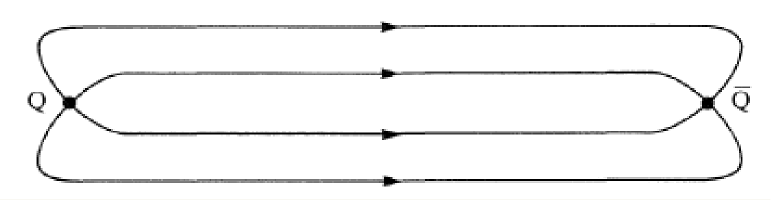
The Role Of Magnetic Monopoles In Quark Confinement (Field Decomposition Approach)

IPM school and workshop on recent developments in Particle Physics (IPP11)

2011, Tehran, Iran



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Outline

- Introduction
- Dual Superconductor
- Magnetic Monopoles In Abelian and Non-Abelian Theories
- Field Decomposition (Cho Approach)
- Zwanziger Formalism
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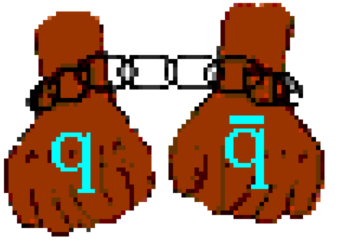
- Quark Confinement Potential
- Conclusion

Quark Confinement

- Quarks are not observed even in very high energy collisions!
- The theory describes Quarks must be such that they can never be free!
- The theory in its nonperturbative domain has defied analytical approach despite the best efforts for more than 40 years.
- It is an outstanding problem of theoretical physics.

