Lorentz Violating Coefficients from Noncommutative Standard Model

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Preface:

- Lorentz & CPT Symmetry
- Standard Model Extension
- Noncommutative space-time
- Noncommutative Standard Model
- Lorentz Violating Coefficients
- Bounds on Noncommutative Scale
- Conclusion and discussion

Lorentz Symmetry

- Lorentz transformations come in two basic types, **rotations** and **boosts**.

There are three possible basic types of rotation, one about each of the three spatial directions.

A boost is a change of velocity. There are also three possible basic types of boost, one along each of the three spatial directions.

CPT Symmetry

The CPT transformation is formed by combining three transformations:

- Charge conjugation converts a particle into its antiparticle.
- Parity transforms an object into its mirror image but turned upside down.
- Time reversal changes the direction of flow of time.

CPT Theorem

If a theory is Lorentz Invariant, the theory has CPT symmetry; and if a theory violates CPT symmetry the theory will be Lorentz Violating.

Greenberg, O.W., Phys. Rev. Lett., 89, 2316021, 2002

Lorentz Violating Extension of Standard Model

The Standard Model (SM) is Lorentz Invariant.

Obtaining Extended SM which violates Lorentz Symmetry: Adding any possible terms to the SM Lagrangian. Terms which are even/odd under CPT transformation and maintain gauge invariance and renormalizability.

Standard Model

- $$SU(3)_C \times SU(2)_L \times U(1)$$

The SM action

$$\mathcal{L}_{lepton} = \frac{1}{2} i\overline{L_A}\gamma^\mu D_\mu L_A + \frac{1}{2} i\overline{R_A}\gamma^\mu D_\mu R_A$$

$$\mathcal{L}_{Yukawa} = -[(G_L)_{AB}\overline{L}_A \phi R_B + (G_D)_{AB}\overline{Q_A}\phi^c D_B] + h.c.$$

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$$

$$\mathcal{L}_{gauge} = -\frac{1}{2}Tr(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}Tr(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4*}Tr(f_{\mu\nu}f^{\mu\nu})$$

SME Lagrangian

$$\mathcal{L}_{lepton}^{CPT-even} = \frac{i}{2}(c_L)_{\mu\nu}\overline{L}\gamma^{\mu}D^{\nu}L + \frac{i}{2}(c_R)_{\mu\nu}\overline{R}\gamma^{\mu}D^{\nu}R$$

$$\mathcal{L}_{lepton}^{CPT-odd} = -(a_L)_{\mu AB}\overline{L}_A\gamma^{\mu}L_B - (a_R)_{\mu AB}\overline{R}_A\gamma^{\mu}R_B$$

Fermion sector

$$\mathcal{L}_{gauge}^{CPT-even} = -\frac{1}{2}(k_W)_{\mu\nu\rho\sigma}Tr(W^{\mu\nu}W^{\rho\sigma})] - \frac{1}{2}(k_F)_{\mu\nu\rho\sigma}Tr(F^{\mu\nu}F^{\rho\sigma})$$

$$\begin{aligned}\mathcal{L}_{gauge}^{CPT-odd} &= (k_3)_{\kappa}\epsilon^{\kappa\lambda\mu\nu}Tr(G_{\lambda}G_{\mu\nu} + \frac{2}{3}igG_{\lambda}G_{\mu}G_{\nu}) \\ &+ (k_2)_{\kappa}\epsilon^{\kappa\lambda\mu\nu}Tr(B_{\lambda}B_{\mu\nu} + \frac{2}{3}igB_{\lambda}B_{\mu}B_{\nu}) \\ &+ (k_1)_{\kappa}\epsilon^{\kappa\lambda\mu\nu}Tr(A_{\lambda}A_{\mu\nu}) + (k_0)_{\kappa}B^{\kappa}\end{aligned}$$

gauge sector

$$\mathcal{L}_{Higgs}^{CPT-even} = \frac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_{\mu}\phi^{\dagger})D_{\nu}\phi + h.c. - \frac{1}{2}(k_{\phi B})^{\mu\nu}\phi^{\dagger}\phi B_{\mu\nu} - \frac{1}{2}(k_{\phi W})^{\mu\nu}\phi^{\dagger}W_{\mu\nu}\phi$$

$$\mathcal{L}_{HIggs}^{CPT-odd} = i(k_{\phi})^{\mu}\phi^{\dagger}D_{\mu}\phi + h.c.$$

Higgs sector

$$L_{Yukawa}^{CPT-even} = \frac{-1}{2}[(H_L)_{\mu\nu}\overline{L}\phi\sigma^{\mu\nu}R + h.c.$$

Yukawa sector

SME could emerge from any fundamental theory that generates SM and contains Spontaneous Lorentz and CPT violation.

