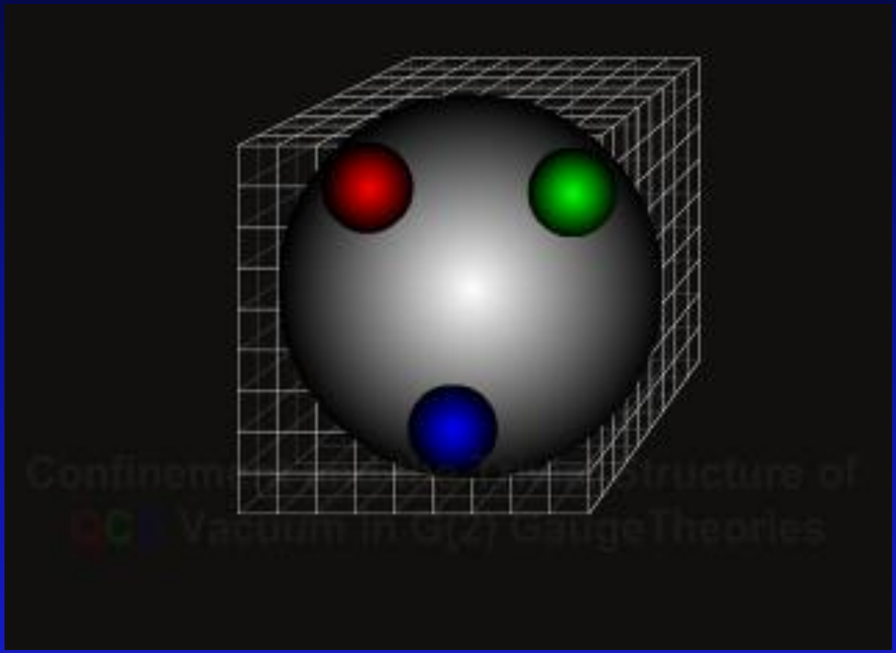
Confinement and the Trivial Structure of QCD Vacuum in G(2) Gauge Theories



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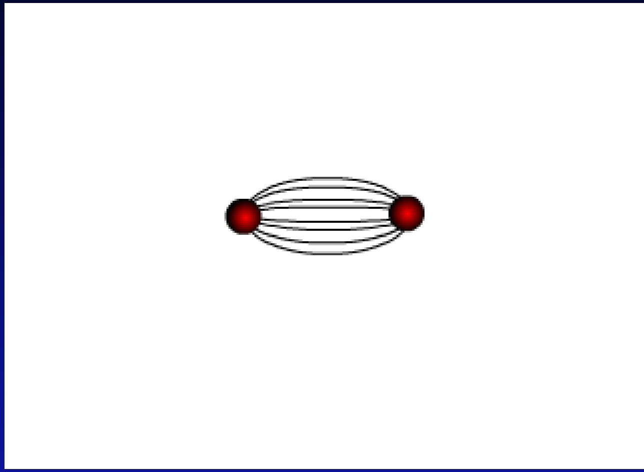
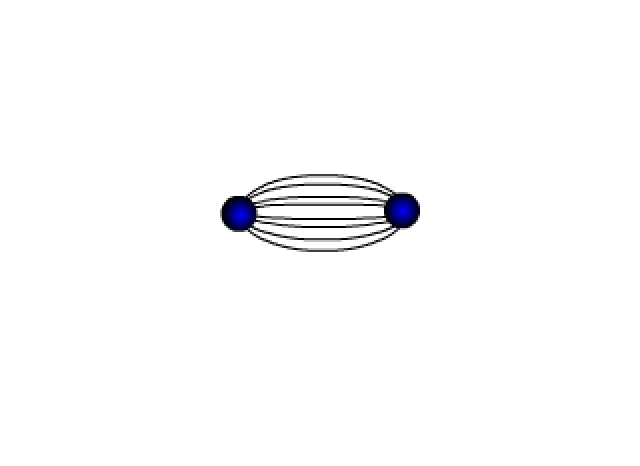
Confinement and the Trivial Structure of QCD Vacuum in G(2) Gauge Theories

- Introduction
- Thick Center Vortex Model
- The Trivial Center Effect in Vacuum Structure
- G(2) Gauge Theory and Thick Center Vortex
- Conclusion

