

Neutron stars with Quark Core (Hybrid Stars)

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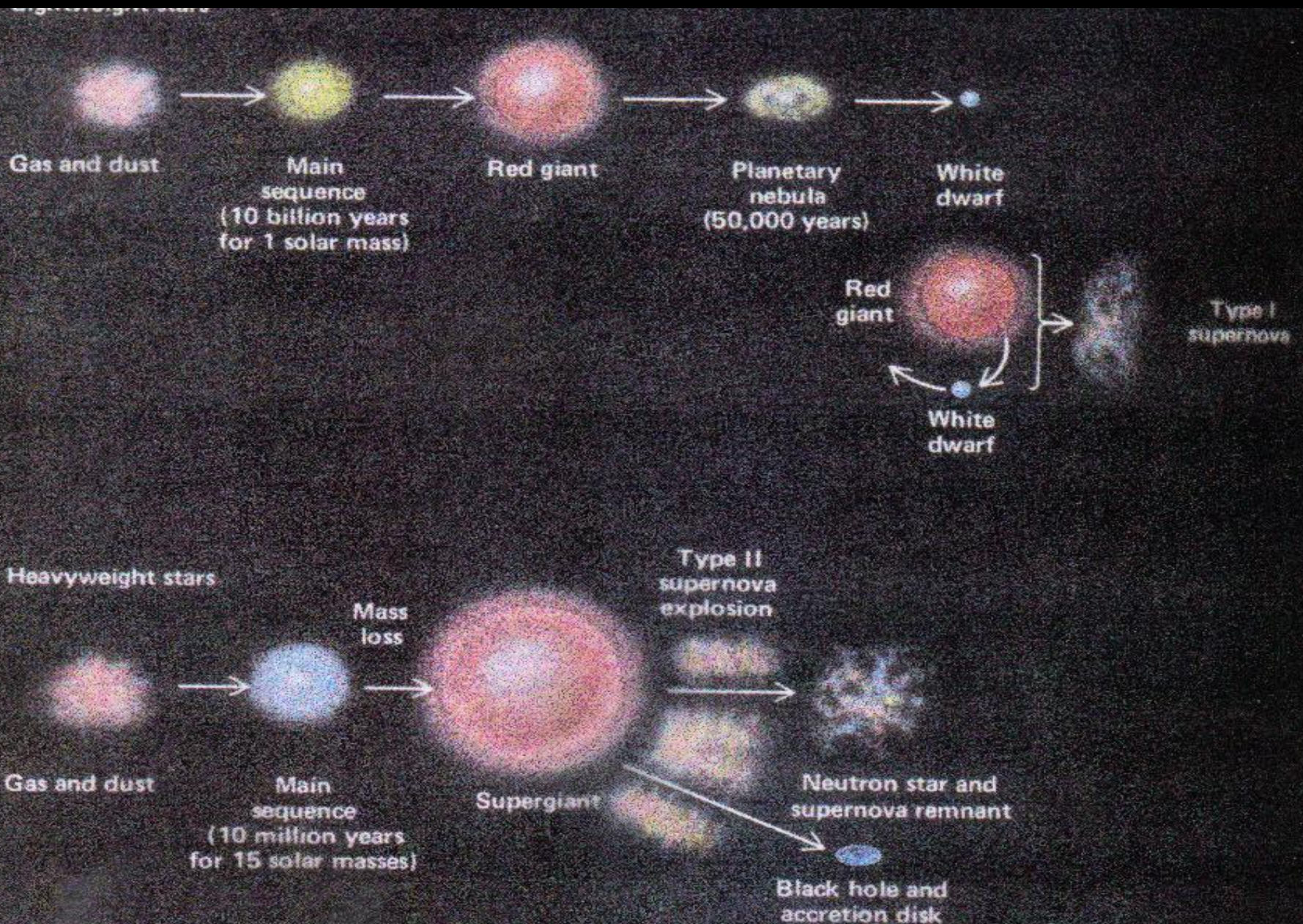