Noncommutative space-time

Noncommutative space-time ⟹ Distinguished direction in space

In Noncommutative space we have

$$[x^\mu, x^\nu] \equiv x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$$

The NC scale is said to be

$$\theta_{\mu\nu} = \frac{1}{\Lambda_{NC}^2}$$

The Star product

$$(f \star g)(x) = \exp\left(\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\right) f(x)g(y)\Big|_{y\to x},$$

$$f \star g = f \cdot g + \frac{i}{2}\theta^{\mu\nu}(x)\partial\mu f \cdot \partial\nu g + \mathcal{O}(\theta^2)$$

Noncommutative Standard Model

- The NCSM action is written as

$$
\begin{aligned}
S_{NCSM} \;=\; & \int d^4x \sum_{i=1}^{3} \overline{\widehat{\Psi}}_L^{(i)} \star i\widehat{\not{D}}\widehat{\Psi}_L^{(i)} + \int d^4x \sum_{i=1}^{3} \overline{\widehat{\Psi}}_R^{(i)} \star i\widehat{\not{D}}\widehat{\Psi}_R^{(i)} \\
& - \int d^4x \frac{1}{2g'} \mathbf{tr_1} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} - \int d^4x \frac{1}{2g} \mathbf{tr_2} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\
& - \int d^4x \frac{1}{2g_S} \mathbf{tr_3} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} + \int d^4x \Big(\rho_0(\widehat{D}_\mu \widehat{\Phi})^\dagger \star \rho_0(\widehat{D}^\mu \widehat{\Phi}) \\
& - \mu^2 \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) - \lambda \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \star \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \Big) \\
& + \int d^4x \Big(- \sum_{i,j=1}^{3} W^{ij} \Big((\overline{\widehat{L}}_L^{(i)} \star \rho_L(\widehat{\Phi})) \star \widehat{e}_R^{(j)} + \overline{\widehat{e}}_R^{(i)} \star (\rho_L(\widehat{\Phi})^\dagger \star \widehat{L}_L^{(j)}) \Big) \\
& - \sum_{i,j=1}^{3} G_u^{ij} \Big((\overline{\widehat{Q}}_L^{(i)} \star \rho_{\bar{Q}}(\widehat{\bar{\Phi}})) \star \widehat{u}_R^{(j)} + \overline{\widehat{u}}_R^{(i)} \star (\rho_{\bar{Q}}(\widehat{\bar{\Phi}})^\dagger \star \widehat{Q}_L^{(j)}) \Big) \\
& - \sum_{i,j=1}^{3} G_d^{ij} \Big((\overline{\widehat{Q}}_L^{(i)} \star \rho_Q(\widehat{\Phi})) \star \widehat{d}_R^{(j)} + \overline{\widehat{d}}_R^{(i)} \star (\rho_Q(\widehat{\Phi})^\dagger \star \widehat{Q}_L^{(j)}) \Big) \Big),
\end{aligned}
$$

Calmet, X., Jurco, B., Schupp, P., Wess, K. and Wohlgenannt, M., Eur. Phys. J. C 23, 363, 2002

Electroweak Sector

- Fermions Sector

$$S_{matter,fermion} = \int d^4x \sum \overline{L} i\gamma_\mu D^\mu L - \frac{1}{4}\theta^{\mu\nu} \int d^4x \sum \overline{L}(\acute{g} f_{\mu\nu} + gF_{\mu\nu}) i\gamma_\mu D^\mu L$$
$$-\frac{1}{2}\theta^{\mu\nu} \int d^4x \sum \overline{L}\gamma^\alpha(\acute{g} f_{\mu\alpha} + gF_{\mu\alpha}) iD_\nu L + \int d^4x \sum \overline{R} i\gamma_\mu D^\mu R$$
$$-\frac{1}{4}\theta^{\mu\nu} \int d^4x \sum \overline{R}(\acute{g} f_{\mu\nu}) i\gamma_\mu D^\mu R - \frac{1}{2}\theta^{\mu\nu} \int d^4x \sum \overline{R}\gamma^\alpha(\acute{g} f_{\mu\alpha}) iD_\nu R$$