Quark Confinement



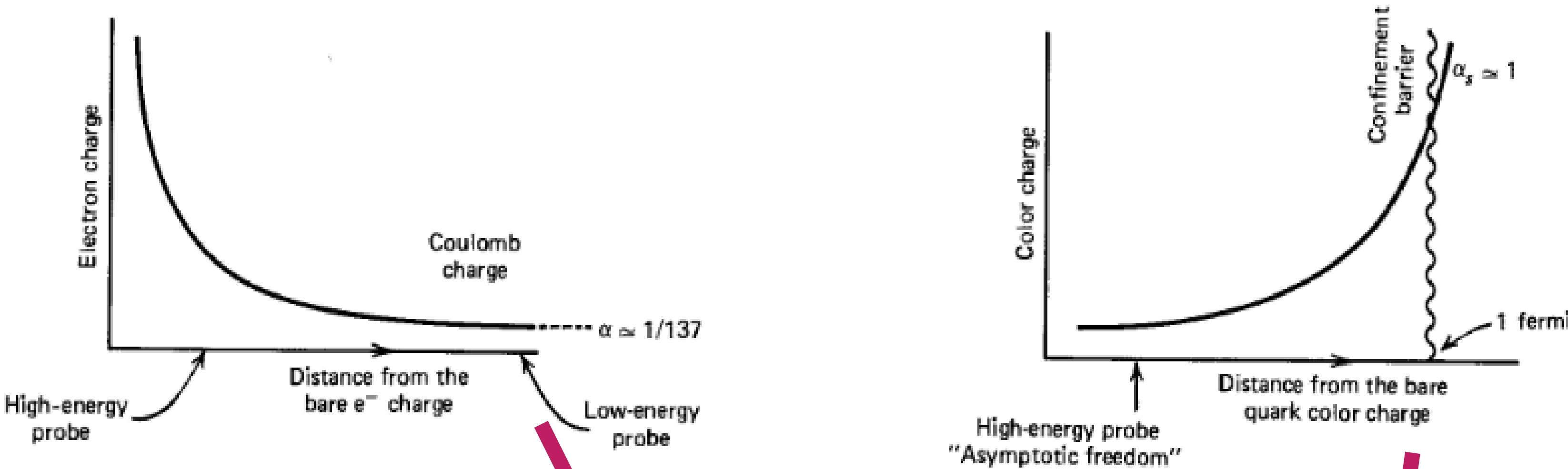


- In QCD the electric lines must form into tubes of conserved flux (Regge Trajectories).
- A flux tube is a real physical object carrying a finite energy per unit length.
- This is the storage medium for the linearly rising inter quark potential.
- Confinement : despite the possibility of existence of free quark in Lagrangian of QCD ,it is not observed freely in the spectrum .

- In usual electromagnetism the electric field lines thus produced spread and give rise to the inverse square law Coulombic field.
- If in our theory we can somehow eliminate massless field, then a Coulombic spreading will no longer be a solution to the equation.
- A simple model for this phenomenon is a type two superconductor containing magnetic monopole impurities

Linear potential → Quark Confinement

Screening of the electric and color charge in QFT



Perturbative QFT

Non perturbative

Lattice gauge Theory
Phenomenological Models

Picture from "Quarks and Leptons,F.Halzen,A.Martin"

Phenomenological Models

1-Center vortex model of t'Hooft

G.'t Hooft, Nucl. Phys. B**138** (1978) **1**.

G. 't Hooft, Nucl. Phys. B**153** (1979) 141.

2-Generalized to thick center vortex model (casimir scaling due this thickness)

M. Faber, J. Greensite, and ˇS. Olejn´ık, Phys. Rev. D**57** (1998) 2603, hep-lat/9710039.

Vortex

✓Briefly, a center vortex is a topological field configuration which is line-like (in D=3 dimensions) or surface-like (in D=4 dimensions) having some finite thickness.

✓Creation of a center vortex can be regarded, outside the line-like or surface-like "core," as a discontinuous gauge transformation of the background, with a discontinuity associated with the gauge group center.

Creation of a center vortex linked to a Wilson loop, in the fundamental representation of SU(N), has the effect of multiplying the Wilson loop by an element of the gauge group center, i.e.

$$W(C) \longrightarrow e^{\frac{i2\pi n}{N}} W(C), \quad n = 1, 2, \ldots, N-1$$

✓The vortex theory, in essence, states that the area law for Wilson loops is due to quantum fluctuations in the number of center vortices linking the loop.