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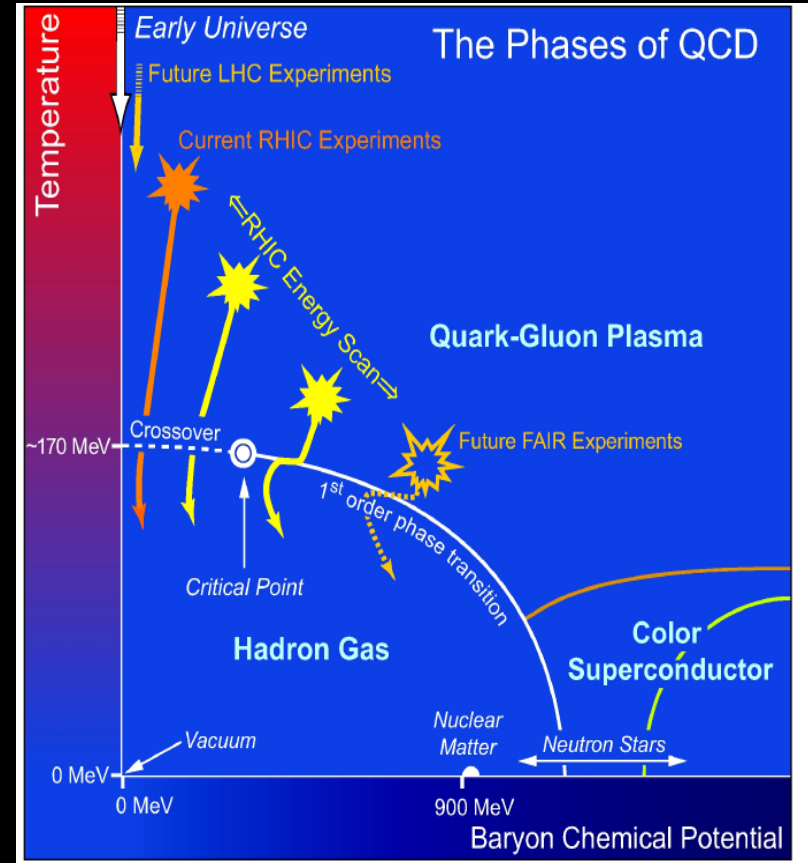
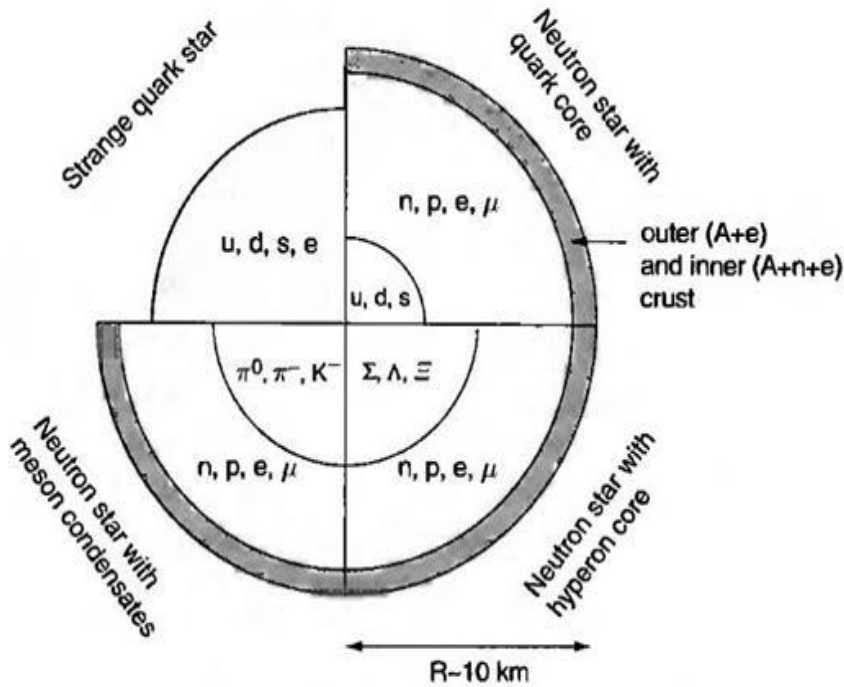
OUTLINE

