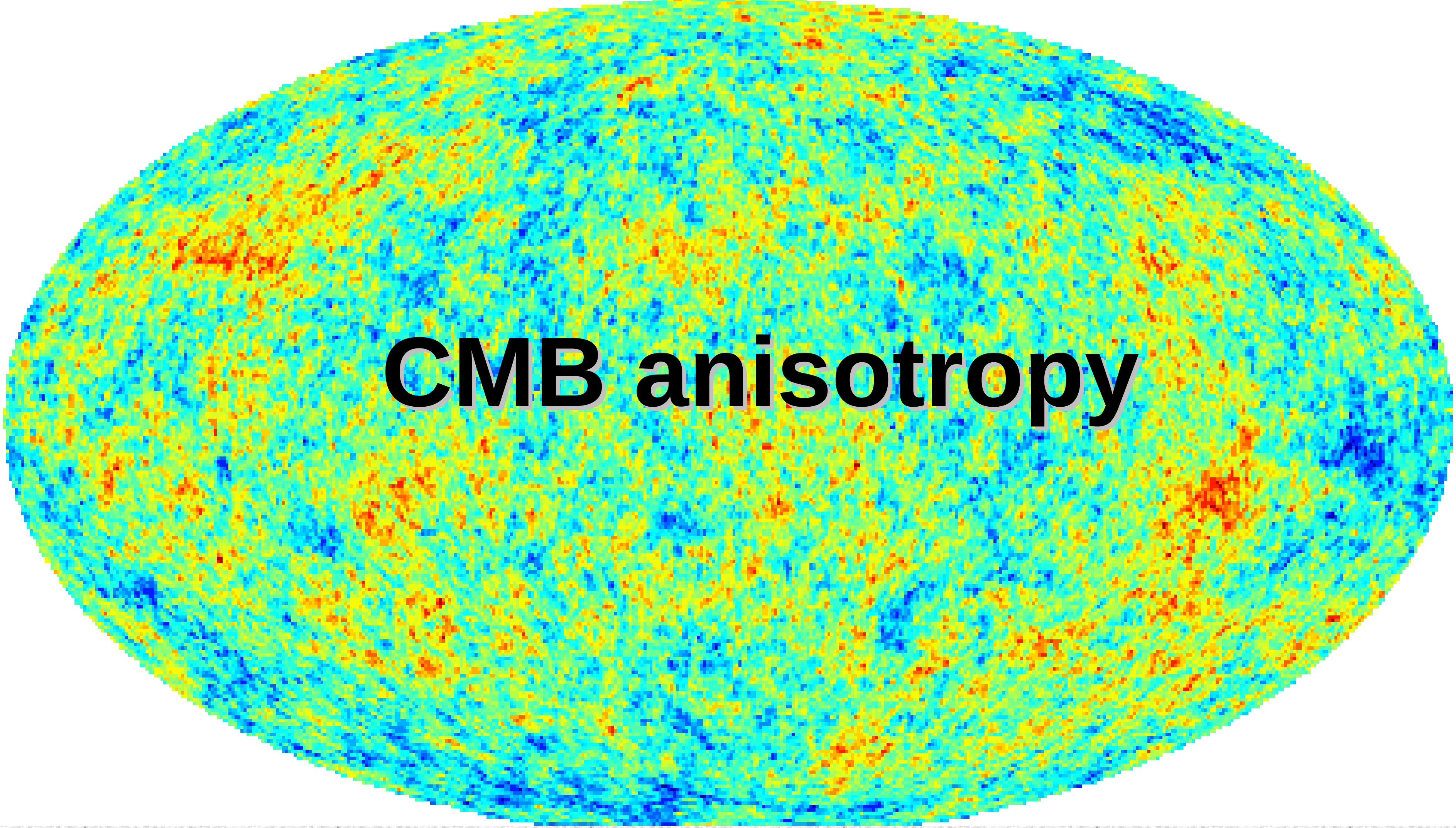


CMB anisotropy

ICSW-07

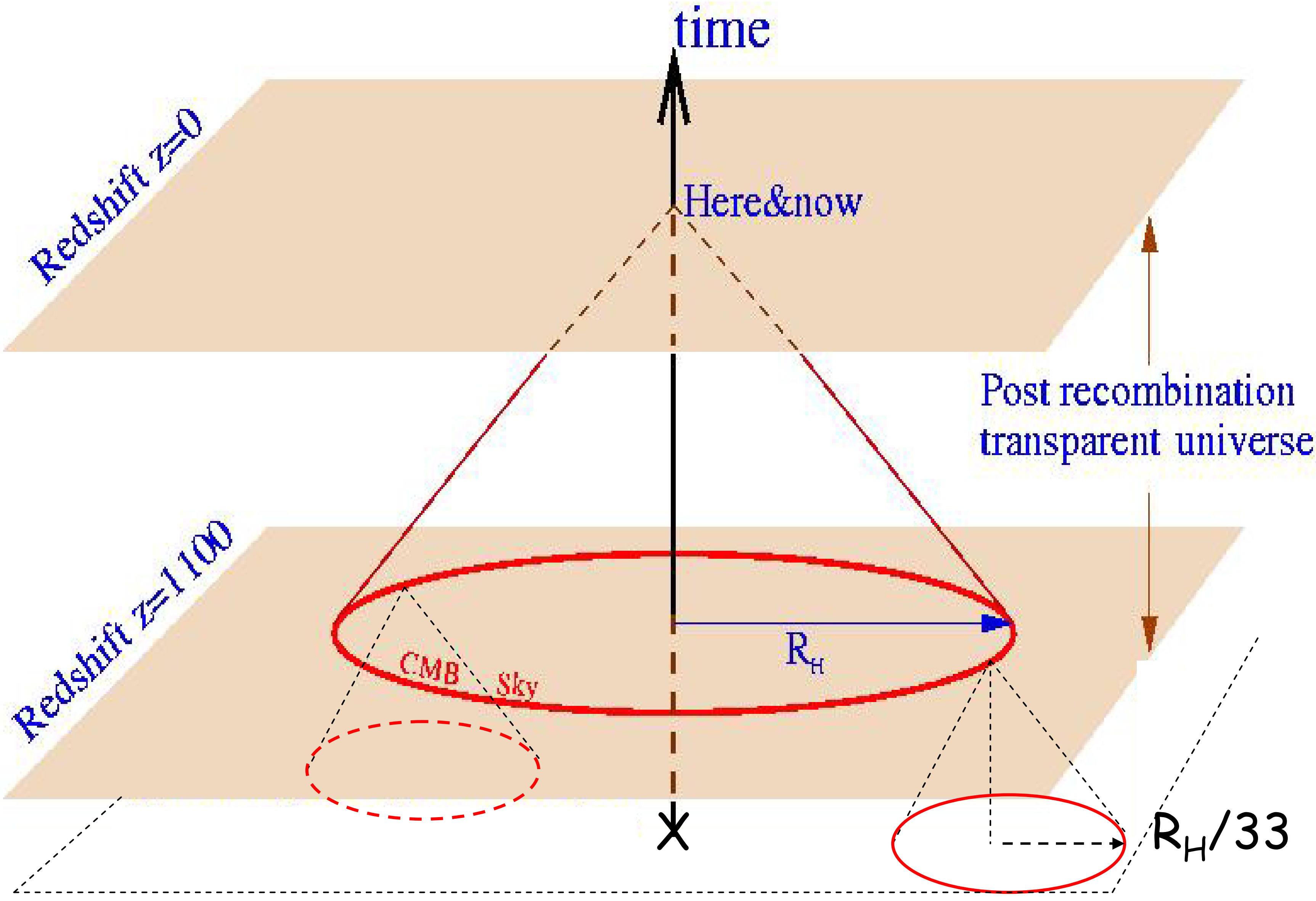
IPM, Tehran

(Jun 2-9, 2007)

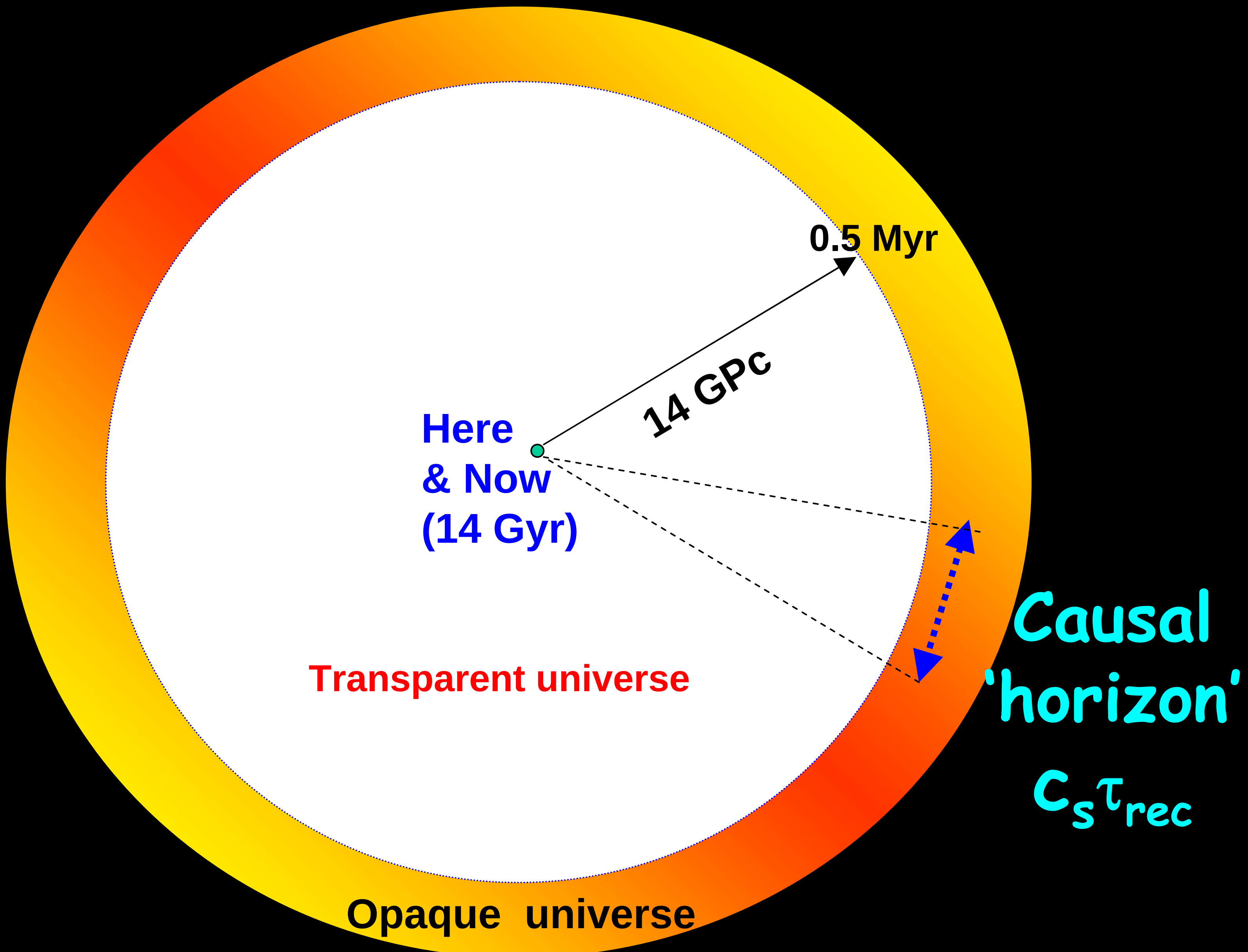
Tarun Souradeep

I.U.C.A.A, Pune, India

Cosmic Microwave Background

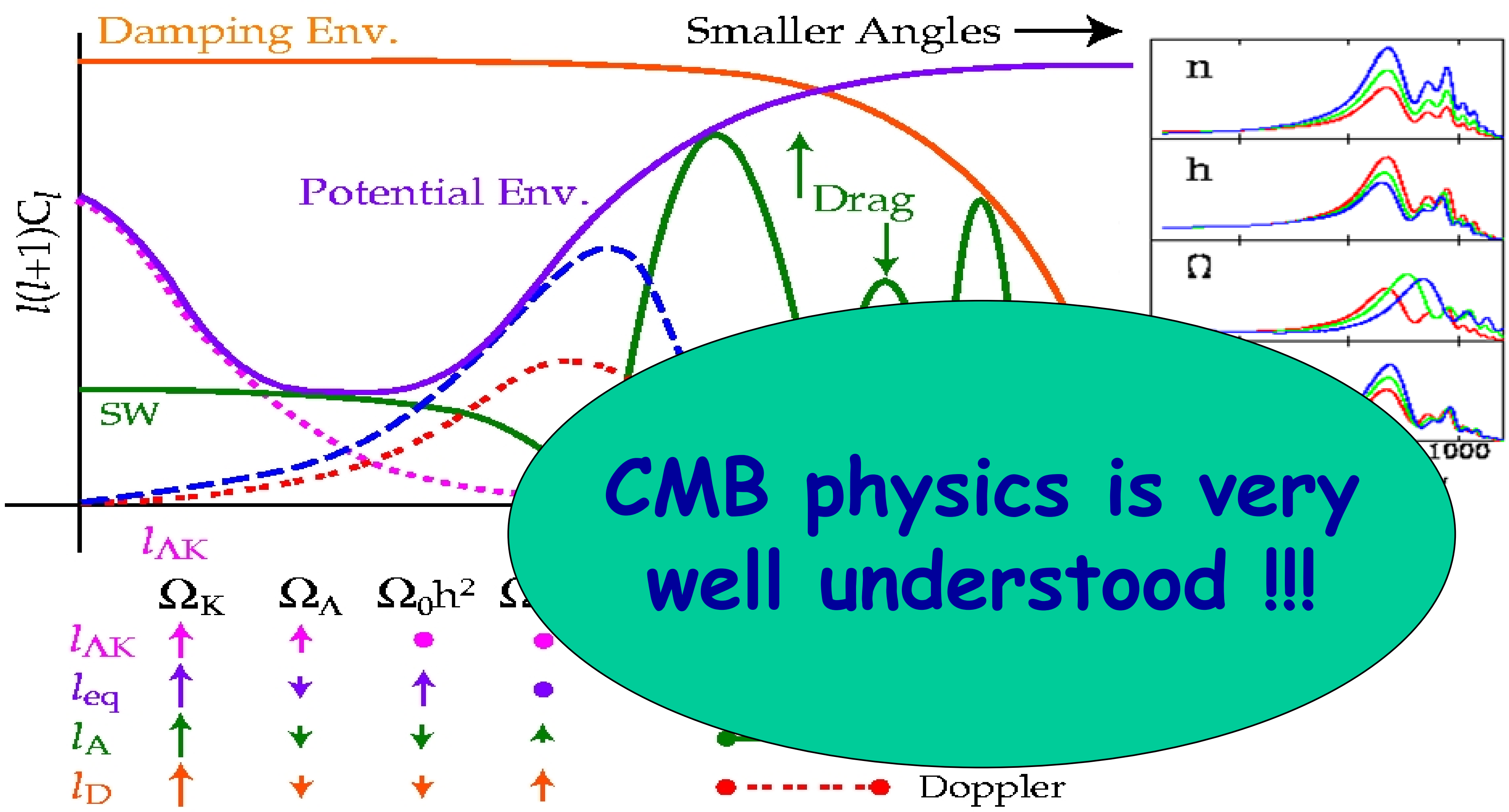


Cosmic "Super-IMAX" theater



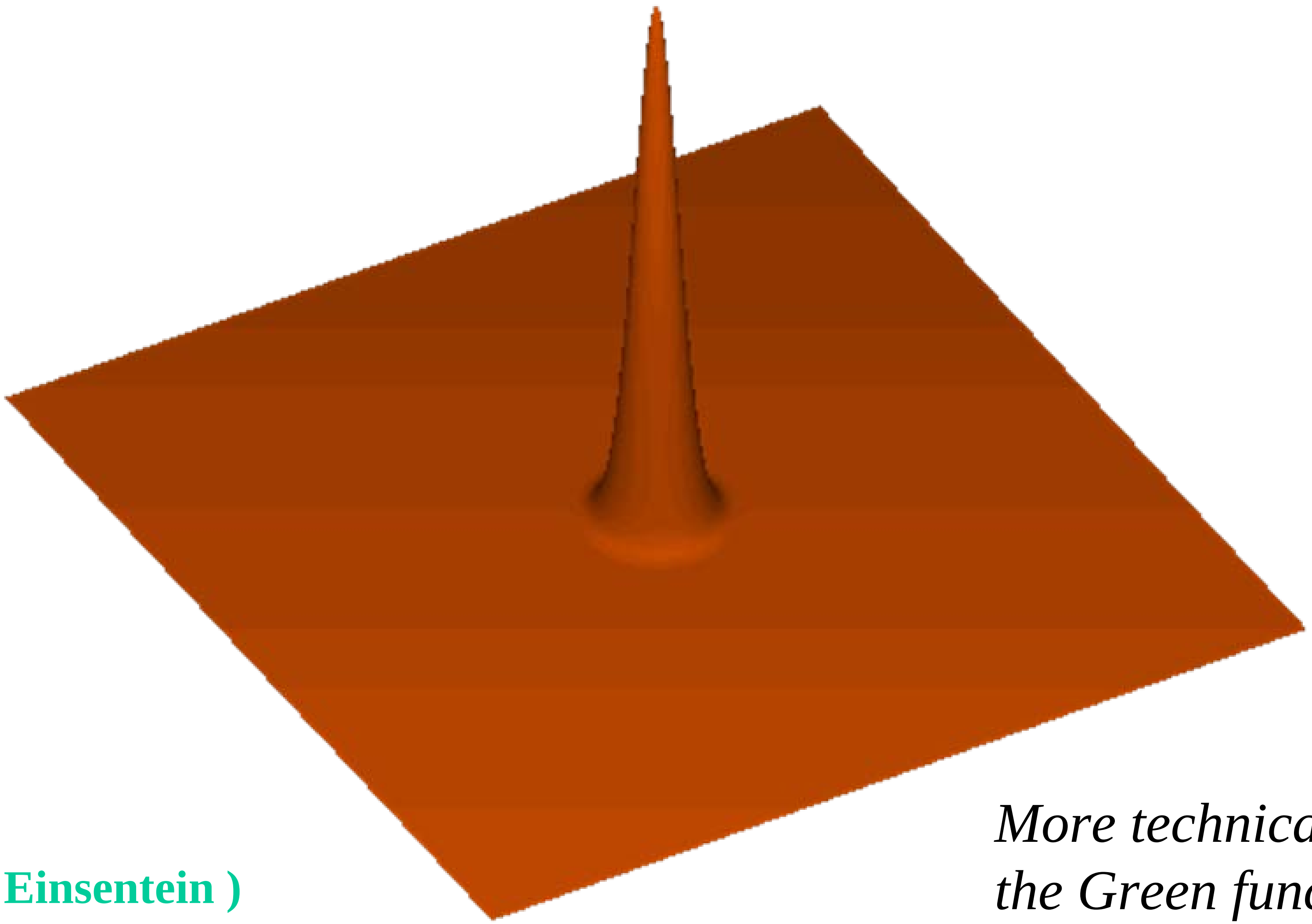
Dissected CMB Angular power spectrum

- **Low multipole : Sachs-Wolfe plateau**
- **Moderate multipole : Acoustic "Doppler" peaks**
- **High multipole : Damping tail**



Music of the Cosmic Drum

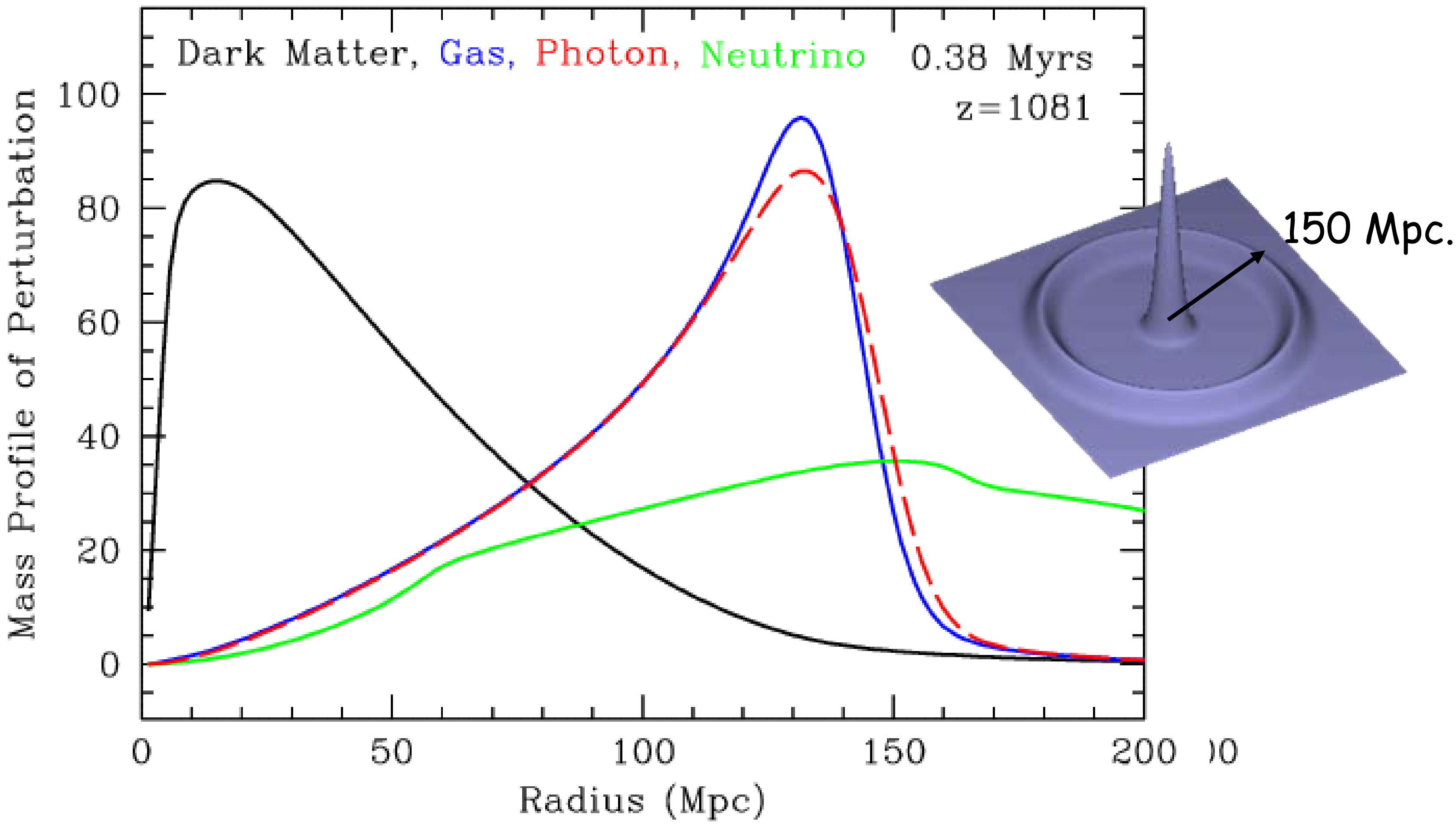
Ping the 'Cosmic drum'



More technically,
the Green function

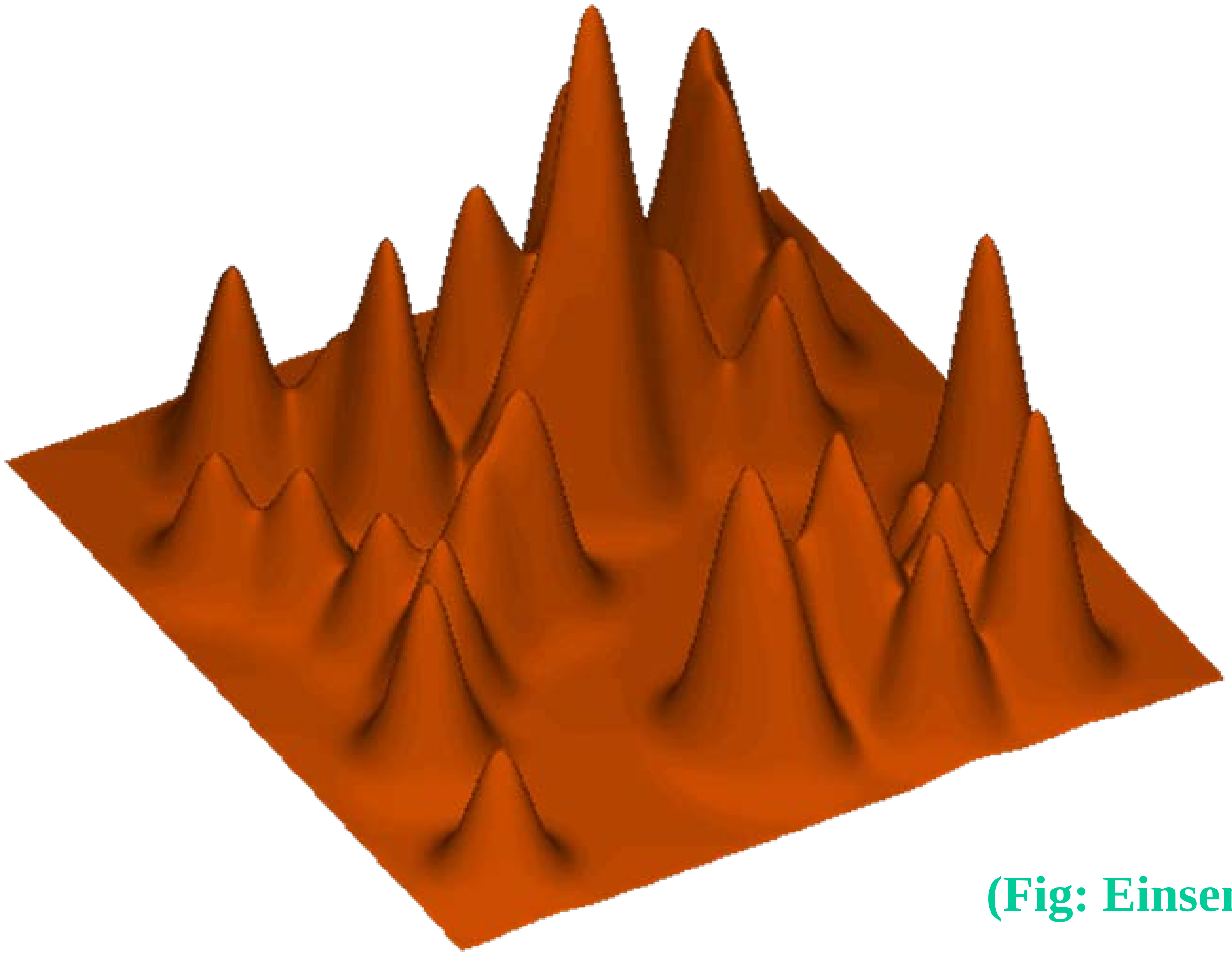
(Fig: Einsentein)

Ripples in the different constituents



(Einsentein et al. 2005)

Perturbed universe: superposition of random `pings'



(Fig: Einsentein)