✓Acceptable results for casimir scaling in intermediate region and true N-ality in asymptotic region

J. Greensite, Prog. Part. Nucl. Phys. 51(2003) 1.

S. Deldar, Phys. Rev. D62 (2000) 034509

S. Deldar, S. Rafibakhsh, Phys.Rev.D76: (2007) 094508

✓ Non trivial center element was considered

✓ What about gauge group with out non trivial center

Greensite, K. Langfeld, ˇS. Olejn´ık, H. Reinhardt and T. Tok, Phys. Rev. D 75, (2007) 034501

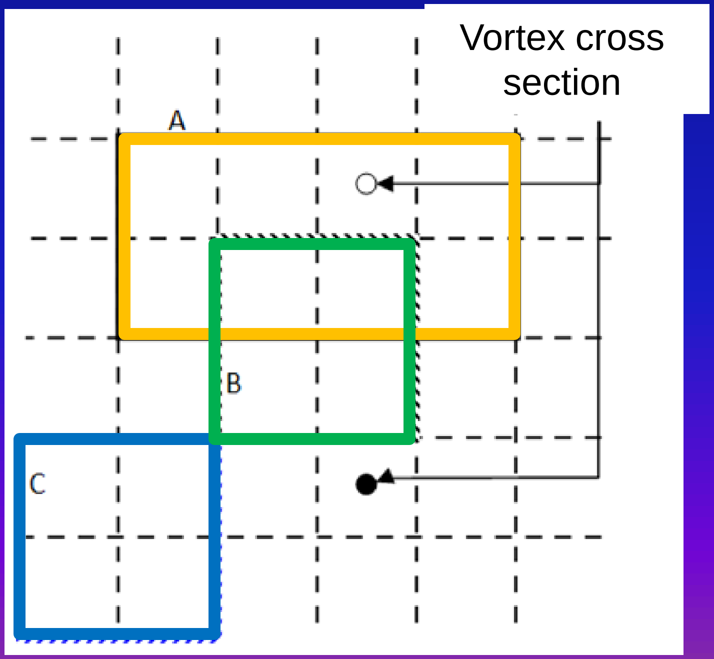
Center Vortex Model

- If, for the moment, we ignore the finite radii of the vortex tubes , then the effect of vortices on a Wilson loop is simple: For every instance where the minimal surface is pierced by a center vortex, insert a center element –I somewhere along the loop ,i.e.

$$W(C) = Tr(UU...U) \longrightarrow Tr(UU...(-\mathrm{I})...U)$$

Wilsoon loop

Link varible



[2] M. Faber, J. Greensite, and ˇS. Olejn´ık, Phys. Rev. D**57** (1998) 2603, hep-lat/9710039.

The effect of creating a center vortex piercing the minimal area of a Wilson loop may be represented by the insertion of a unitary matrix G at some point along the loop