- Introduction
- EOS of Nuclear matter and Neutron Star
- EOS of Quark matter
- Hadron-quark phase transition
- Structure of Hybrid stars
- Summary

Stellar evolution



Possible internal structures and compositions of four different types of compact stars



A semi-quantitative phase diagram on $T - \mu$ plane

EOS

Equation of states(EOS) of nuclear matter at high density

* Hadron matter

- * Nuclear many body theories: Brueckner-Bethe-Goldstone(BBG),....
- * Relativistic mean field theory

* Quark matter

- * MIT bag model
- * Dyson-Schwinger equations (DSE)
- * Nambu - Jona-Lasino (NJL) model (color superconductivity)
- * ...

Eos of Hadron Matter

We have calculated the equation of state of hadronic matter, we use statistical Thomas-Fermi approach by employing Myers-Swiatecki density dependent potential.

$$H = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{1 \leq i \leq j \leq A} V_{ij} ,$$

$$H = H_0 + \Delta H ,$$

$$H_0 = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i=1}^A V_i ,$$

$$H_0 = H_{0n} + H_{0p}$$

$$V_{12} = -2T_F \rho_0^{-1} f\left(\frac{r_{12}}{a}\right) \left\{ \frac{1}{2}(1 \mp \xi) \alpha - \frac{1}{2}(1 \mp \zeta) \times \left[\beta \left(\frac{p_{12}}{p_F}\right)^2 - \gamma \left(\frac{p_F}{p_{12}}\right) + \sigma \left(\frac{2\bar{\rho}}{\rho_0}\right)^{\frac{2}{3}} \right] \right\} .$$

$$f\left(\frac{r_{12}}{a}\right) = \frac{1}{4\pi a^3} \frac{\exp\left(-\frac{r_{12}}{a}\right)}{\frac{r_{12}}{a}}$$

Relativistic Star

$$ds^2 = e^{\nu(r)} c^2 dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \longrightarrow \text{Metric}$$

$$T_{\nu}^{\mu} = (p + \epsilon) u^{\mu} u_{\nu} + p \delta_{\nu}^{\mu} \longrightarrow \text{Energy-Momentum Tensor (EOS)}$$

$$G_{\nu\mu} = 8\pi T_{\nu\mu} \longrightarrow \text{Einstein's Equation}$$

$$\frac{dp}{dr} = - \frac{[p(r) + \epsilon(r)][m(r) + 4\pi r^3 p(r)]}{r(r - 2m(r))} \longrightarrow \text{TOV equation (General Relativistic Hydrostatic equilibrium)}$$

$$m(r) = 4\pi \int_0^r dr r^2 \epsilon(r) \longrightarrow \text{Stellar mass}$$

$$r \rightarrow 0 \quad m(r) \rightarrow 0 \quad P \rightarrow P_c \quad \epsilon \rightarrow \epsilon_c$$

can easily be solved numerically for given EoS $\epsilon(p)$

initial value: $p(r = 0) = p_0$

surface of the star: $p(r = R) = 0$

gravitational mass of the star: $M \equiv m(R)$

EOS of Quark Matter (MIT Bag Model)

$$P_q = -B + P_q^{kin} + P_q^{int}$$

$$\varepsilon_q = B + \varepsilon_q^{kin} + \varepsilon_q^{int}$$

$$\Omega = \Omega_u + \Omega_d + \Omega_s + \Omega_e + B$$

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2}$$

$$\begin{aligned} \Omega_q = & -\frac{3m_q^4}{8\pi^2} \left[\frac{\eta_q x_q}{3} (2x_q^2 - 3) + \ln(x_q + \eta_q) \right] \\ & + \frac{3m_q^4 \alpha_s}{4\pi^3} \left\{ 2 \left[\eta_q x_q - \ln(x_q + \eta_q) \right]^2 - \frac{4}{3} x_q^4 + 2 \ln(\eta_q) \right. \\ & \left. + 4 \ln\left(\frac{\sigma_{ren}}{m_q \eta_q}\right) \left[\eta_q x_q - \ln(x_q + \eta_q) \right] \right\} \end{aligned}$$

$$\mu_s = \mu_d \equiv \mu, \quad \mu_u = \mu - \mu_e,$$

$$\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0$$

m_q, μ_q : q quark mass and chemical potential.

