Minimal Supersymmetric Standard Model

Vector super-multiplet

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A mini review of gauge theory

- Covariant derivative

$$\mathcal{D}_m\phi = (\partial_m - igA^a_m t^a_R)\phi$$

- Group algebra

$$[t^a_R, t^b_R] = if^{abc}t^c_R$$

- Adjoint represebtaion

$$(t^a_G)_{bc} = if^{bac} \qquad \mathcal{D}_m\Phi^a = \partial_m\Phi^a + gf^{abc}A^b_m\Phi^c$$

- Field strength

$$F^a_{mn} = \partial_m A^a_n - \partial_n A^a_m + gf^{abc}A^b_m A^c_n$$

Vector supermultiplet

Field content

$$(A_m^a, \lambda^a, D^a)$$

Massless gauge boson
(2 bosonic degrees of freedom)

Left handed massless fermion
and antifermion

Auxiliary field

Supersymmetry transformation

$$\delta_\xi A^{am} = [\xi^\dagger \overline{\sigma}^m \lambda^a + \lambda^{\dagger a} \overline{\sigma}^m \xi]$$

$$\delta_\xi \lambda^a = [i\sigma^{mn} F^a_{mn} + D^a]\,\xi$$

$$\delta_\xi D = -i[\xi^\dagger \overline{\sigma}^m \mathcal{D}_m \lambda^a - \mathcal{D}_m \lambda^{\dagger a} \overline{\sigma}^m \xi]$$

where

$$\sigma^{mn} = (\sigma^m \overline{\sigma}^n - \sigma^n \overline{\sigma}^m)/4$$

$$[\delta_\xi, \delta_\eta] = 2i(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi)\partial_m + \delta_\alpha$$

Gauge transformation

Gauge vector super-multiplet Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F^a_{mn})^2 + \lambda^{\dagger a} i\overline{\sigma} \cdot \mathcal{D}\lambda^a + \frac{1}{2}(D^a)^2$$

Gaugino (adjoint rep.) — Covariant derivative

SU(2) × U(1)

What are the interactions of gauginos with gauge bosons?

Gauge interaction of chiral super multiplet

$$\mathcal{L} = \mathcal{D}^m \phi^* \mathcal{D}_m \phi + \psi^\dagger i\overline{\sigma} \cdot \mathcal{D}\psi + F^* F$$
$$-\sqrt{2} g(\phi^* \lambda^{aT} t^a c\psi - \psi^\dagger c\lambda^{a*} t^a \phi) + gD^a \phi^* t^a \phi$$

$$D^a = -g\phi^* t^a \phi$$

Integrating out the auxiliary field

$$V_D = +\frac{1}{2} g^2 \left(\sum_k \phi_k^* t^a \phi_k \right)^2$$

Zero or positive

An illustrative example

- U(1) gauge theory ϕ_+, ϕ_-, X (+1,-1,0)
- Super-potential $W = \lambda(\phi_+\phi_- - v^2)X$

$$V_F = \sum_i |\frac{\partial W}{\partial \phi_i}|^2 = \lambda^2 \left(|\phi_+\phi_- - v|^2 + |\phi_+ X|^2 + |\phi_- X|^2\right)$$

Minimizing the potential

$$\phi_+\phi_- - v^2 = 0 \quad \phi_+ X = 0 \quad \phi_- X = 0$$

solution

$$X = 0 \quad \phi_+ = v/y \quad \phi_- = vy$$

- y can be made real

D-term

$$V_D = \frac{g^2}{2}\sum_a |\sum_b \phi_b^\dagger t^a \phi_b|^2 = \frac{g^2}{2}|\phi_+^\dagger \phi_+ - \phi_-^\dagger \phi_-|^2$$

- Remember that $\phi_+ = v/y \quad \phi_- = vy$
- Minimizing D-term $y = 1$

Mass terms

Covariant kinetic term for chiral field

$$\mathcal{L} = \phi_+^\dagger(-\mathcal{D}^2)\phi_+ + \phi_-^\dagger(-\mathcal{D}^2)\phi_- = \phi_+^\dagger(g^2A^2)\phi_+ + \phi_-^\dagger(g^2A^2)\phi_-$$

- Mass for gauge field

$$m^2 = 4g^2v^2$$

- Scalar dynamical modes

$$\delta\phi_+ = \eta/\sqrt{2} \quad \delta\phi_- = -\eta/\sqrt{2}$$

- Mass for this scalar mode comes from

$$\frac{g^2}{2}|\phi_+^\dagger\phi_+ - \phi_-^\dagger\phi_-|^2 \Longrightarrow m^2 = 4g^2v^2$$

