Modified Gravity: Dark Energy & Dark Matter

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Lecture I: Standard model of cosmology and acceleration of universe

- Cosmological principle
- FRW metric and equations
- Geometry properties of metric
- Comparison with observations
- Evidence for cosmic acceleration

Cosmological principles

From the observations of large scales:

- Universe is isotrope for us in large scales
- We are not located in a special place of the universe.

In scales larger than few hundred Mpc universe is homogenous.

Observation by Hubble: Universe is expanding

metric of universe

- Isotrope metric
- Spatial part of metric should expand uniformly

$$ds^2 = -dt^2 + a^2(t)\left[f(r)dr^2 + r^2 d\Omega\right]$$

- We demand homogenous universe, by means that curvature of spatial part of metric to be a constant value.

FRW metric

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$$dl^2 = f(r)dr^2 + r^2 d\Omega$$

$$f(r) = \frac{1}{1 - kr^2}$$

Geometry of spatial part of metric

k >0 Newtonian equivalent E<0 (close)

k=0 Newtonian equivalent E=0 (flat)

k<0 Newtonian equivalent E>0 (open)

By coordinate transformation:

$$ds^2 = -dt^2 + a^2(d\chi^2 + \left\{ \begin{array}{l} Sin(\chi)^2 \\ \chi^2 \\ Sinh(\chi)^2 \end{array} \right\} d\Omega)$$

Observation: redshift-apparent magnitude

$$\chi = \int \frac{dt}{a(t)} = \int_0^z \frac{dz}{H(z)}$$

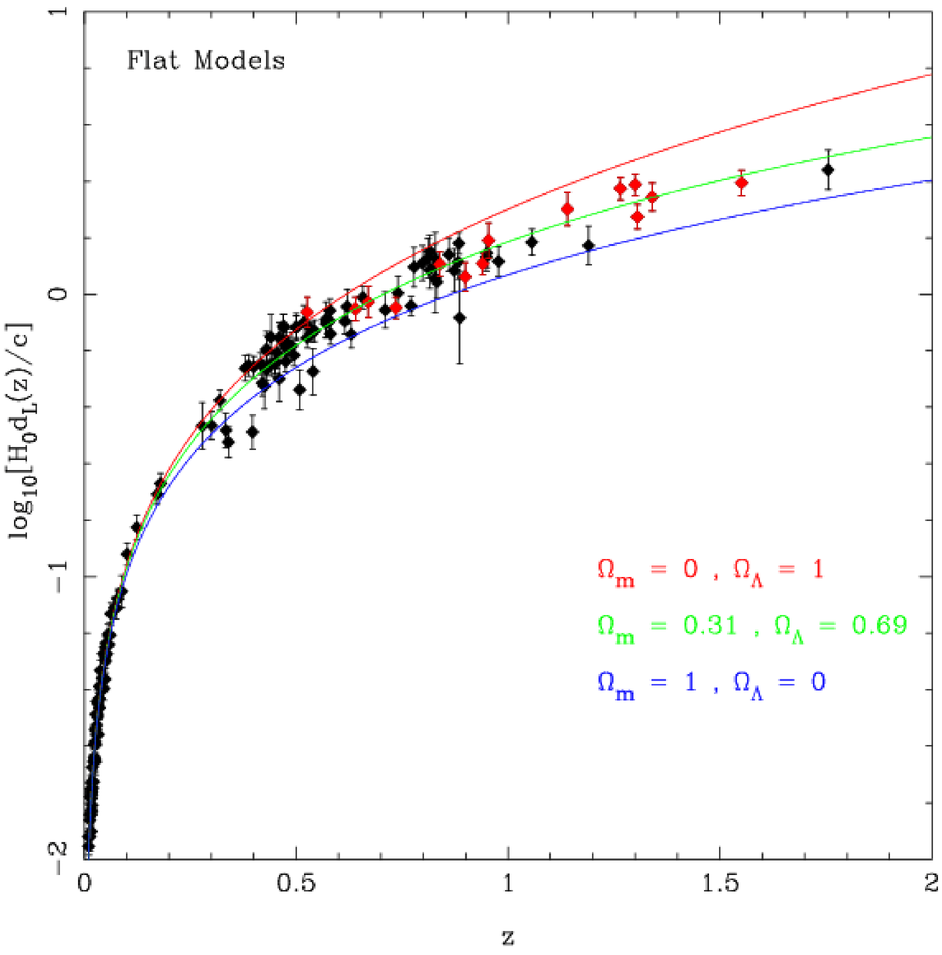
From observation we have the redshift of cosmological objects that knowing the cosmological parameters we will have χ

$$F = \frac{I_0}{4\pi d_L{}^2}$$

$$d_L = C(\chi)(1+z)$$

$$m = C + 5\log(d_L)$$

Standard Candle



Why SNIa seems dimmer

Supernovas are farther than what we expect from CDM model.

