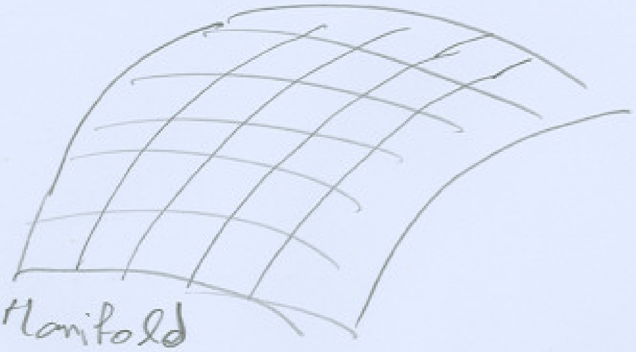
palatini formalism in modified gravity:



$\to g_{\mu\nu}$ (length measurement)

$\to \hat{\Gamma}^{\alpha}_{\mu\nu}$ (parallel trans.)

Manifold

$$\hat{R}^{\alpha}{}_{\mu\nu\beta} = \hat{\Gamma}^{\alpha}_{\mu\nu,\beta} - \hat{\Gamma}^{\alpha}_{\mu\beta,\nu} + \hat{\Gamma}^{\lambda}_{\mu\nu}\hat{\Gamma}^{\alpha}_{\beta\lambda} - \hat{\Gamma}^{\lambda}_{\mu\beta}\hat{\Gamma}^{\alpha}_{\nu\lambda}$$

$$\hat{R}_{\mu\beta} = g^{\nu}{}_{\alpha}\hat{R}^{\alpha}{}_{\mu\nu\beta} \quad , \quad \hat{R} = \hat{R}_{\mu\beta}\, g^{\mu\beta}$$

$$S = \int f(\hat{R})\sqrt{-g}\, dx^4 + k\int \mathcal{L}_M \sqrt{-g}\, dx^4$$

(A) varying S with respect to $g_{\mu\nu}$

$$\frac{\delta S}{\delta g_{\mu\nu}} = \int f'(\hat{R})\, \frac{\delta(\hat{R}^{\alpha\beta} g_{\alpha\beta}\sqrt{-g})}{\delta g_{\mu\nu}}\, dx^4 + k\int \frac{1}{\sqrt{-g}}\frac{\delta(\mathcal{L}_M\sqrt{-g})}{\delta g_{\mu\nu}}\sqrt{-g}\, dx^4 = 0$$

$$f'(\hat{R})R^{\mu\nu} - \frac{1}{2}f(\hat{R})g^{\mu\nu} = kT^{\mu\nu}, \quad T^{\mu\nu} = -\frac{1}{\sqrt{-g}}\frac{\delta(\mathcal{L}_M\sqrt{-g})}{\delta g_{\mu\nu}}$$

(B) varying with respect to Γ

$$\delta S = \int f'(R)\,\delta(R^{\mu\nu} g_{\mu\nu}\sqrt{-g})\, dx^4 = 0$$

$$\nabla_\alpha[f'\sqrt{-g}\, g^{\mu\nu}] - \frac{1}{2}\nabla_\sigma[f'\sqrt{-g}\, g^{\sigma\mu}]\delta^{\nu}_{\alpha} - \frac{1}{2}\nabla_\sigma[f'\sqrt{-g}\, g^{\sigma\nu}]\delta^{\mu}_{\alpha} = 0$$

Contracting over α and μ

$$\nabla_\mu[f'\sqrt{-g}\, g^{\mu\nu}] - \frac{1}{2}\nabla_\sigma[\sqrt{-g}\, f' g^{\sigma\mu}]\delta^{\nu}_{\mu} - \frac{1}{2}\times 4\,\nabla_\sigma[f'\sqrt{-g}\, g^{\sigma\nu}]$$

$$\to \nabla_\mu[f'\sqrt{-g}\, g^{\mu\nu}] = 0 \to \nabla_\alpha(f'\sqrt{-g}\, g^{\mu\nu}) = 0$$