$$x_q = \sqrt{\mu_q^2 - m_q^2} / m_q$$

$$\eta_q = \sqrt{1 + x_q^2} = \mu_q / m_q$$

α_s : QCD fine structure constant

$$\rho_q = -\frac{\partial \Omega_q}{\partial \mu_q}$$

$$\varepsilon_Q = \sum_q (\Omega_q + \mu_q \rho_q) + B$$

$$P_Q = -\sum_q \Omega_q - B$$

$$B(\rho) = B_\infty + (B_0 - B_\infty) \exp \left[-\beta \left(\frac{\rho}{\rho_0} \right)^2 \right]$$

with $B_\infty = 60 \text{ MeV fm}^{-3}$, $B_0 = 400 \text{ MeV fm}^{-3}$, and $\beta = 0.17$.

Hadron-quark phase transition

Gibbs condition at zero temperature
between hadron phase and quark phase

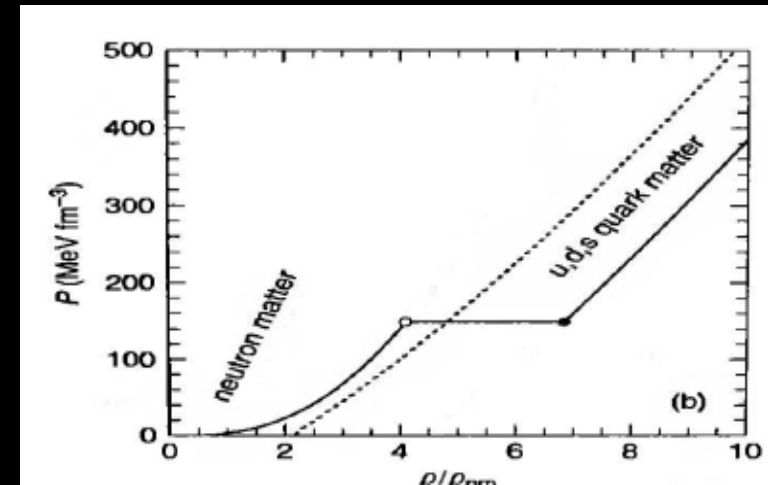
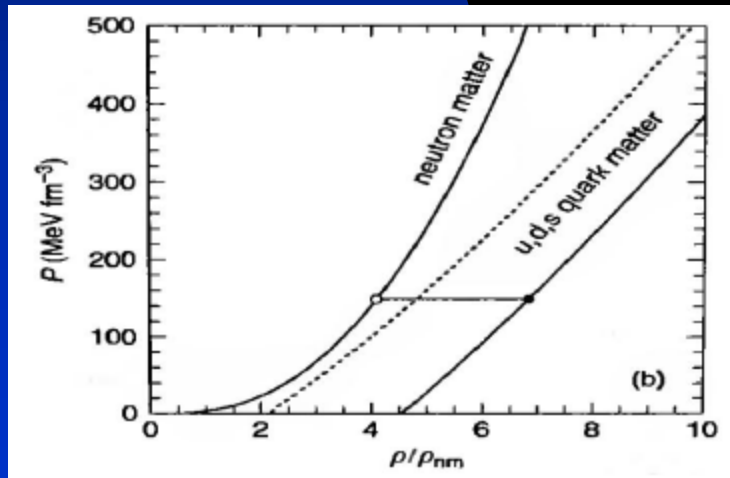
$$P_{HP}(\mu_n, \mu_e) = P_{QP}(\mu_n, \mu_e).$$