$$ds^2 = 0$$

$$\chi = \int \frac{dt}{a(t)} = \int \frac{dz}{H(z)}$$

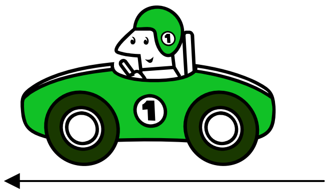
We can simulate light travel with a car

Car $$X = \int_{t_i}^{t_0} v(t)\,dt$$

$$dt \rightarrow dz$$

Light $$\chi = \int_0^z \frac{dz}{H(z)}$$

$$v(t) \rightarrow \frac{1}{H(z)}$$



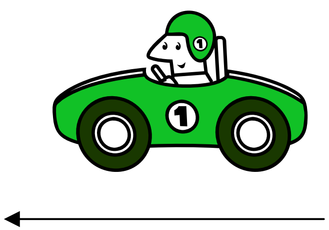
Expecting

$$v_0, t = 0$$

$$Ma = -F_f$$

$$t = -t_1$$

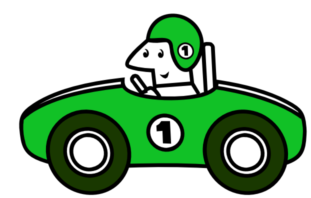
$$v = v_i$$



Observing

$$v_0, t = 0$$

$$Ma' = -F_f + F'$$



$$t = -t_1$$

$'$

$$v = v_i$$

So the acceleration of the second car is more than the first one

$$a' > a$$

$$v'_i > v_i \rightarrow H_{obs}(z_i) < H_{CDM}(z_i)$$

Let's compare the motion of a car with the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_0 a^{-3})$$

CDM

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_0 a^{-3}) + F'$$

Speed up the expansion

One of the possible solutions is using cosmological constant

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} - \Lambda g_{\mu\nu}$$

We can interpret the cosmological constant as a fluid, moving it to the right hand side of the equation. In this case the corresponding density and pressure Is:

$$\rho_\Lambda = \frac{\Lambda}{\kappa}$$

$$p_\Lambda = -\frac{\Lambda}{\kappa}$$

$$\Rightarrow \quad p_\Lambda = -\rho_\Lambda$$

The density and pressure is constant during the expansion of the universe

FRW equations:

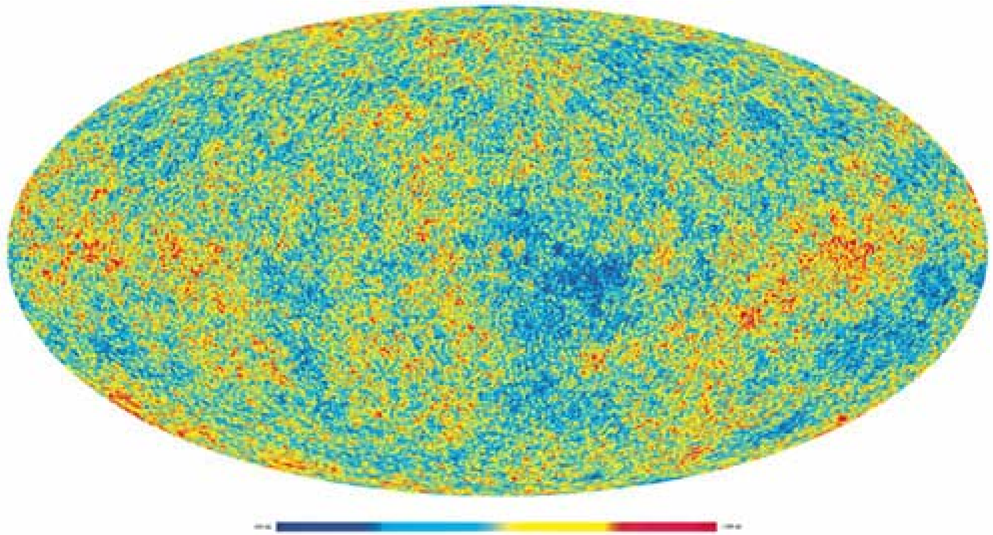
From the continuety equation, for the cosmological constant we have:

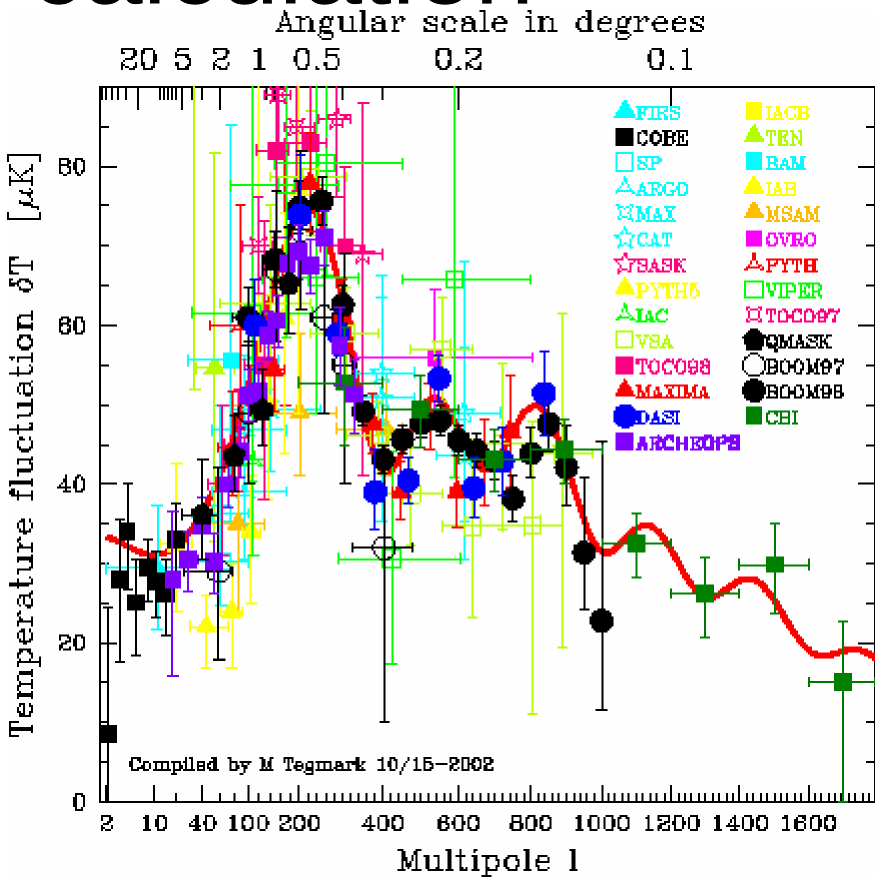
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_0 a^{-3} + (\rho_\Lambda + 3p_\Lambda)\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_0 a^{-3} - 2\frac{\Lambda}{8\pi G}\right]$$