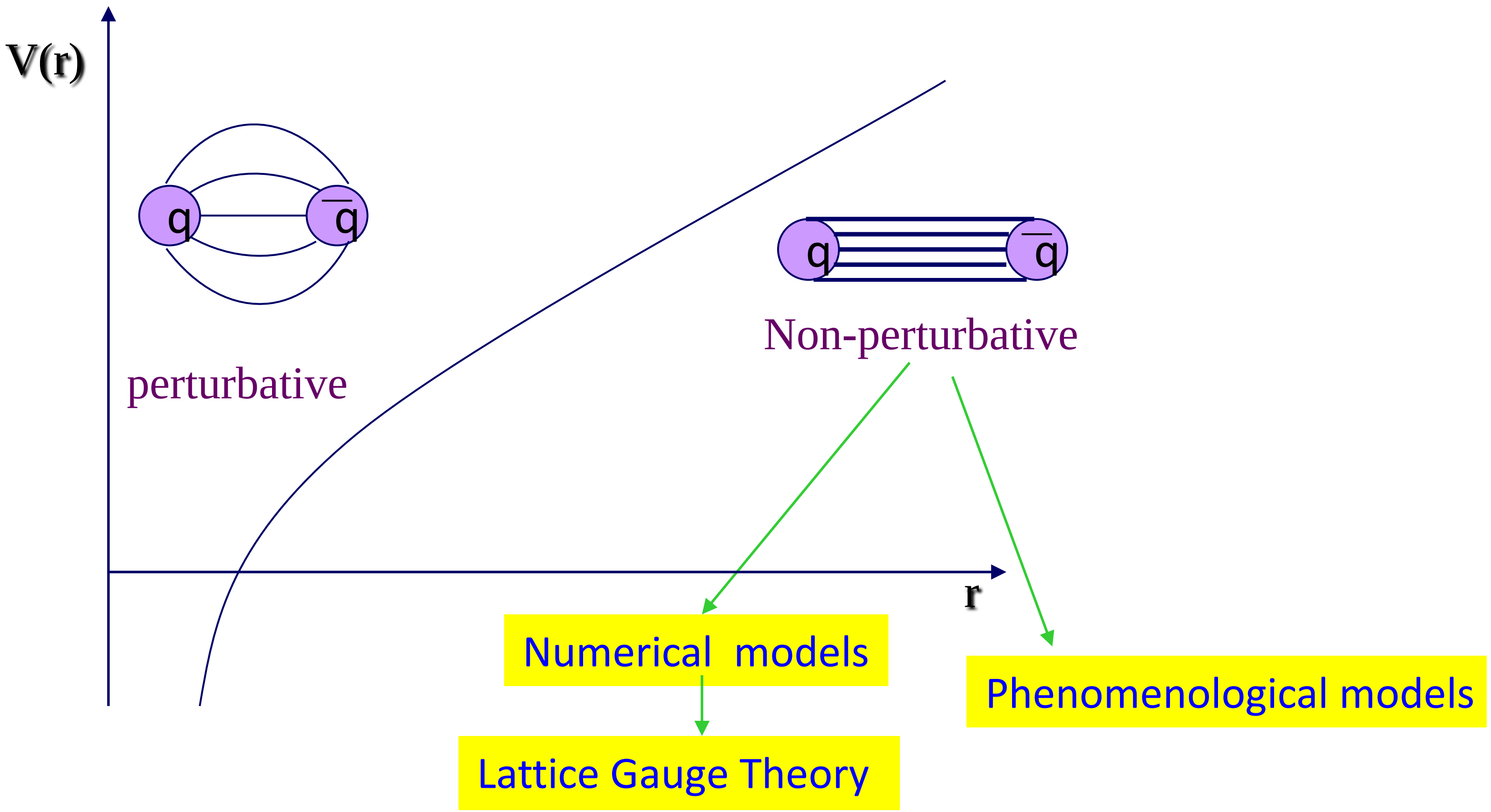


- **High energy regime:**

Short distances ⟶ Asymptotic freedom ⟶ Perturbative methods

- **Low energy regime:**

Large distances ⟶ Confinement ⟶ Non-perturbative methods



Phenomenological models

- The general idea is that the Low energy regime of QCD can be described, in a dual language, in terms of topologically non trivial excitations like Vortices, Monopoles, Instantons, Merons and Calerons.
- There exist two main proposals for these excitations, both due to ’t Hooft.

    1) Vortices (Center vortex Model)

    2) Monopoles (Dual Superconductor Model)