the total electrical charge is

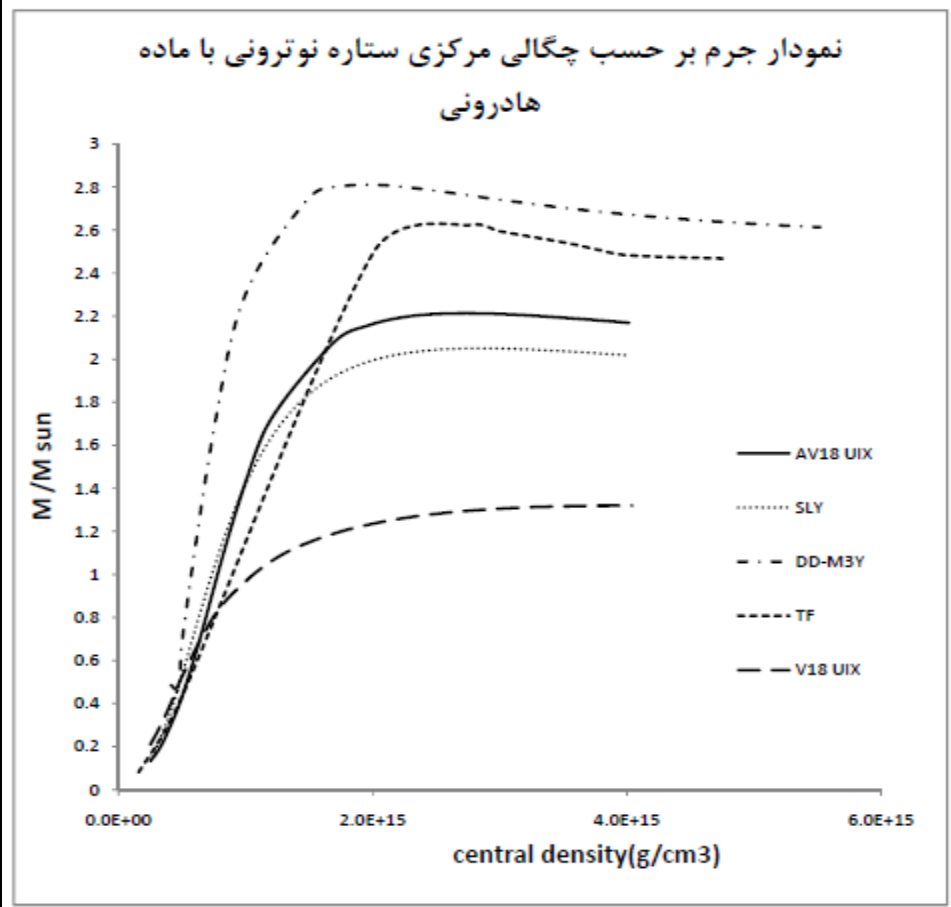
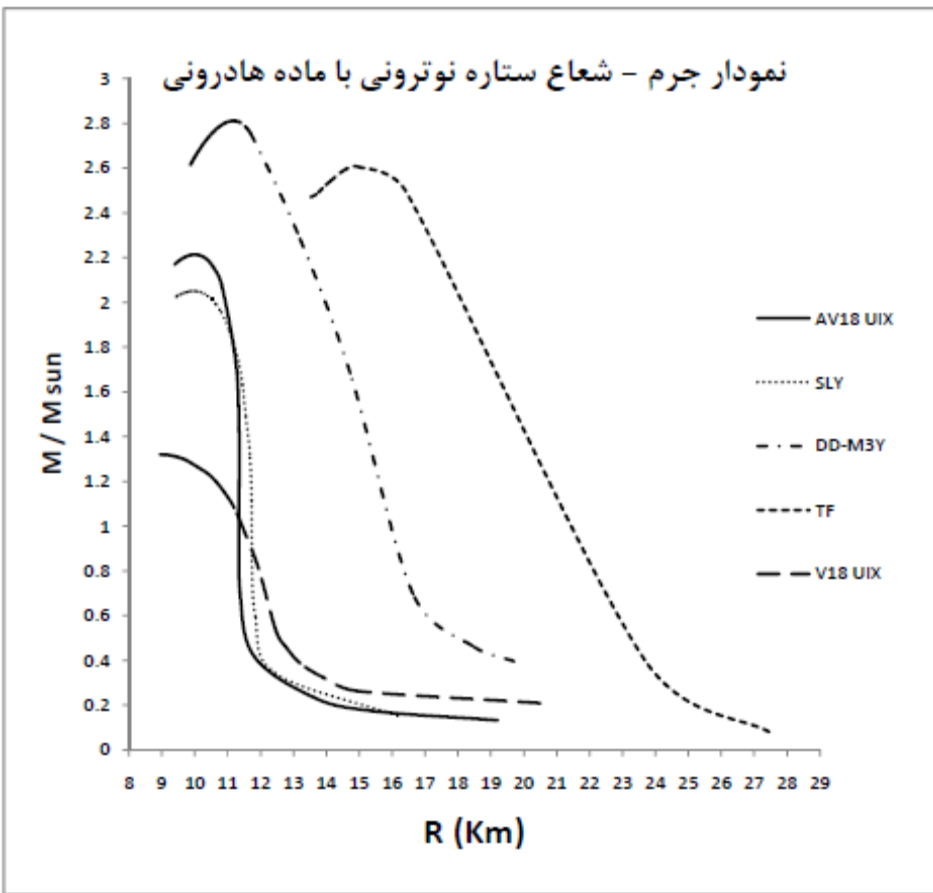