- Gauge Sector

$$\begin{aligned} S_{gauge} &= -\int d^4x \frac{1}{2g'} \mathbf{tr_1} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} - \int d^4x \frac{1}{2g} \mathbf{tr_2} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} - \int d^4x \frac{1}{2g_S} \mathbf{tr_3} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\ &= -\frac{1}{4} \int d^4x \, f_{\mu\nu} f^{\mu\nu} \\ &\quad -\frac{1}{2} \mathrm{Tr} \int d^4x \, F^L_{\mu\nu} F^{L\mu\nu} - g\, \theta^{\mu\nu} \, \mathrm{Tr} \int d^4x \, F^L_{\mu\rho} F^L_{\nu\sigma} F^{L\rho\sigma} \\ &\quad -\frac{1}{2} \mathrm{Tr} \int d^4x \, F^S_{\mu\nu} F^{S\mu\nu} + \frac{1}{4} g_S \, \theta^{\mu\nu} \, \mathrm{Tr} \int d^4x \, F^S_{\mu\nu} F^S_{\rho\sigma} F^{S\rho\sigma} \\ &\quad -g_S \, \theta^{\mu\nu} \, \mathrm{Tr} \int d^4x \, F^S_{\mu\rho} F^S_{\nu\sigma} F^{S\rho\sigma} + \mathcal{O}(\theta^2). \end{aligned} \tag{31}$$

- Higgs Sector

$$\begin{aligned} S_{Higgs} &= \int d^4x \Bigg((D_\mu^{SM}\phi)^\dagger D^{SM\mu}\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)(\phi^\dagger\phi) \Bigg) \\ &+ \int d^4x \Bigg((D_\mu^{SM}\phi)^\dagger \left(D^{SM\mu}\rho_0(\phi^1) + \frac{1}{2}\theta^{\alpha\beta}\partial_\alpha V^\mu \partial_\beta\phi + \Gamma^\mu\phi \right) \\ &+ \left(D_\mu^{SM}\rho_0(\phi^1) + \frac{1}{2}\theta^{\alpha\beta}\partial_\alpha V_\mu \partial_\beta\phi + \Gamma_\mu\phi \right)^\dagger D^{SM\mu}\phi \\ &+ \frac{1}{4}\mu^2\theta^{\mu\nu}\phi^\dagger(g' f_{\mu\nu} + gF^L_{\mu\nu})\phi - \lambda i\theta^{\alpha\beta}\phi^\dagger\phi(D_\alpha^{SM}\phi)^\dagger(D_\beta^{SM}\phi) \Bigg) + \mathcal{O}(\theta^2) \end{aligned}$$

where

$$\Gamma_\mu = -iV_\mu^1 = i\frac{1}{4}\theta^{\alpha\beta}\{g'\mathcal{A}_\alpha + gB_\alpha, g'\partial_\beta\mathcal{A}_\mu + g\partial_\beta B_\mu + g' f_{\beta\mu} + gF^L_{\beta\mu}\}$$

- Yukawa Sector

$$
\begin{aligned}
S_{Yukawa} &= S^{SM}_{Yukawa} - \int d^4x \Bigg(\sum_{i,j=1}^{3} W^{ij} \Big((\bar{L}^i_L \phi) e^{1j}_R + (\bar{L}^i_L \rho_L (\phi^1)) e^j_R \\
&+ (\bar{L}^{1i}_L \phi) e^j_R + i\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha L^i_L \partial_\beta \phi e^j_R + \bar{e}^i_R (\phi^\dagger L^{1j}_L) \\
&+ \bar{e}^i_R (\rho_L (\phi^1)^\dagger L^j_L) + \bar{e}^{1i}_R (\phi^\dagger L^j_L) + i\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha e^i_R \partial_\beta \phi^\dagger L^j_L \Big) \\
&- \sum_{i,j=1}^{3} G^{ij}_u \Big((\bar{Q}^i_L \bar{\phi}) u^{1j}_R + (\bar{Q}^i_L \rho_{\bar{Q}} (\bar{\phi}^1)) u^j_R + (\bar{Q}^{1i}_L \bar{\phi}) u^j_R \\
&+ i\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha Q^i_L \partial_\beta \bar{\phi} u^j_R + \bar{u}^i_R (\bar{\phi}^\dagger Q^{1j}_L) + \bar{u}^i_R (\rho_{\bar{Q}} (\bar{\phi}^1)^\dagger Q^j_L) \\
&+ \bar{u}^{1i}_R (\bar{\phi}^\dagger Q^j_L) + i\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha u^i_R \partial_\beta \bar{\phi}^\dagger Q^j_L \Big) \\
&- \sum_{i,j=1}^{3} G^{ij}_d \Big((\bar{Q}^i_L \phi) d^{1j}_R + (\bar{Q}^i_L \rho_Q (\phi^1)) d^j_R + (\bar{Q}^{1i}_L \phi) d^j_R \\
&+ i\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha Q^i_L \partial_\beta \phi d^j_R + \bar{d}^i_R (\phi^\dagger Q^{1j}_L) + \bar{d}^i_R (\rho_Q (\phi^1)^\dagger Q^j_L) \\
&+ \bar{d}^{1i}_R (\phi^\dagger Q^j_L) + i\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha \bar{d}^i_R \partial_\beta \phi^\dagger Q^j_L \Big) \Bigg) + \mathcal{O}(\theta^2),
\end{aligned}
$$

