$u_k^{(n)}=\frac{e^{+ik\tau}}{a\sqrt{2k}}(1-\frac{i}{k\tau})$$

$$<\Phi^2>=\frac{1}{(2\pi)^3}\int\left|u_k^{(p)}\right|d^3k-\frac{1}{(2\pi)^3}\int\left|u_k^{(n)}\right|d^3k=0 \quad\rightarrow\quad P_\phi=0$$

Adding quantum metric fluctuation to QFT in kren space:

$$<G_T(x,x')>=\frac{-1}{8\pi}\sqrt{\frac{\pi}{2\left\langle\sigma_1^2\right\rangle}}\exp(-\frac{\sigma_0^2}{2\left\langle\sigma_1^2\right\rangle})+\frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}$$

$$<\phi^2>=c \quad\rightarrow\quad P_\phi(k)=ke^{-\alpha k^2} \qquad \alpha=\frac{4\pi}{c^2}$$

We have :

c is related to the density of gravitons.

The Anisotropies of CMB in krein space

Second Perspective: The gravitational field appears as an enternal phenomenon imposed on the structure of spacetime.

The negative energy modes equation and solution:

$$v''_k + \frac{k^2}{a^2} v_k = 0 \qquad v_k^{(n)} = \frac{e^{+ikz}}{a\sqrt{2a}}$$

$$\Phi(x) = (2\pi)^{\frac{-3}{2}} \int d^3k \left\{ \left[a(k) u_k^{(p)}(t) e^{+iK.x} + a^+(k) u_k^{*(p)}(t) e^{-iK.x} \right] + \left[b(k) v_k^{(n)}(t) e^{-iK.x} + b^+(k) v_k^{*(n)}(t) e^{iK.x} \right] \right\}$$

$$< \Phi^2 > = \frac{1}{(2\pi)^3} \int \left| u_k^{(p)} \right|^2 d^3k - \frac{1}{(2\pi)^3} \int \left| v_k^{(n)} \right|^2 d^3k$$

$$< \Phi^2 > = \frac{1}{(2\pi)^3} \int \frac{d^3k}{k} \left(\frac{1}{2a^2} + \frac{H^2}{2k^2} \right) - \frac{1}{(2\pi)^3} \int \frac{d^3k}{2ka^2} = \frac{1}{(2\pi)^3} \int \frac{d^3k}{k} \left(\frac{H^2}{2k^2} \right)$$

Comparing with: $< \Phi^2 > = \int_0^\infty P_\Phi(k) \frac{dk}{k}$

we have $P_\Phi(k) = \dfrac{H^2}{4\pi^2}$

Conclusions

-The problem of singularity in the QFT (the regularization and renormalization procedure)

-The problem of renormalizability in Quantum gravity

-The anomally may be in the QFT, not in the general relativity (which is not renormalized in the quantum level)

-The problem may be solved in the :

Quantum Field Theory with the N.F.S.

The new method of quantization using unphysical negative frequency states results in an automatically renormalized theory.

Conclusions

- Bunch-Davis Vacuum: $P_0(k) = \frac{H^2}{4\pi^2}$
- Alpha Vacuum: $P(k) = P_0(k)[1 - \frac{H}{\Lambda}\sin(\frac{2\Lambda}{H})]$
- Gupta-Bleuler Vacuum: $P(k) = ke^{-\alpha k^2}$ $\alpha = \frac{4\pi}{c^2}$

References

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