$\hat{\Gamma}^{\mu}_{\nu\lambda}$ can be defined as the christoffel symbol of $h_{\mu\nu} = f'(R) g_{\mu\nu}$

$$\hat{\Gamma}^{\mu}_{\nu\lambda} = h^{\mu\alpha}\left(h_{\nu\alpha,\lambda} + h_{\lambda\alpha,\nu} - h_{\nu\lambda,\alpha}\right)$$

$$\hat{\Gamma}^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(g) + \frac{1}{2}\left[(\ln f')_{,\nu}\,\delta^{\mu}_{\lambda} + (\ln f')_{,\lambda}\,\delta^{\mu}_{\nu} - (\ln f')_{,\alpha}\, g^{\mu\alpha} g_{\nu\lambda}\right]$$

$$\hat{\hat{\Gamma}}^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda} + C^{\mu}_{\nu\lambda}$$

we can calculate $\hat{R}_{\mu\nu}$ in terms of $R_{\mu\nu}$:

$$\hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2}\frac{f'_{,\mu} f'_{,\nu}}{f'^2} - \frac{\nabla_\mu \nabla_\nu f'}{f'} - \frac{1}{2} g_{\mu\nu}\frac{\nabla_\alpha \nabla^\alpha f'}{f'}$$

And for Ricci Scalar:

$$\hat{R} = R + \frac{3}{2}\frac{f'_{,\alpha} f'^{,\alpha}}{f'^2} - 3\,\frac{\nabla_\alpha \nabla^\alpha f'}{f'}$$

for the FRW metric:

$$\hat{R} = R - \frac{3}{2}\left(\frac{\dot{f}'}{f'}\right)^2 - \frac{3}{f'}\left(f'^{,0}_{,0} + \Gamma^{\alpha}_{\mu\alpha} f'^{,\mu}\right)$$

$\Gamma^{\alpha}_{\alpha 0} = 3\dot{a}/a$

$$\longrightarrow \hat{R} = R - \frac{3}{2}\left(\frac{\dot{f}'}{f'}\right)^2 + 3\frac{\ddot{f}'}{f'} + \frac{9}{f'}\frac{\dot{a}}{a}\dot{f}'$$

$$\hat{R} = R - \frac{3}{2}\left(\frac{\dot{f}'}{f'}\right)^2 + 3\frac{\ddot{f}'}{f'} + 9H\frac{\dot{f}'}{f'}$$

For the trace of Modified Einstein equation:

$$f'(\hat{R})\hat{R} - 2f(\hat{R}) = \kappa T \rightarrow \hat{R} - 2\frac{f}{f'} = \frac{\kappa}{f'}T$$

for a perfect fluid: $T^{\mu\nu} = (\rho+p)u^\mu u^\nu + p g^{\mu\nu}$ $(-,+,+,+)$

$$T^\mu_\mu = -(\rho+p) + 4p = 3p - \rho$$

$$\hookrightarrow R - \frac{3}{2}\left(\frac{\dot{f}'}{f'}\right)^2 + 3\frac{\ddot{f}'}{f'} + 9H\frac{\dot{f}'}{f'} - 2\frac{f}{f'} = \frac{\kappa}{f'}(3p-\rho) \quad (A)$$

for (00) component of Modified gravity equation:

$$R_{00} + \frac{3}{2}\left(\frac{\dot{f}'}{f'}\right) - \frac{3}{2}\left(\frac{\ddot{f}'}{f'}\right) - \frac{3}{2}H\frac{\dot{f}'}{f'} + \frac{1}{2}\frac{f}{f'} = \frac{\kappa}{f'}\rho \quad (B)$$

Combining (A) and (B) results in:

$$\boxed{\left(H + \frac{1}{2}\frac{\dot{f}'}{f'}\right)^2 = \frac{\kappa}{6f'}(\rho+3p) + \frac{f}{6f'}}$$

For Einstein-Hilbert action, $\hat{R} = R$, Then:

$$H^2 = \frac{\kappa}{6}(\rho+3p) + \frac{1}{6}(12H^2 + 6\dot{H})$$

$$-H^2 - \dot{H} = \frac{\kappa}{6}(\rho+3p) \longrightarrow \boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho+3p)}$$