Mass term for fermions

$$\delta\psi_+ = \chi/\sqrt{2} \quad \delta\psi_- = -\chi/\sqrt{2}$$

$$\mathcal{L} = -\sqrt{2}g(\phi_+^\dagger \lambda^T c\psi_+ - \phi_-^\dagger \lambda^T c\psi_-) + h.c.$$

⇓

$$m = 2gv$$

Counting the new degrees of freedom

- Four bosonic degrees of freedom with Mass

$$m = 2gv$$

Three degrees of freedom in vector boson plus the scalar

Four fermionic degrees of freedom with mass

$$m = 2gv$$

A full Dirac field

What about the rest of degrees of freedom?

Consistency

- Taking the transformation we should check that

$$[\delta_\xi, \delta_\eta] = 2i(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi)\partial_m + \delta_\alpha$$

- And all pieces of Lagrangian that we wrote remains invariant.
- Straightforward but tedious

Superspace approach

- Reminder: Chiral superfield

$$\overline{D}_\alpha \Phi = 0$$

$$\Phi(x,\theta) = \phi(x) + \sqrt{2}\theta^T c\psi(x) + \theta^T c\theta\, F(x)$$

- General real valued superfield

$$V(x,\theta,\overline{\theta})$$

Wess-Zumino Gauge

- To remove linear terms in θ_α and $\bar{\theta}_\alpha$

$$\delta V(x,\theta,\bar{\theta}) = \frac{-i}{g}(\Lambda - \Lambda^*)$$

$$\Lambda(x,\theta,\overline{\theta}) = \Lambda(x + i\overline{\theta}\overline{\sigma}\theta, \theta) = \lambda(x) + \cdots + i\overline{\theta}\overline{\sigma}^m\theta\partial_m\lambda + \cdots$$

The same

$$V = \cdots + 2\ \overline{\theta}\overline{\sigma}^m\theta\ A_m + \cdots$$

Gauge transformation

$$\delta A_m = \frac{1}{g}\partial_m(\mathrm{Re}\ \lambda)\ .$$

Field Content in Wess-Zumino gauge

$$V(x,\theta,\overline{\theta}) = 2\overline{\theta}\sigma^m\theta\ A_m + 2\overline{\theta}^2\theta^T c\lambda - 2\theta^2\overline{\theta}^T c\lambda^* + \theta^2\overline{\theta}^2 D$$

$$(A_m, \lambda, D)$$

$$\int d^2\theta d^2\overline{\theta}\ \Phi^\dagger e^{gV}\Phi$$

→ Lagrangian we want

Obviously susy invariant because

Susy transformation

$$\mathcal{Q}_\xi = (-\frac{\partial}{\partial\theta} - i\overline{\theta}\overline{\sigma}^m\partial_m)\xi + \xi^\dagger(\frac{\partial}{\partial\overline{\theta}} + i\overline{\sigma}^m\theta\partial_m)$$

Total derivative

Pure gauge Lagrangian

- Fermion anticommutation

$$\overline{D}_\alpha \, \overline{D}^2 X = 0$$

$$D_\alpha = \frac{\partial}{\partial \theta_\alpha} - i(\bar\theta \sigma^m)_\alpha \partial_m$$

$$\bar{D}_\alpha = -\frac{\partial}{\partial \bar\theta_\alpha} + i(\sigma^m \theta)_\alpha \partial_m$$

Let us define

$$W_\alpha = -\frac{1}{8} \overline{D}^2 (Dc)_\alpha V$$

$$W_\alpha = \lambda_\alpha + [(i\sigma^{mn} F_{mn} + D)\theta]_\alpha + \theta^2 [\partial_m \lambda^* \;\; i\overline{\sigma}^m c]_\alpha$$

$$\int d^2\theta \; \frac{1}{2} W^T c W$$

Non-Abelian Gauge transformation

- Gauge transformation

$$\Phi \to e^{i\Lambda^a t^a}\Phi \qquad \Phi^\dagger \to \Phi^\dagger e^{-i\Lambda^{a*} t^a}$$

$$e^{gV^a t^a} \to e^{i\Lambda^{*a} t^a} e^{gV^a t^a} e^{-i\Lambda^a t^a}$$

Gauge field strength

$$W_\alpha^a t^a = -\frac{1}{8g}\bar{D}^2 e^{-gV^a t^a}(Dc)_\alpha e^{gV^a t^a}$$