CMB Angular power spectrum

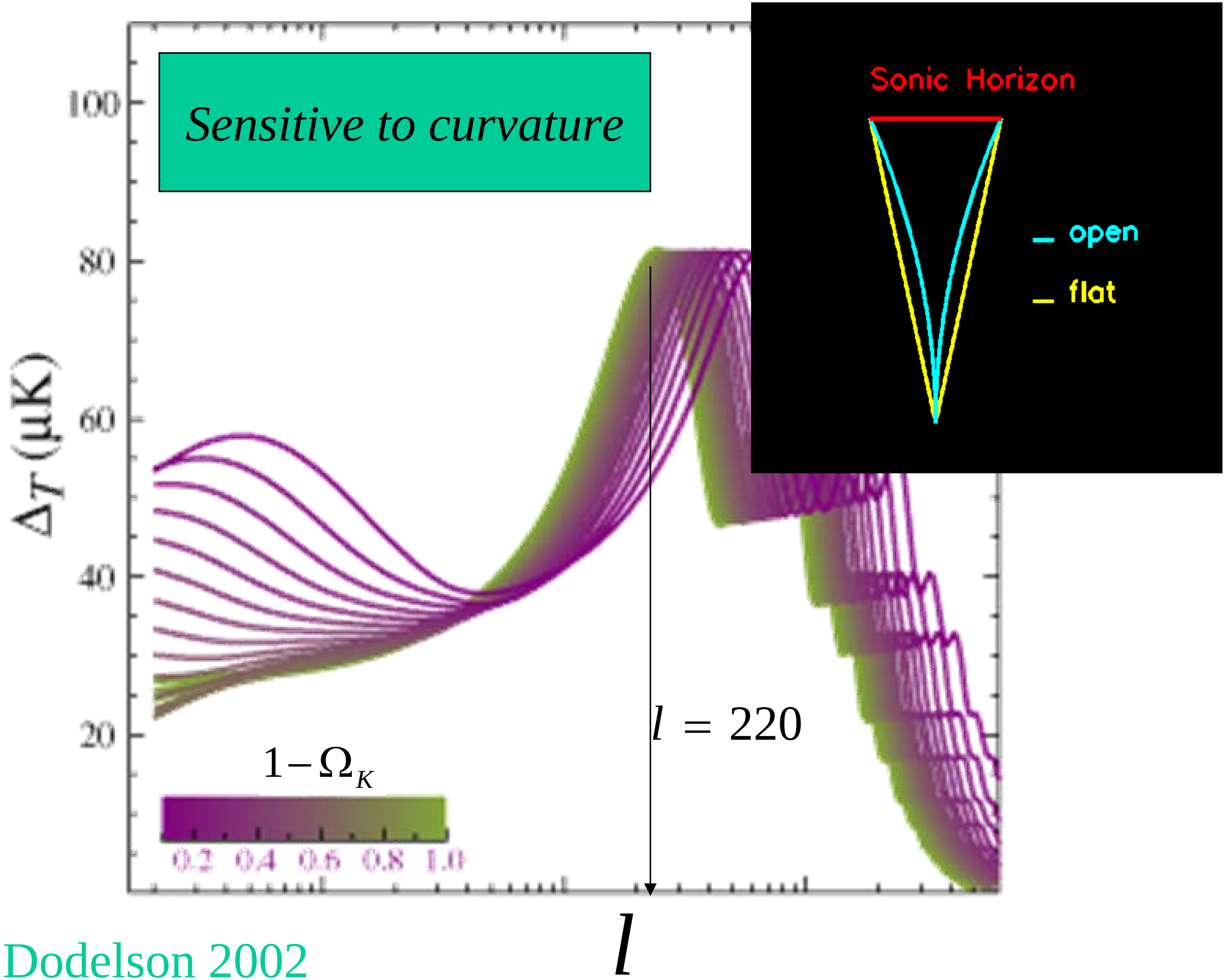


Fig:Hu & Dodelson 2002

CMB Angular power spectrum

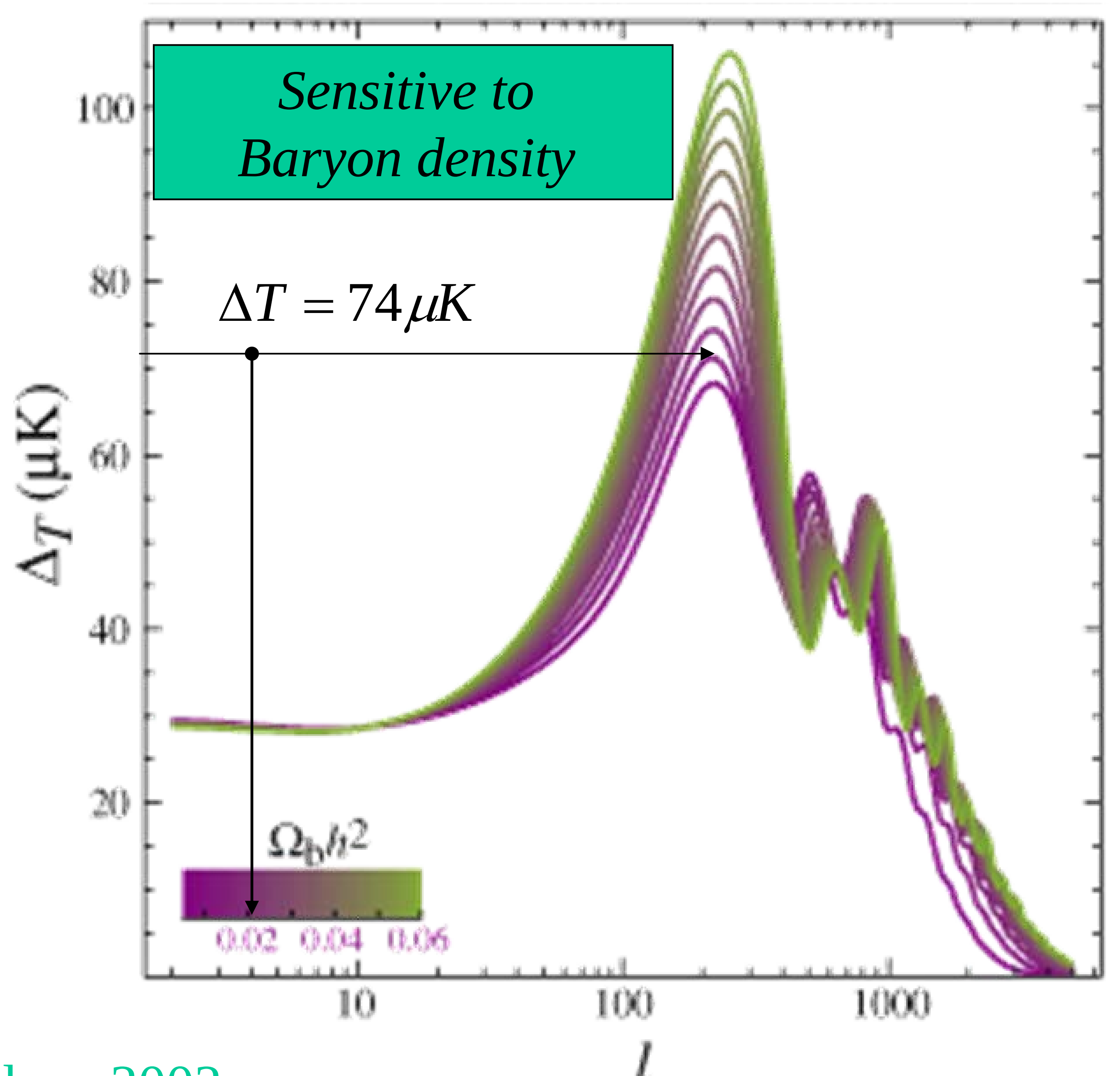
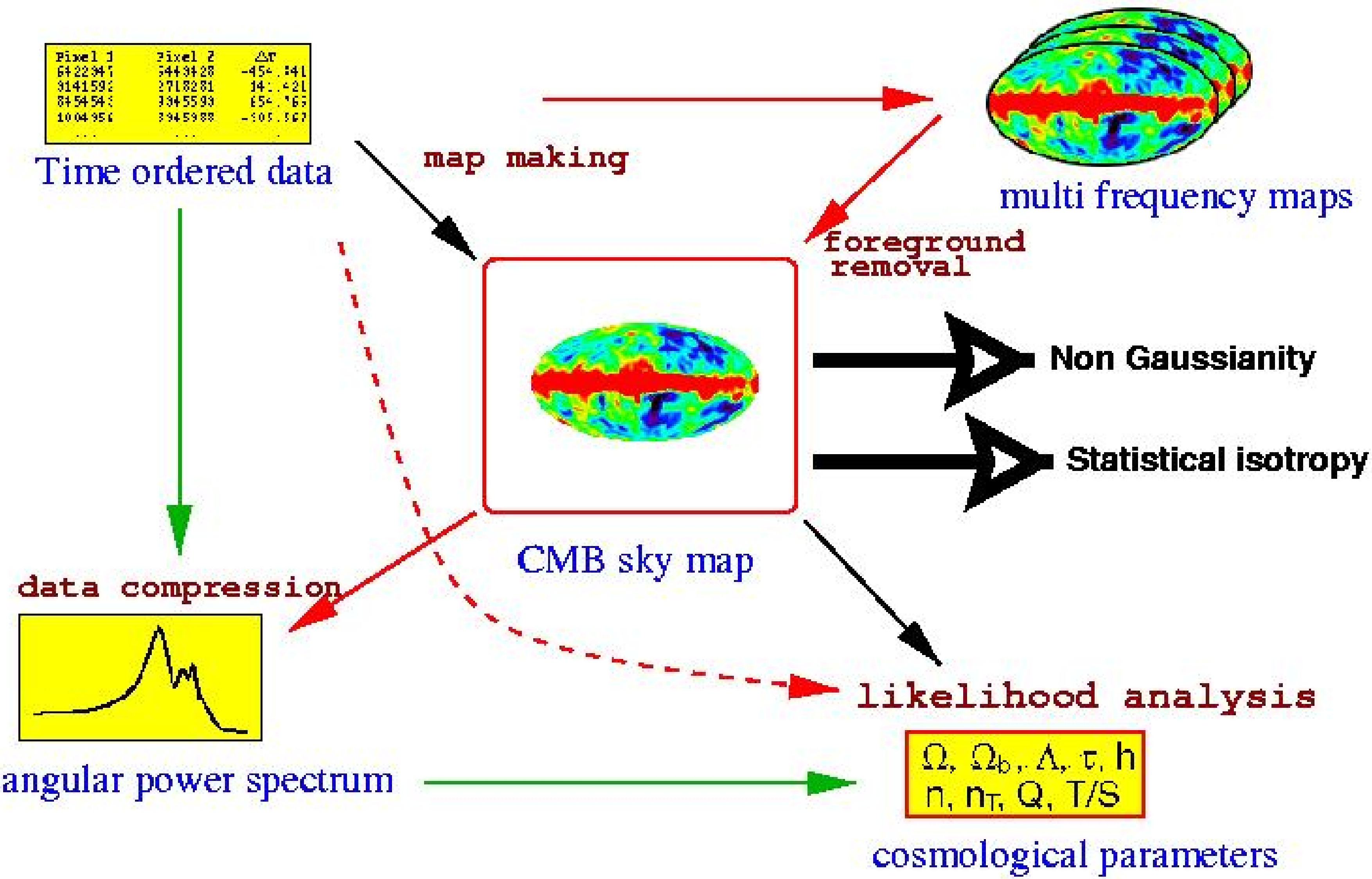
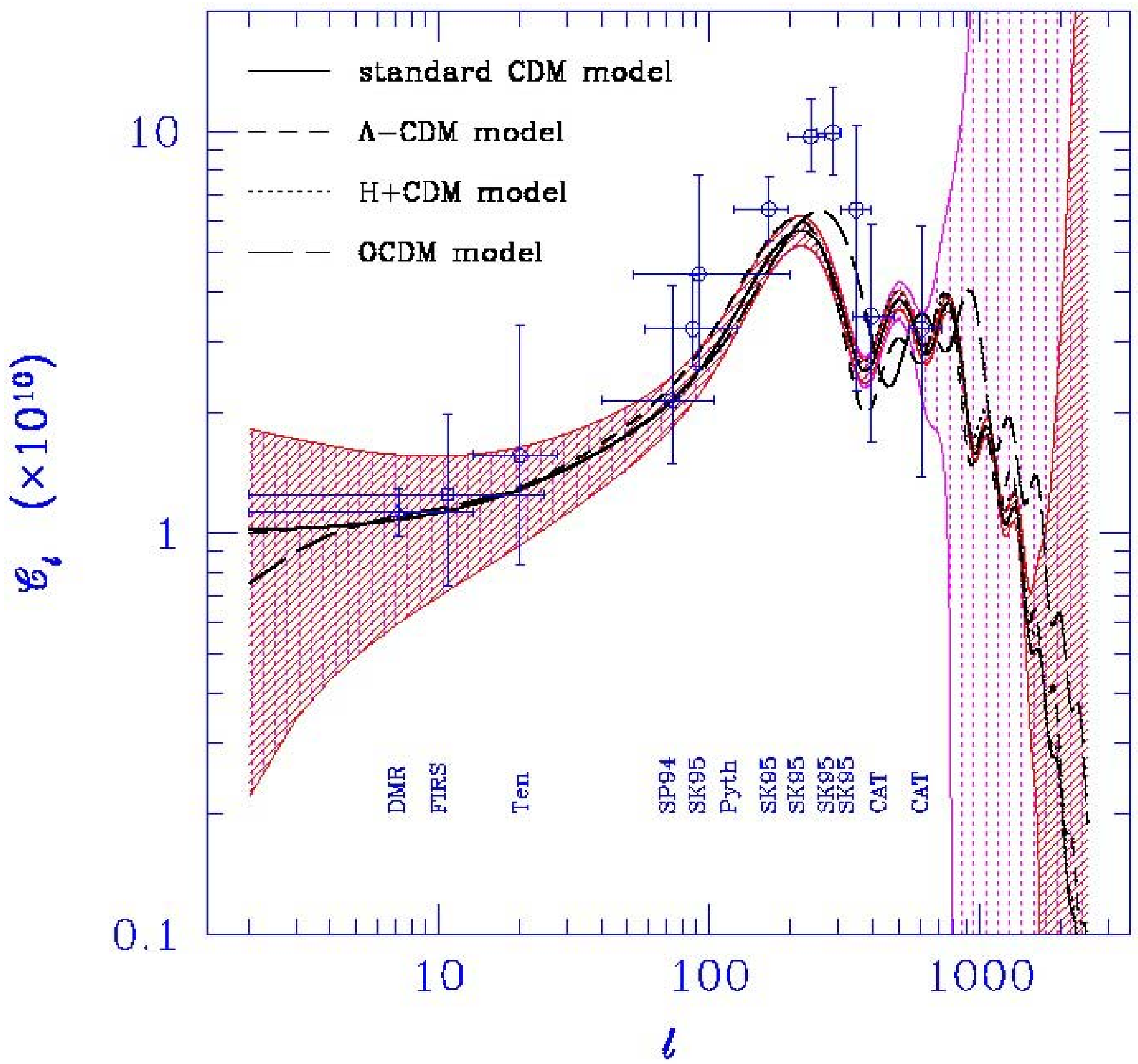


Fig:Hu & Dodelson 2002

Analysis of CMB data



Estimating the Angular power spectrum



(Souradeep 1998)

Estimating the Angular power spectrum

Cosmic Variance of the unbiased estimator

Homo. , Uncorrelated noise:

$$C_l^N = \frac{4\pi}{N_{\rm pix}} \sigma_{\rm pix}^2 \equiv \sigma_N^2$$

Inevitable error for one sky

Gaussian beam :

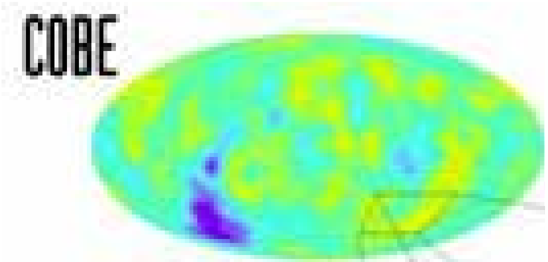
$$B(\theta) = \exp\left(-\frac{\theta^2}{2\sigma_\theta^2}\right), \quad B_l = \exp\left(-\frac{1}{2} l^2 \sigma_\theta^2\right), \quad \sigma_\theta = \frac{1}{\sqrt{8 \ln 2}} \theta_{\rm fwhm}$$

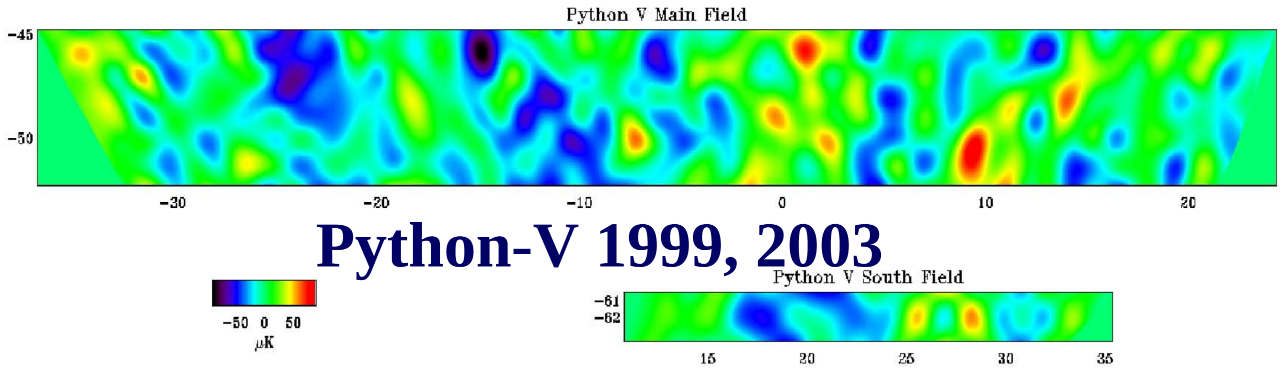
$$\mathrm{var}\left(\tilde{C}_l\right) = \frac{2}{(2l+1) f_{\rm sky}} \left[C_l^S + \sigma_N^2 \exp(l^2 \sigma_\theta^2)\right]^2$$

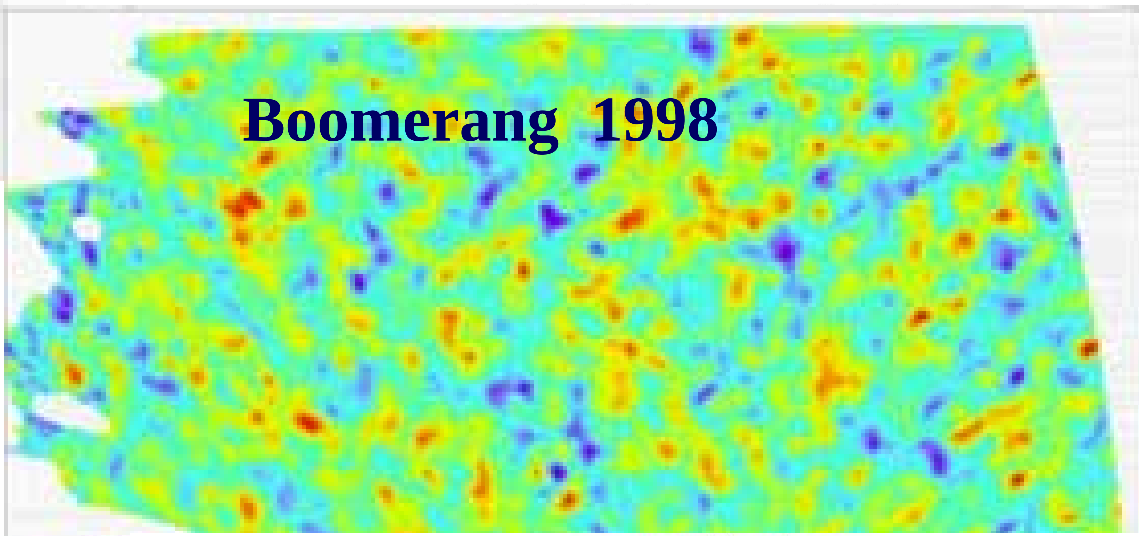
crude account of incomplete sky

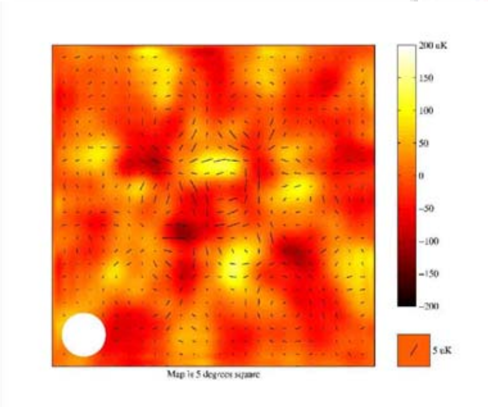
Noise term dominates beyond beam width

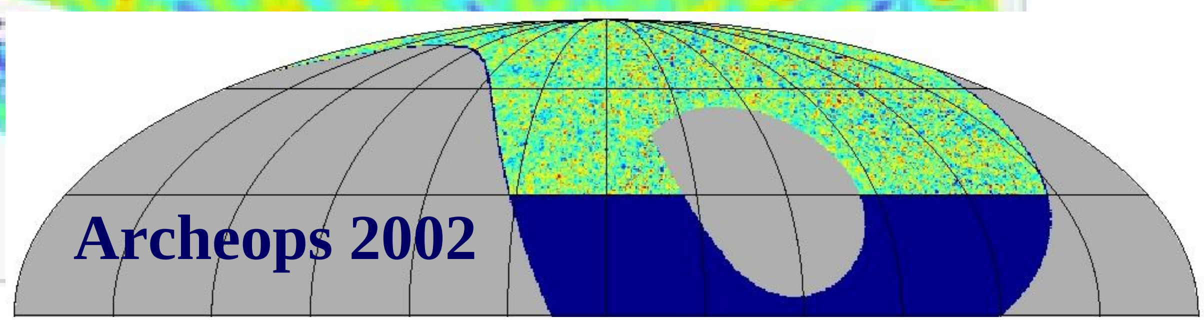
Post-COBE Ground & Balloon Experiments







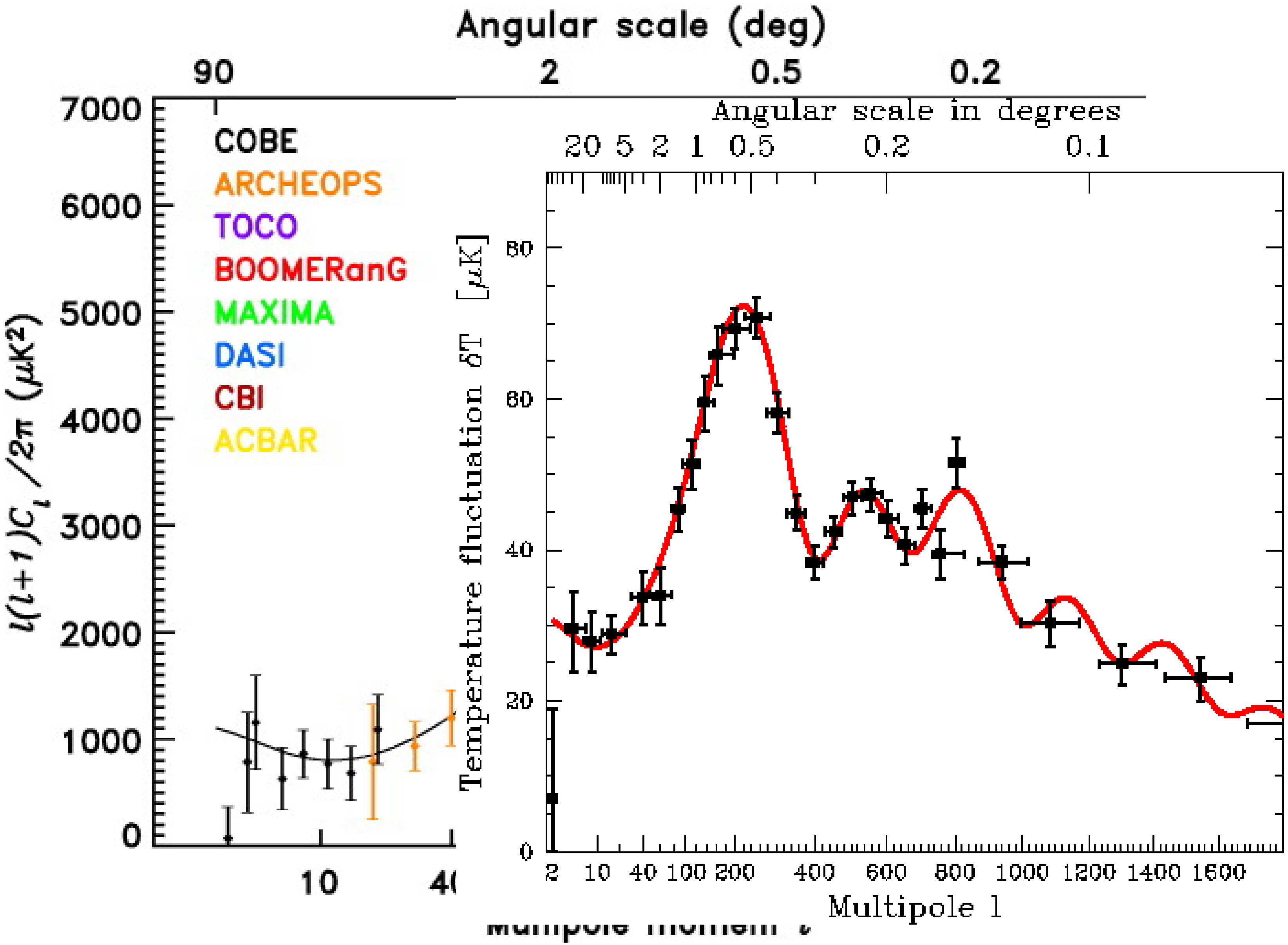




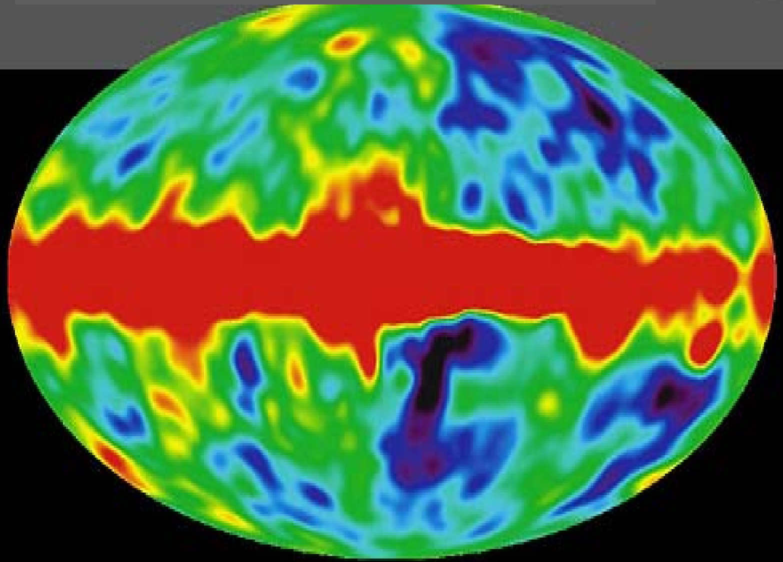
DASI 2002

(Degree Angular scale Interferometer)

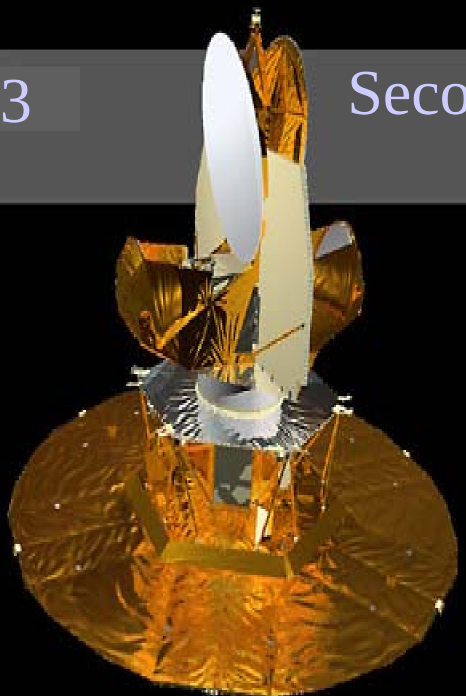
Highlights of CMB Anisotropy Measurements (1992- 2002)

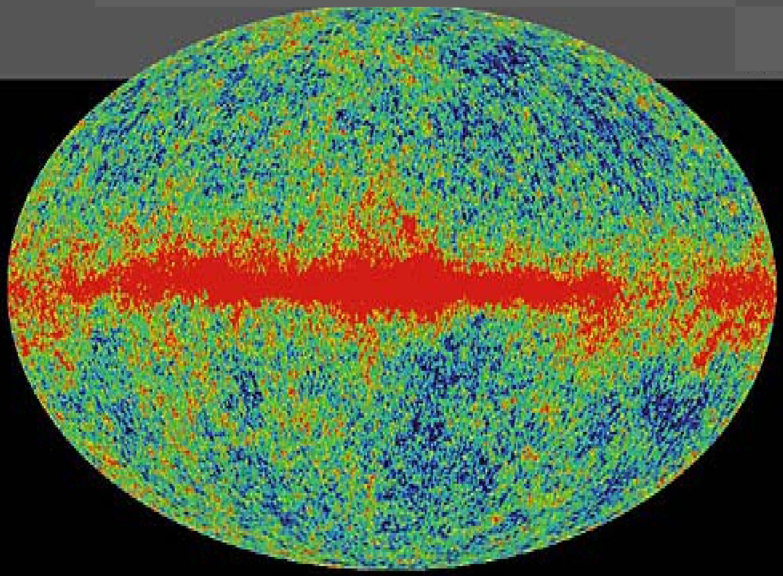


1992 First NASA CMB Satellite mission COBE



2003 Second NASA CMB Satellite mission MAP





CMB space mission : WMAP

Wilkinson **Microwave Anisotropy Probe**

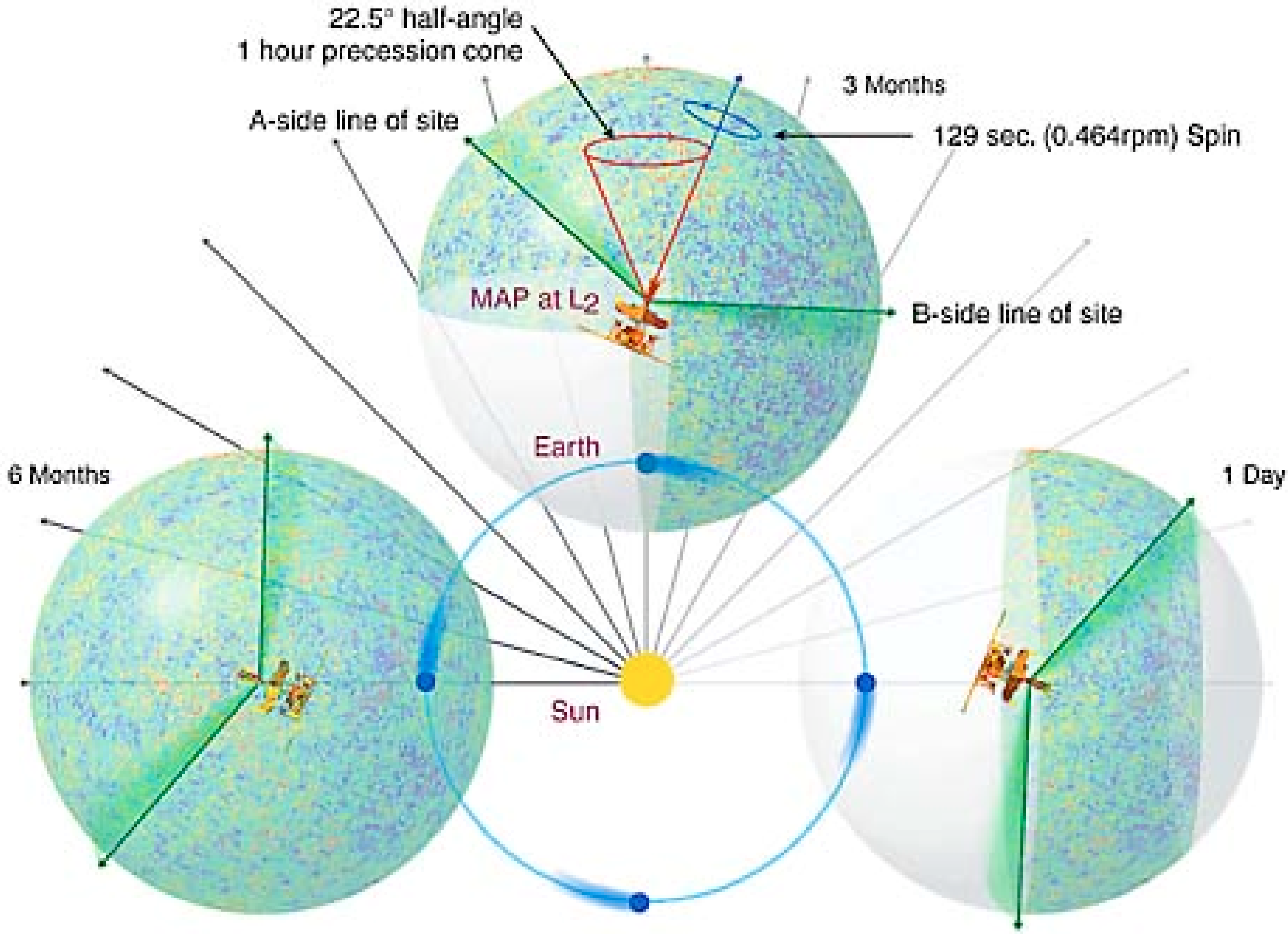
NASA : Launched July 2001

WMAP: 3-year results announced on Mar, 2006 !