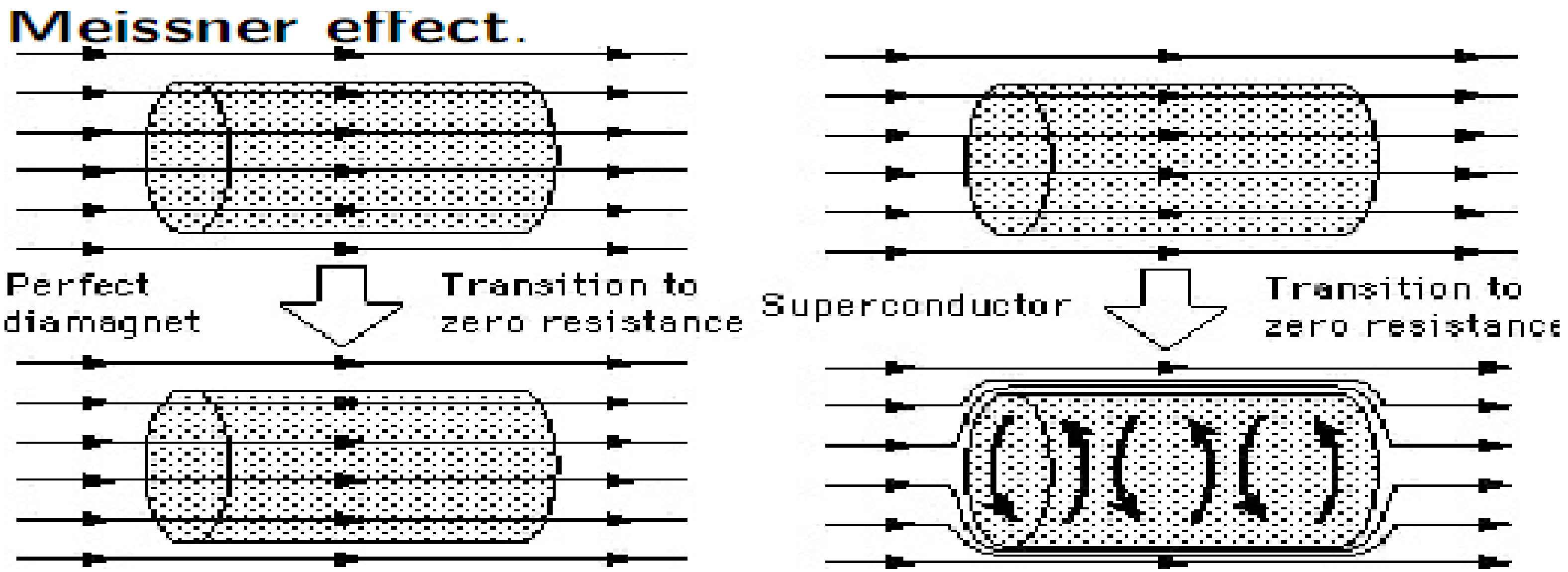
Dual Superconductor

- More than three decades ago, the dual superconductor (DS) picture of confinement in quantum chromodynamics (QCD) was proposed by Nambu, Mandelstam and ’t Hooft in an analogy to type II superconductivity. In DS picture, color and quarks are confined within hadrons due to the dual Meissner effect.

[1] Y. Nambu, Phys. Rev. D 10 (1974) 42624268; G. 't Hooft, in: High Energy Physics, edited by A. Zichichi (Editorice Compositori, Bologna, 1975); S. Mandelstam, Phys. Report 23 (1976) 245249.

Dual Meissner effect:

- condensation of magnetic monopoles
- quarks are sources of (colored) electric field
- Along the tube connecting quarks the magnetic condensate is destroyed
- electric field squeezed inside the tube
  = Abrikosov–Nielsen–Olesen vortex



Usual Superconductor	Dual Superconductor
Cooper pair condensation	Magnetic Monopole Condensation
Magnetic field flux tube	Color Electric field flux tube
Magnetic monopole Confinement	Quark Confinement

Dual Superconductor ingredients

- Vacuum structure of QCD is like a dual superconductor
- Magnetic Monopoles can appear in non-Abelian theories like QCD
- Magnetic Monopoles can condensate and monopole condensation makes quarks to be confined.

Magnetic Monopole existence is necessary for this picture

The Dirac Monopole (Wu-Yang)

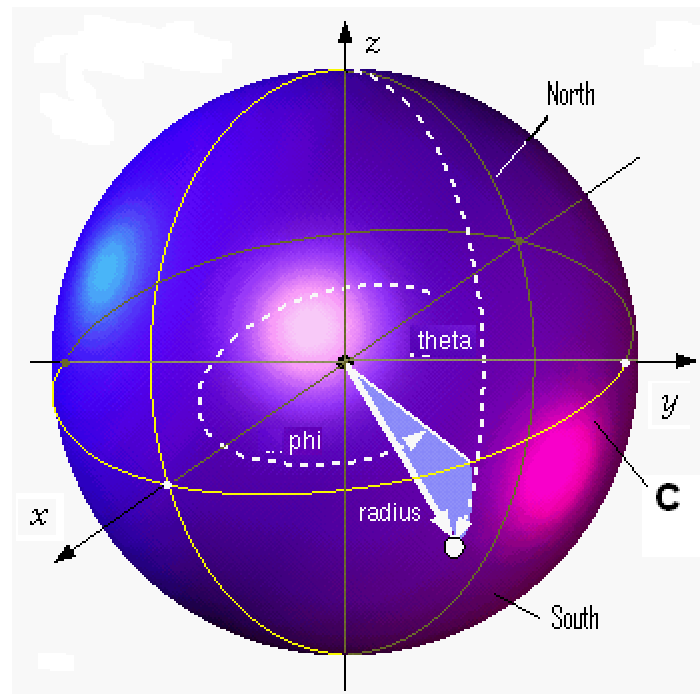
$$\mathbf{B} = \frac{g}{r^3}\mathbf{r} = \text{curl}\,\mathbf{A}$$