Gauge kinetic term

$$\int d^2\theta \ \mathrm{tr}[W^2]$$

Supersymmetrizing Standard Model (SM)

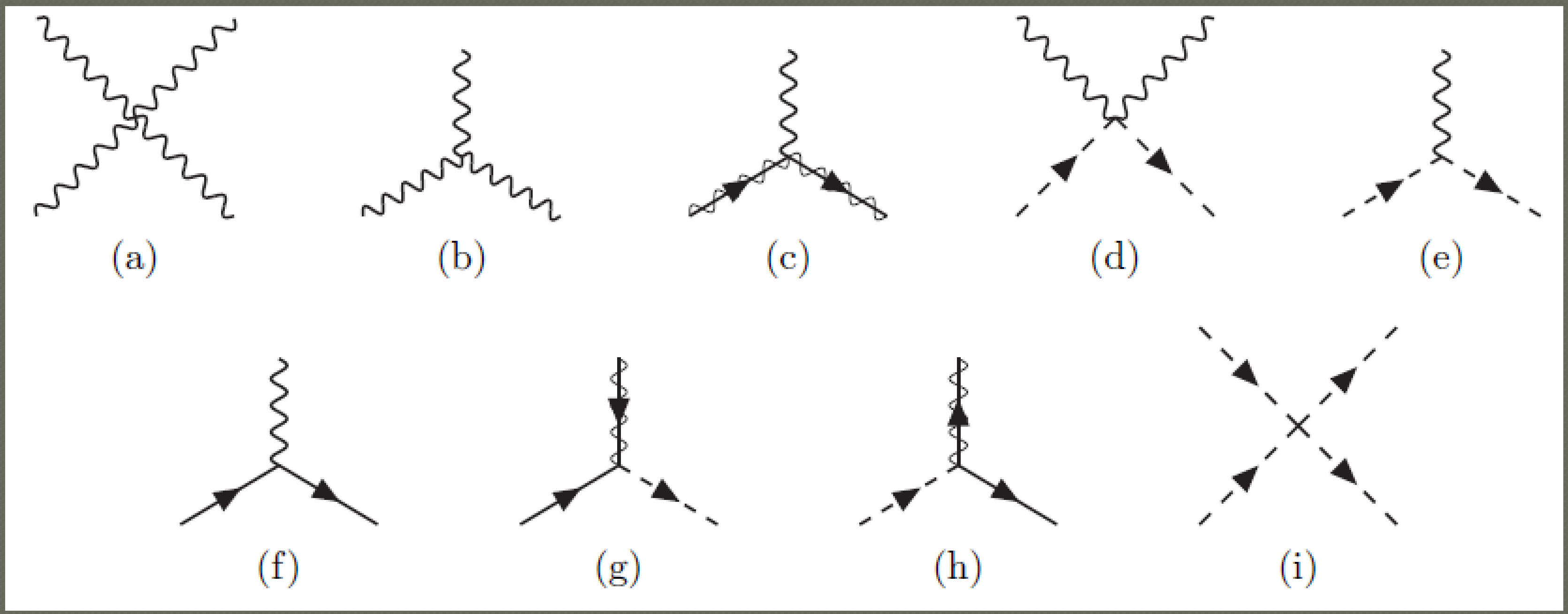
- Remember that that whole fields have to be supersymmterized.

Supersymmetrizing Standard Model (SM)

- The gauge sector
- SU(3)×SU(2)×U(1)

$$
\begin{aligned}
U(1) &: \quad B_m \to B_m, \widetilde{b} \\
SU(2) &: \quad W_m^a \to W_m^a, \widetilde{w}^a \\
SU(3) &: \quad A_m^a \to A_m^a, \widetilde{g}
\end{aligned}
$$

Vertices from gauge interactions



What to do with the fermions?