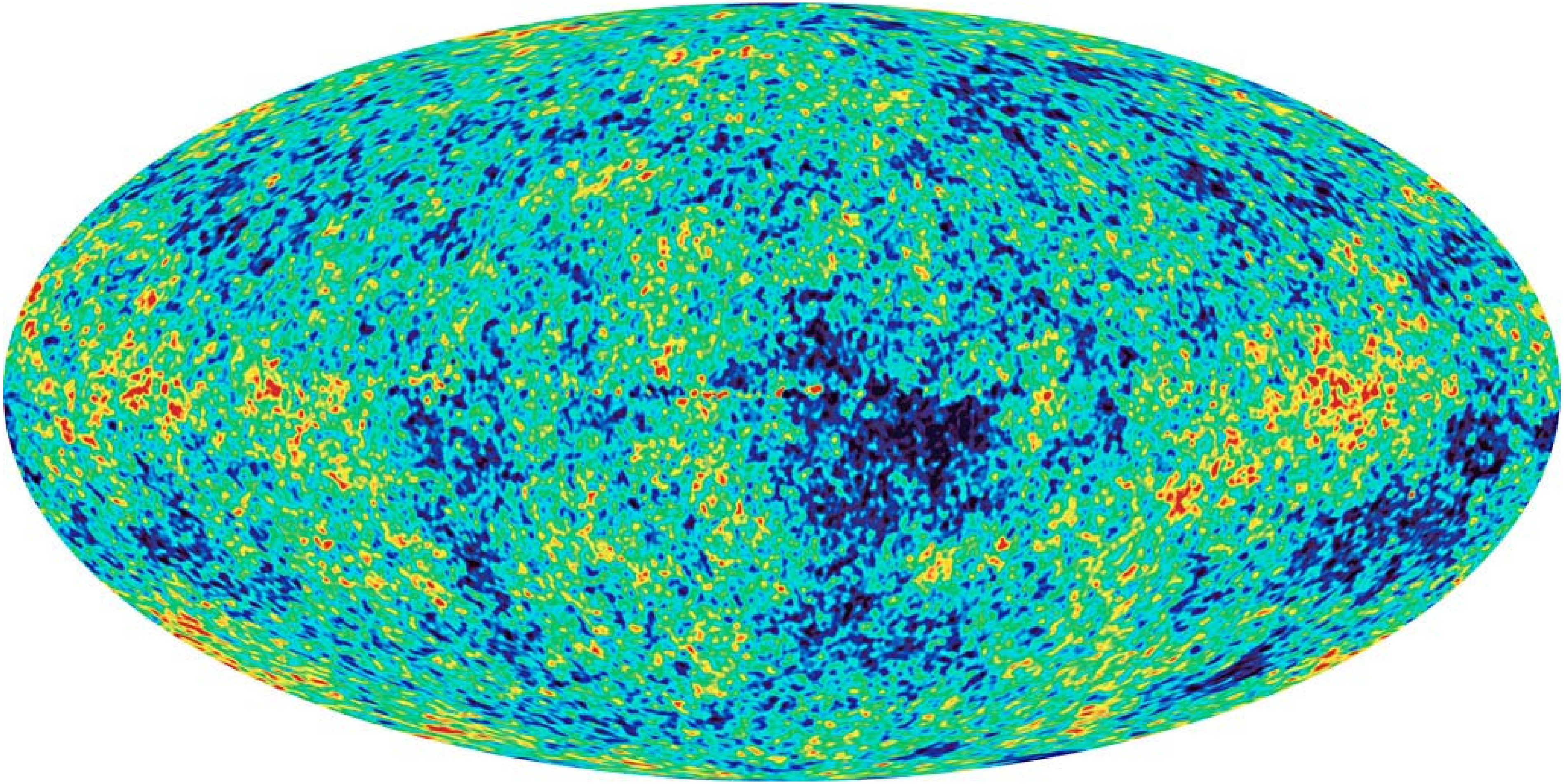
NASA/WMAP science team

WMAP: Full sky coverage

30% sky daily, Whole sky every 6 months

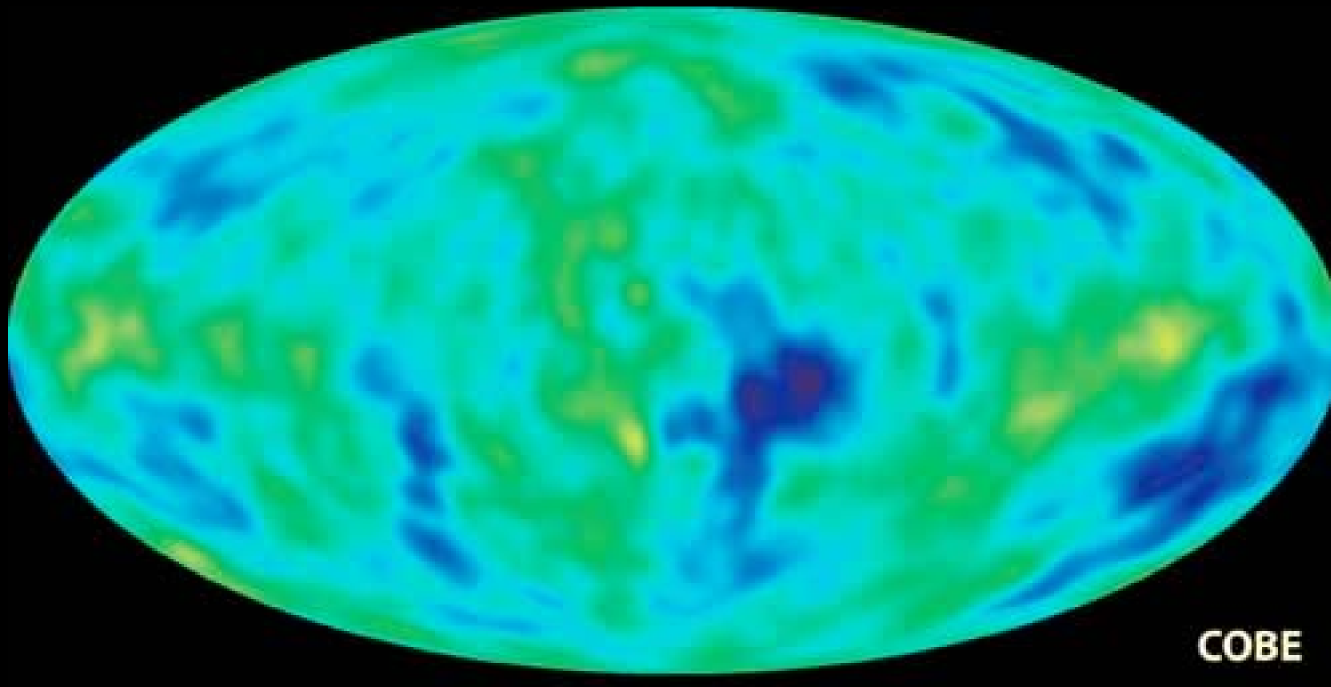
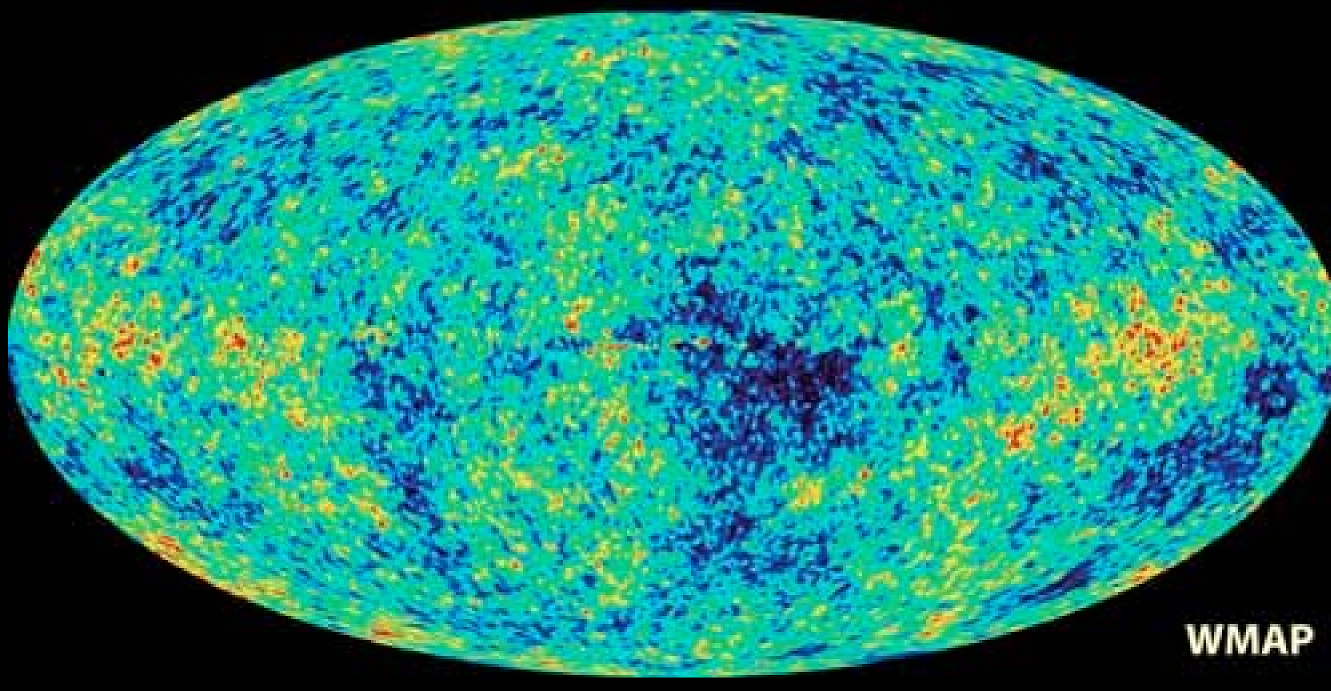


WMAP map of CMB anisotropy



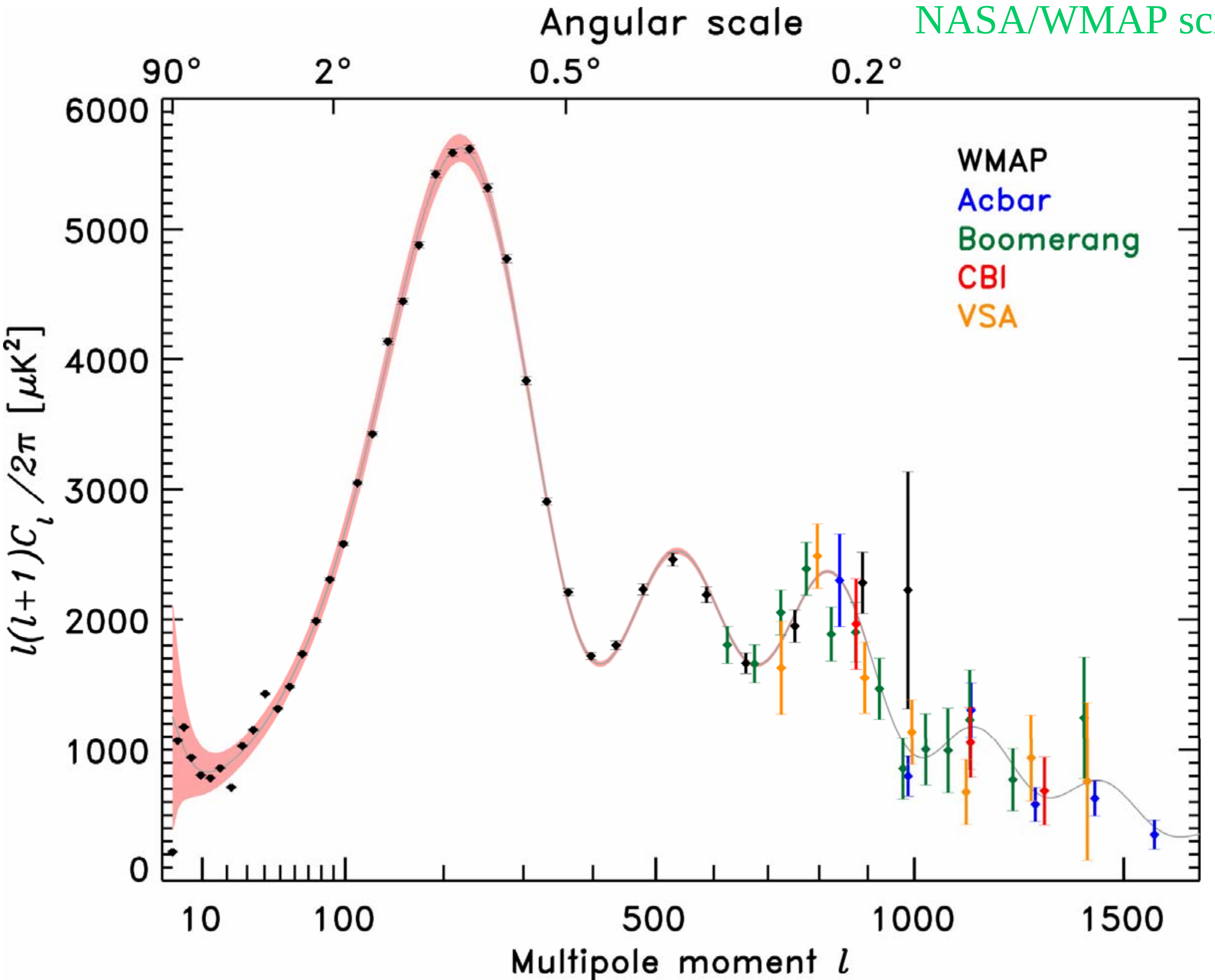
CMB temperature
T_{cmb} = 2.725 K

-200 μK < ΔT < 200 μK
ΔT_{rms} ¼ 70μK

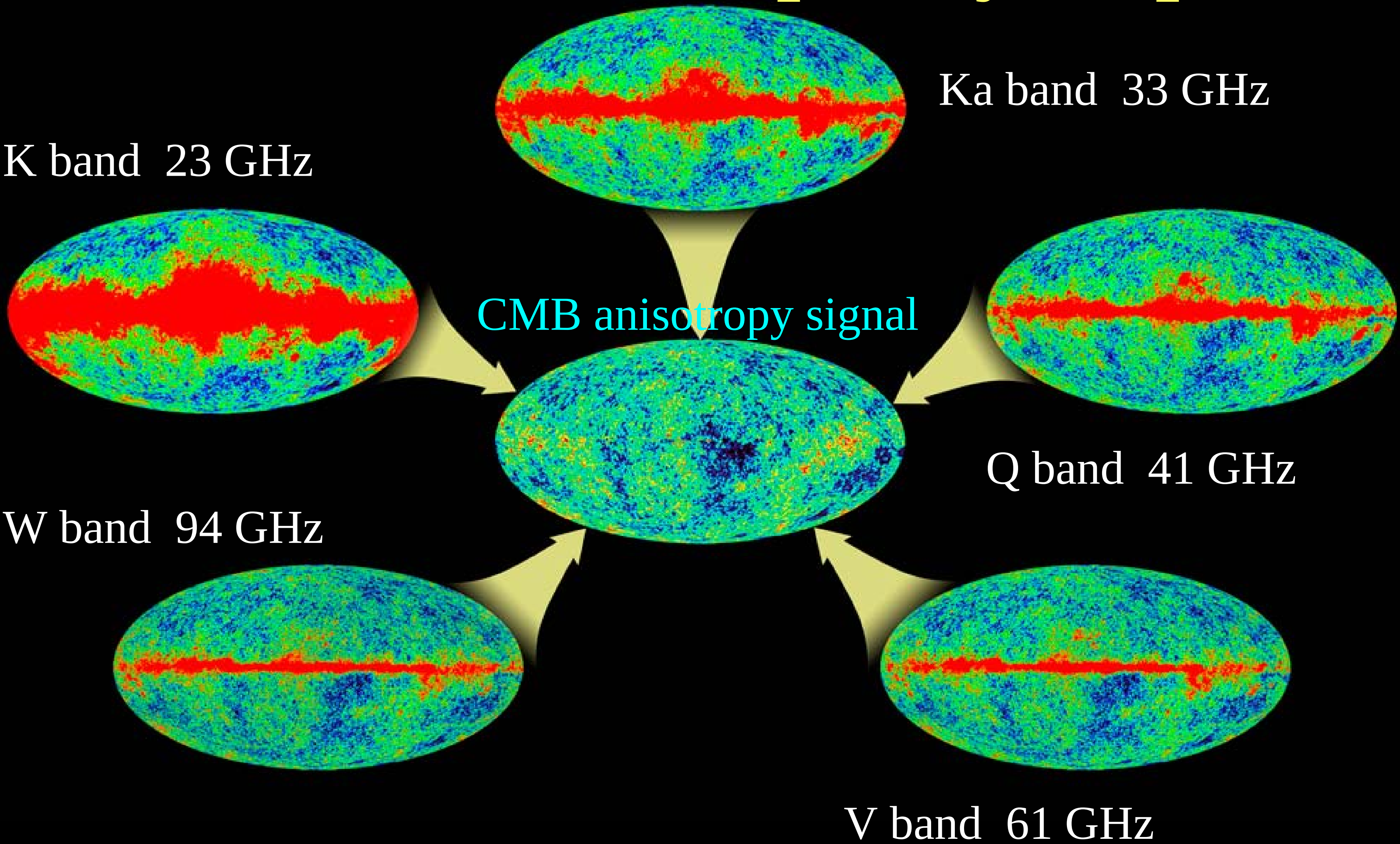

COBE

WMAP

WMAP-3yr: Angular power spectrum

NASA/WMAP science team



WMAP multi-frequency maps



CMB anisotropy signal is frequency independent

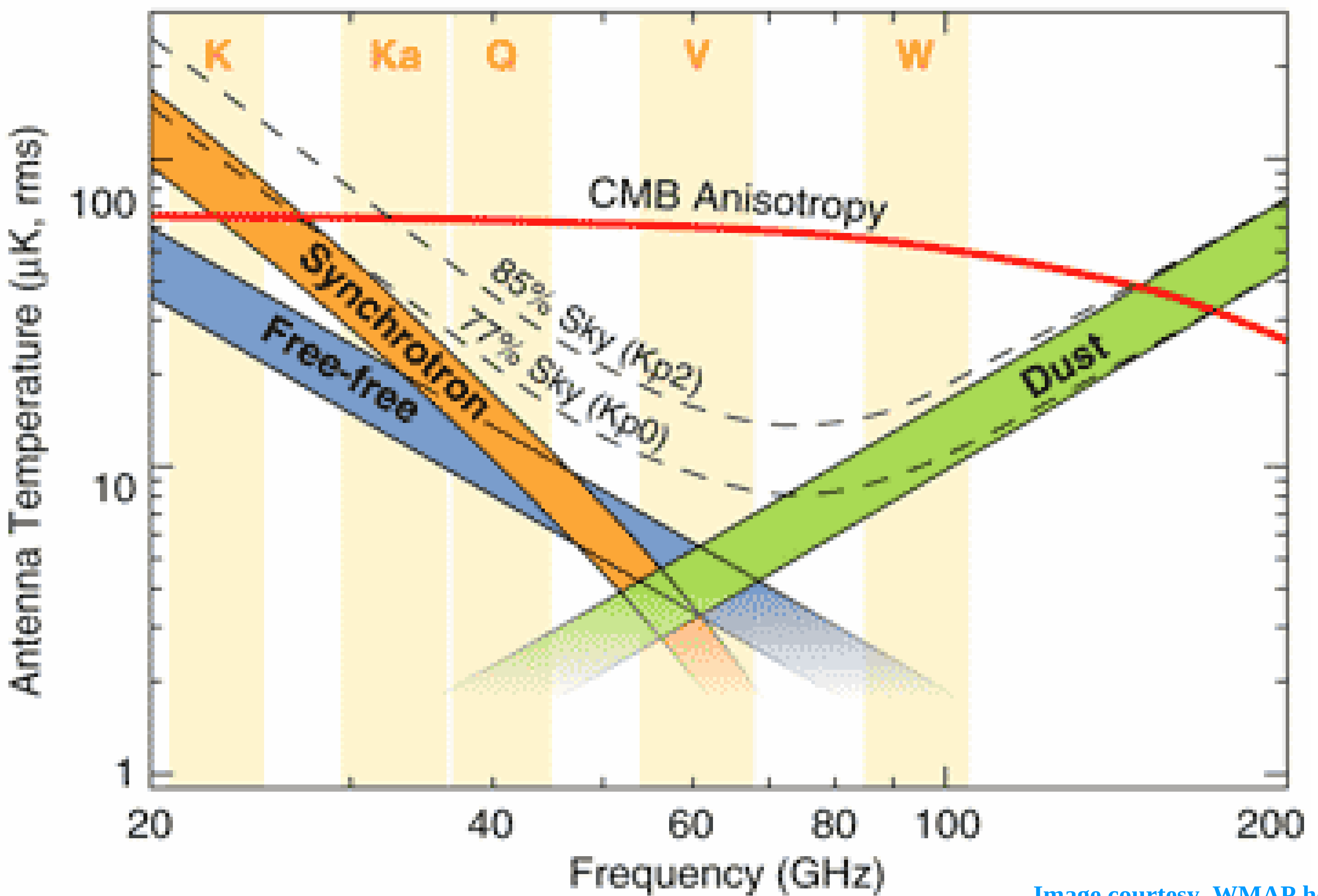
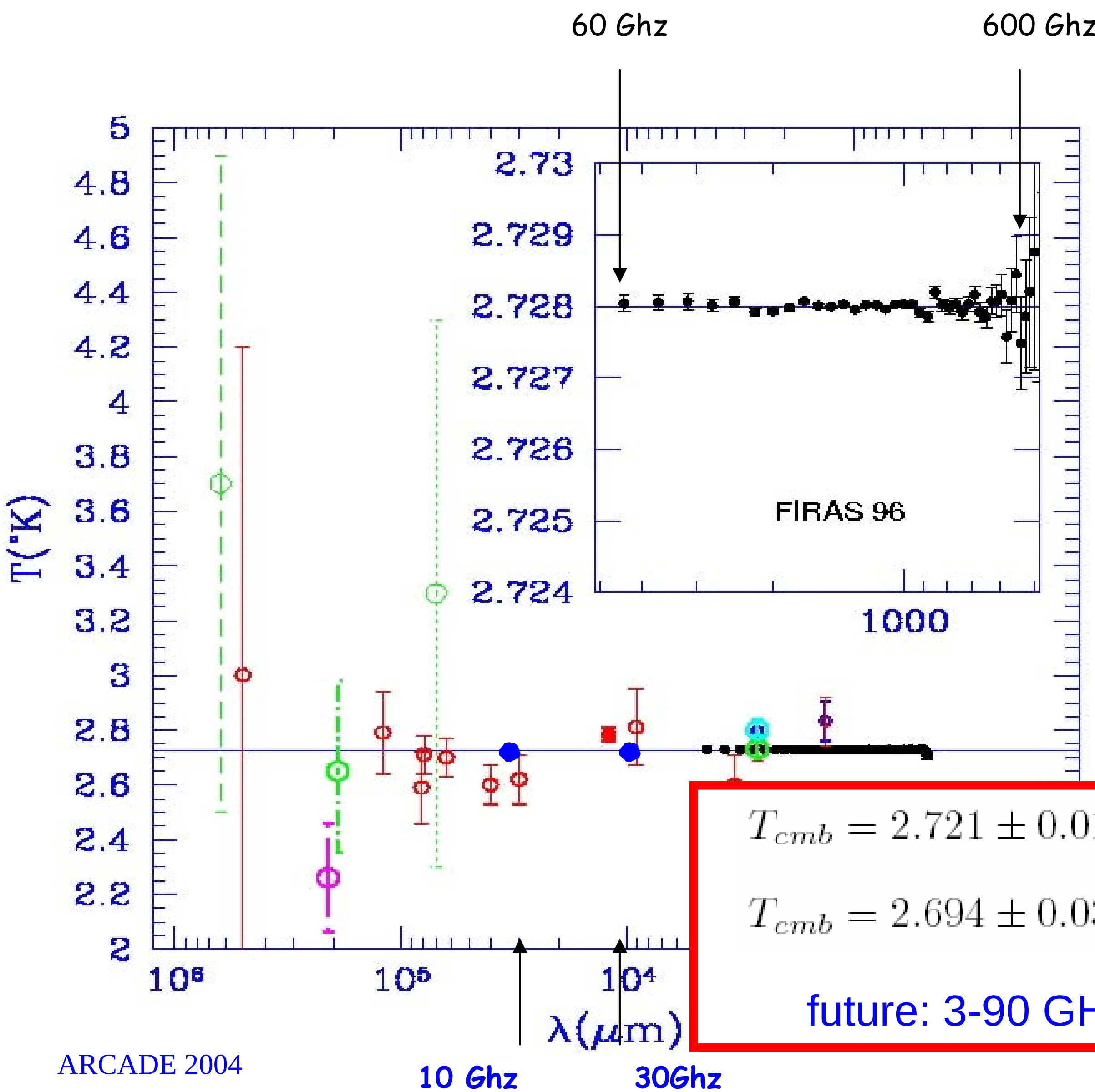


Image courtesy WMAP homepage

Cosmic Microwave Background



(fig: Bond, 1996)