$$A_r^a = A_\theta^a = 0, \quad A_\phi^a = \frac{g}{r}\frac{1-\cos\theta}{\sin\theta}.$$

$$A_r^b = A_\theta^b = 0, \quad A_\phi^b = -\frac{g}{r}\frac{1+\cos\theta}{\sin\theta}.$$

$$A_\phi^b = A_\phi^a - \frac{2g}{r\sin\theta} = A_\phi^a - \frac{i}{e}S\nabla_\phi S^{-1}$$

$$S = \exp(2ige\phi) \Rightarrow eg = \frac{1}{2}n$$



't Hooft-Polyakov monopoles

In the mean time, it is discovered that magnetic monopoles will occur naturally in a non-Abelian model like Georgi-Glashow model without Dirac magnetic monopole difficulties, where the non-abelian local symmetry is broken down by the Higgs mechanism into an electromagnetic U(1) symmetry.

$$\mathcal{L} = -\tfrac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu a} + \tfrac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \frac{m^2}{2}\phi^a\phi^a - \lambda(\phi^a\phi^a)^2$$

- Φ^a are scalar fields in the adjoint representation of SU(2) (a=1,2,3).

$$F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\varepsilon^{abc}A^b_\mu A^c_\nu,$$

$$D_\mu\phi^a = \partial_\mu\phi^a + e\varepsilon^{abc}A^b_\mu\phi^c.$$

't Hooft, G., Nucl. Phys. B79, 276 (1974); Polyakov, A.M., JETP Lett. 20, 194. (1974)

Boundary Conditions

- r→∞, need vacuum state, $D_\mu \phi^a = 0$

$$\phi^a = F\frac{r^a}{r} \quad (r \to \infty) \qquad A_i^a = -\varepsilon_{iab}\frac{r^b}{er^2} \quad (r \to \infty)$$

- Shown by 't Hooft that F_μ^a and $F_{\mu\nu}$ are related by ($\Phi = \{\phi^a\}$):

$$F_{\mu\nu} = \frac{1}{|\phi|}\phi^a F_{\mu\nu}^a - \frac{1}{e|\phi|^3}\varepsilon_{abc}\phi^a(D_\mu\phi^b)(D_\nu\phi^c).$$

- Working out electric and magnetic fields using the r→∞ limit

$$F_{0i} = 0, \quad F_{ij} = -\frac{1}{er^3}\varepsilon_{ijk}r^k$$

$$B_k = \frac{r^k}{er^3}.$$

Quantization of Magnetic Charge

The conserved magnetic charge is

$$M = \frac{1}{4\pi}\int K^0 \, d^3x$$

$$= -\frac{1}{8\pi e}\oint_{S^2} \varepsilon_{ijk}\varepsilon_{abc}\hat{\phi}^a \partial_j \hat{\phi}^b \partial_k \hat{\phi}^c (d^2S)_i = \frac{d}{e}, \quad d \text{ integer.}$$

- This is same as Dirac quantization condition $eg = \frac{1}{2}n$ with n = 2d!
- Considering all possible solutions of the equations of motion for this theory, it can be shown we get the Dirac quantization condition with n even
- Agrees with result for possible homotopy classes

$\pi_2(SO(3)/U(1))$ = additive group of even integers ⇒ $eg = n$

Mass of Monopoles

- 't Hooft showed $M_{mon} \approx \frac{4\pi M_V}{e^2}$, where M_V is gauge boson mass
- Monopoles arise in GUTs, with mass of the scale of the symmetry breaking of the theory
- $SU(5) \xrightarrow{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow{M_W} SU(3) \times U(1)$
- GUT monopoles are too massive to be produced in accelerators, but could have been produced in early universe
- Monopole searches focus on accelerators, cosmic rays, and on monopoles possibly bound in matter
- Up to Now, nothing found