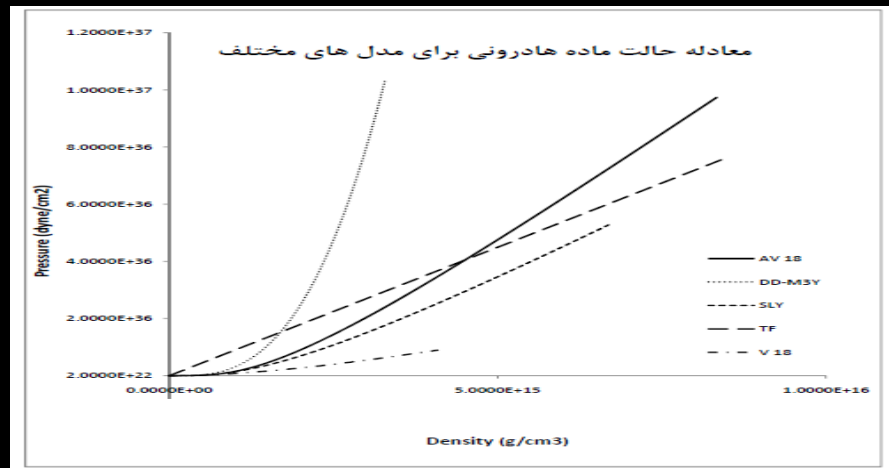
$$0 = \frac{Q}{V} = \chi q_{QP} + (1 - \chi)q_{HP}$$