ARCADE 2004

$T_{cmb} = 2.721 \pm 0.010 K$ 10 GHz

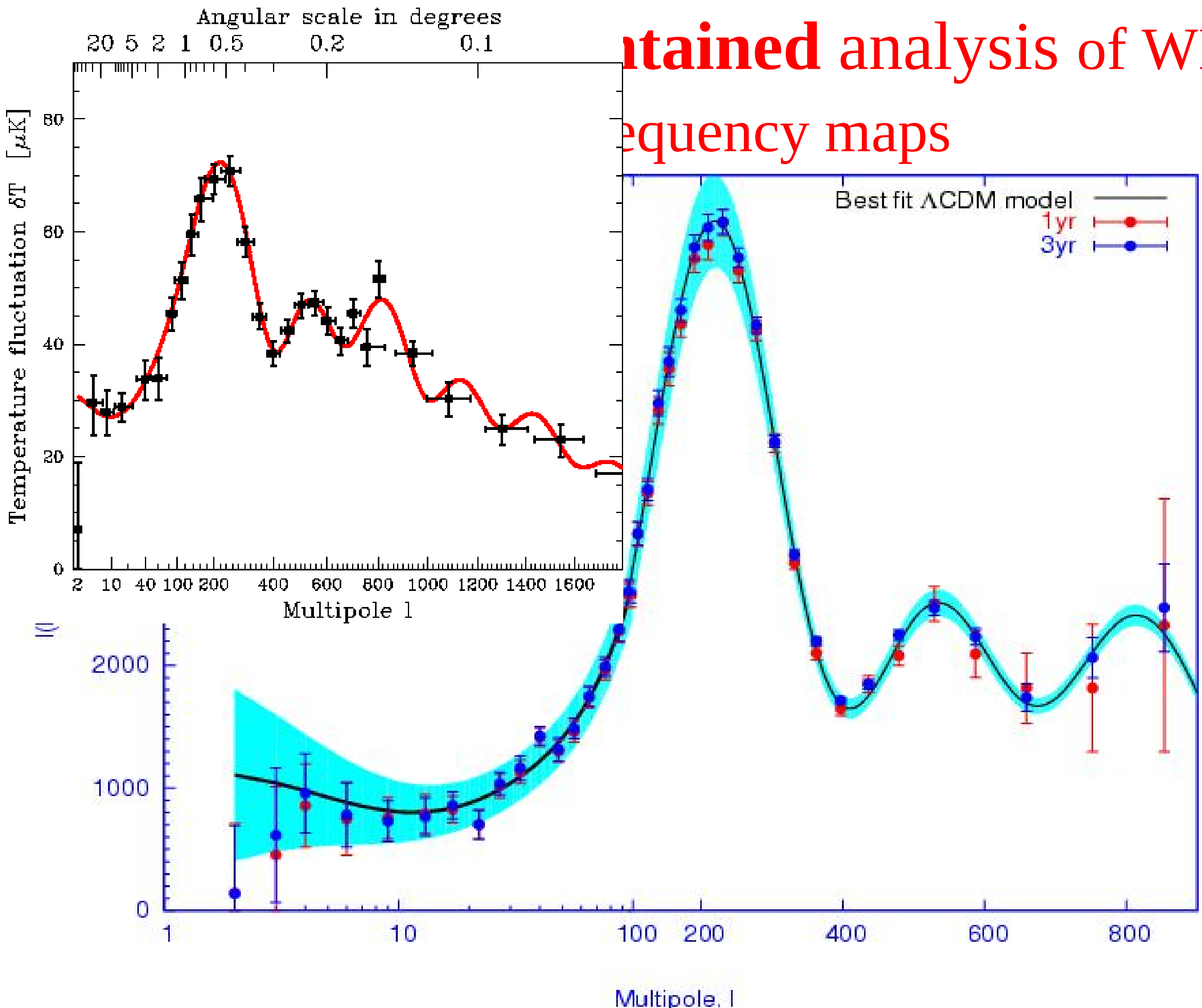
$T_{cmb} = 2.694 \pm 0.032 K$ 30 GHz

future: 3-90 GHz

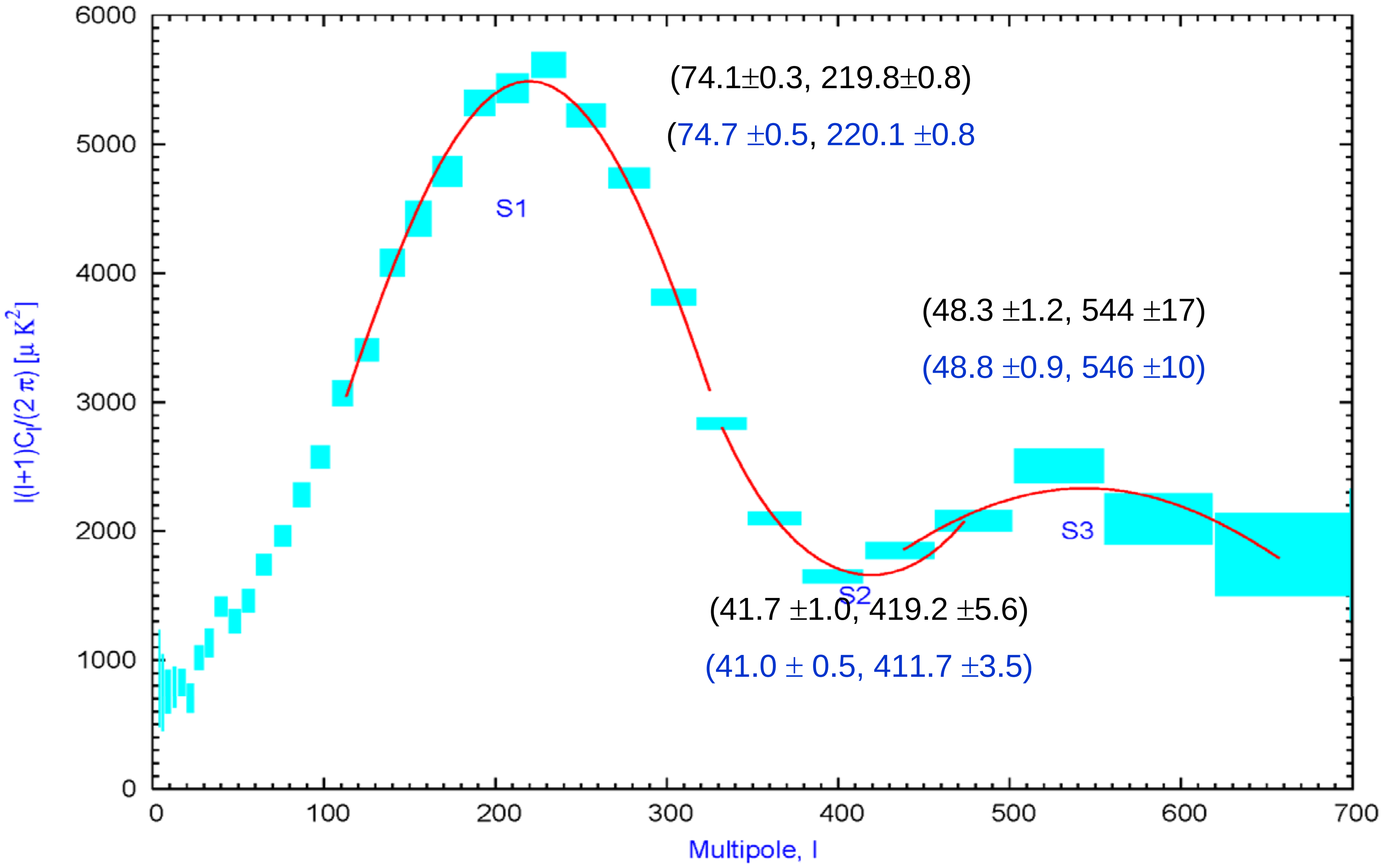
WMAP: Angular power spectrum

ntained analysis of WMAP

equency maps



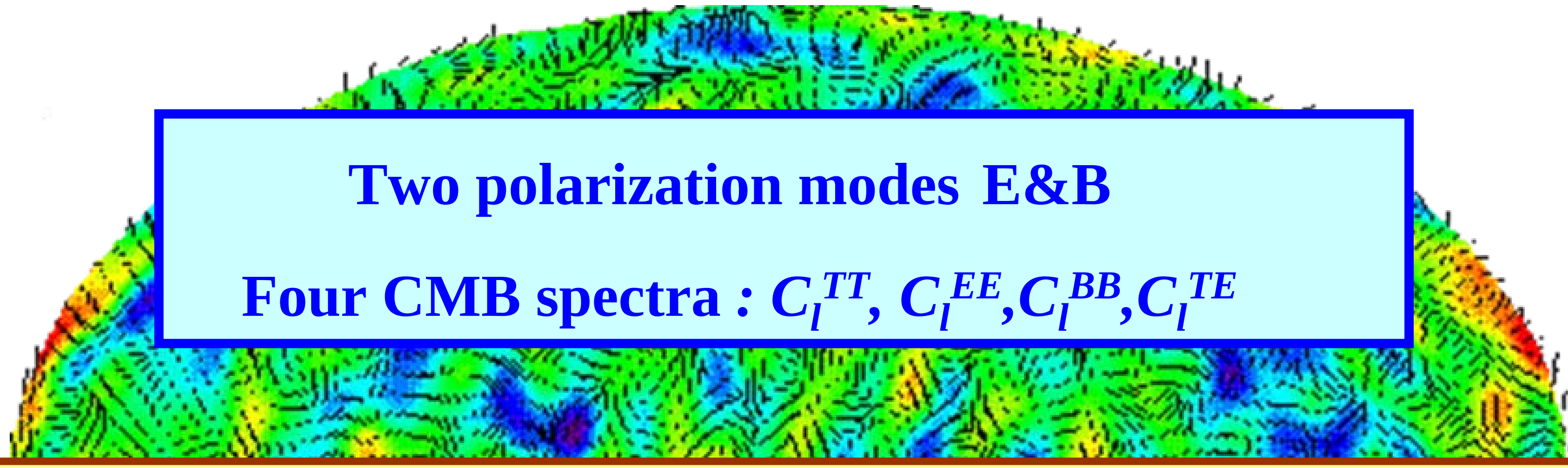
Peaks of the angular power spectrum



(Saha, Jain, Souradeep Apj Lett 2006)

CMB Polarization

Thompson scattering at redshift z=1100 (surface of last scattering) generates **a linear polarization pattern in the CMB sky.**



Two polarization modes E&B

Four CMB spectra : C_l^{TT}, C_l^{EE}, C_l^{BB}, C_l^{TE}

- **Density (scalar) perturbations generate only E mode polarization.**

E-mode ¼ 5 --10 μ K

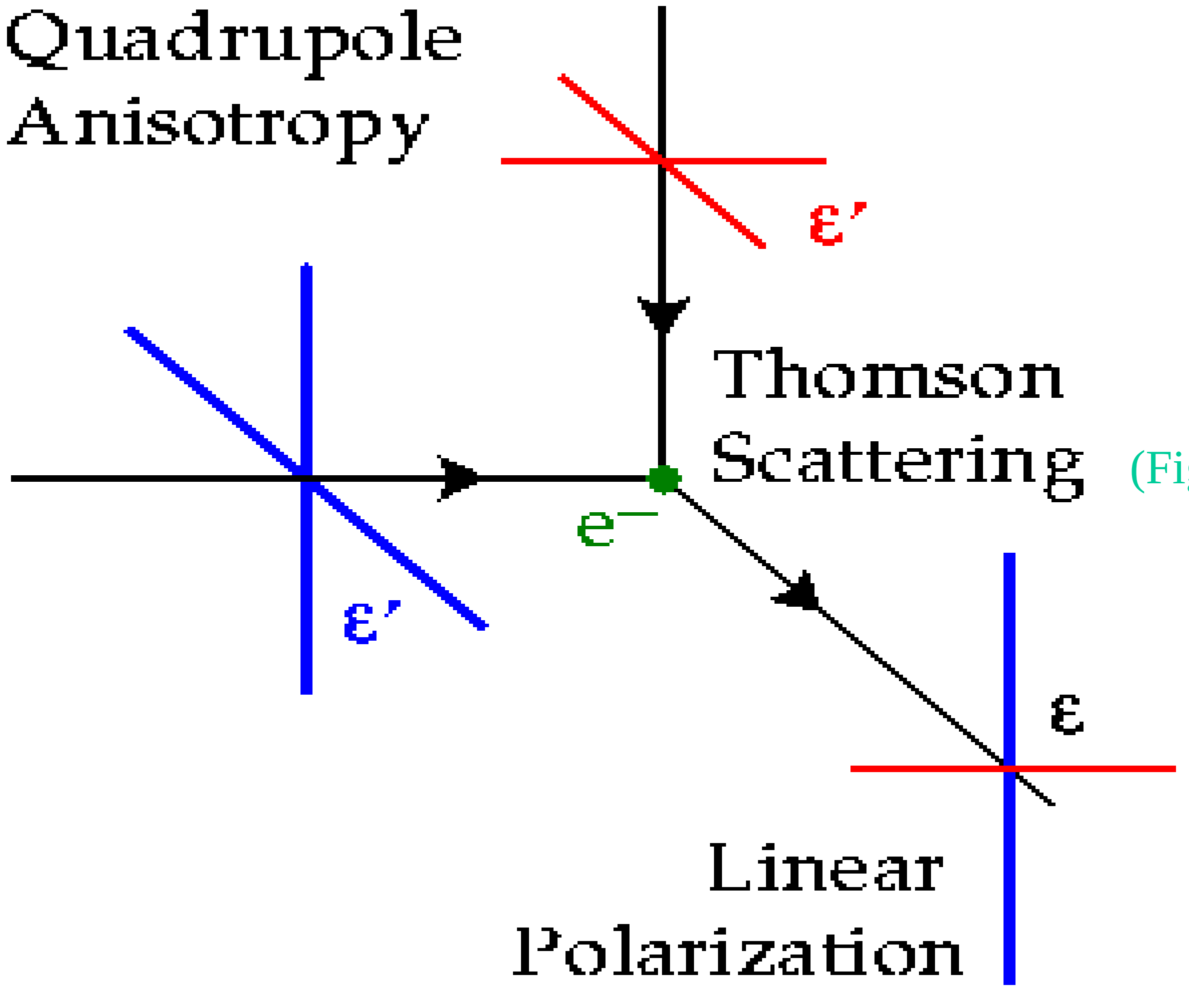
- **Gravitational waves generate both the modes in comparable amounts .**

B-mode ¼ 0.05 – 0.1 μ K

- **B-mode measures cosmic gravity wave background.**

CMB Polarization

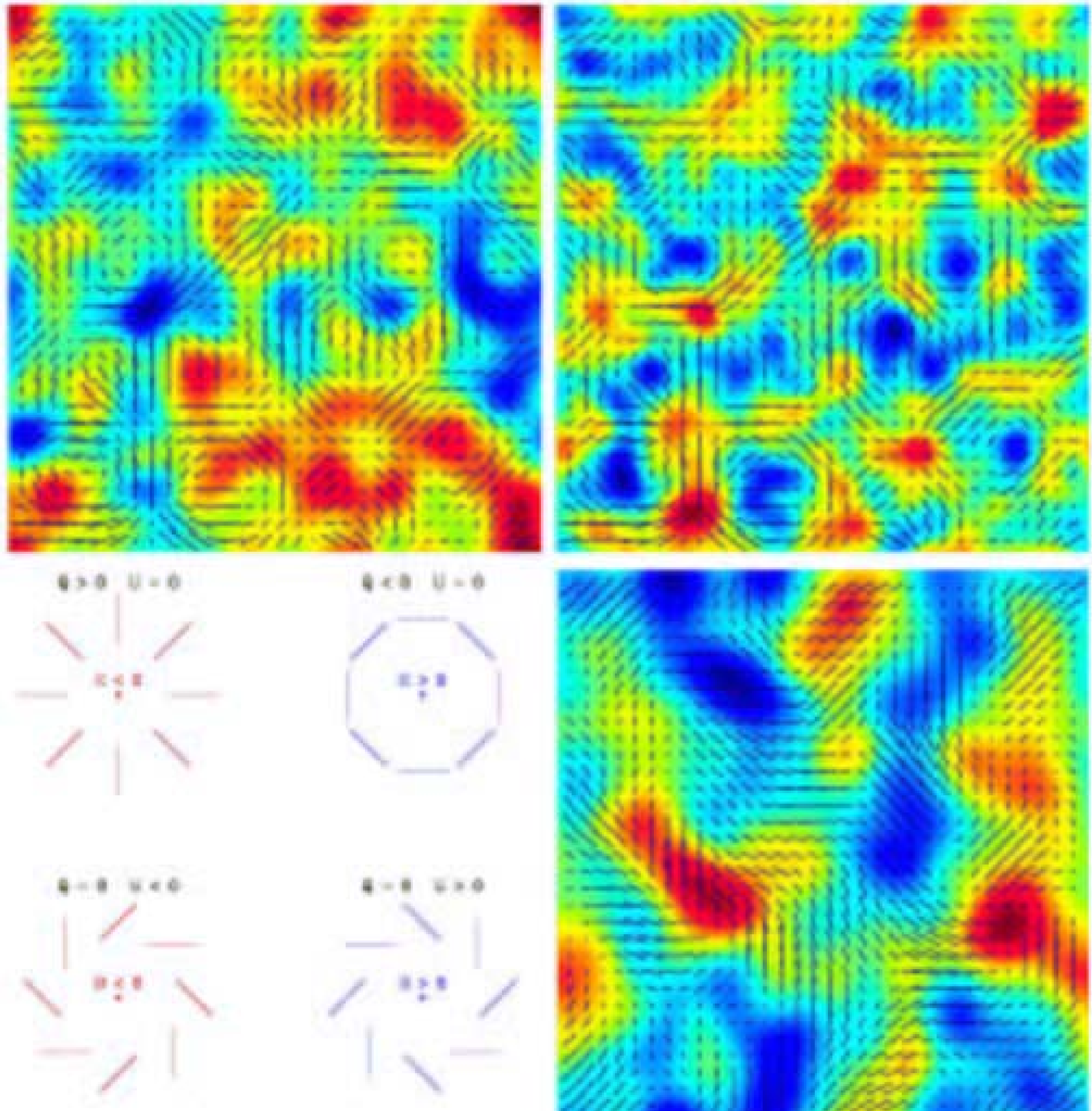
Thompson scattering of the CMB anisotropy quadrupole at the surface of last scattering generates a linear polarization pattern in the CMB.



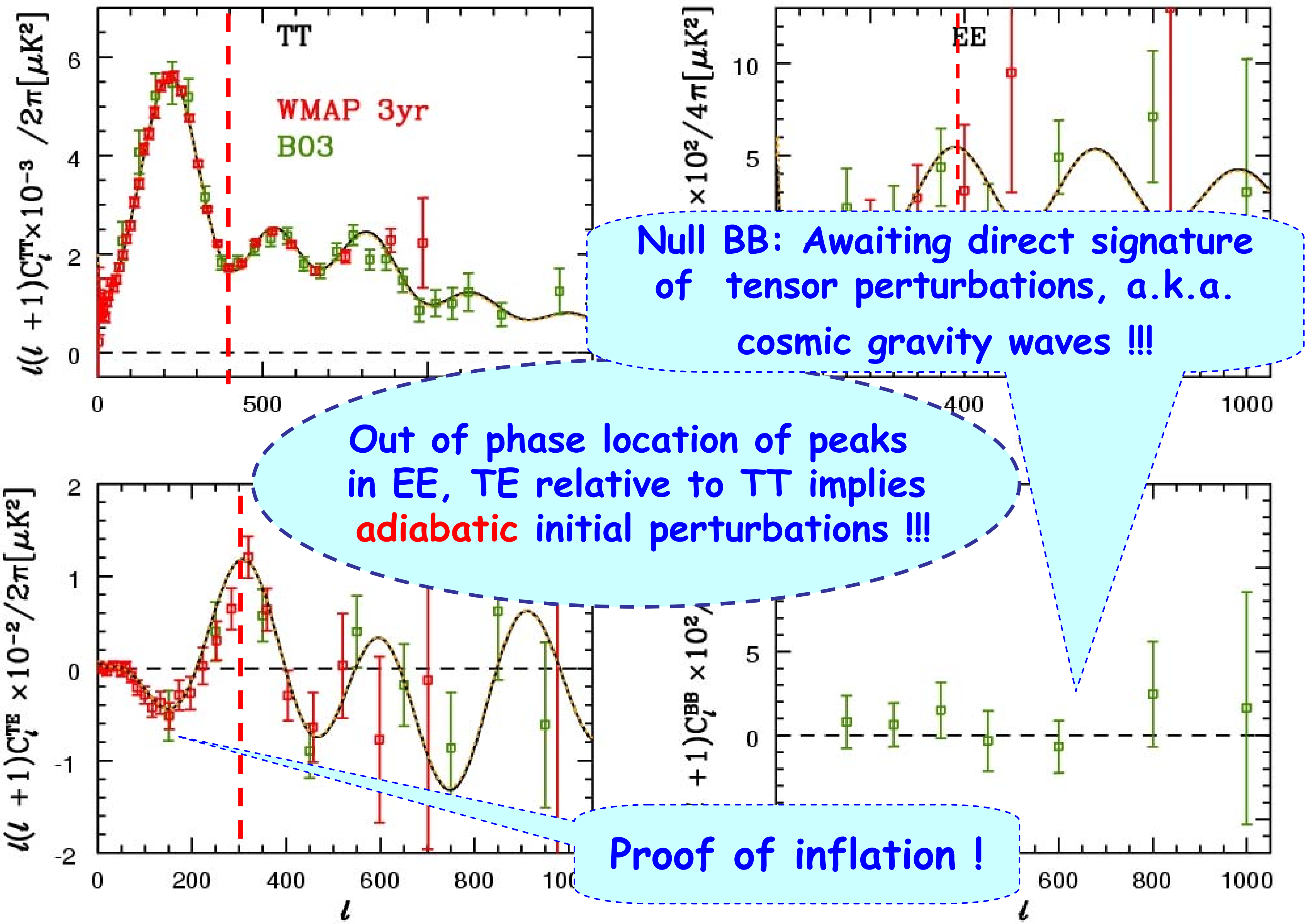
(Fig:Hu & White , 97)

E modes
(Gradient)

B modes
(Curl)

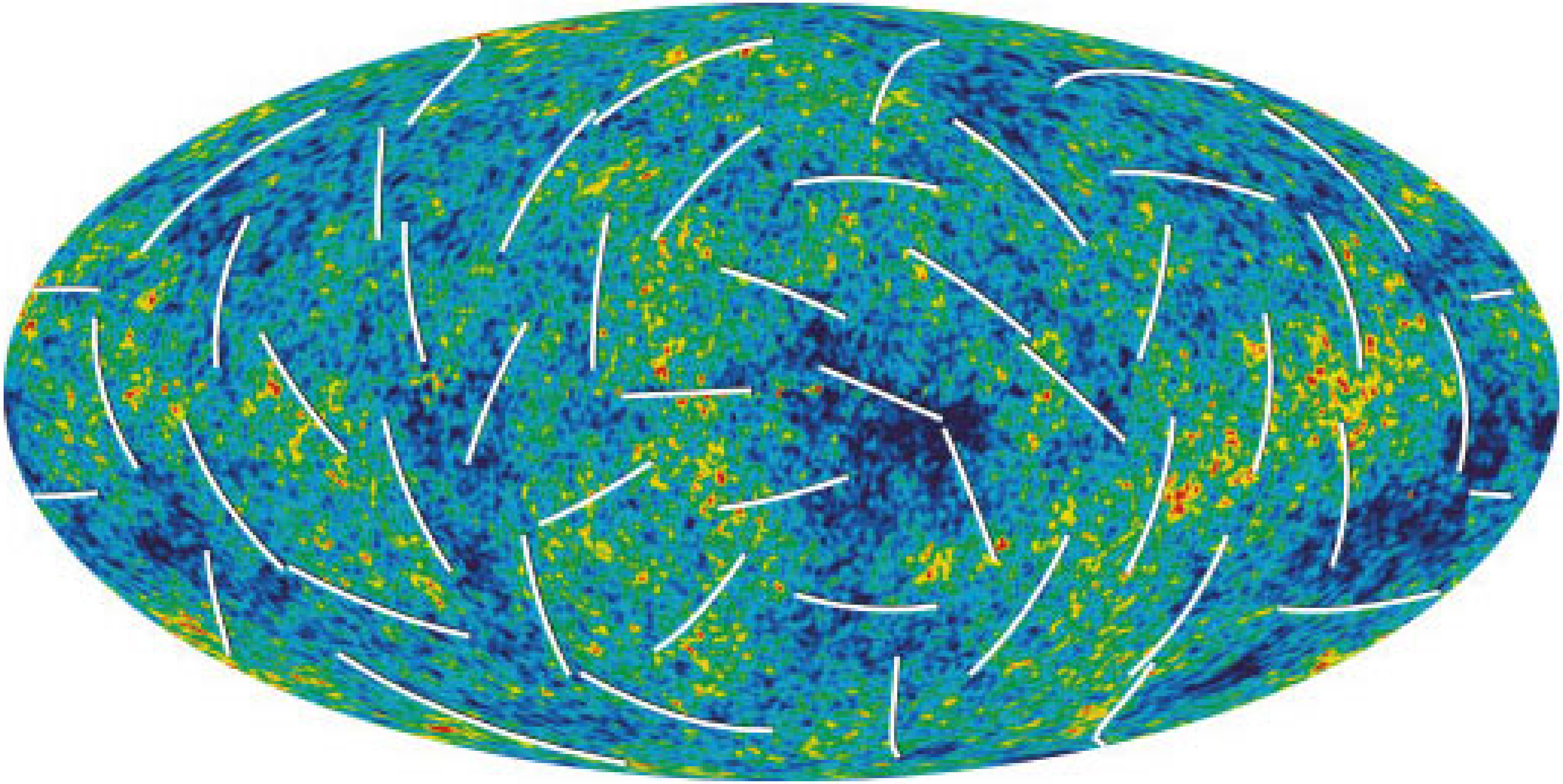


Current status of CMB Spectra



(Boomerang 2003, WMAP-3)