't Hooft-Polyakov Monopoles are useless in dual superconductor picture

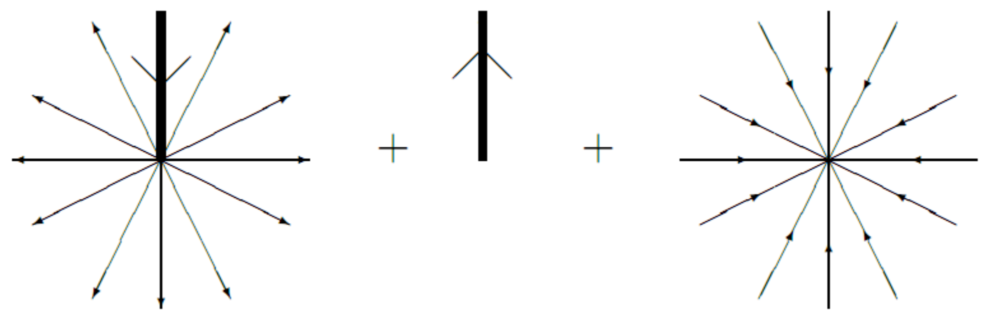
- 't Hooft-Polyakov monopoles are Massive and can not become superconducting in order to explain confinement. Furthermore, In pure Yang-Mills theory there is no matter fields. Therefore a 't Hooft-Polyakov monopole, that is, a localized solution of the classical field equations, does not exist in a pure gauge theory. However, there might exist some large-scale field configurations that, in some gauge, could look “like a monopole”.
- In absence of matter fields, two methods have so far been known in extracting magnetic monopole degrees of freedom in Yang-Mills theory:

  1) Abelian projection which is a partial gauge fixing proposed by 't Hooft.

  2) Field decomposition, new variables proposed by Cho and Faddeev-Niemi.

Abelian Projection

- The first method (Abelian projection) leads to the Abelian dominance. In fact, the first method is nothing but a gauge-fixed version of the second method.



$$G^{\Omega}_{\mu\nu} = \underbrace{\partial_\mu A^{\Omega}_\nu - \partial_\nu A^{\Omega}_\mu + \frac{i}{e}\Omega[\partial_\mu, \partial_\nu]\Omega^\dagger}_{\text{Abelian Projected QCD}} + ie[A^{\Omega}_\mu, A^{\Omega}_\nu]$$

G. t Hooft, Nucl.Phys. B 190 [FS3] (1981) 455478.

Field decomposition

- Field decomposition enables us to answer the following questions to establish the dual superconductivity in Yang-Mills theory.

  1) How to extract the Abelian part responsible for quark confinement from the non-Abelian gauge theory in the gauge-invariant way without losing important features of non-Abelian gauge theory.

  2) How to define the magnetic monopole to be condensed in Yang-Mills theory in the gauge-invariant way even in absence of any fundamental scalar field, in sharp contrast to the Georgi-Glashow model.

Cho Decomposition For SU(2) QCD

- In Cho formulation an extra symmetry, magnetic symmetry, is imposed to the theory by making a unit vector field in adjoint representation and the theory is restricted by this condition

$$D_\mu \hat{m} = \partial_\mu \hat{m} + g\overrightarrow{B}_\mu \times \hat{m} = 0 \quad \Longrightarrow \quad \overrightarrow{B}_\mu = A_\mu \hat{m} - \frac{1}{g}\hat{m} \times \partial_\mu \hat{m}$$

- The unrestricted part A is called electric and the other part which is restricted is called magnetic.