Question(I): How is the Conservation of energy Momentum tensor,

$$\nabla_\nu T^{\mu\nu} = ?$$

In Barraco et al. (PRD 60, 044012 (1999)) Claim, that it is Zero and PRD (2004) ("Invariant Lagrangian under Coordinate transformation and Conservation of Charge")

if so then for energy momentum tensor of a point like particle,

$$T^{\mu\nu}(y^n) = mc \int \frac{\delta^4[(y^n - x^n(\tau)]}{\sqrt{-g}} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} d\tau$$

$$\nabla_\nu T^{\mu\nu} = 0 \rightarrow \frac{d^2 x^\mu}{d\tau} + \left\{ \begin{matrix} \mu \\ \nu\lambda \end{matrix} \right\} \left(\frac{dx^\nu}{d\tau}\right)\left(\frac{dx^\lambda}{d\tau}\right) = 0$$

Then Particles move along the geodesic of space-time

if $\nabla_\nu T^{\mu\nu} \neq 0$, the equation of motion is given by parallel transfer. (Under Construction ...

For the FRW space with the matter of $p = \omega\rho$

$$\dot{\rho} + 3H\rho(1+\omega) = 0 \rightarrow \rho = \rho_0 \, a^{-3(1+\omega)}$$

Trace of Modified gravity: $f'(R)R - 2f(R) = k\rho(1-3\omega)$

$$a = \left(\frac{2f - f'R}{k\rho_0 (1-3w)} \right)^{-\frac{1}{3(1+w)}}$$

Question (II): Can we find an explicit solution for $H = H(a)$?

Yes (Amarzguioui astro-ph/0510519).

1: Continuity equation: $\dot{\rho} = -3H(1+w)\rho$

Trace of equation: $f'R - 2f = k\rho(1-3w)$

$$\longrightarrow \quad k\dot{\rho} = \frac{1}{k(1-3w)} \left(f''\dot{R}R + f'\dot{R} - 2f'\dot{R} \right) = \frac{\dot{R}}{k(1-3w)} \left(f''R - f' \right) = -3H\rho(1+w)$$

$$\boxed{\dot{R} = -3H\rho \frac{(1+w)(1-3w)}{f''R - f'}}$$

Now we can write $\dot{f}' = \frac{df'}{dR}\dot{R} = f''\dot{R}$

$\longrightarrow$ then for $w=0$

$$\begin{cases} H^2 = \dfrac{1}{6f'} \dfrac{3f - Rf'}{\left(1 - \frac{3}{2} \dfrac{f''(Rf' - 2f)}{f'(Rf'' - f')}\right)^2} \\[2ex] a = \left(\dfrac{2f - Rf'}{k\rho_0} \right)^{-1/3} \end{cases}$$

Eliminating $\hat{R}$, we can obtain $H = H(a)$

Observational tests:

1) CMB Shift parameter

2) SNIa → Background effect

3) Baryonic Oscillation

4) Large Scale structure → { Spherical Collapse / perturbation theory

An example of Comparing a Modified gravity Model with the observation will be present with Mr. Baghram

Ref: Baghram et al. phys. Rev. D 75, 087703 (2007) and arXiv: 0705

Movahed et al. arXiv: 0705.0889

Hope to Solve Anomalies in gravity:

Anomalies in gravity:

1) Pioneer Anomaly $a \sim -8 \times 10^{-10}\ m/s^2$

2) rotation Curve of galaxies (Dark Matter) $a_{MOND} \sim 10^{-10}\ m/s^2$

3) Dark energy $a_H = d_H \ddot{a} \simeq 10^{-10}\ m/s^2$

Some advatages of palatini

Unification of Inflation with late time acceleration.

Inflation $\quad f(R) = R^3/\beta^2 + R - \frac{\varepsilon^2}{3R}$

vacuum solution $\quad R^2 = \frac{\beta^2}{2}\left[1 \pm \sqrt{1 - 4(\varepsilon/\beta)^2}\right]$

if $\beta \gg \varepsilon \longrightarrow \begin{cases} R_v^{(1)} = \beta \rightarrow \text{inflation era} \\ R_v^{(2)} = \varepsilon \rightarrow \text{late time acceleration} \end{cases}$

It is possible to add an extra term to have inflation

$$f(R) = \sqrt{R^2 - R_0^2} + R^3/\alpha^2 \quad , \quad \alpha \gg R_0 \rightarrow \begin{cases} R_v^{(1)} = \alpha \\ R_v^{(2)} = \sqrt{2}\, R_0 \end{cases}$$