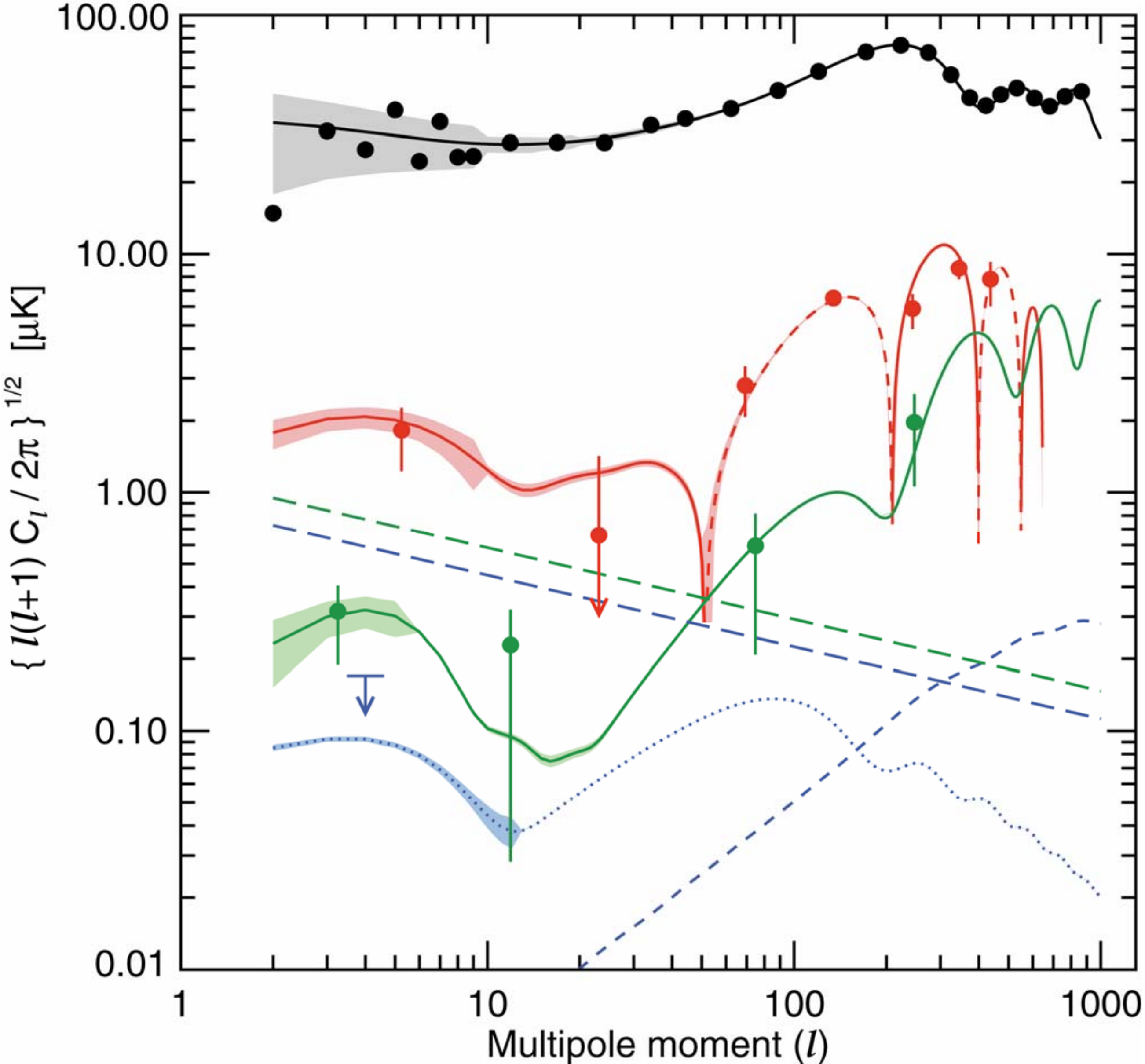
WMAP map of CMB Polarization



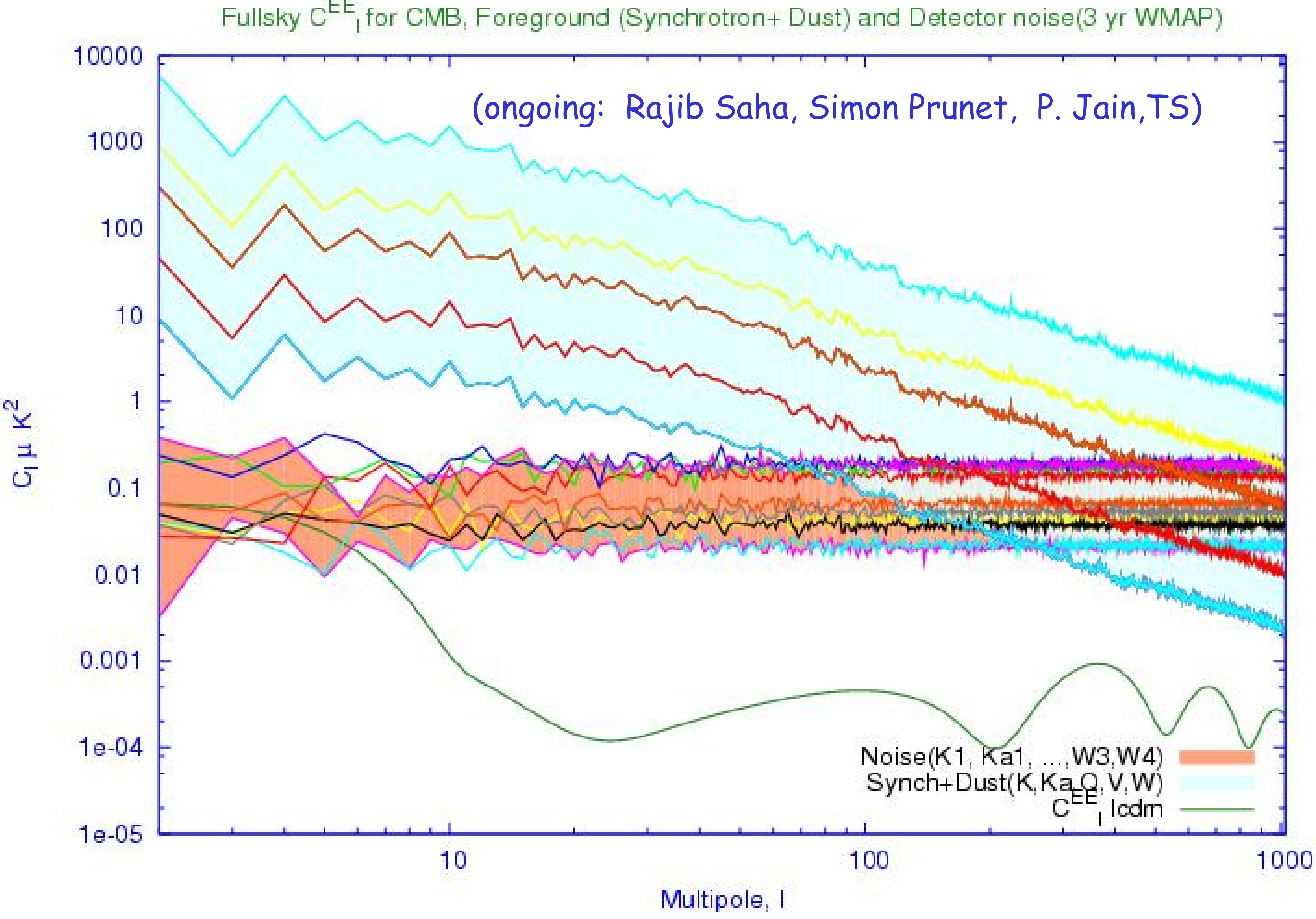
NASA/WMAP science team 2006

WMAP-3yr: Angular power spectra

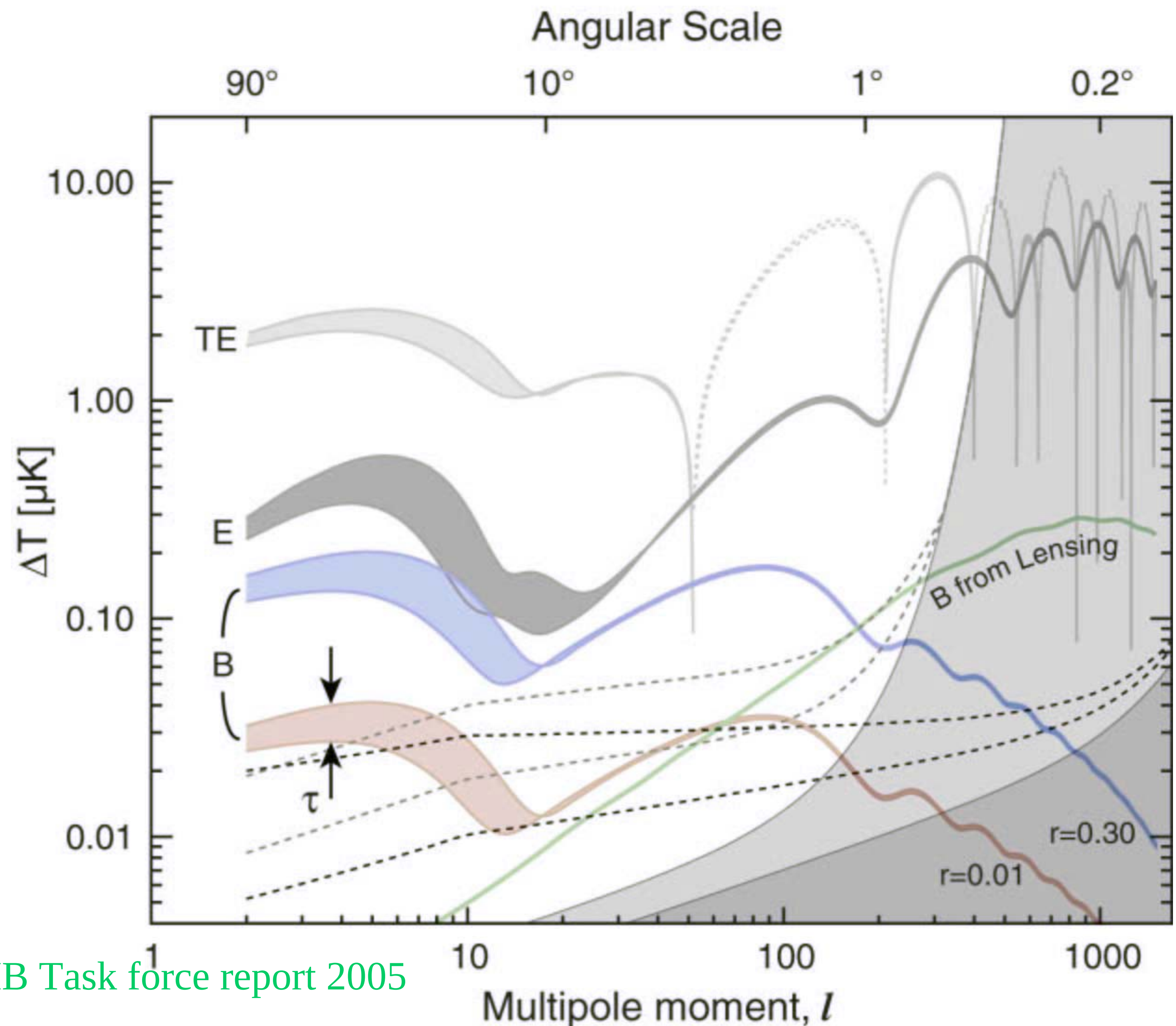
NASA/WMAP science team



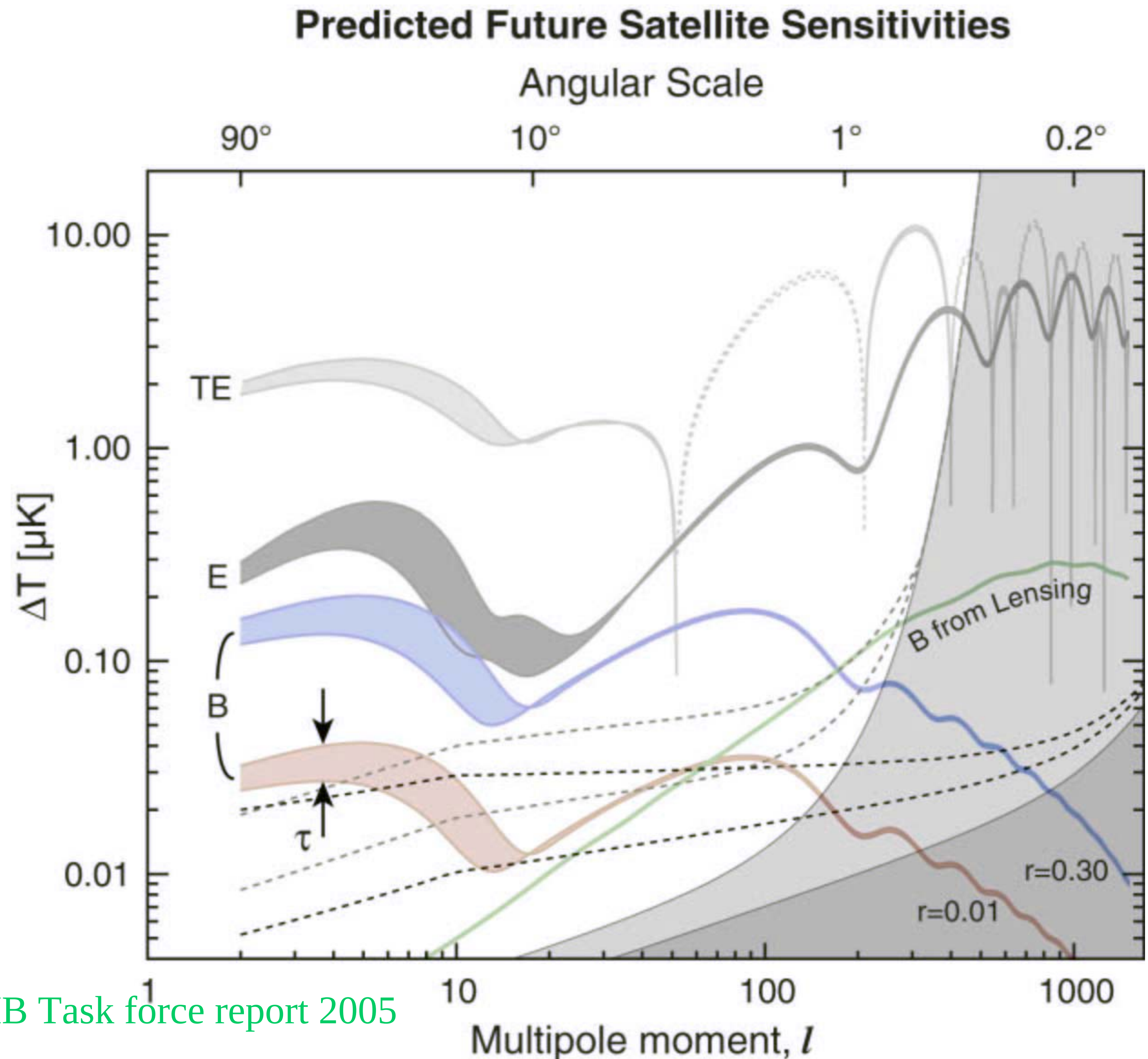
WMAP CMB Polarization challenge



Predicted Future Satellite Sensitivities

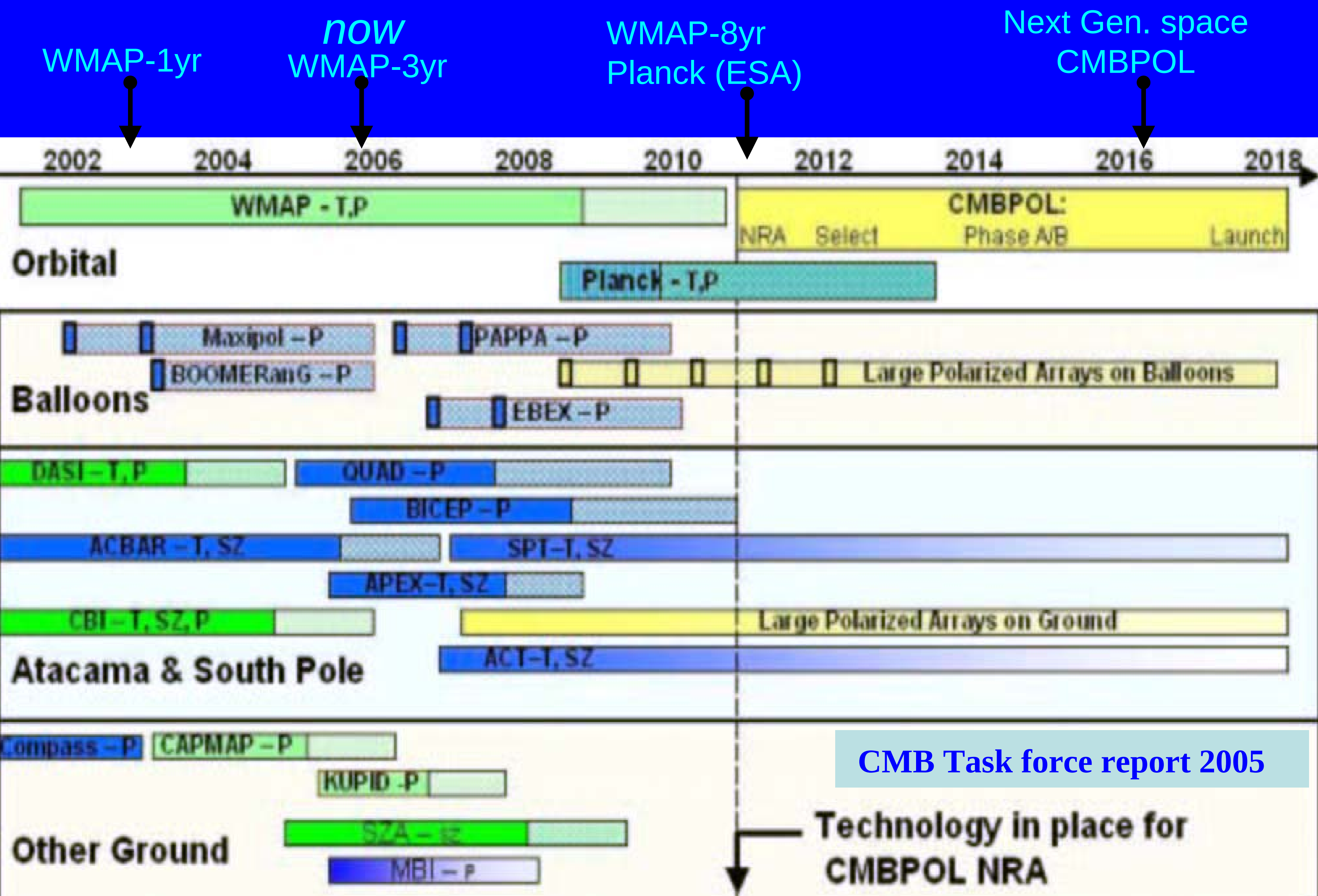


CMB Task force report 2005



CMB Task force report 2005

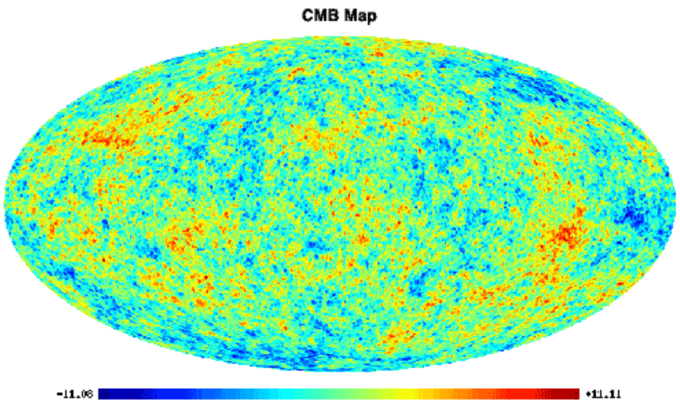
Timeline of CMB experiments



Gravitational Instability ?

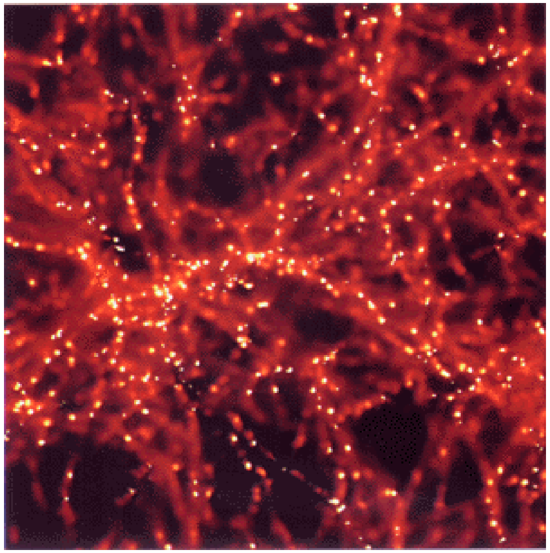
Mildly Perturbed universe at z=1100

Present universe at z=0

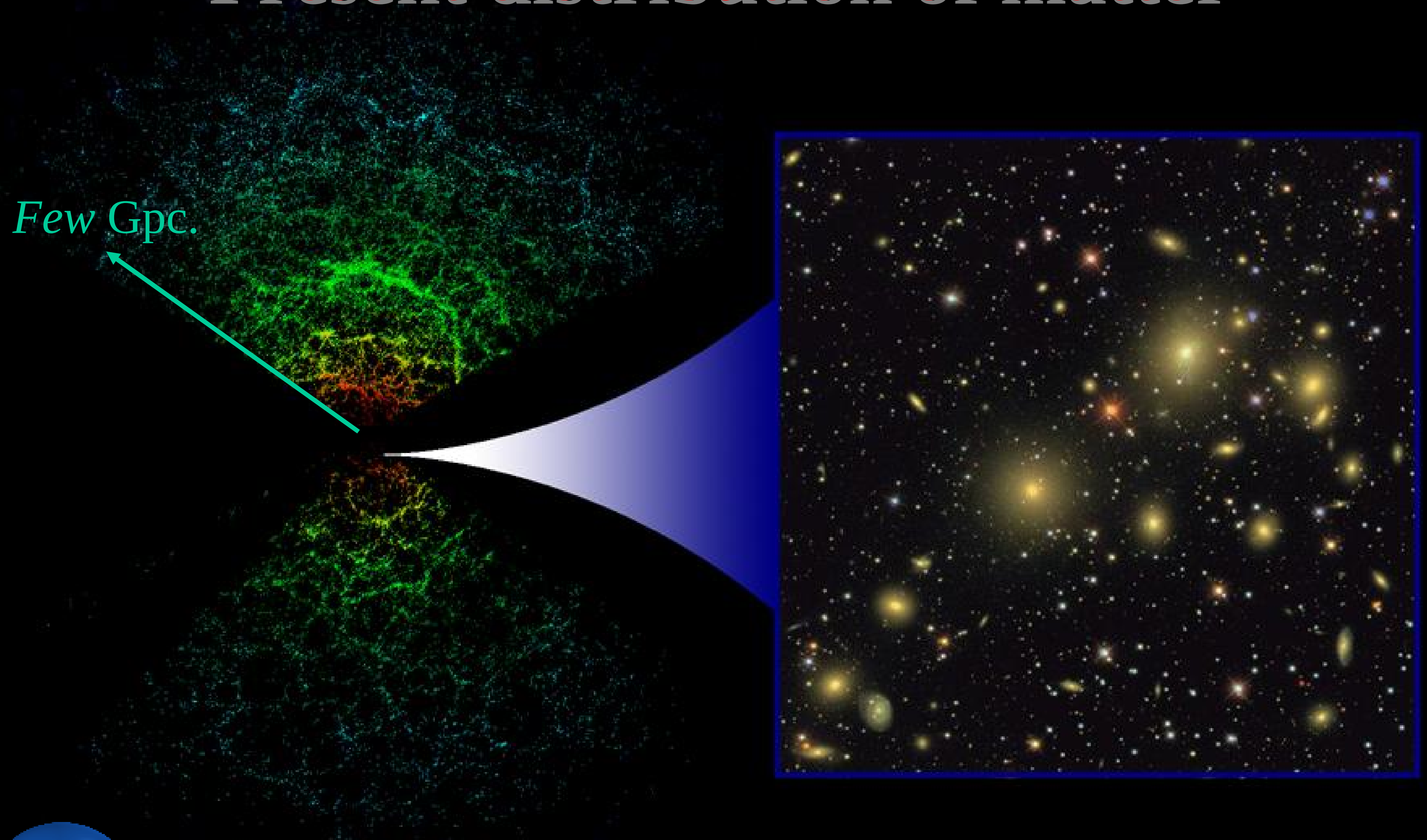


Cosmic matter content

$$\Omega_{tot}$$
$$\Omega_{b}$$
$$\Omega_{DM}$$
$$\Omega_{\Lambda}$$
$$H_{0}$$



Present distribution of matter



SLOAN DIGITAL SKY SURVEY (SDSS)

One little telltale bump !!

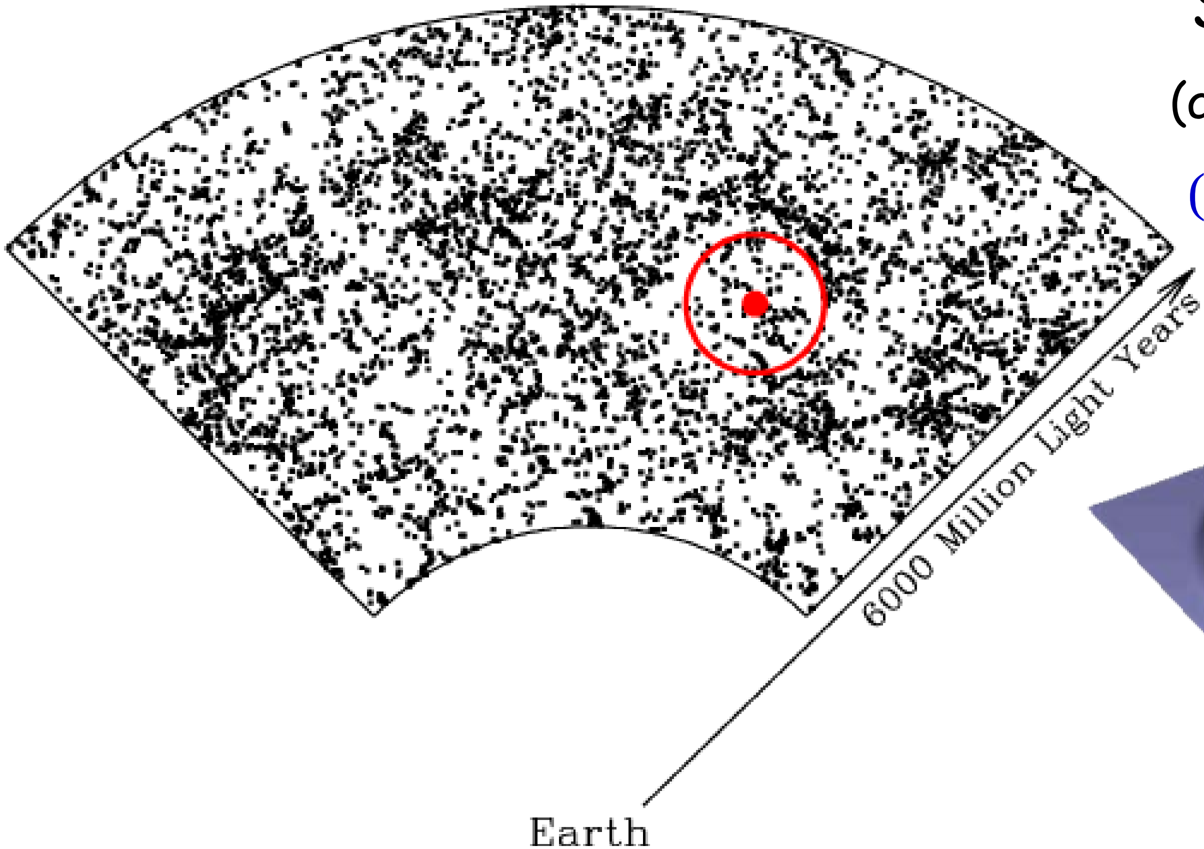
$$\xi(r) = \langle \delta\rho(\vec{r}_1)\delta\rho(\vec{r}_2) \rangle$$

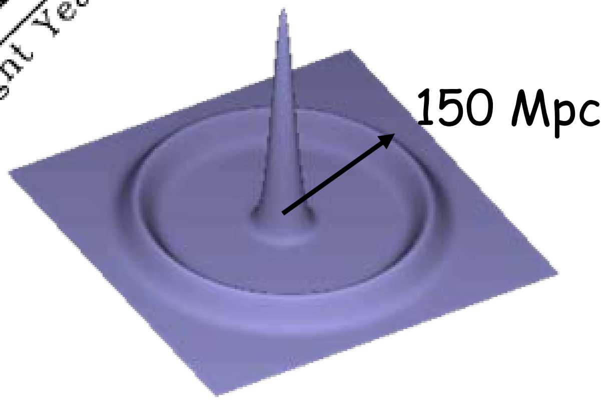
A small excess in correlation at 150 Mpc.!

SDSS survey

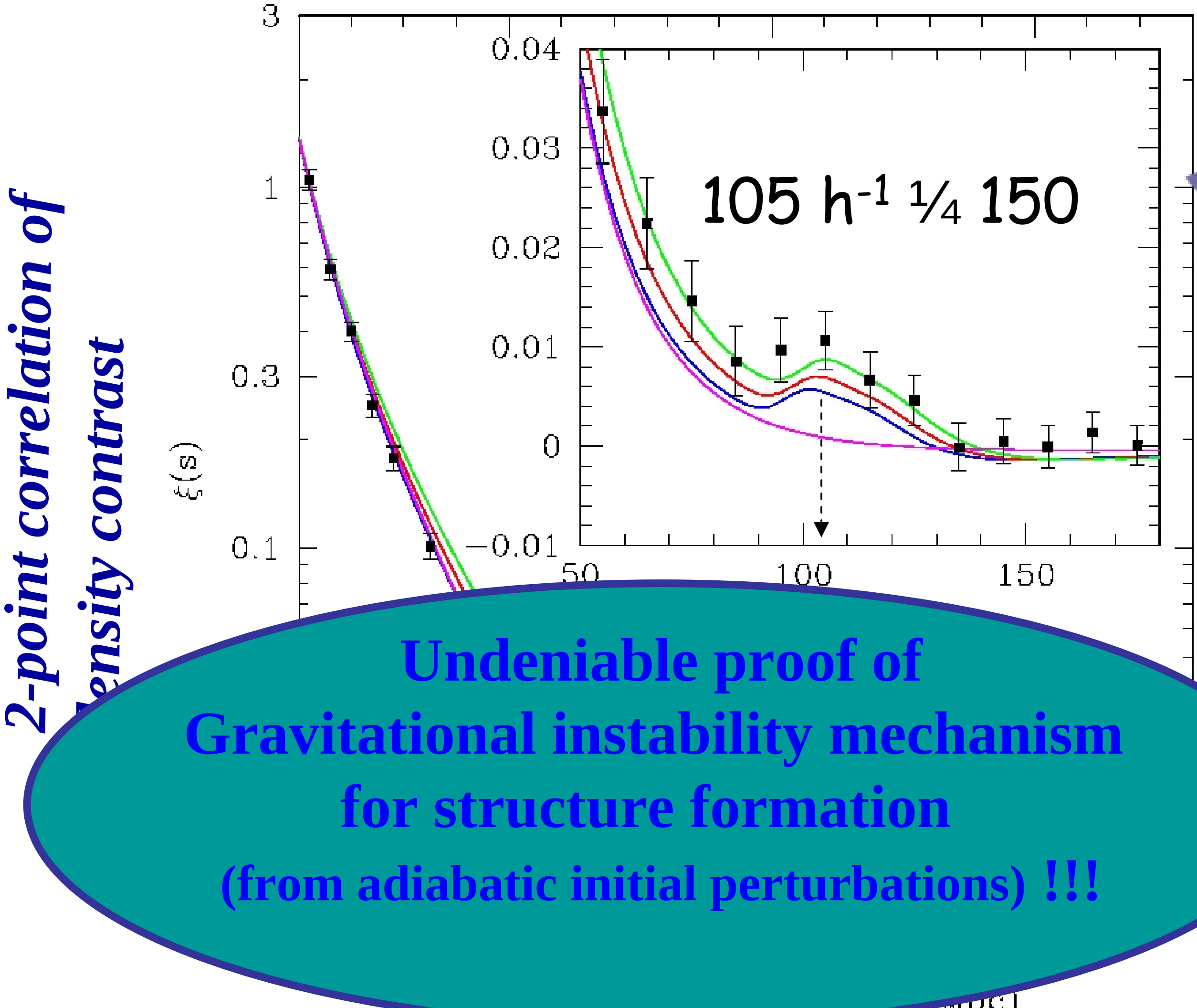
(astro-ph/0501171)

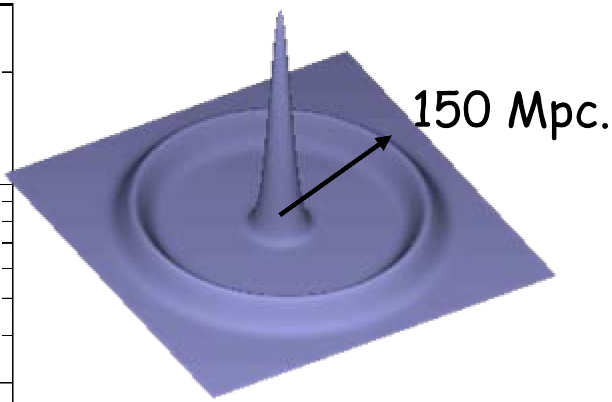
(Einsentein et al. 2005)





Acoustic Baryon oscillations in the matter correlation function !!





The same CMB oscillations at low redshifts !!!