Y. M. Cho, Phys. Rev. D 21 (1980) 10801088; D 23 (1981) 24152426.

- In **Faddeev-Niemi** Decomposition Restriction is removed by a new decomposition

$\mathbf{A}_\mu$ describes six polarization degrees of freedom

$$\Downarrow$$

$$\mathbf{A}_\mu = C_\mu \mathbf{n} + \mathbf{dn} \times \mathbf{n} + \rho \mathbf{dn} + \sigma \mathbf{dn} \times \mathbf{n}$$

L. Faddeev and A. J. Niemi, [hep-th/9807069], Phys. Rev. Lett. 82 (1999) 16241627

Cho Decomposition (continued)

- Having B, one can easily calculate the field strength

$$\overrightarrow{G}_{\mu\nu} = \partial_\mu \overrightarrow{B}_\nu - \partial_\nu \overrightarrow{B}_\mu + g \overrightarrow{B}_\mu \times \overrightarrow{B}_\nu = (F_{\mu\nu} + H_{\mu\nu})\hat{m}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$H_{\mu\nu} = -\frac{1}{g}\hat{m} \,.\, (\partial_\mu \hat{m} \times \partial_\nu \hat{m})$$

$F_{\mu\nu}$ is called electric

$H_{\mu\nu}$ is called magnetic

As we see $\overrightarrow{G}_{\mu\nu}$ is parallel to $\hat{m}$

Cho Decomposition (continued)

It is possible to introduce a magnetic potential C_μ^* corresponding to $H_{\mu\nu}$ as well as we have A_μ for $F_{\mu\nu}$. First we choose hedgehog configuration for $\hat{m}$

$$\hat{m} = \frac{r^a}{r} = \begin{pmatrix} \sin\alpha\, cos\beta \\ \sin\alpha\, sin\beta \\ cos\alpha \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} H_{\mu\nu} &= -\frac{1}{g}\hat{m}\,.\,(\partial_\mu \hat{m} \times \partial_\nu \hat{m}) \\ &= \partial_\mu C_\nu^* - \partial_\nu C_\mu^* \end{aligned}$$

$$C_\mu^* = \frac{1}{g} cos\alpha\, \partial_\mu \beta$$

Cho Decomposition (continued)

- Consider full SU(2) QCD Lagrangian in special gauge with isodoublet spinor source

$$L = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}F_{\mu\nu}H_{\mu\nu} - \frac{1}{4}H_{\mu\nu}^2 + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

$$D_\mu = \partial_\mu + igB_\mu$$

$$B_\mu = (A_\mu + C_\mu^*)\,\frac{1}{2}\sigma_3$$

The above Lagrangian is Abelian with B_μ as an Abelian gauge field.

We must notice that we have monopole current for this Abelian gauge theory (unlike QED)

$$\partial^\mu G_{\mu\nu}^* = \partial^\mu H_{\mu\nu}^* = k_\nu \neq 0$$

Cho Decomposition (continued)

- Monopole current is resulted from magnetic potential C_μ^* . Magnetic potential has some unusual Features like having Dirac string and being spacelike.

- Since in field theory we have a field for every particle, we must introduce a field for magnetic monopole as a particle in Lagrangian. Moreover, there is some undesirable properties related to C_μ^* , mentioned above, in Lagrangian. So, for a field-theoretic description, it's necessary to remove these undesirable features. The dual magnetic potential C_μ and a complex scalar field for monopole can complete the Lagrangian.

$$L = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}F_{\mu\nu}H_{\mu\nu} - \frac{1}{4}{H_{\mu\nu}^*}^2 + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi + |(\partial_\mu + i\frac{4\pi}{g}B_\mu^*)\phi|^2 - V(\phi^*\phi)$$

- **There are still spacelike potentials, with string in the above Lagrangian.**

Zwanziger Formalism

- A local Lagrangian of electric and magnetic charges which leads to local field equations without unphysical singularities like Dirac string, depending on a pair of four-potentials and an arbitrary fixed four-vector is constructed by Zwanziger.