and the total energy density is

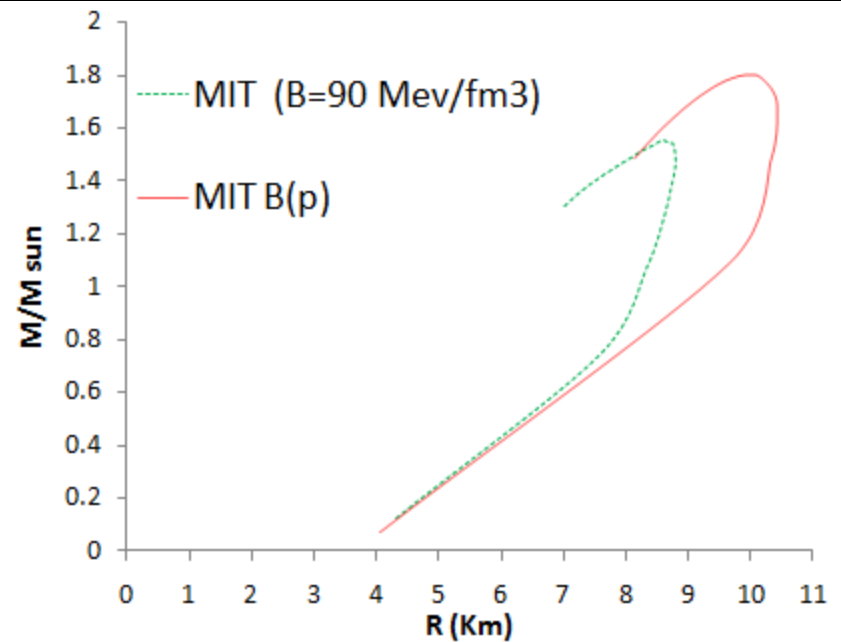
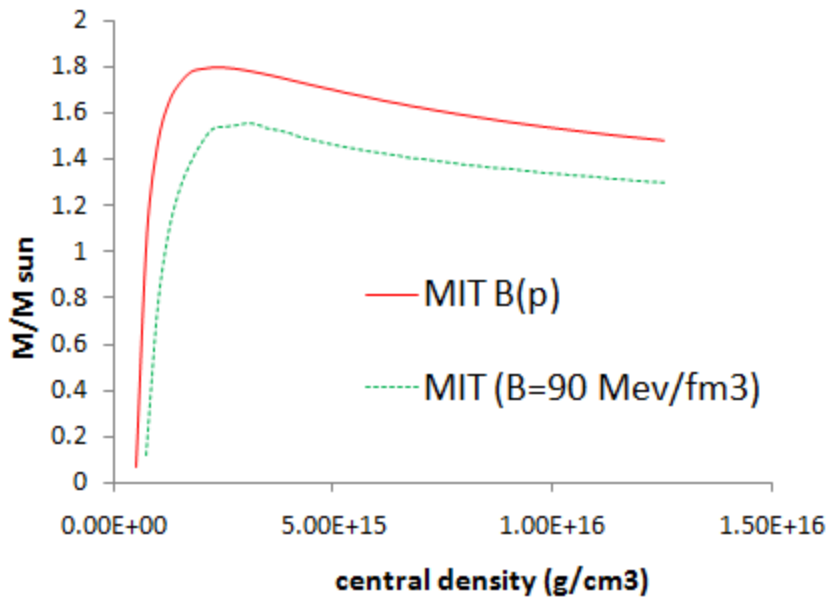
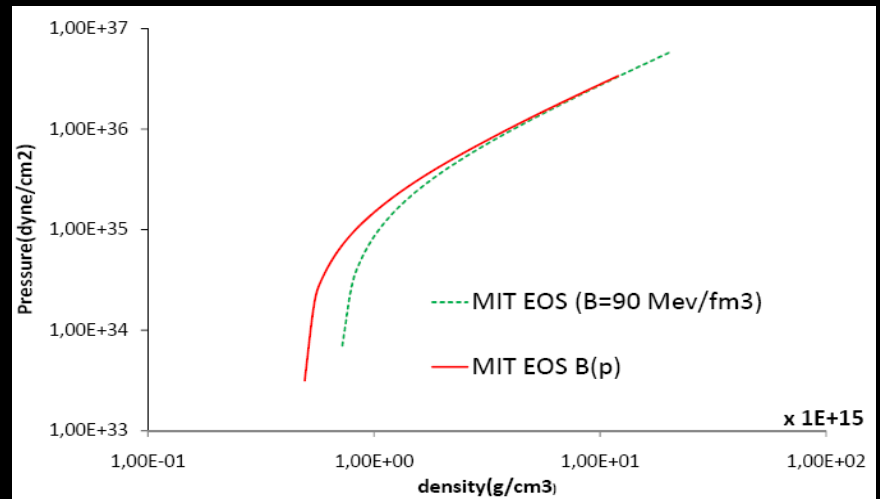
$$\epsilon = \frac{E}{V} = \chi \epsilon_{QP} + (1 - \chi)\epsilon_{HP}$$