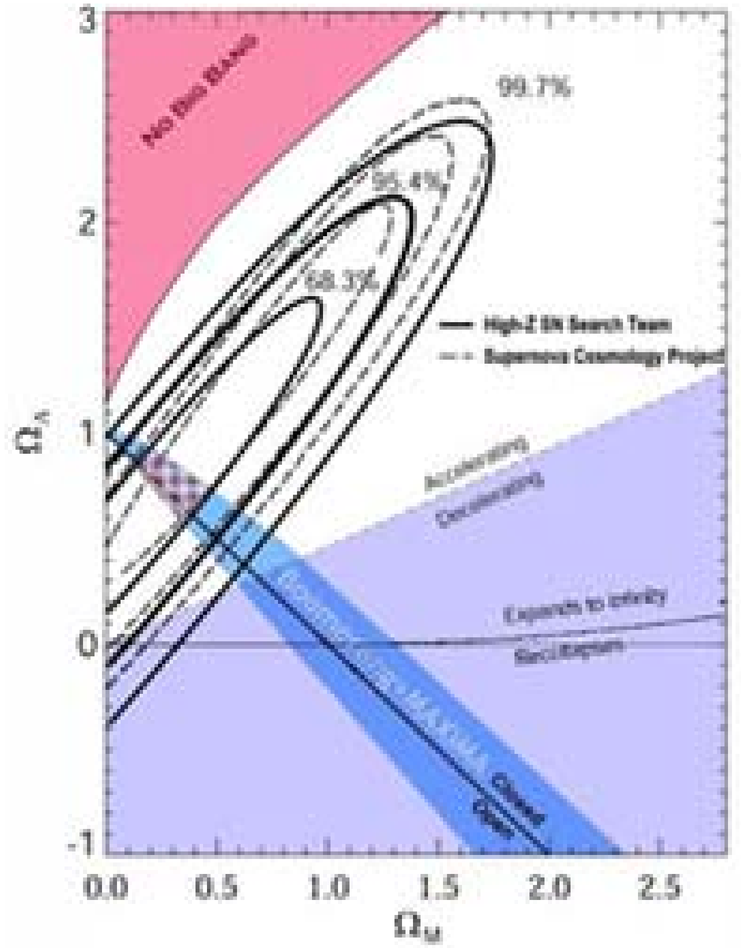
$$F' = \frac{\Lambda}{3}$$

Standard ruler from CMB fluctuations: Hereafter we use flat universe in our calculation





Result: Cosmological constant dominates universe at the present time



Problems with Cosmological constant

$$\rho_\Lambda \approx \rho_M{}^{(0)} = \frac{3H_0{}^2}{8\pi G} = 10^{-30} g/cm^3$$

Comparing with the Planck energy density at the early universe:

$$\frac{\rho_\Lambda}{M_{pl}{}^4} = 10^{-123}$$

Possible Solutions

- Quintessence models (Using a scalar field)
- Variable equation of state
- Modified gravity

Are the Quintessence and parameterized equation of state are equivalent

Starting from parameterized equation of state

$$\omega = \omega(a)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho[1+\omega(a)] = 0$$

$$\rho_{DE} = \rho_{DE}{}^{(0)} a^{-3[1+\overline{\omega}(a)]}$$

$$\overline{\omega}(a) = \frac{\int d\ln(a)\omega(a)}{\int d\ln(a)}$$

For a given ◆(a), we have density of dark energy as a function scale factor then using FRW equations we have

$$H = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_{DE} (1+z)^{3(1+\overline{\omega}(a))}}$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$\dot{\varphi}^2 = (\frac{d\varphi}{dz})^2 H^2 (1+z)^2 = \rho_{DE}(z)(1+\omega(z)) \rightarrow \varphi = \varphi(z)$$

$$V(\varphi) = \rho_{DE}(z)(1-\omega(z)) \rightarrow V = V(z)$$

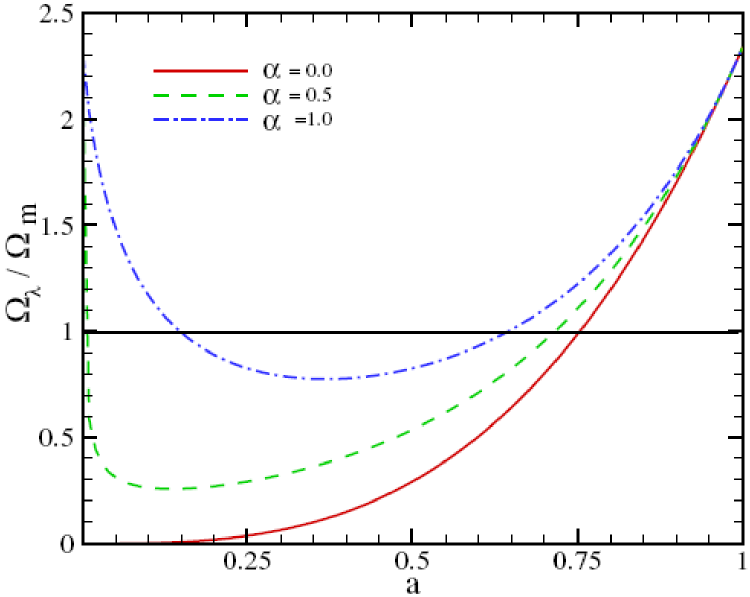
Eliminating redshift in favor of scalar field results in: $V = V(\varphi)$