Lorentz Violating Coefficients

- Fermions Sector coefficients

$$(c_L)_{\mu\nu} = -\frac{1}{2}\theta_\nu{}^\alpha\,(\acute{g} f_{\mu\alpha} + g F_{\mu\alpha})$$

$$(c_R)_{\mu\nu} = -\frac{1}{2}\theta_\nu{}^\alpha\,(\acute{g} f_{\mu\alpha})$$

Combining the 2 above

$$c_{\mu\nu} = \frac{1}{2}c_L + \frac{1}{2}c_R \qquad d_{\mu\nu} = \frac{1}{2}c_R - \frac{1}{2}c_L$$

$$c_{\mu\nu} = -\frac{1}{2}\theta_\nu{}^\alpha\,(2\,\acute{g} f_{\mu\alpha} + g F_{\mu\alpha})$$

$$d_{\mu\nu} = \frac{1}{2}\theta_\nu{}^\alpha g F_{\mu\alpha}$$

After SSB ⇒

$$c_{\mu\nu} = -\frac{3}{2}\frac{\theta_\nu^\alpha g \acute{g} A^b_{\mu\alpha}}{\sqrt{g^2 + \acute{g}^2}}$$

$$d_{\mu\nu} = \frac{1}{2}\frac{\theta_\nu^\alpha g \acute{g} A^b_{\mu\alpha}}{\sqrt{g^2 + \acute{g}^2}}$$

Gauge Sector

-

$$\left(k^{W^+W^-}\right)_{\mu\nu\rho\sigma} = \left(k^{W^-W^+}\right)_{\mu\nu\rho\sigma} = \theta_{\mu\nu}A^{b}_{\rho\sigma}$$

$$(k_F)_{\alpha\beta\mu\nu} = \{-q\, f_{\alpha}{}^{\lambda}\,\theta_{\lambda\mu}\,\eta_{\beta\nu} - (\alpha \leftrightarrow \beta)\}$$

$$+\{\frac{1}{2} q\, f_{\alpha\mu}\,\theta_{\beta\nu} - (\mu \leftrightarrow \nu)\} + \{-\frac{1}{4} q\, f_{\alpha\beta}\,\theta_{\mu\nu} + (\alpha\beta \leftrightarrow \mu\nu)\}$$

Higgs Sector

- Higgs Sector

$$(k_{\phi\phi})_{\mu\nu} = -\theta^{\alpha}_{\mu} D_{\nu} Z_{\alpha} - i\theta^{\alpha\beta} Z_{\alpha} V_{\beta} \eta_{\mu\nu} + \frac{i}{2}\theta^{\alpha\beta} Z_{\alpha} Z_{\beta} \eta_{\mu\nu} + \theta^{\alpha}_{\mu} \partial_{\alpha} V_{\mu} - 2i\lambda\phi^{\dagger}\phi\theta_{\mu\nu} + 2i\theta^{\alpha}_{\mu} Z_{\alpha} V_{\nu} - i\theta^{\alpha}_{\mu} V_{\nu} Z_{\alpha}$$

$$(k_{\phi})^{\dagger}_{\mu} = i\Gamma^{\mu} + \frac{1}{2}\theta^{\alpha\beta} D_{\mu} Z_{\alpha} V_{\beta} + \frac{1}{2}\theta^{\alpha\beta} Z_{\alpha} D_{\mu} V_{\beta} - \frac{1}{4}\theta^{\alpha\beta} D_{\mu}(Z_{\alpha} Z_{\beta}) - \frac{1}{2}\theta^{\alpha\beta}\partial_{\alpha} V_{\mu} V_{\beta} - \frac{i}{2}\theta^{\alpha}_{\mu}\mu^2 Z_{\alpha} - i\lambda\theta^{\alpha}_{\mu} Z_{\alpha}\phi^{\dagger}\phi$$

$$Z_{\mu} = \acute{g}\mathcal{A}_{\mu} + gB_{\mu}$$

$$(k_{\phi B})_{\mu\nu} = -\frac{1}{2}\mu^2 \acute{g}\theta_{\mu\nu} \qquad (k_{\phi W})_{\mu\nu} = -\frac{1}{2}\mu^2 g\theta_{\mu\nu}$$