SDSS survey

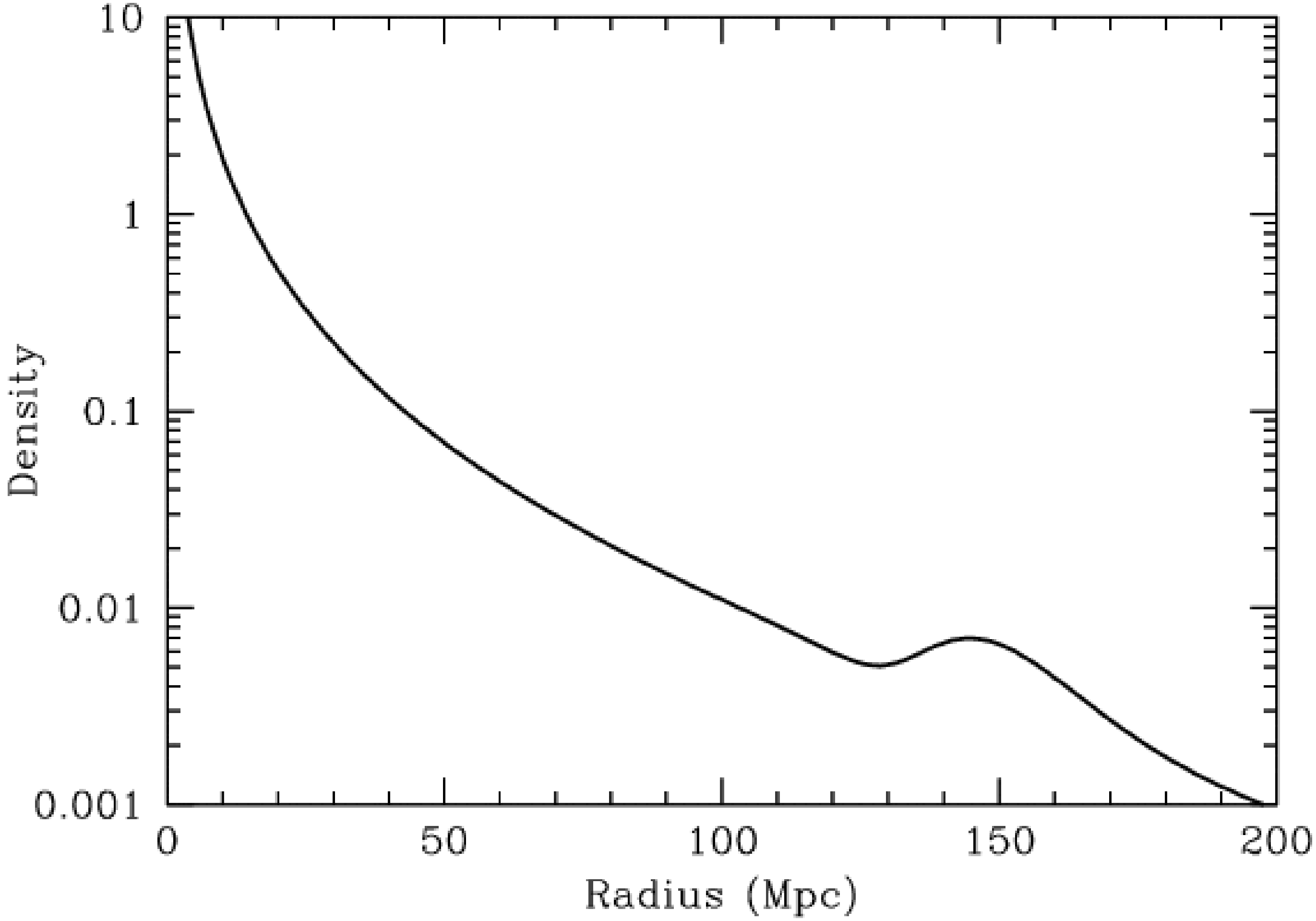
(astro-ph/0501171)

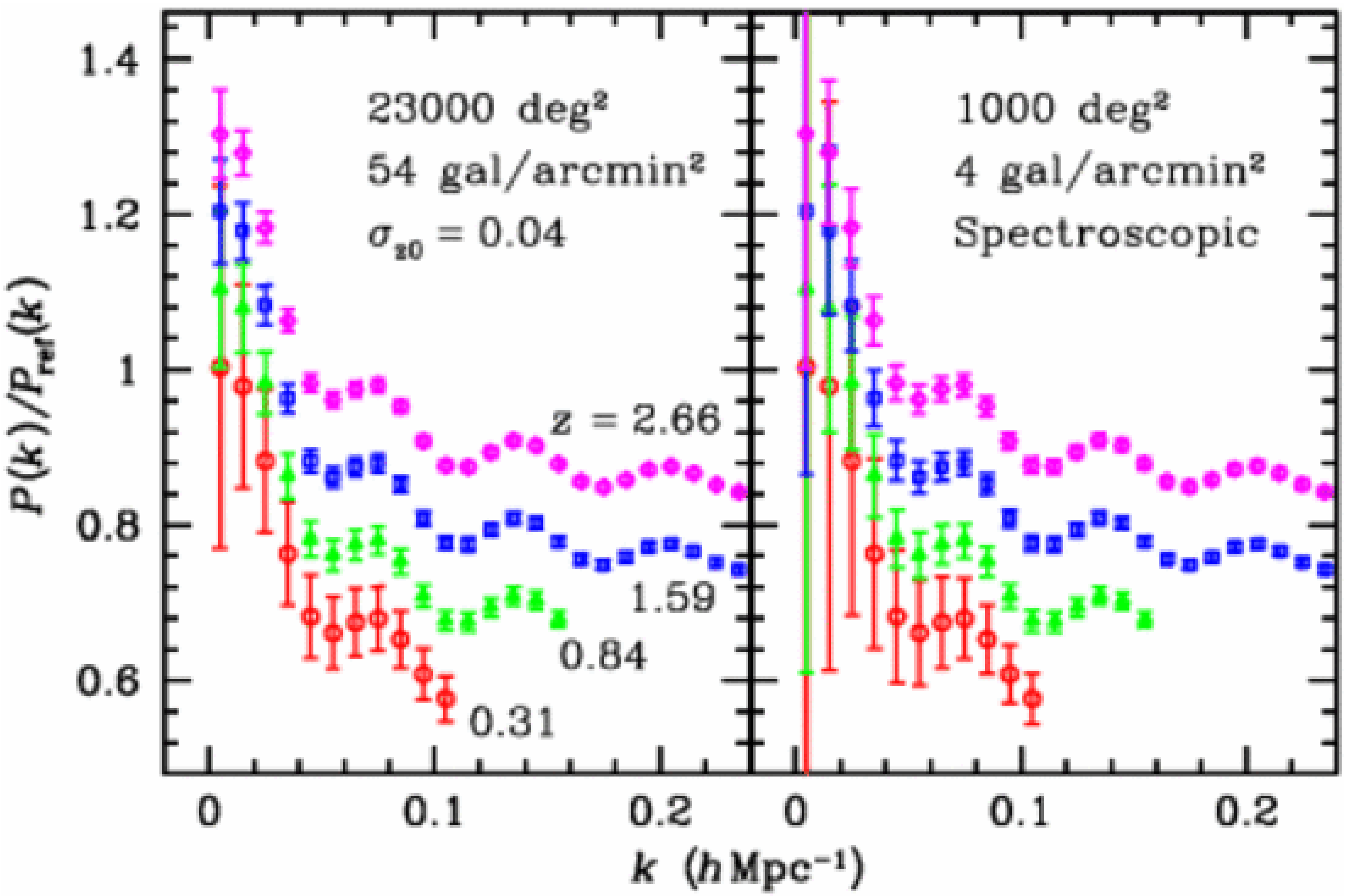
Undeniable proof of Gravitational instability mechanism for structure formation

(from adiabatic initial perturbations) !!!

(Einsentein et al. 2005)

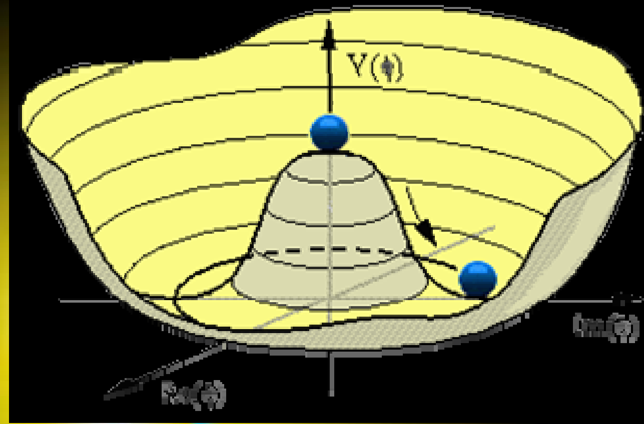
Ripples in the different constituents





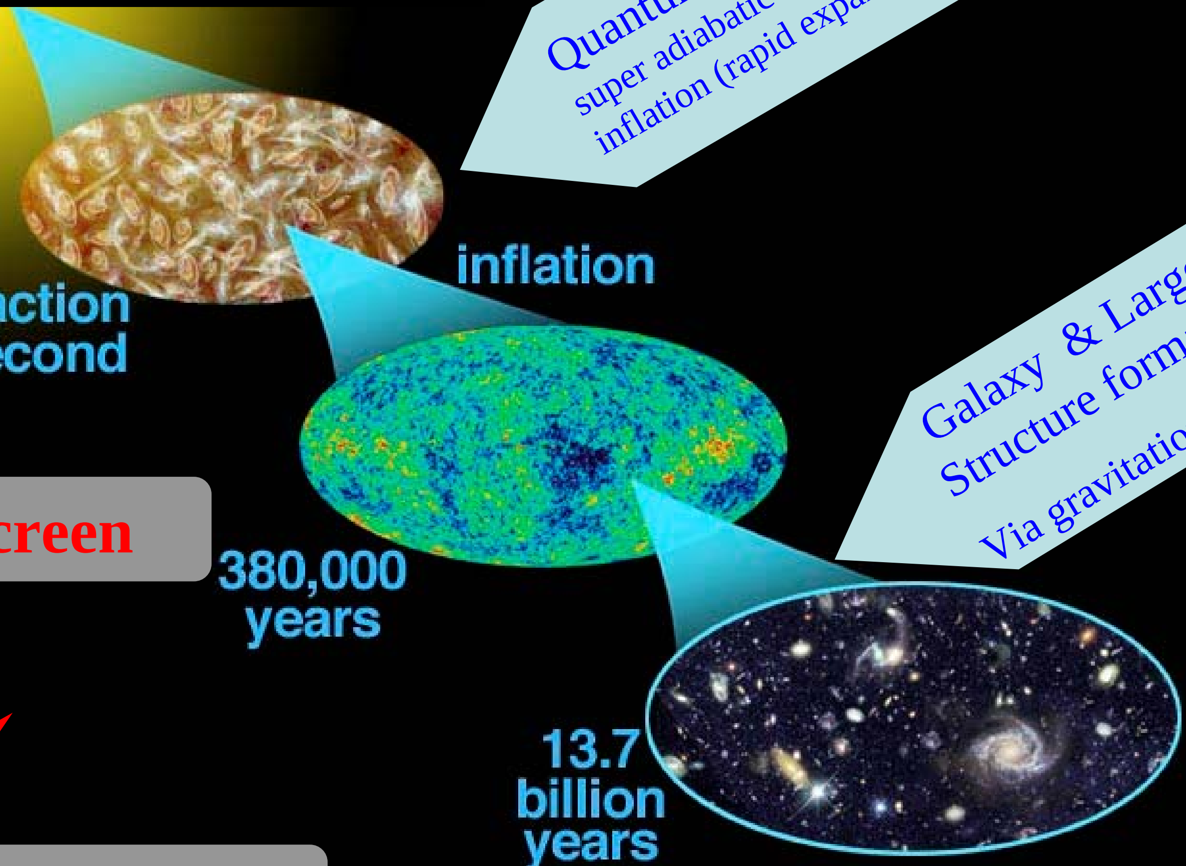
1.4
1.2
1
0.8
0.6
P(k)/P_ref(k)
23000 deg²
54 gal/arcmin²
σ_z0 = 0.04
z = 2.66
1.59
0.84
0.31
1000 deg²
4 gal/arcmin²
Spectroscopic
0
0.1
0.2
0
0.1
0.2
k (h Mpc⁻¹)

Who pinged the Cosmic drum ?



Quantum fluctuations
super adiabatic amplified by inflation (rapid expansion)

Early Universe



The Cosmic screen

Galaxy & Large scale Structure formation
Via gravitational instability

Present Universe

Generic Inflation model

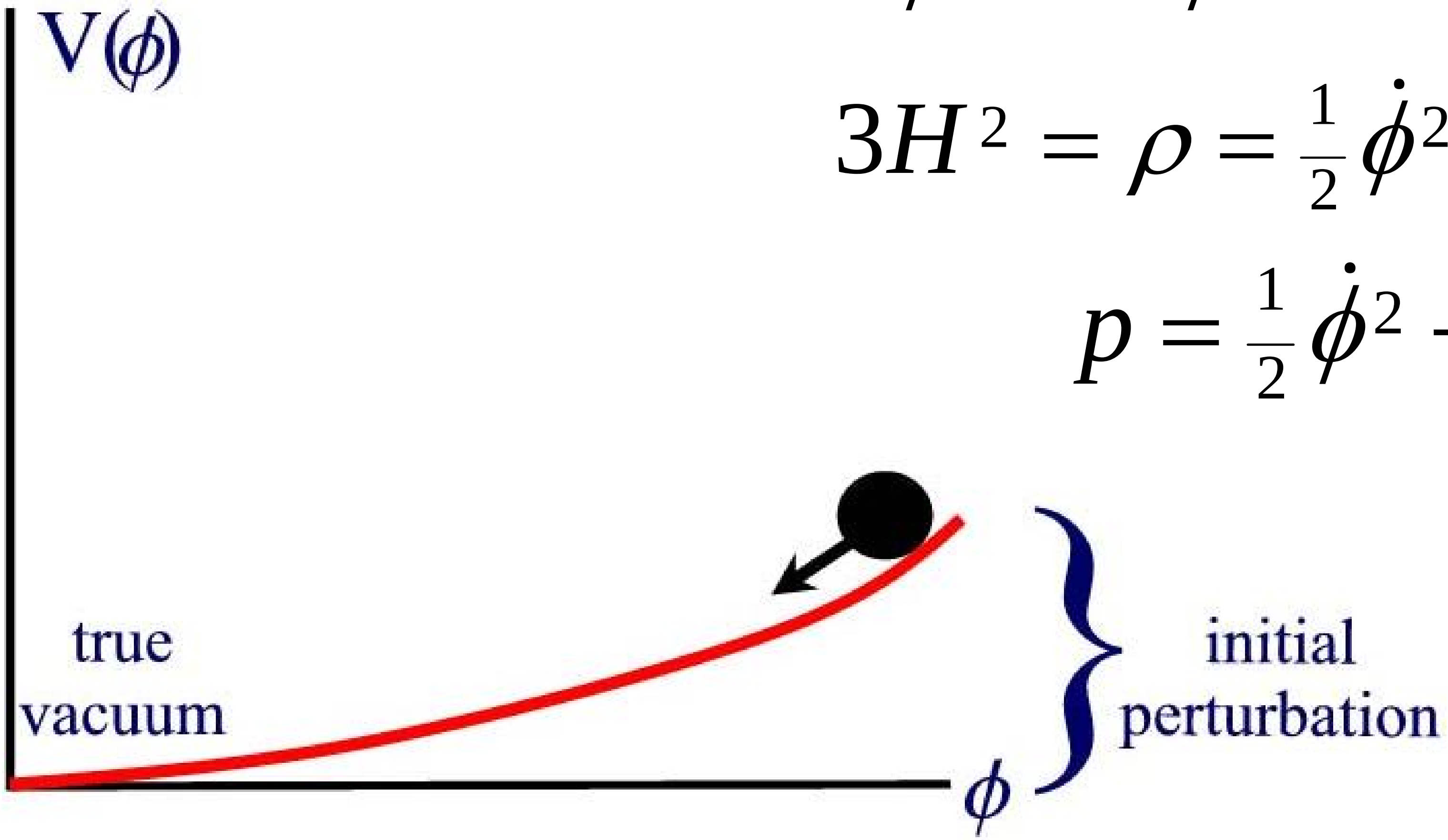
A scalar field displaced from the minima of its potential

Linde's chaotic inflation

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$3H^2 = \rho = \frac{1}{2}\dot{\phi}^2 + V$$

$$p = \frac{1}{2}\dot{\phi}^2 - V$$



Generic Inflation model

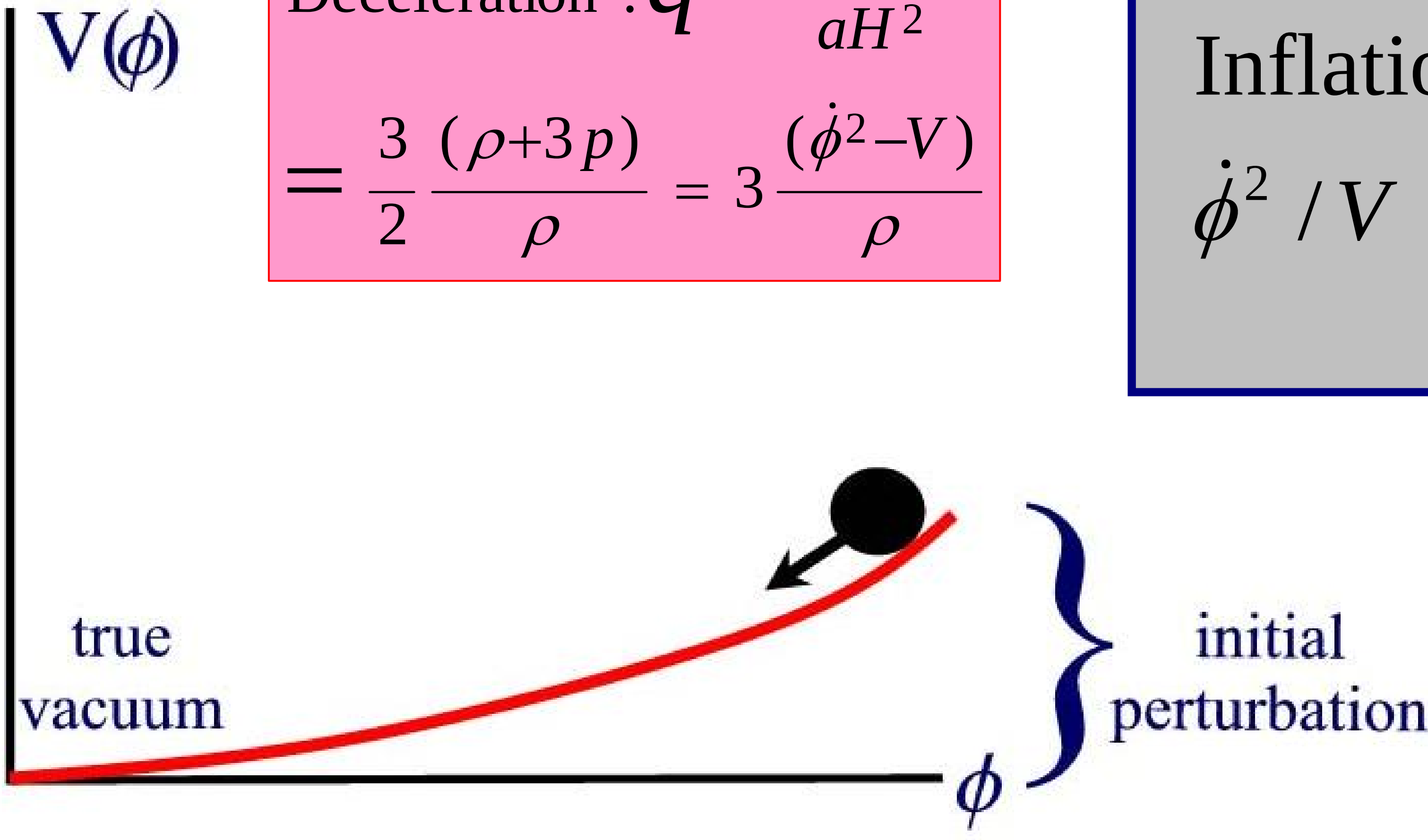
A scalar field displaced from the minima of its potential

$$\text{Deceleration}: q = \frac{-\ddot{a}}{aH^2}$$

$$= \frac{3}{2}\frac{(\rho + 3p)}{\rho} = 3\frac{(\dot{\phi}^2 - V)}{\rho}$$

Inflation

$$\dot{\phi}^2 / V < 1$$

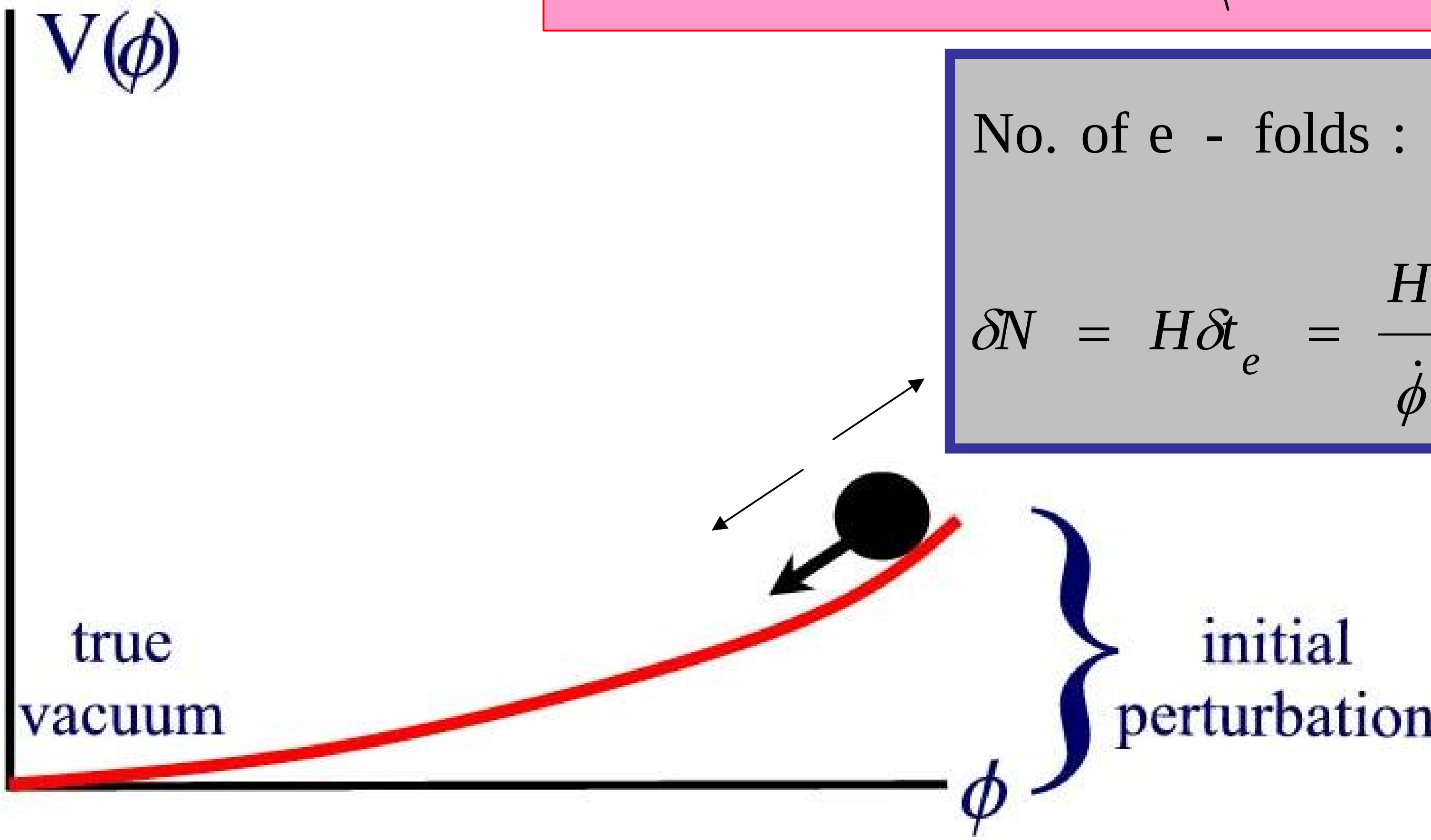


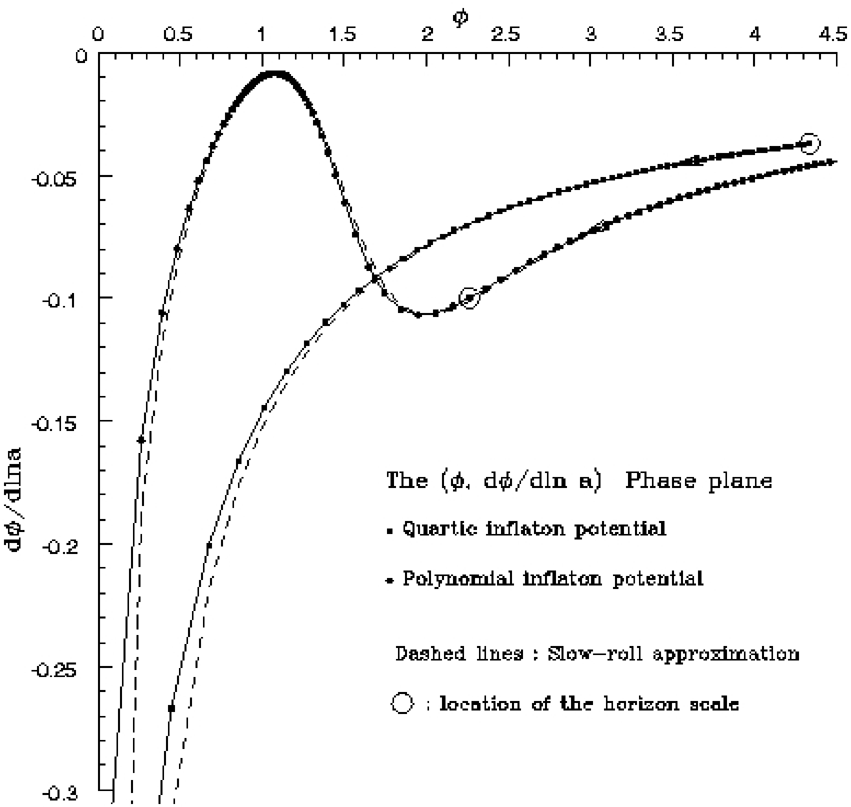
Generation of fluctuations

$$\text{"DeSitter" q. fluc.:}\ \delta\phi \equiv \left\langle (\delta\phi)^2 \right\rangle^{\frac{1}{2}} \approx H$$

$$\text{No. of e - folds : N} = \int \text{H dt}$$

$$\delta N = H\delta t_e = \frac{H}{\dot{\phi}}\ \delta\phi$$





(Souradeep, Thesis 1995)

Slow – roll parameters

$$\varepsilon \equiv \frac{\frac{1}{2}\dot{\phi}^2}{H^2} = \left(\frac{4\pi}{m_P^2}\frac{d\ln H}{d\phi}\right)^2$$

$$\delta \equiv \frac{\ddot{\phi}}{\dot{\phi}H} = \frac{d\ln\left(\frac{1}{2}\dot{\phi}^2\right)}{d\ln a}$$

$$\varepsilon + \delta = \frac{4\pi}{m_P^2}\frac{d^2\ln H}{d\phi^2}$$

Scalar & Tensor perturbations

$$u_k \equiv a\delta\phi_k, \qquad v_k \equiv a h_k$$

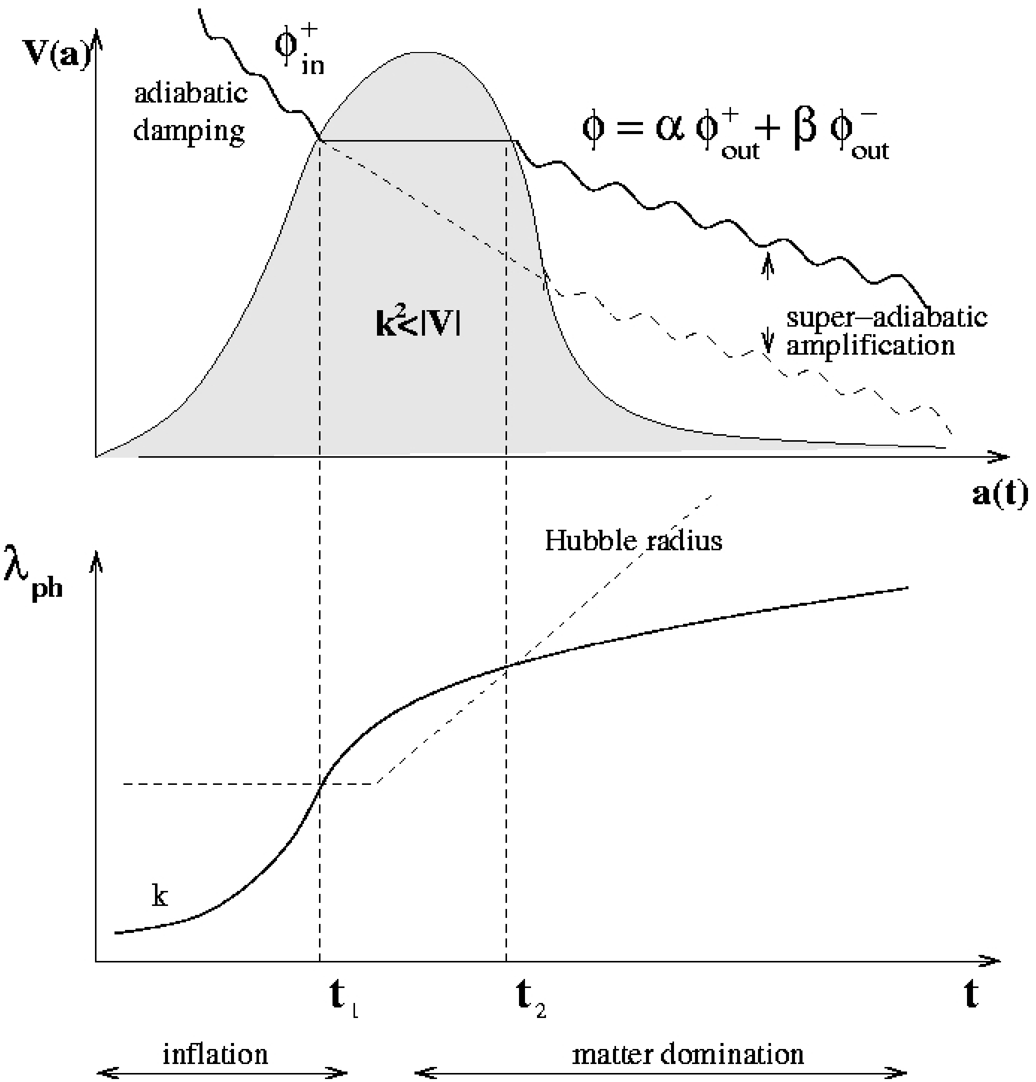
$$u_k'' + \left[k^2 - V_S(\eta)\right] u_k = 0$$

$$v_k'' + \left[k^2 - V_T(\eta)\right] v_k = 0$$

$$V_S = \frac{a''}{a} - \frac{m_{eff}^2}{H^2}, \quad V_T = \frac{a''}{a}$$

$$\frac{m_{eff}^2}{H^2} = (\varepsilon + \delta)(\delta + 3) + \frac{\dot{\varepsilon} - \dot{\delta}}{H}$$

$$\approx \frac{4\pi}{m_P^2} \frac{d^2 \ln H}{d\phi^2}$$



(Fig:Souradeep, Thesis 1995)

Early Universe in CMB

The Background universe

- Homogeneous & isotropic space: ***Cosmological principle*** ?
- Flat (Euclidean) Geometry ... but global topology?

The nature of initial/primordial perturbations

- **Power spectrum :** *'Nearly' Scale invariant /scale free form*

  ... but are there features ?

- **Spin characteristics: (*Scalar) Density perturbations***

  ... cosmic (Tensor) Gravity waves ?