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad \overline{e} \quad Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \overline{u} \quad \overline{d}$$

$$(1, 2, -\frac{1}{2}) \qquad \begin{pmatrix} \widetilde{\nu} \\ \widetilde{e} \end{pmatrix} , \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$(1, 1, +1) \qquad \widetilde{\overline{e}} , \overline{e}$$

$$(3, 2, +\frac{1}{6}) \qquad \begin{pmatrix} \widetilde{u} \\ \widetilde{d} \end{pmatrix} , \begin{pmatrix} u \\ d \end{pmatrix}$$

$$(\overline{3}, 1, -\frac{2}{3}) \qquad \widetilde{\overline{u}} , \overline{u}$$

$$(\overline{3}, 1, +\frac{1}{3}) \qquad \widetilde{\overline{d}} , \overline{d}$$

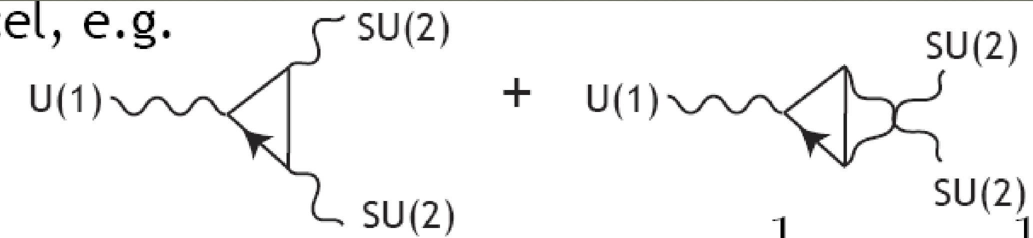
Higgs

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{\phi}^+ \\ \tilde{\phi}^0 \end{pmatrix}$$

OK???

Miraculous anomaly cancelation in the SM

cancel, e.g.



$$= \mathrm{tr}[Y\{T^a, T^b\}] = \frac{1}{2}\delta^{ab} \cdot (-\frac{1}{2} + 3 \cdot \frac{1}{6}) = 0$$

To maintain anomaly cancelation we need more than one Higgs doublet

:

$$(1,2,+\frac{1}{2}) \qquad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \ , \ \widetilde{h}_u = \begin{pmatrix} \widetilde{h}_u^+ \\ \widetilde{h}_u^0 \end{pmatrix}$$

$$(1,2,-\frac{1}{2}) \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \ , \ \widetilde{h}_d = \begin{pmatrix} \widetilde{h}_d^+ \\ \widetilde{h}_d^0 \end{pmatrix}$$

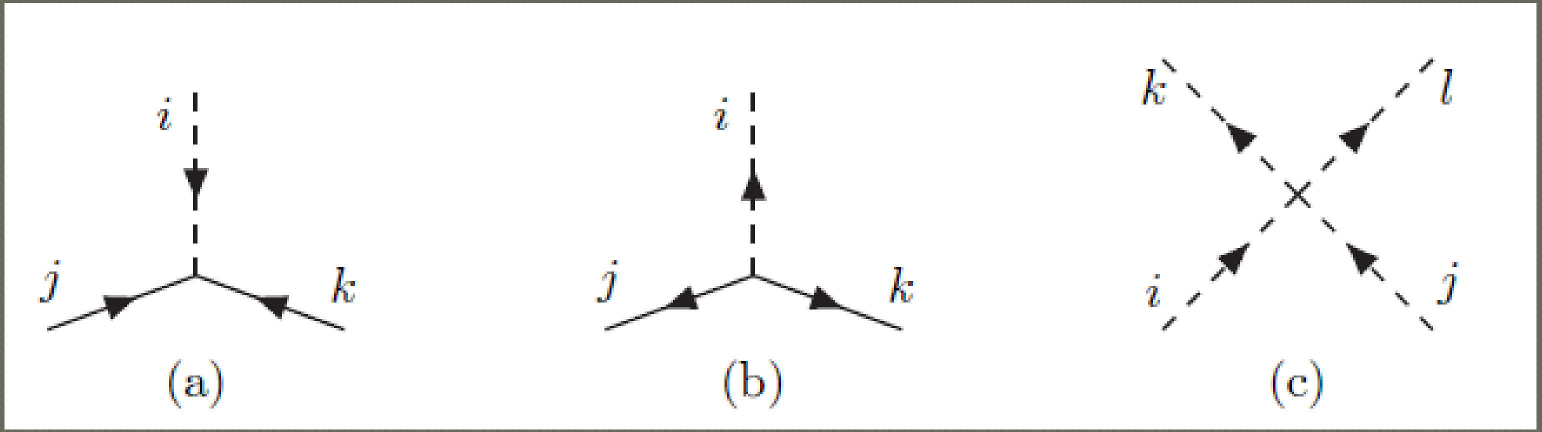
Yukawa interactions

$$W = y_d^{ij}\overline{d}^i H_{d\alpha}\epsilon_{\alpha\beta}Q_\beta^j + y_e^{ij}\overline{e}^i H_{d\alpha}\epsilon_{\alpha\beta}L_\beta^j - y_u^{ij}\overline{u}^i H_{u\alpha}\epsilon_{\alpha\beta}Q_\beta^j$$

$$y_d = W_d Y_d V_d^\dagger \quad y_e = W_e Y_e V_e^\dagger \quad y_u = W_u Y_u V_u^\dagger$$