Yukawa Sector

$$S_{Yukawa} = S_{Yukawa}^{SM} - \int d^4x \Big(\sum_{i,j=1}^{3} W^{ij} \Big((Y_1 \bar{L}_L^i \phi) e_R^j + (Y_2^\mu \bar{L}_L^i \phi) D_\mu e_R^j$$

$$+ \quad (Y_3^{\mu\nu} D_\mu \bar{L}_L^{1i} D_\nu \phi) e_R^j + D_\mu \bar{L}_L^i \phi e_R^j Y_4^\mu + (Y_5^\mu \bar{L}_L^i D_\mu \phi) e_R^j \Big)$$

Where

$$Y_1 = -i\theta^{\mu\nu}(\frac{1}{4} g\acute{g} \mathcal{A}_\mu B_\nu) \qquad Y_2^\nu = \frac{-1}{2} \acute{g} \theta^{\mu\nu} \mathcal{A}_\mu$$

$$Y_3^{\mu\nu} = \frac{i}{2}\theta^{\mu\nu} \qquad Y_4^\mu = -\frac{1}{2}\theta^{\mu\nu}(\acute{g}\mathcal{A}_\mu)$$

$$Y_5^\mu = \theta^{\mu\nu}(\acute{g}\mathcal{A}_\mu + gB_\mu)$$

Bounds on Noncommutative Scale

Coefficient	Exp. Value	System	Background Field	Λ
$(c_{tt})_{\mu\nu}$	10^{-15}	Collider	4-8 T	0.42-0.6 GeV
$(c_{xx})_{\mu\nu}$	10^{-11}	1S-2S Transition	mG-0.5 T	0.67keV-1.5 MeV
$c_{\mu\nu}$	2×10^{-16}	Microwave Resonator	0.5 T	0.33 GeV
$c_{\mu\nu}$	8×10^{-15}	Optical Resonator	1.7 T	0.1 GeV
$c_{\mu\nu}$	10^{-15}	Astrophysics	G	1 MeV
$c_{\mu\nu}$	10^{-15}	Astrophysics(SN)	10^{15}G	100 TeV
$d_{\mu\nu}$	10^{-17}	Astrophysics	G	10 MeV
$d_{\mu\nu}$	10^{-25}	Atomic Clock (Cs)	mT	0.39 TeV
$(c_L)_{\mu\nu}$	10^{-27}	ICE CUBE	mG-G	54 GeV-1.7 TeV
$(k_f)_{\mu\nu\rho\sigma}$	10^{-16}	Microwave Resonator	0.5 T	0.10 GeV
$(k_f)_{\mu\nu\rho\sigma}$	10^{-31}	CMB	μG	10 TeV
$(k_f)_{\mu\nu\rho\sigma}$	10^{-37}	Astrophysics(GRB)	10^{16}G	10^{16} GeV

Data from hep-ph/0801.0287

Coefficient	Exp. Value	System	Background Field	Λ
$k_{\phi B}$	10^{-16}	Cosmological	-	10^6 Tev
$k_{\phi W}$	10^{-16}	Cosmological	-	10^6 Tev
k_{ϕ}	10^{-31}	Xe-He Masor	1 T	10^3 Tev
$k_{\phi\phi}$	10^{-27}	Clock Comparison	1 T	10^{12} Tev
$k_{\phi\phi}$	10^{-13}	H^- ion, $\overline{p}$ comparison	1 T	10^5 Tev

Conclusion

- NC field theory has been used as a source to find LV coefficients.
- From experimental data on LV coefficients various bounds has been found for NC scale.
- The bound on NC scale varies for different kind of experiment.
- Data results from ICE CUBE leads to a favorite bound on NC scale
- Data results for Higgs Sector give NC scale about the Planck Scale
- There are LV terms in Yukawa Sector which differ from predicted coefficients in SME; while the SME coefficient may be found using loop calculations
- There is a new coefficient in CPT-even part of Higgs Lagrangian
- There is no CPT-odd term as was expected

Thanks