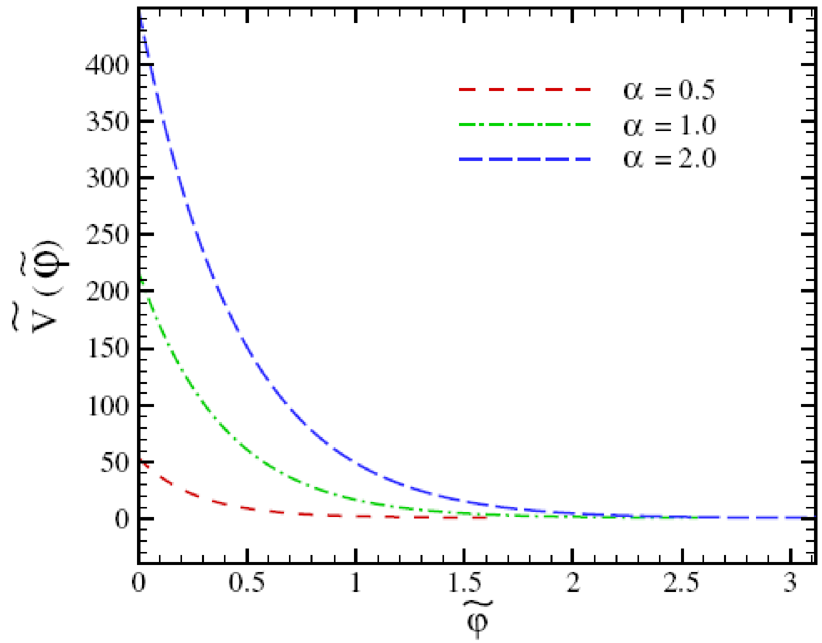
An example: Power law dark energy model

$$\overline{\omega} = \omega_0 a^{-\alpha}$$

Rahvar and Movahed,

PHYSICAL REVIEW D **75,** 023512 (2007)





Starting from a potential of scalar field

$$V = k\varphi^{-\alpha}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$\ddot{\varphi} + 3H\dot{\varphi} - k\alpha\varphi^{-\alpha-1} = 0$$

Considering $a \propto t^q$ and $\varphi \propto t^p$ we can obtain the equation of state of dark energy.

(Ref: V. Sahni and A. Starobinsky , astro-ph/9904398)

$$\frac{\rho_\varphi}{\rho_M} = t^{\frac{4}{2+\alpha}}$$

We can obtain as least numerically the corresponding equation of state

Second Part

Modified Gravity

Using a generalized action to speed up universe at the present time

$$S = \int (f(R) + 2\kappa L_m)\sqrt{-g}\,dx^4$$

$$\int \frac{\delta(f(R)\sqrt{-g})}{\delta g_{\mu\nu}}dx^4 = \kappa \int \frac{-2}{\sqrt{-g}} \frac{\delta(L_m\sqrt{-g})}{\delta g_{\mu\nu}} \sqrt{-g}\,dx^4$$

Varying with respect to the metric results in:

$$f'(R)R_{\mu\nu} - \frac{1}{2} f(R)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}\nabla_\lambda \nabla^\lambda) f'(R) = \kappa T_{\mu\nu}$$

Can extra-terms can play the role of dark energy ?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{f'} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\alpha \nabla^\alpha) f' + \frac{1}{2} g_{\mu\nu} (\frac{f}{f'} - R) + \kappa T_{\mu\nu} (\frac{1}{f'} - 1) + \kappa T_{\mu\nu}$$

$$T_{\mu\nu}(D.E.) = \frac{1}{f'} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\alpha \nabla^\alpha) f' + \frac{1}{2} g_{\mu\nu} (\frac{f}{f'} - R) + \kappa T_{\mu\nu} (\frac{1}{f'} - 1)$$

The motivation in modified gravity is to choose action in such a way that we have Quintessence like dynamics

Famous proposed actions:

$$f(R) = R - \frac{c}{(R-\Lambda_1)^n} + b(R-\Lambda_2)^m$$

$$f(R) = R + n\ln(\frac{R}{\mu^2}) + aR^m$$

Vacuum solution: means that the contribution of the matter is negligible

$$f'(R)R - 2f(R) = 0$$

$$f(R) = R - \frac{\mu^4}{R}$$

$$\begin{cases} t \rightarrow \infty \\ R \rightarrow 3\mu^2 \end{cases}$$

In these models universe asymmpotically goes to de Sitter space with exponential expansion.