- **Type of scalar perturbation:** *Adiabatic -- no entropy fluctuations*

- **Underlying statistics:** *Gaussian*

Detecting the relic GW background :

Energy scale & mechanism of inflation

Early Universe in CMB

- **Tensor to scalar ratio is crucial discriminant of EU scenarios**

Scalar --- Density perturbations

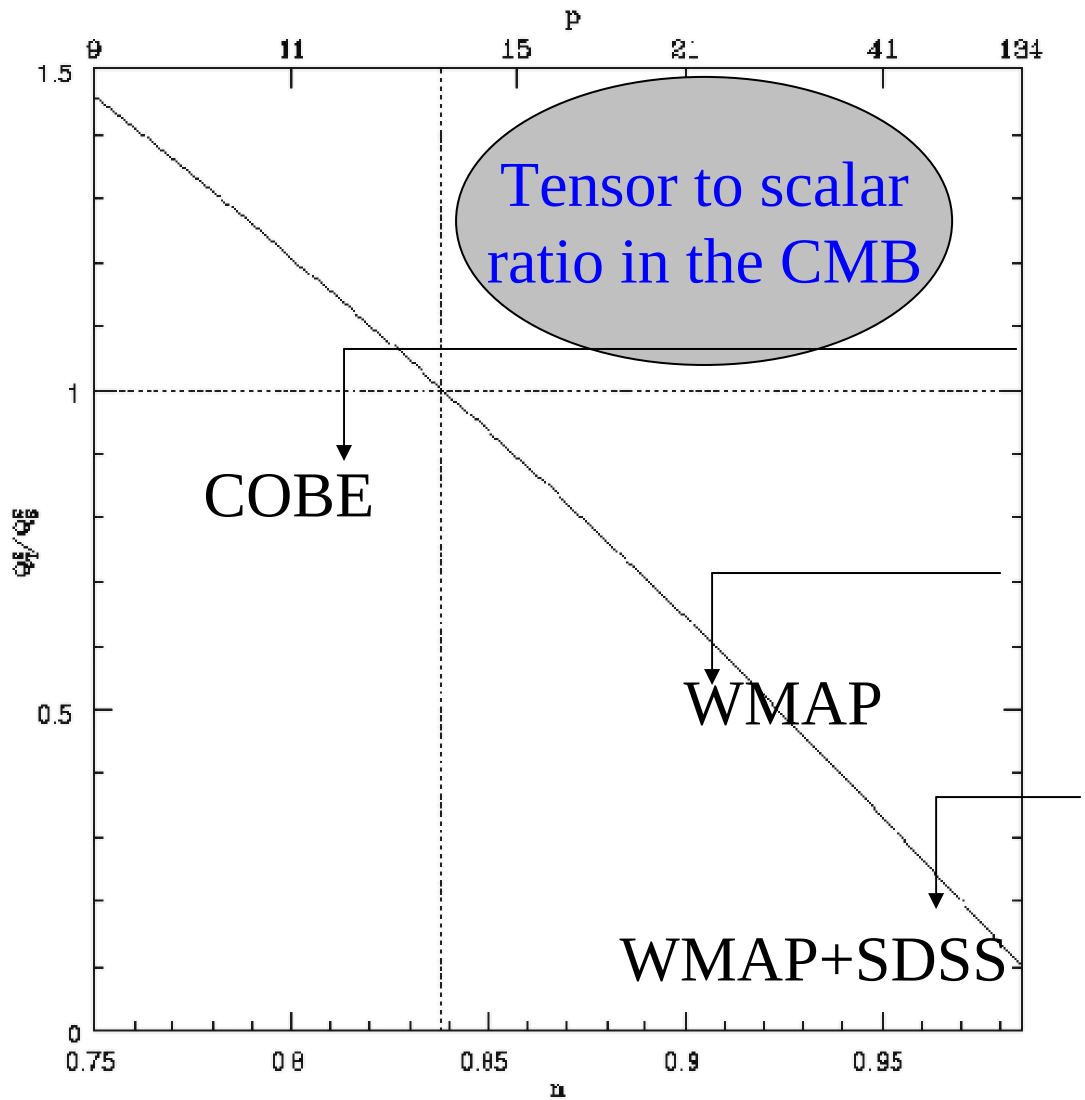
→ **Large scale structure in galaxy clustering**

Tensor --- Gravitational waves

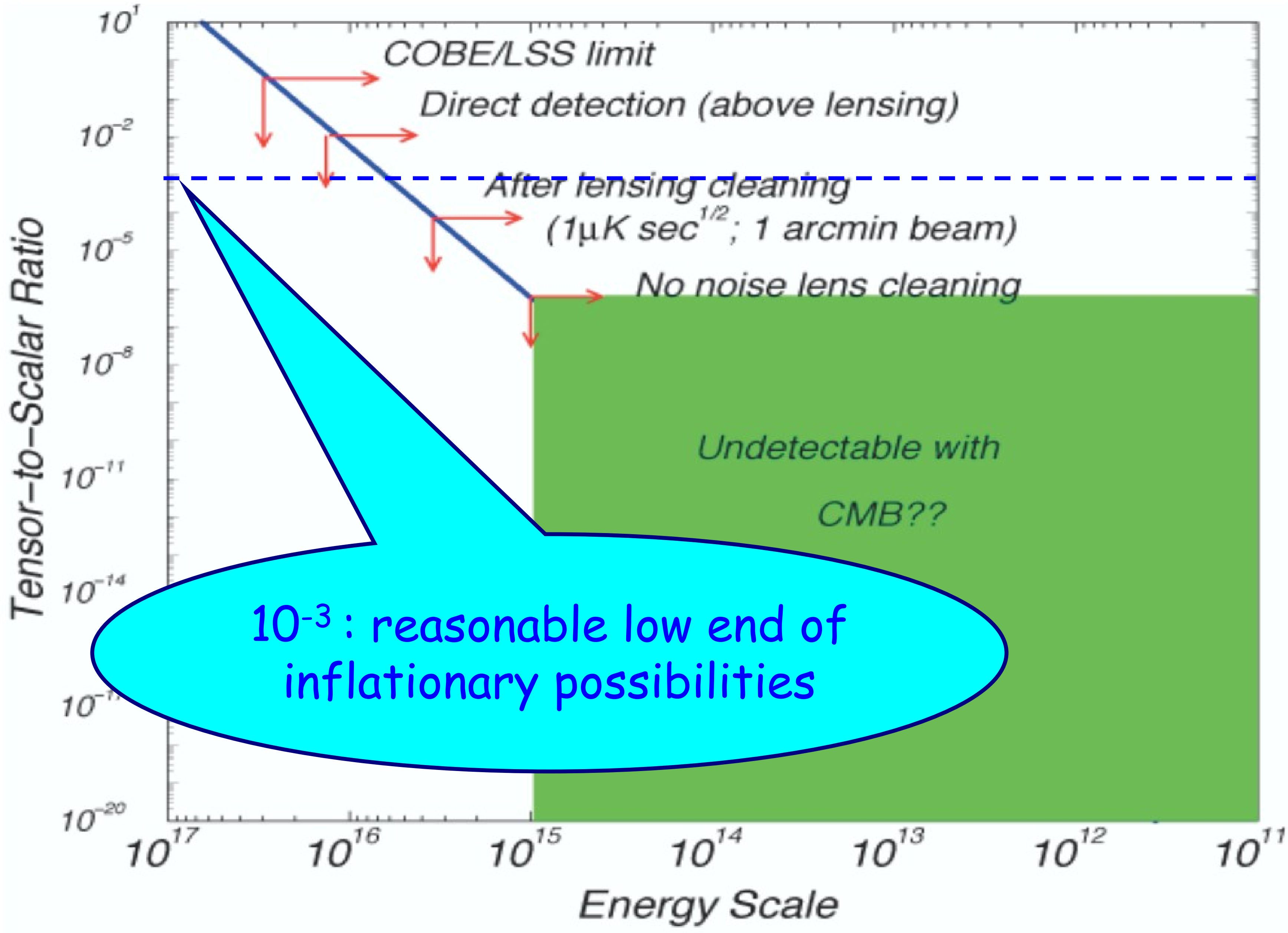
→ **Relic stochastic GW background**

Vector --- rotational modes

→ **Perhaps unimportant, but primordial magnetic field**

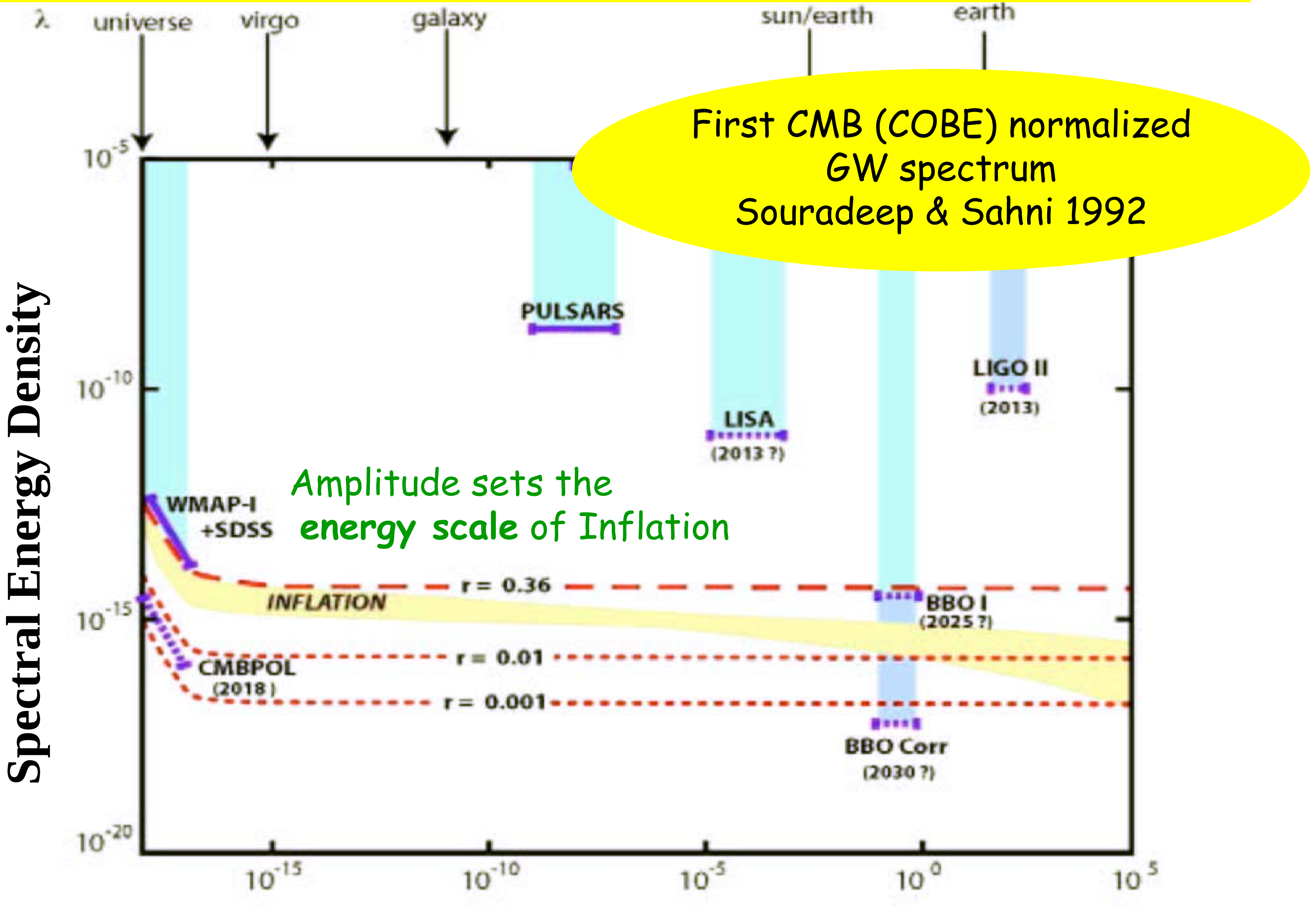


(Souradeep & Sahni, 1992, Souradeep, Ph.D.thesis, 1995)



Courtesy: A. Coorey (EPIC)

Cosmic Gravity wave background



CMB Task force report 2005

Measuring the primordial power spectrum:

Features as signatures of new physics

Primordial Power spectrum ?

CMB anisotropy has two independent aspects:

$$C_l = \int \frac{dk}{k} P(k) G_\ell(k)$$

$P(k)$

Primordial power spectrum from Early universe

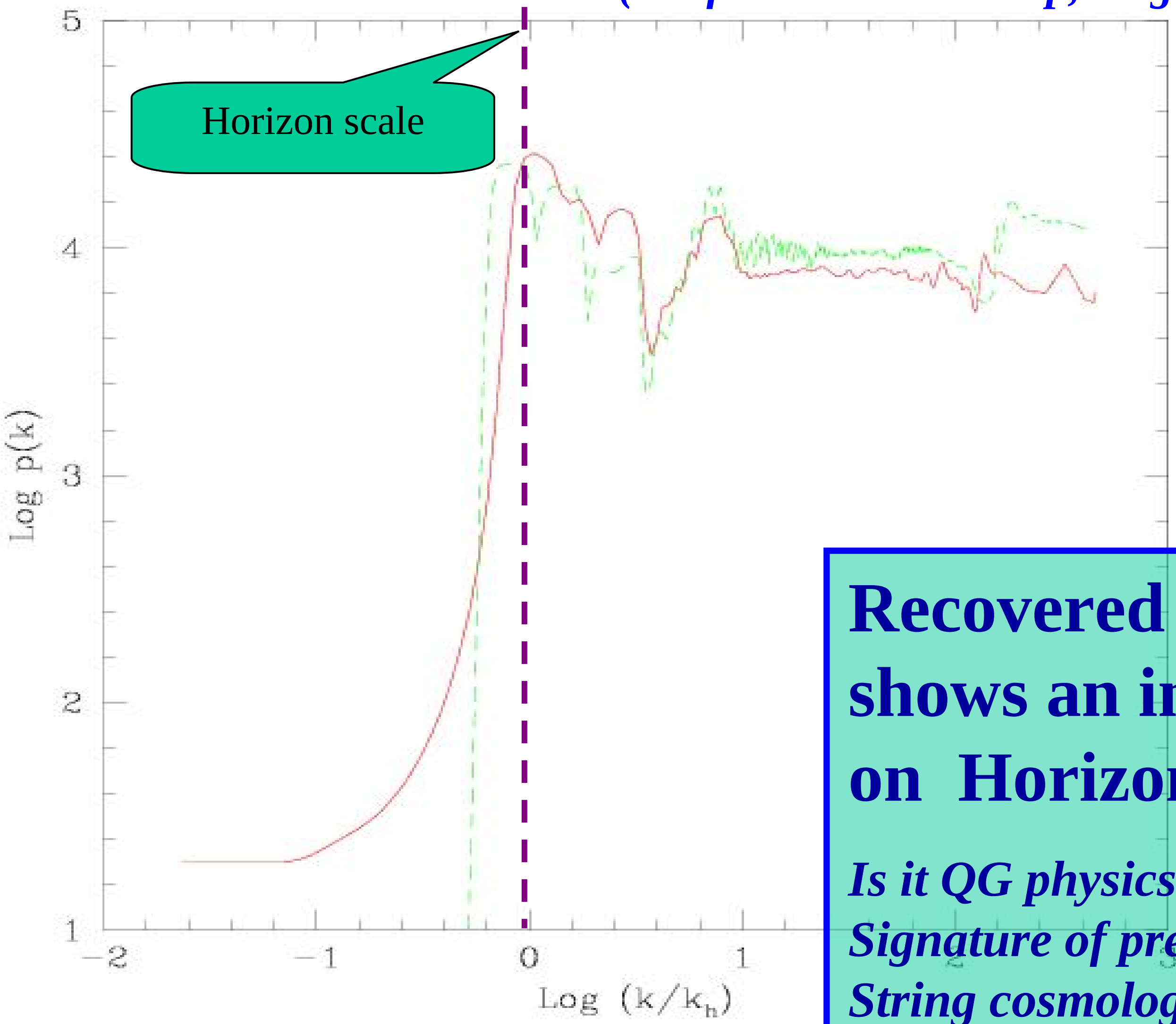
$G_l(k)$

Post recombination Radiation transport in a given cosmology

Primordial power spectrum from WMAP

(Shafieloo & Souradeep, 2004 PRD)

(Shafieloo & Souradeep, ongoing for WMAP-3)



$$C_l = \int \frac{dk}{k} P(k) G_l(k)$$

Improved Richardson-Lucy deconvolution method (Error sensitive)

Recovered spectrum shows an infra-red cut-off on Horizon scale !!!

Is it QG physics ? cosmic topology ?
Signature of pre-inflationary phase ?
String cosmology ? ….

Primordial Power spectrum ?

The Grand program :

$$C_\ell^{TT} = \int \frac{dk}{k} P(k) \quad G_\ell^{TT}(k)$$

$$C_\ell^{EE} = \int \frac{dk}{k} P(k) \quad G_\ell^{EE}(k)$$

$$C_\ell^{BB} = \int \frac{dk}{k} P(k) \quad G_\ell^{BB}(k)$$

$$C_\ell^{TE} = \int \frac{dk}{k} P(k) \quad G_\ell^{TE}(k)$$

$$P_S(k), P_T(k), P_{iso}(k)$$

Primordial power spectra from Early universe

$$G_\ell^{TT}(k), G_\ell^{EE}(k), G_\ell^{BB}(k), G_\ell^{TE}(k)$$

Post recombination Radiative transport kernels in a given cosmology

Beyond C_l : Measuring correlation patterns in CMB

Bipolar Sph. Harmonic rep.

(BiPS: Bipolar power spectra)

‘Anomalies’ in the WMAP CMB maps

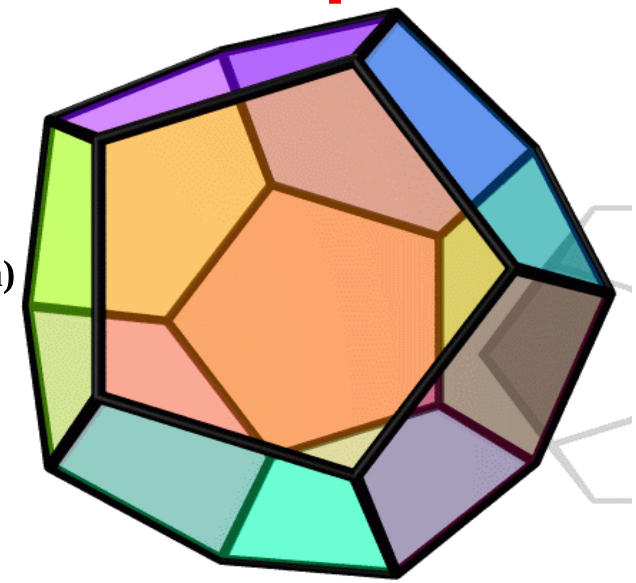
North-South asymmetry

Eriksen, et al. 2004,2006; Hansen et al. 2004 (in local power)
Larson & Wandelt 2004 … , Park 2004 (genus stat.)

.
.

Cosmic topology

(Poincare Dodecahedron)

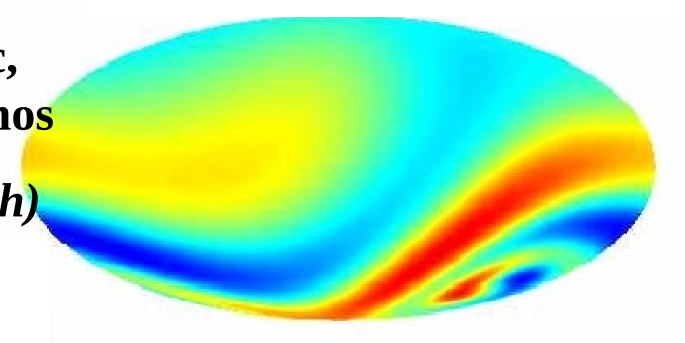


Special directions (“Axis of Evil”)

Tegmark et al. 2004 (*l*=2,3 aligned), 2006
Copi et al. 2004 (multipole vectors), … ,2006
Land & Magueijo 2004 (cubic anomalies), …
Prunet et al., 2004 (mode coupling)
Bernui et al. 2005 (separation histogram)
Wiaux et al. 2006

Anisotropic,
rotating cosmos

(Bianchi VIIh)



Underlying patterns

T.Jaffe et al. 2005,2006

.
.

Statistical properties are **not** ***invariant*** *under rotation of the sky*

Breakdown of statistical isotropy

(cosmological principle) ?

SI violation, or ... Correlation patterns

Can we measure

correlation patterns?

the *COSMIC CATCH* is

there is only one CMB sky !

Advanced Statistics of CMB

$\Delta T(n)$: smooth Gaussian random function on a sphere (sky map), i.e., Completely specified by the **two-point correlation**

$$C(\hat{n}_1, \hat{n}_2) \equiv \langle \Delta T(\hat{n}_1) \Delta T(\hat{n}_2) \rangle$$

Statistical isotropy implies C_l is sufficient

$$C(\hat{n}_1, \hat{n}_2) = C(\hat{n}_1 \bullet \hat{n}_2) = \sum_l \frac{2l+1}{4\pi} C_l \quad P_\ell(\hat{n}_1 \bullet \hat{n}_2)$$

Most general,
Bipolar
Spherical
Harmonic
Expansion

$$C(n_1, n_2) \not\equiv C(n_1 \bullet n_2)$$

$$= \sum_{l_1 l_2 LM} A^{LM}_{l_1 l_2} \{ Y_{l_1}(n_1) \otimes Y_{l_2}(n_2) \}_{LM}$$

Recall: Coupling of angular momentum states

$\langle l_1 m_1 l_2 m_2 \mid \ell M \rangle \qquad |l_1 - \ell| \le l_2 \le l_1 + \ell, \quad m_1 + m_2 + M = 0$

BiPoSH coefficients :

$$A^{\ell M}_{l_1 l_2} = \sum_{m_1} \left\langle a_{l_1 m_1} a^{*}_{l_2\; M+m_1} \right\rangle \; C^{\ell M}_{l_1 m_1 l_2\; M+m_1}$$

- Complete,Independent linear combinations of off-diagonal correlations.

BiPS:

rotationally invariant

$$\kappa^{\ell} \equiv \sum_{M, l_1, l_2} | A^{\ell M}_{l_1 l_2} |^2 \ge 0$$

Statistical Isotropy $\Rightarrow \kappa^{\ell} = \kappa^{0} \delta_{\ell 0}$

Spherical harmonics	Bipolar spherical
a_{lm}	$A^{\ell M}_{ll'}$
Spherical Harmonic coefficents	BiPoSH coefficents
C_l	κ^{ℓ}
Angular power spectrum	**BiPS**

Bipolar Power spectrum (BiPS) :

A Generic Measure of Statistical Anisotropy

Spherical harmonics	**Bipolar spherical harmonics**
a_{lm}	$A^{\ell M}_{ll'}$
Spherical Harmonic Transforms	BipoSH Transforms
C_l	κ^{ℓ}
Angular power spectrum	**BiPS** **(Bipolar Power Spectrum)**

Bipolar Power spectrum (BiPS) :

A Generic Measure of Statistical Anisotropy

Spherical harmonics	**Bipolar spherical harmonics**
a_{lm}	$A^{\ell M}_{ll'}$
Spherical Harmonic Transforms	BipoSH Transforms
C_l	κ^{ℓ}
Angular power spectrum	**BiPS**

Statistical Isotropy
i.e., NO Patterns

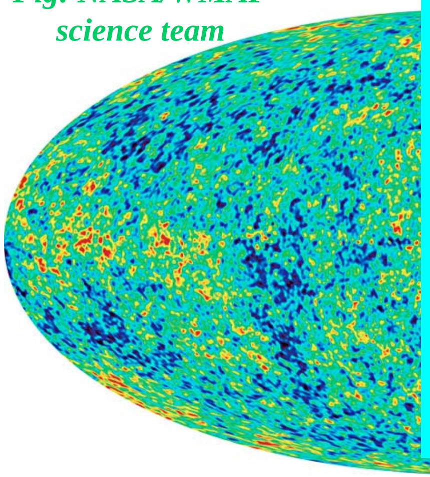
$$\Rightarrow \kappa^{\ell} = \kappa^{0} \delta_{\ell 0}$$

Statistical Isotropy of CMB Anisotropy

Amir Hajian thesis 2006

WMAP CMB anisotropy map

Fig: NASA/WMAP science team



WMAP Angular power spectrum

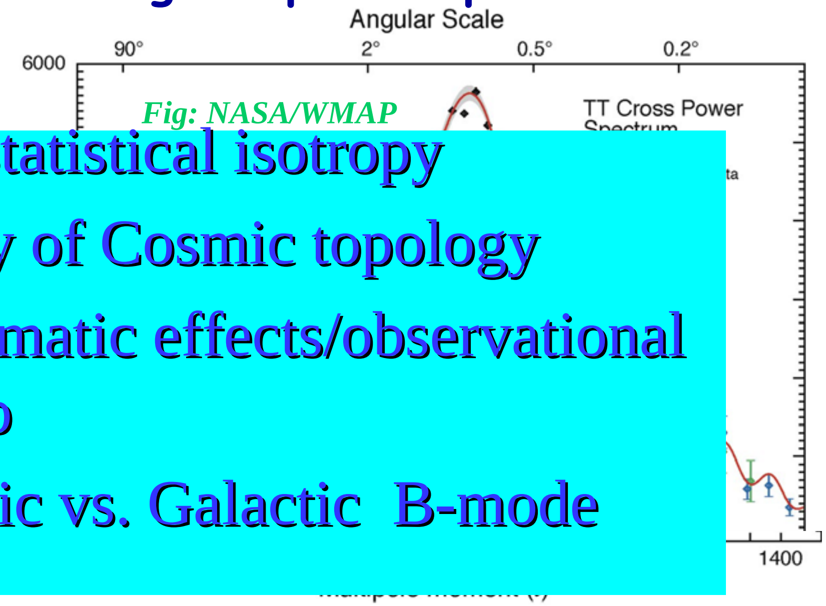
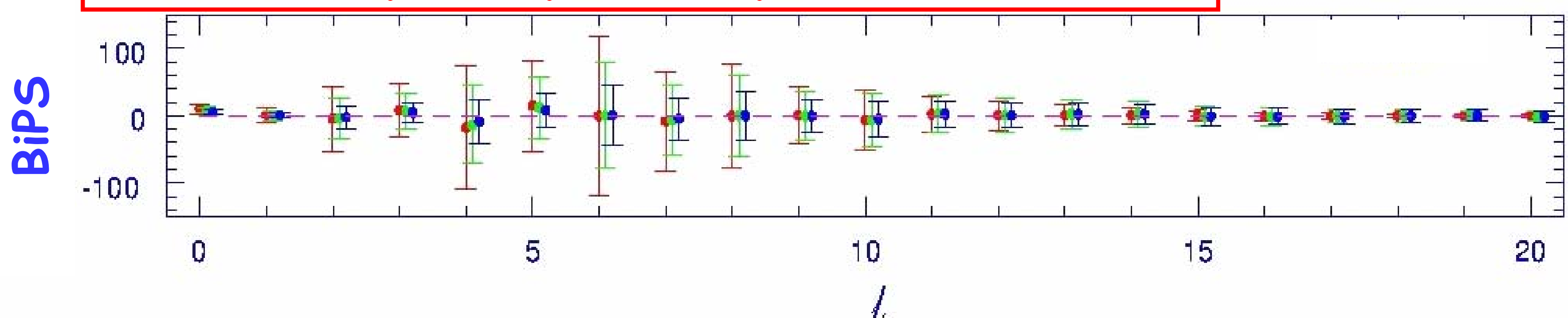


Fig: NASA/WMAP

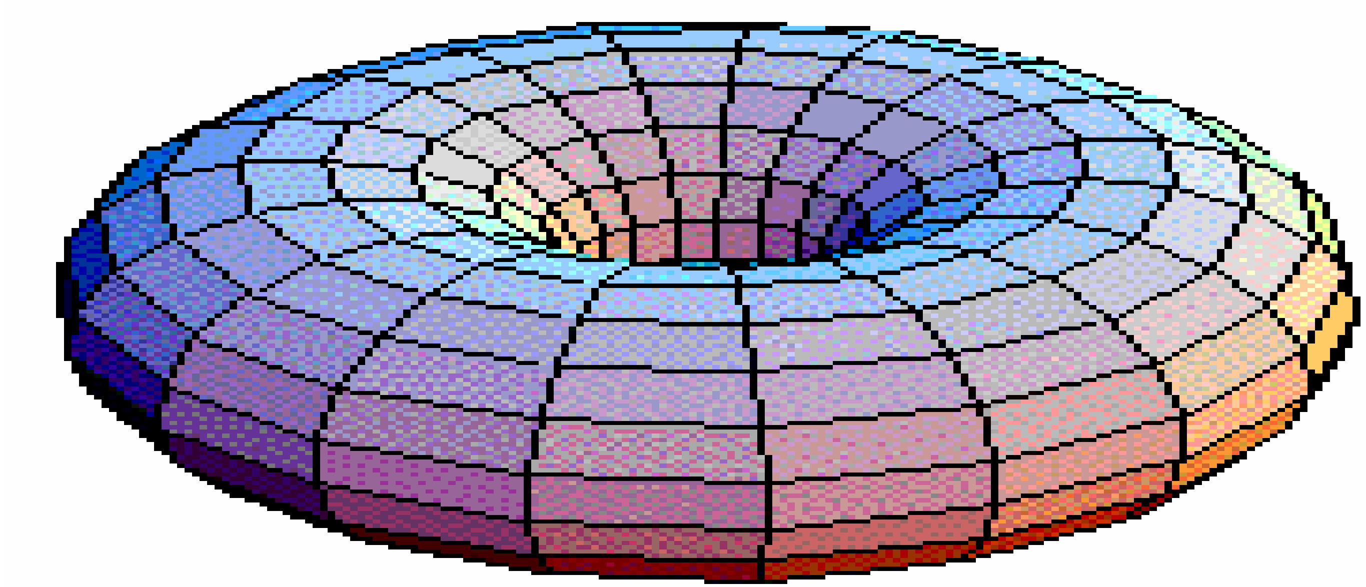
- BiPS: Measure of statistical isotropy
- BiPS: Spectroscopy of Cosmic topology
- Diagnostic of systematic effects/observational artifacts in the map
- Differentiate Cosmic vs. Galactic B-mode polarization

WMAP Bipolar power spectrum (BiPS)

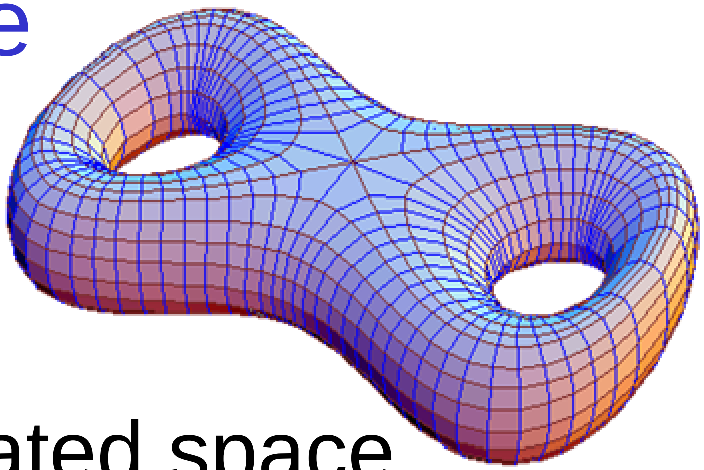


Hajian & Souradeep ApJ. Lett 2003, ApJ. Lett. 2005, New Astron Rev. 2006, PRD 2006,

Is the Universe compact ?