$$\{H_i, i = 1, \ldots, N-1\}$$

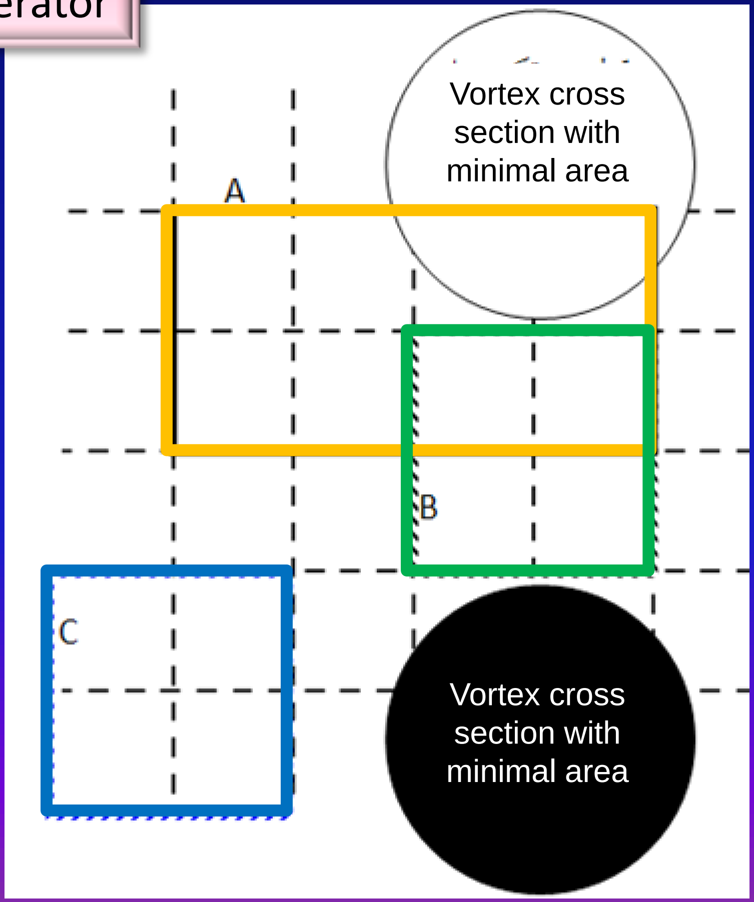
Cartan subalgebra generator

Changes between a maximum and non trivial center element

$$G_r[\vec{\alpha}] = \frac{1}{d_r} Tr \exp[i\, \vec{\alpha}.\vec{H}\,]$$

Vortex profile

Dimension of representation



$$W(C) = Tr(UU \ldots U) \longrightarrow Tr(UU \ldots G \ldots U)$$

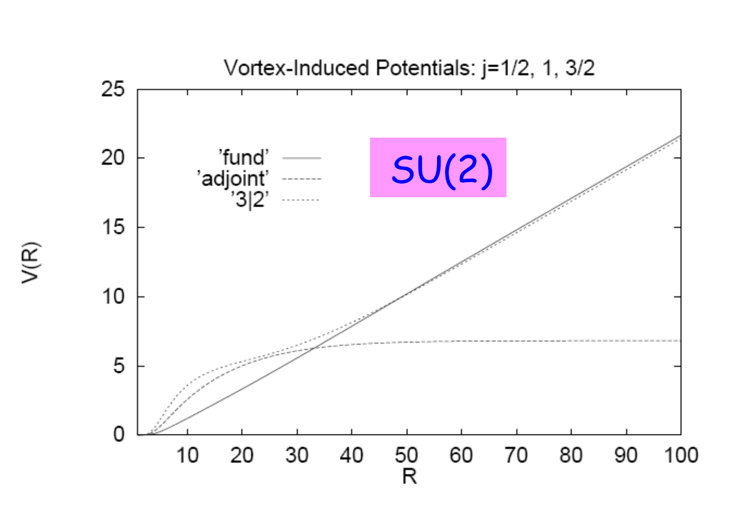
$$V_r(R) = -\lim_{T\to\infty} \log\left[\frac{W_r(R,T+1)}{W_r(R,T)}\right]$$

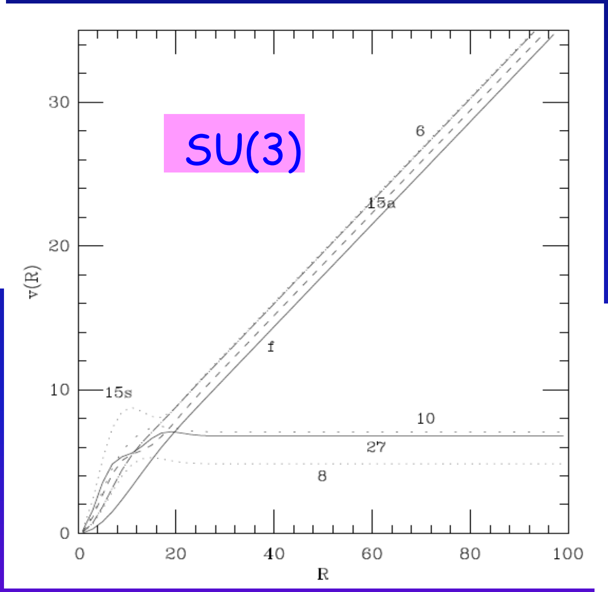
the probability that any given unit area is pierced by a vortex