$$6\dot{H} + 12H^2 = R_0$$

$$\frac{H - \sqrt{R_0/12}}{H + \sqrt{R_0/12}} = c\exp(-\alpha t)$$

$$H \to \sqrt{R_0/12}$$

$$a \propto \exp(Ht)$$

Modified gravity as dark energy

$$f'(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\lambda \nabla^\lambda) f'(R) = \kappa T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} + \kappa T_{\mu\nu}(D.E)$$

$$\kappa T_{\mu\nu}(D.E) = \frac{1}{2}(f'/f - R) g_{\mu\nu} + \frac{1}{2f'} \hat{H}_{\mu\nu} f' + \kappa T_{\mu\nu}(f^{-1} - 1)$$

FRW universe

$$\kappa\rho_{DE} = 3H^2(1-F) - 3H\dot{F} + \frac{1}{2}(RF - f)$$

$$kp_{DE} = \ddot{F} + 2H\dot{F} - H^2(1-2q)(1-F) - \frac{1}{2}(RF - f)$$

$$\ddot{F} - H\dot{F} + 2\dot{H}F + \kappa\rho_m = 0$$

For a given action, one can calculate the dynamics and finally obtain corresponding density and pressure of dark energy (rahvar and Sobouti, arxiv:0704.0680

Equivalence of Modified gravity with the dark energy

Quintessence models $\longleftrightarrow$ Parameterized dark energy models

What equivalence of modified gravity with the dark energy models

Modified gravity is equivalent to a Einstein-Hilbert + scalar field

$$S = \int \left[f(A) + f'(A)(R - A) \right] \sqrt{-g}\, dx^4 + 2k \int L_m \sqrt{-g}\, dx^4$$

$$\frac{\delta S}{\delta A} = f'(A) + f''(A)(R - A) - f'(A) = 0$$

$$f''(A) \neq 0, A = R$$

$$g^{\mu\nu} = e^{\varphi} g'^{\mu\nu}$$

Let’s do a conformal transformation:

$$\varphi = -\ln(f'(A))$$

$$S = \int [R' - \frac{3}{2} \varphi_{,\mu} \varphi^{,\nu} - V(\varphi)] \sqrt{-g'}\, dx^4$$

$$V(\varphi) = -\frac{A}{f'(A)} - \frac{f(A)}{f'(A)^2}$$

Interpretation:

Modified gravity can be considered as Einstein-Hilbert action + scalar field.

The question is that which frame is the physical one, Einstein or Jordan ?

Advantages: We can make acceleration universe at the present time and also is it possible to make inflation era.

Problems with modified gravity

- It is fourth order differential equation in contrast to the field equations in physics that are second order.

- We have instability problem: For instance the amplitude of gravity wave approach to infinity in the vacuum space

Solution:

Using Palatini approach:

There are two independent parameters describe a Manifold (Metric and Connections). So we do variation with respect to those:

Connection is the Christoffel symbol of $h_{\mu\nu} = f'(R)g_{\mu\nu}$

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0 \rightarrow f'(\hat{R})\hat{R}_{\mu\nu} - \frac{1}{2} f(\hat{R})g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\frac{\delta S}{\delta \Gamma} = 0 \rightarrow \nabla_{\lambda}(\sqrt{-g}\, f'(\hat{R})g_{\mu\nu}) = 0$$

$$\hat{\Gamma}^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} + \frac{1}{2}\frac{f'_{,\nu}}{f'}\delta^{\alpha}_{\mu} + \frac{1}{2}\frac{f'_{,\mu}}{f'}\delta^{\alpha}_{\nu} - \frac{1}{2}\frac{f'^{,\alpha}}{f'} g_{\mu\nu}$$

In the FRW universe:

$$\hat{R}_{00} = -3\frac{\ddot{a}}{a} + \frac{3}{2}\left(\frac{f'_{,0}}{f'}\right)^2 - \frac{3}{2} f'^{-1}\,\overline{\nabla}_0\overline{\nabla}_0 f'$$

$$\hat{R}_{ij} = [a\ddot{a} + 2\dot{a}^2 + f'^{-1}\, f'_{,0} + \frac{a^2}{2} f'^{-1}\,\overline{\nabla}_0\overline{\nabla}_0 f'$$

$$6H^2 + 6Hf'^{-1}\, f'_{,0} + \frac{3}{2}\left(\frac{f'_{,0}}{f'}\right)^2 = \frac{\kappa(\rho + 3p) + f}{f'}$$