CKM matrix: $V_d^\dagger V_u = V_{CKM}$

Vertices from Yukawa couplings



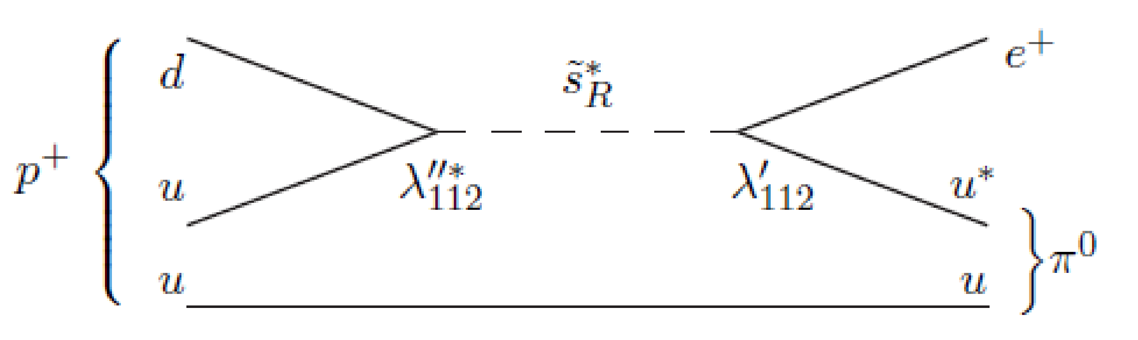
Mu term

$$W_\mu = -\mu H_{d\alpha}\epsilon_{\alpha\beta}H_{u\beta}$$

~~$\mu = 0,$~~

R-parity violating terms

$$W_{\not R} = \eta_1 \epsilon_{ijk} \overline{u}_i \overline{d}_j \overline{d}_k + \eta_2 \overline{d} \epsilon_{\alpha\beta} L_\alpha Q_\beta$$
$$+\eta_3 \overline{e} \epsilon_{\alpha\beta} L_\alpha L_\beta + \eta_4 \epsilon_{\alpha\beta} L_\alpha H_{u\beta}$$



R-parity

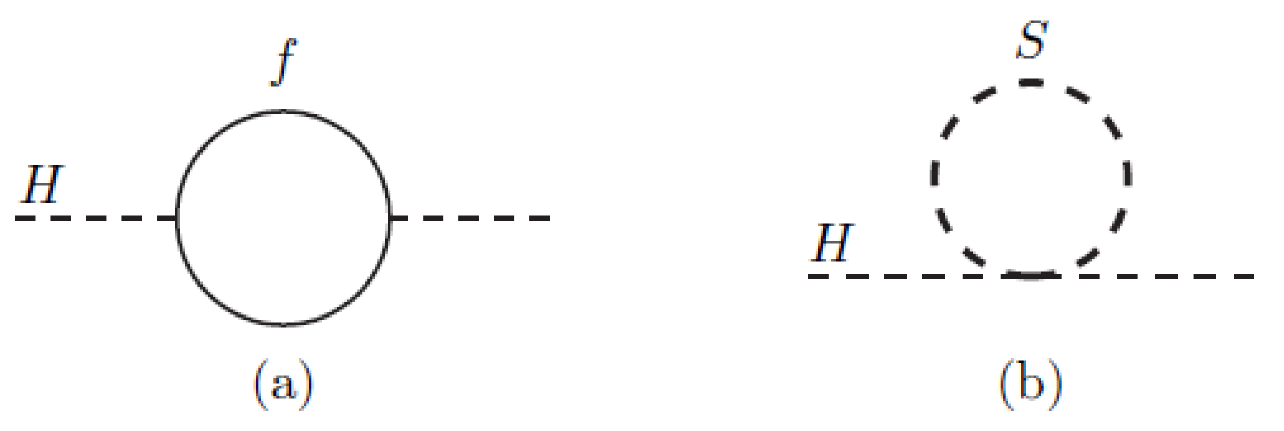
$$R = (-1)^{3B+L+2J}$$

- Higgs boson and all SM particles are parity even.
- The supersymmetric particles are all parity odd.

Dark matter candidate

- LSP is dark matter
- LSP cannot be charged or colored.

Correction to Higgs mass



What are f and S?