$$\partial_\mu G^{\mu\nu} = j^\nu \qquad \partial_\mu G^{*\mu\nu} = k^\nu$$

$$\Rightarrow \quad G = (\partial \wedge A) - (n.\partial)^{-1}(n \wedge k)^* \qquad G^* = (\partial \wedge C) + (n.\partial)^{-1}(n \wedge j)^*$$

$$\mathfrak{L} = \mathfrak{L}_\gamma + \mathfrak{L}_I \quad \left\{ \begin{aligned} &\mathfrak{L}_I = -j_e \cdot A - j_g \cdot B \\ &j_e{}^\mu = \sum_n e_n \bar{\psi}_n \gamma^\mu \psi_n\,, \quad j_g{}^\mu = \sum_n g_n \bar{\psi}_n \gamma^\mu \psi_n \\ &\mathfrak{L}_\gamma = -(1/2n^2)[n\cdot(\partial \wedge A)]\cdot[n\cdot(\partial \wedge B)^d] \\ &\qquad +(1/2n^2)[n\cdot(\partial \wedge B)]\cdot[n\cdot(\partial \wedge A)^d] \\ &\qquad -(1/2n^2)[n\cdot(\partial \wedge A)]^2 - (1/2n^2)[n\cdot(\partial \wedge B)]^2 \end{aligned} \right.$$

D. Zwanziger, Phys. Rev. D 3 (1971) 880

Applying Zwanziger Formalism to Field Decomposition

Applying Zwanziger formalism to Cho decomposition, finally the local Lagrangian will be

$$L = -\frac{1}{2n^2}[n.(\partial \wedge A)]^\nu [n.(\partial \wedge C)^*]_\nu + \frac{1}{2n^2}[n.(\partial \wedge C)]^\nu [n.(\partial \wedge A)^*]_\nu$$
$$- \frac{1}{2n^2}[n.(\partial \wedge A)]^2 - \frac{1}{2n^2}[n.(\partial \wedge C)]^2 + \overline{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu \tau_3 - m)\Psi$$
$$+ |(\partial_\mu + i\frac{4\pi}{g} C_\mu)\phi|^2 - V(\phi^* \phi)$$

Where

$$V(\phi^*\phi) = M^2 \phi^* \phi + \lambda(\phi^*\phi)^2$$

Applying Zwanziger Formalism (continued)

- After symmetry breaking and approximating the monopole field as the constant mean field that exterminate $V(\phi^*\phi)$ we get the final Lagrangian which is the same as Dual Ginsburg Landau (DGL) Lagrangian

$$L = -\frac{1}{2n^2}[n.(\partial \wedge A)]^\nu [n.(\partial \wedge C)^*]_\nu + \frac{1}{2n^2}[n.(\partial \wedge C)]^\nu [n.(\partial \wedge A)^*]_\nu$$
$$- \frac{1}{2n^2}[n.(\partial \wedge A)]^2 - \frac{1}{2n^2}[n.(\partial \wedge C)]^2 + \overline{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu \tau_3 - m)\Psi$$
$$+ \frac{1}{2} m_C^2 C_\mu^2$$

Where $$m_C^2 = \frac{-M^2}{\lambda}(\frac{4\pi}{g})^2$$

H. Suganuma, S. Sasaki, H. Toki, Nucl. Phys. B 435 (1995) 207-240

QUARK CONFINEMENT POTENTIAL

- The static potential between heavy quarks can be obtained from the vacuum energy where the static quark and antiquark exist. Here, we take a quench approximation, i.e. we neglect the quantum effect of the light quarks. Information on confinement is included in the gluon propagator, which leads to the strong interaction in the infrared region. The vacuum energy V(j) in the presence of the static quark sources j is obtained from

$$Z = \langle 0 | e^{i \int j_\mu A^\mu d^4x} | 0 \rangle = N \int DA_\mu DC_\mu e^{i \int (L + j_\mu A^\mu) d^4x} = e^{-iV(j) \int dt}$$

Integrating out the gauge fields , it becomes

$$L_j = -\frac{1}{2} j_\mu D^{\mu\nu} j_\nu$$

Where $D_{\mu\nu}$ is the propagator of the diagonal gluons in the Lorentz gauge $\partial_\mu A^\mu = 0$.