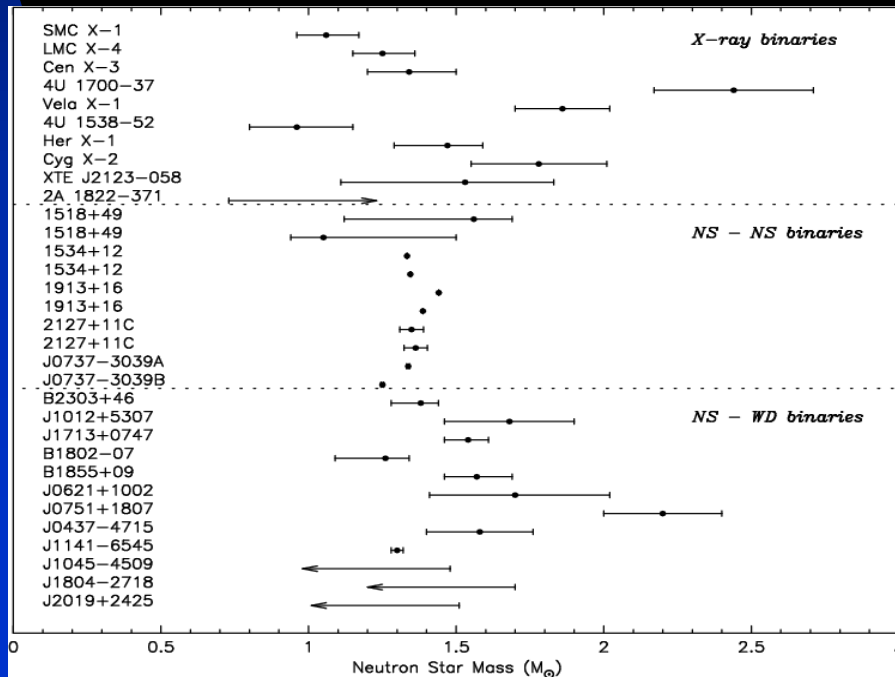
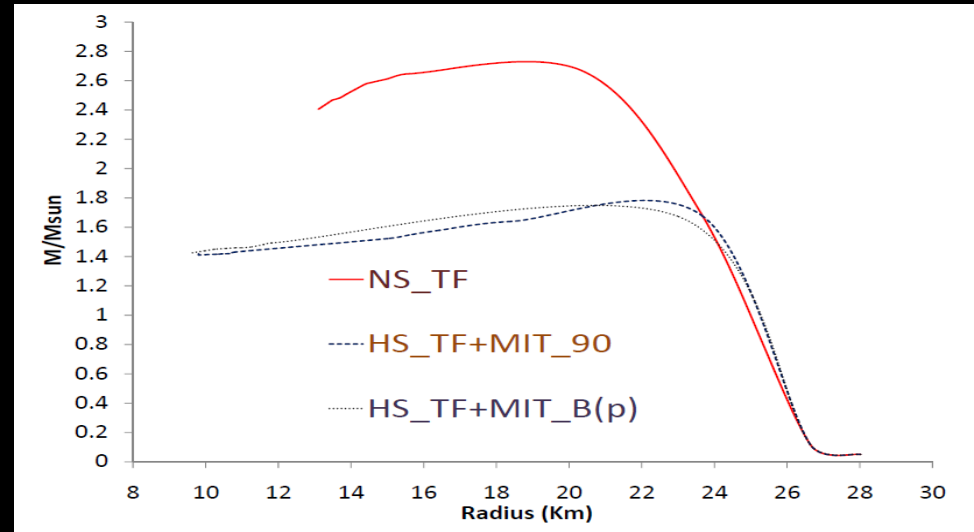
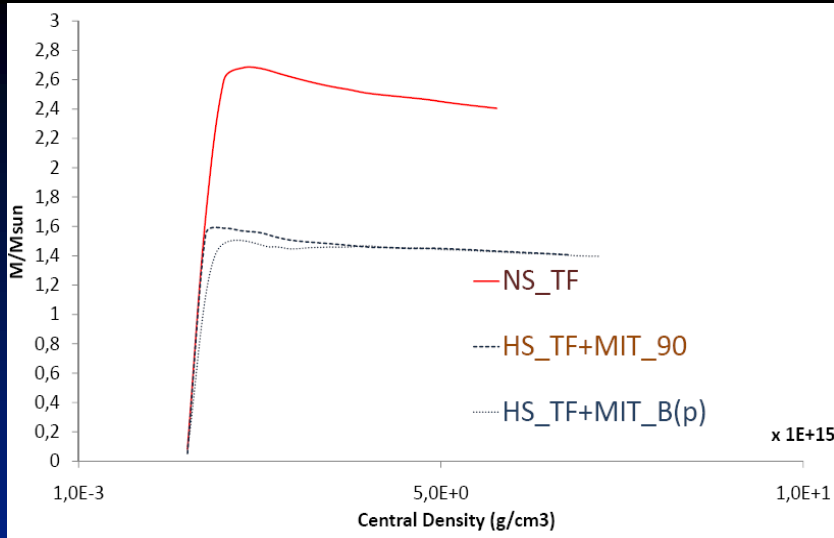
Results of hadron matter



Results of quark matter



Conclusions



References:

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- Weber et al. ArXiv: 0705.2708
- H.R. Moshfegh et al. J.Phys.G38:085102, 2011.

Thank you for your attention.