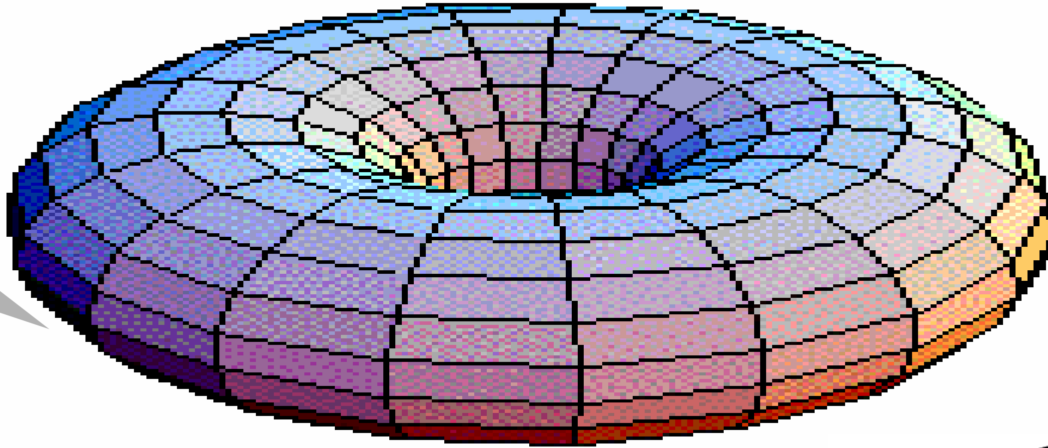


Simple Toroidal space

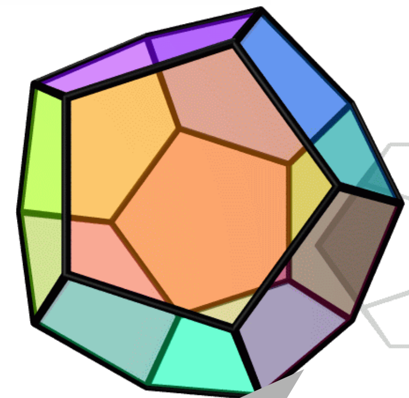


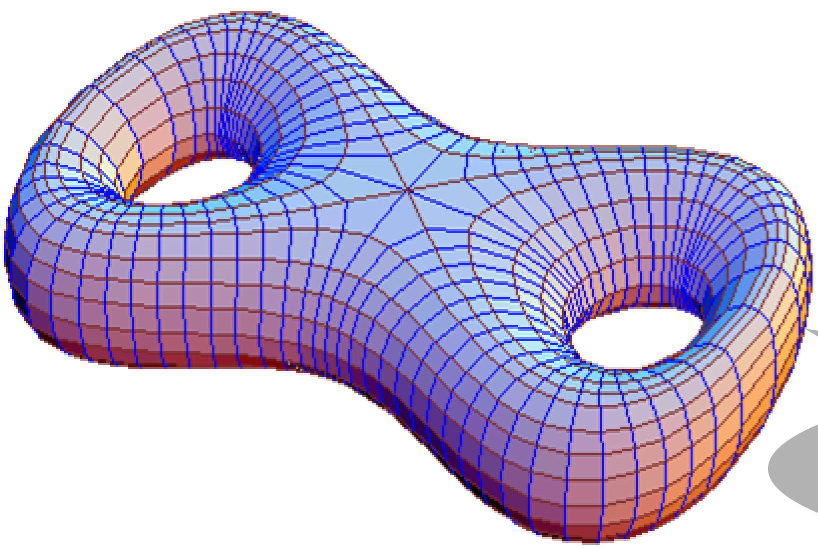
… more complicated space

Simple Torus
(Euclidean)



BiPS ➔ Spectroscopy of Cosmic topology !?!



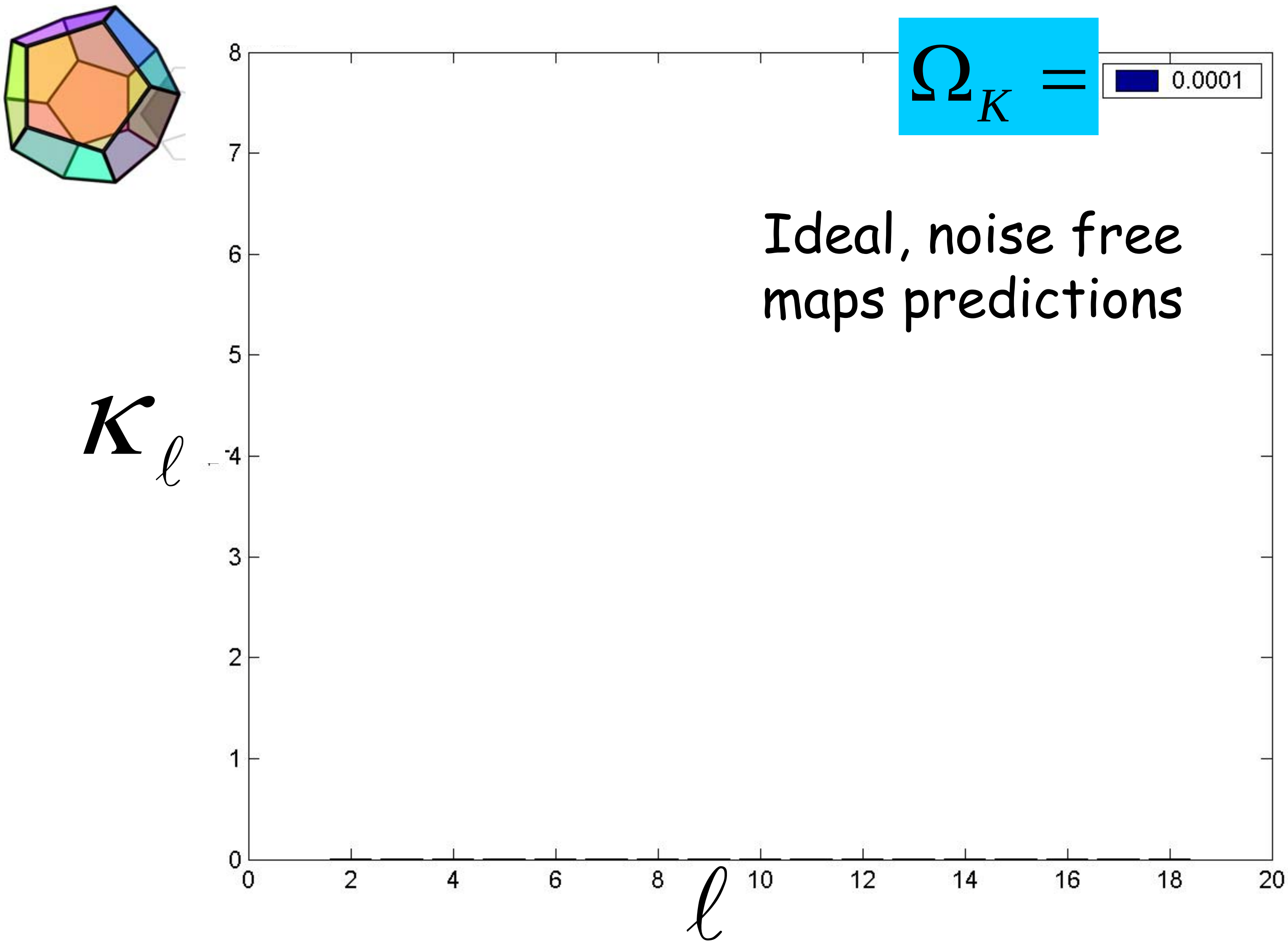


MC spherical space

Compact hyperbolic space

BiPS signature of a "soccer ball" universe

(Hajian, Pogosyan, TS, Contaldi, Bond : in progress.)



BiPS signature of a "soccer ball" universe

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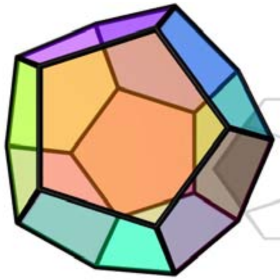
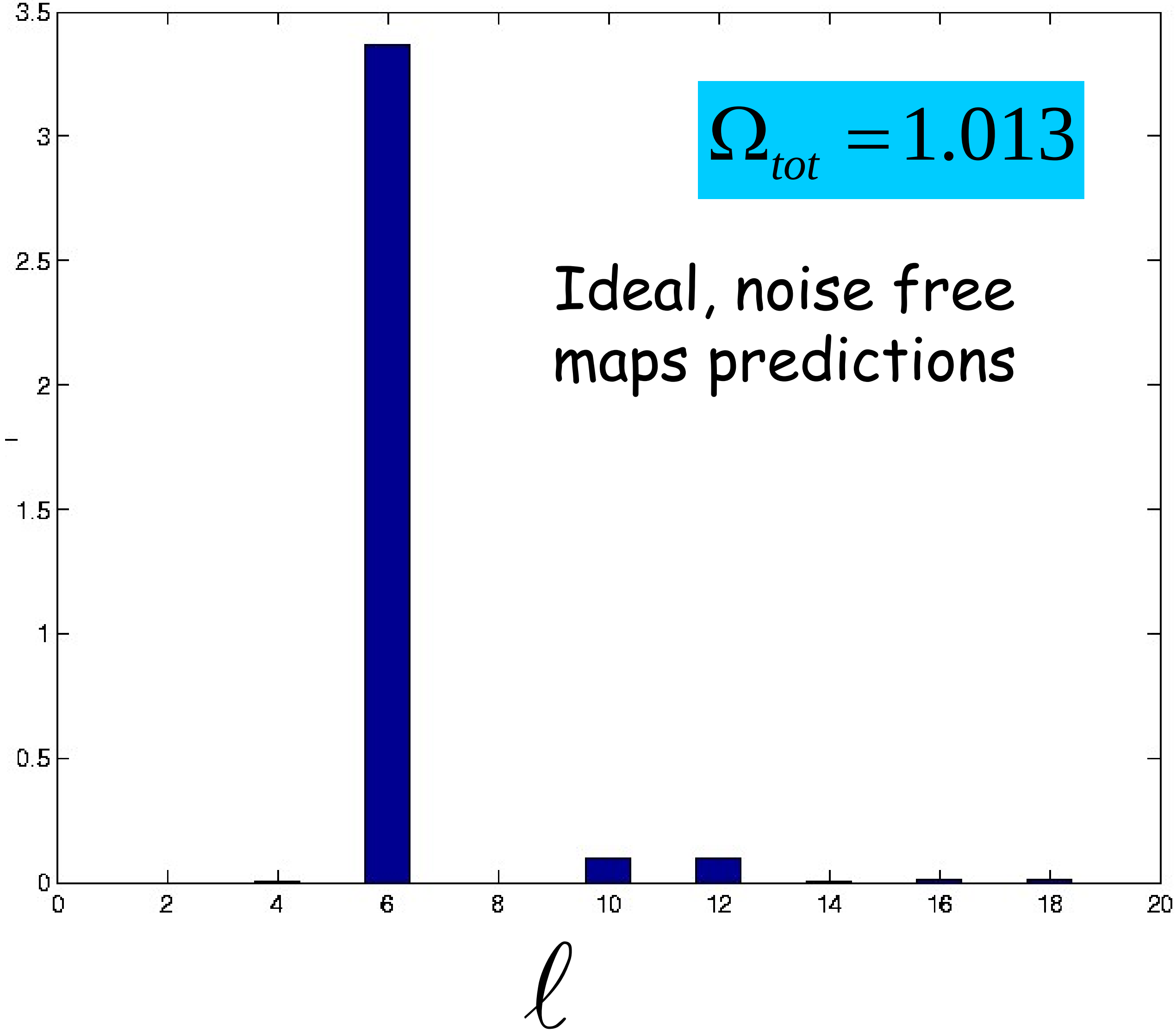
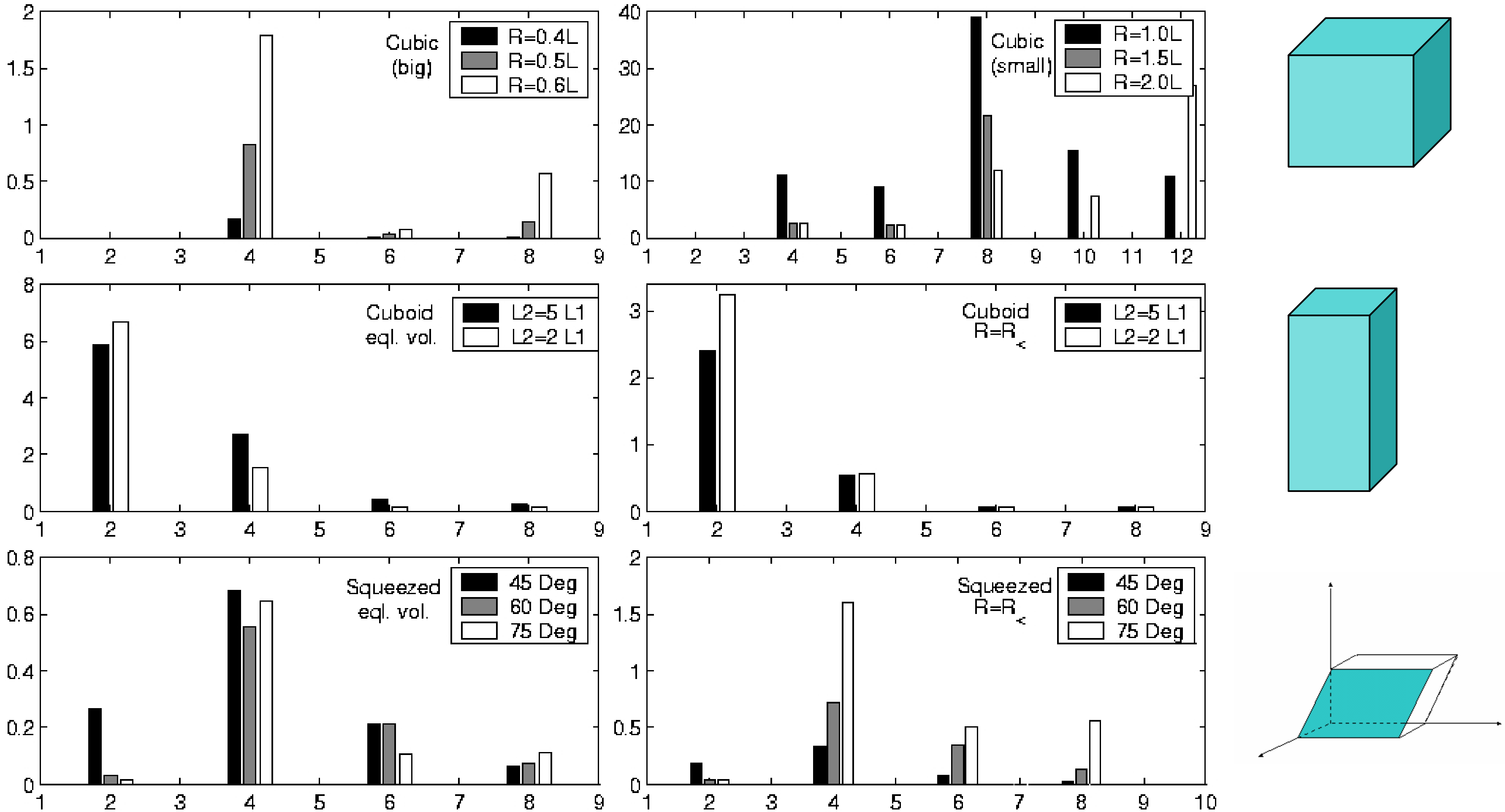




BiPS signature of Flat Torus spaces

κ_ℓ



ℓ

Hajian & Souradeep
(astro-ph/0301590)

How Big is the Observable Universe ?

Relative to the local curvature & topological scales

Model free Foreground removal

- **For each frequency channel, *i* :** $a_{lm} = B_l a^C_{lm} + B_l a^F_{lm} + a^n_{lm}$
- **CMB anisotropy is achromatic :** $a^{C(i)}_{lm} = \text{constant}$
- Linear combinations of maps of different frequency channels, i.

$$a_{lm} = \sum_{i=1}^{i=n} w^i_l \frac{a^i_{lm}}{B^i_l}$$

Such that CMB signal is untouched in the final map) $\sum_{i=1}^{i=n} w^i_l = 1$

- Determine weights that **minimize** total power

$$C_l = \langle a_{lm} a^*_{lm} \rangle = [W][C_l][W]^T$$

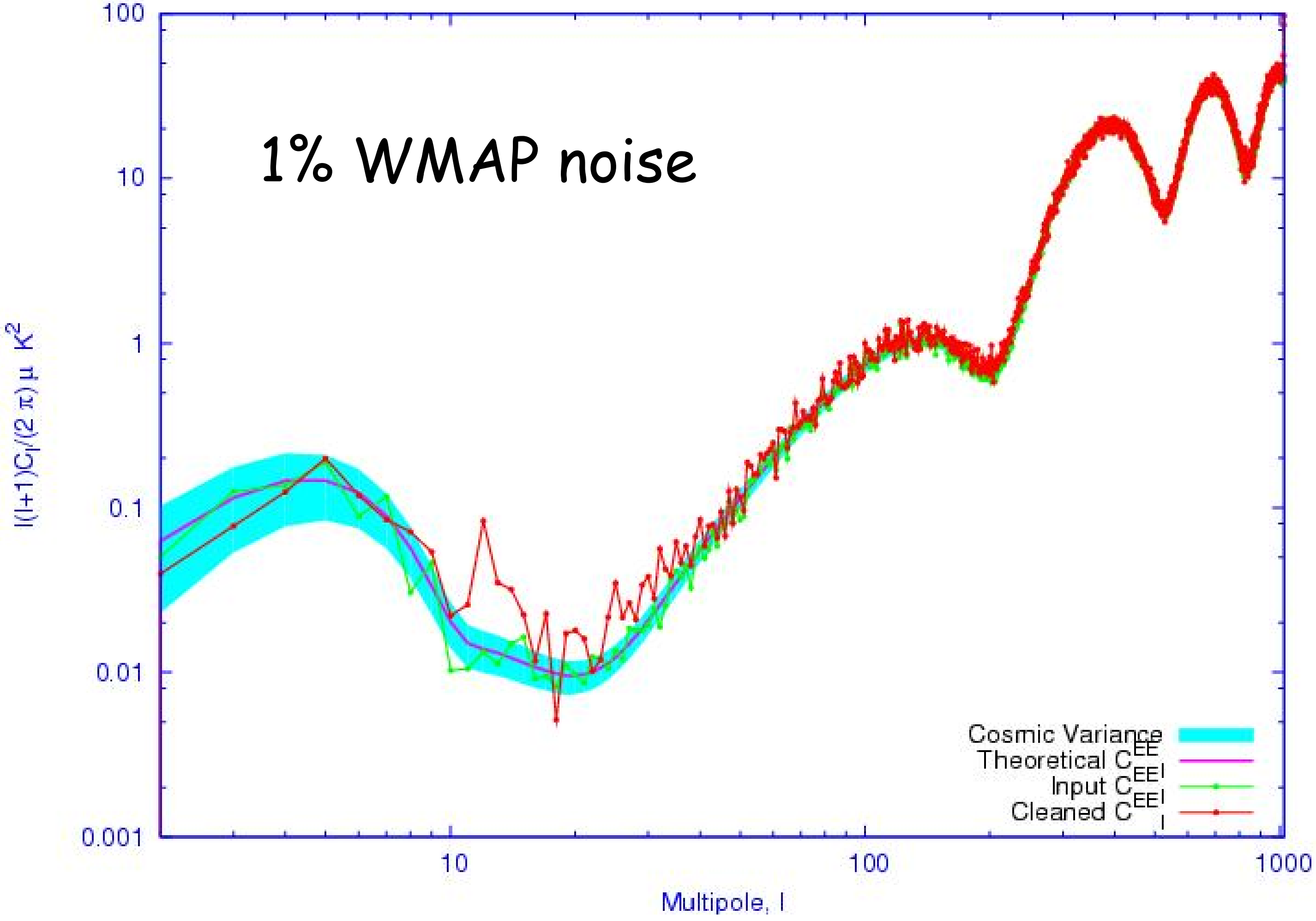
Foreground cleaned map:

$$[W_l]^T = \frac{[C_l][e]^T}{[e][C_l][e]^T}$$

$$[e] = (1 \quad 1 \quad .. \quad .. \quad 1)$$

(Tegmark & Efstathiou 96, Tegmark 2003)

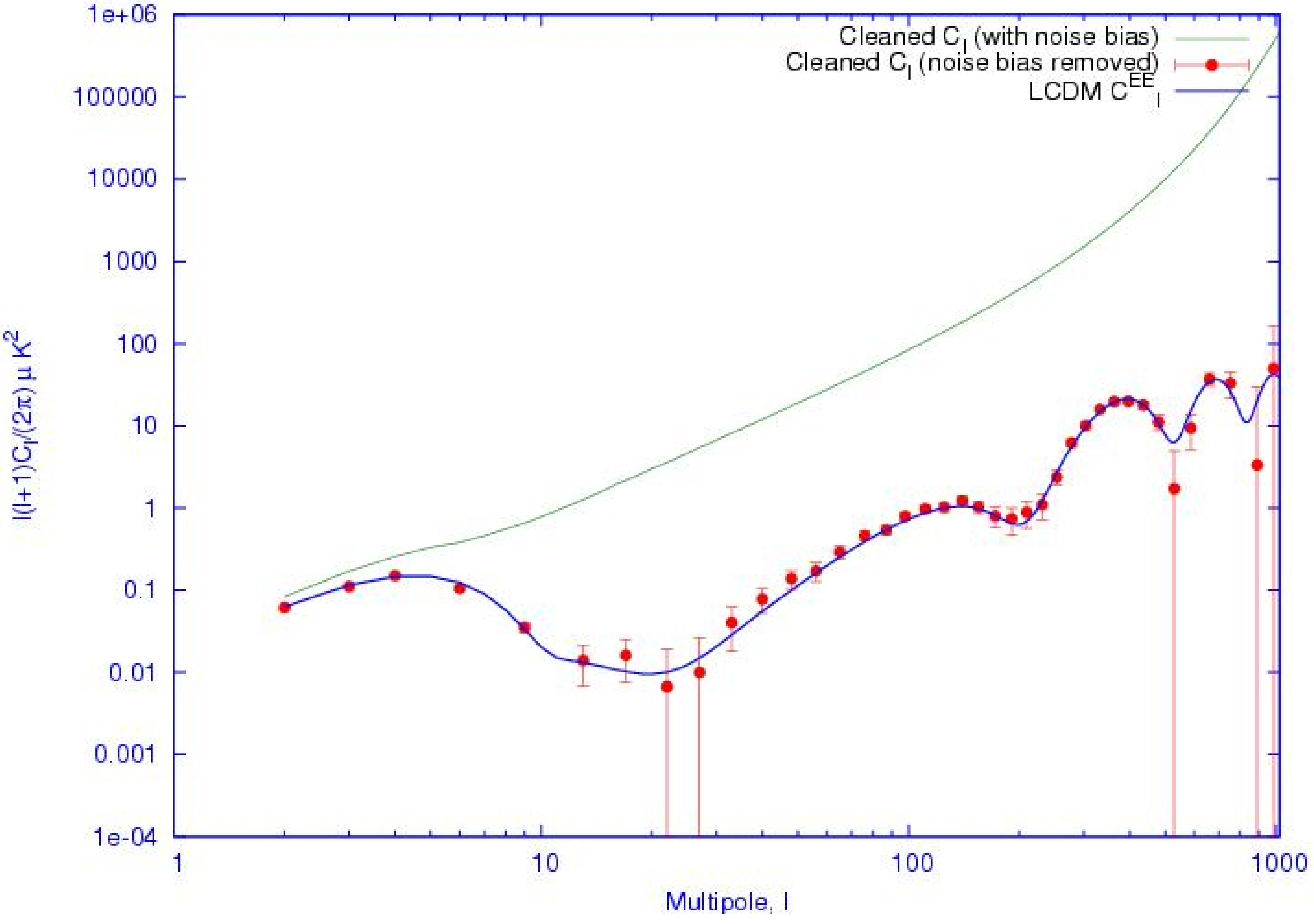
WMAP CMB Polarization challenge



(ongoing: Rajib Saha, Simon Prunet, P. Jain,TS)

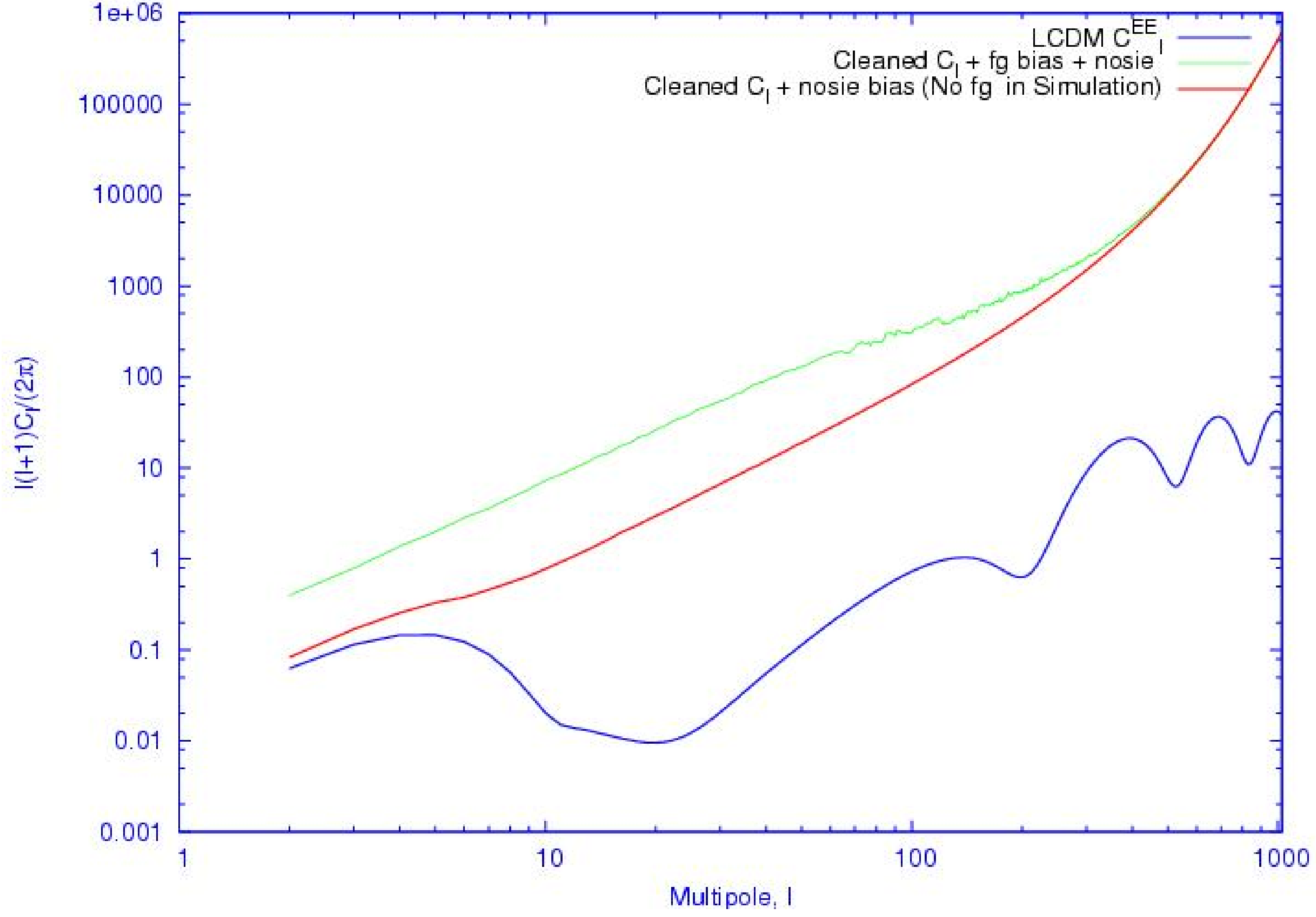
WMAP CMB Polarization challenge

(ongoing: Rajib Saha, Simon Prunet, P. Jain,TS)



WMAP CMB Polarization challenge

(ongoing: Rajib Saha, Simon Prunet, P. Jain,TS)



BiPS Spectroscopy of Cosmic topology

(Himan Mukhopadhyay, TS, in progress.)

Symmetry requirements for even BiPS: Group-theoretic

1. 2n-fold symmetry

There should exist at least one plane characterized by $\hat{n}$ such that

$$R_{\hat{n}}(\pi)\gamma = \gamma R_{\hat{n}}(\pi) \quad \forall \gamma \in \Gamma$$

2. Symmetry under reflection

Let $\vec{x}$ and $\vec{x}'$ be images of each other for the same plane. Then

$$\{d(\vec{x}, \gamma\vec{x})\} \equiv \{d(\vec{x}', \gamma\vec{x}')\} \quad \forall \gamma \in \Gamma$$

Which spaces satisfy 1 &2 ?

✓ *Flat compact spaces*

✓ *Single-action spherical compact spaces*

✗ *No hyperbolic compact spaces*

Discussion with Jeff Weeks, in progress