QUARK CONFINEMENT POTENTIAL (continued)

$$X^{\mu\nu} \equiv \frac{1}{n^2}\epsilon_{\lambda}^{\mu\alpha\beta}\epsilon^{\lambda\nu\gamma\delta}n_\alpha n_\gamma \partial_\beta \partial_\delta$$

$$D_{\mu\nu} = \frac{1}{\partial^2}[g_{\mu\nu} + (\alpha_g - 1)\frac{\partial_\mu \partial_\nu}{\partial^2}] - \frac{1}{\partial^2}\frac{m_C^2}{\partial^2 + m_C^2}\frac{n^2}{(n.\partial)^2}X_{\mu\nu}$$

The nonperturbative effect is included in the second term

The action will be

$$S_j \equiv \int d^4x L_j$$

$$= \int \frac{d^4k}{(2\pi)^4}\frac{1}{2}j^\mu(-k)[\frac{1}{k^2 - m_C^2}g_{\mu\nu} + \frac{-m_C^2}{k^2 - m_C^2}\frac{n^2}{(n.k)^2}(g_{\mu\nu} - \frac{n_\mu n_\nu}{n^2})]j^\nu(k)$$

QUARK CONFINEMENT POTENTIAL (continued)

- Now consider a system of a heavy quark and anti-quark pair with opposite color charge located at a and b, respectively. The quark current is given by

$$j_\mu(x) = Q\, g_{\mu 0}[\delta^3(x-b) - \delta^3(x-a)],$$

$$j_\mu(k) = Q\, g_{\mu 0}\, 2\pi\delta(k_0)(e^{-ik.b} - e^{-ik.a})$$

$$S_j = -Q^2 \int dt \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}(1-e^{ik.r})(1-e^{-ik.r})[\frac{1}{k^2+m_C^2} + \frac{m_C^2}{k^2+m_C^2}\frac{1}{(n.k)^2}]$$

Where n is a unit vector and r = b - a is the relative vector between the quark and the antiquark. then, the static quark potential is divided into two parts

QUARK CONFINEMENT POTENTIAL (continued)

$$V(r;n) = V_{Yukawa}(r) + V_{Linear}(r;n)$$

$$V_{Yukawa}(r) = Q^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2}(1 - e^{ik.r})(1 - e^{-ik.r}) \frac{1}{k^2 + m_C^2} = \frac{-Q^2}{4\pi} \frac{e^{-m_C r}}{r}$$

$$V_{Linear}(r;n) = Q^2 \int \frac{d^3k}{(2\pi)^3} [1 - cos(k.r)] \frac{m_C^2}{k^2 + m_C^2} \frac{1}{(n.k)^2}$$

It should be noted that there appears a physical ultraviolet cutoff in the k_T -integral, and therefore no ultraviolet divergence comes in above equation. Introducing the ultraviolet cutoff m_ϕ corresponding to the core of the flux-tube, we get

$$V_{Linear}(r) = \frac{Q^2 m_C^2}{8\pi^2} \int \frac{dk_r}{k_r^2} [1 - cos(k_r r)] ln \frac{m_\phi^2 + k_r^2 + m_C^2}{k_r^2 + m_C^2}$$

$$= (\frac{Q^2 m_C^2}{8\pi} ln[\frac{m_C^2 + m_\phi^2}{m_C^2}]) r$$

$$V(r) = \frac{-Q^2}{4\pi} \frac{e^{-m_C r}}{r} + \sigma r$$

the string tension between quark and antiquark ➡ $\sigma = \frac{Q^2 m_C^2}{8\pi} ln[\frac{m_C^2 + m_\phi^2}{m_C^2}]$

Conclusion & summary

- Imposing an extra symmetry, called magnetic symmetry, the SU(2) QCD is restricted and magnetic monopoles in the theory are observed.
- We solve the Cho Lagrangian problems by using Zwanaiger dual variables method.
- We obtain a Lagrangian which is the same as the dual Ginsburg Landau Lagrangian, after symmetry breaking and mean field approximation for monopole field.
- Monopole condensation makes magnetic potential C_μ massive and this massive magnetic potential modifies the usual propagator and finally it gives rise to a linear potential.
- One can generalize this method for other gauge groups.
- Using other decompositions like Faddeev-Niemi for gauge fields are also other interesting subject.

The End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