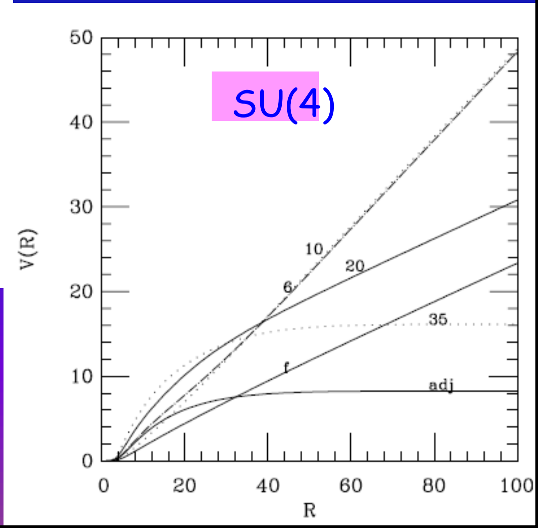
$$V(R) = -\sum_x \ln\left\{1 - \sum_{n=1}^{N-1} f_n \left(1 - \mathrm{Re}\, G_r[\vec{\alpha}_C^n(x)]\right)\right\}$$

Location of center of vortex

- The model of Thick Center Vortices is a simple phenomenological model which has been fairly successful in explaining the linear part of the potentials between static quarks.

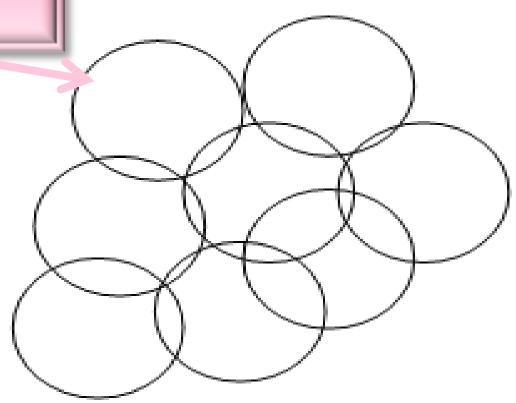




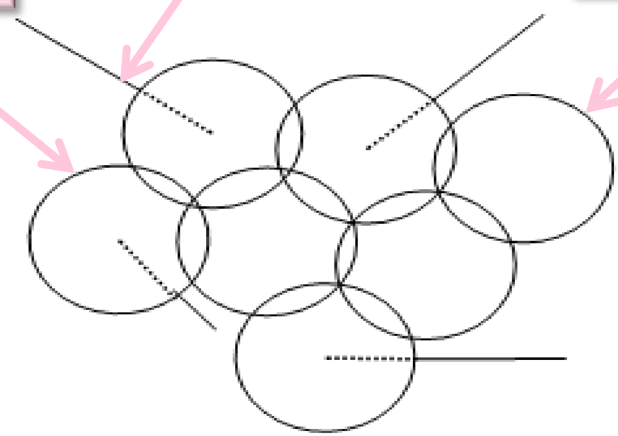


The Trivial Center Effect in Vacuum Structure

- Must consider trivial center.
- They are subject to the constraint that the total magnetic flux, as measured by loop holonomies, corresponds to an element of the gauge group center.
- This picture is sufficient to explain the N-ality dependence of loops which are large compared to the domain size, and Casimir scaling of loops which are small compared to the domain size.



A 2D slice of the D=4 vacuum of G(2) gauge theory. There is only one type of domain, corresponding to the single element of the center subgroup.



A 2D slice of the D=4 Yang-Mills vacuum. Circular regions with (Dirac) lines correspond to *z=−1 domains , circular regions* without lines denote *z = +1 domains.*

- In this model, it is assumed that the effect of a domain (2D cross-section of a vortex) on a planar Wilson loop holonomy is to multiply the holonomy by a group element G.

The probability of pierceing the trivial domain

$f_0 = 0.1$

$$V(R) = -\sum_{n=-\infty}^{\infty} ln\left\{(1-f_0) + f_0 G_r[\vec{\alpha}_R^{\,0}(x)]\right\}$$

$a = 0.05$

$b = 4$

Type of domain

Free parameters

$$\alpha_i^0(x) = N_i^0\left[1 - \tanh\left(ay(x) + \frac{b}{R}\right)\right]$$

The number of cartan generators

The normalisation factor

$$y(x) = \begin{cases} x - R & for \quad |R-x| \le |x| \\ -x & for \quad |R-x| > |x| \end{cases}$$

Cartan subalgebra generators for G(2) gauge group

$$t_3 = \frac{1}{\sqrt{8}}(p_{11} - P_{22} - P_{55} + P_{66})$$

$$(P_{ij})_{\alpha\beta} = \delta_{i\alpha}\delta_{j\beta}$$

$$t_8 = \frac{1}{\sqrt{24}}(p_{11} + P_{22} - 2P_{33} - P_{55} - P_{66} + 2P_{77})$$

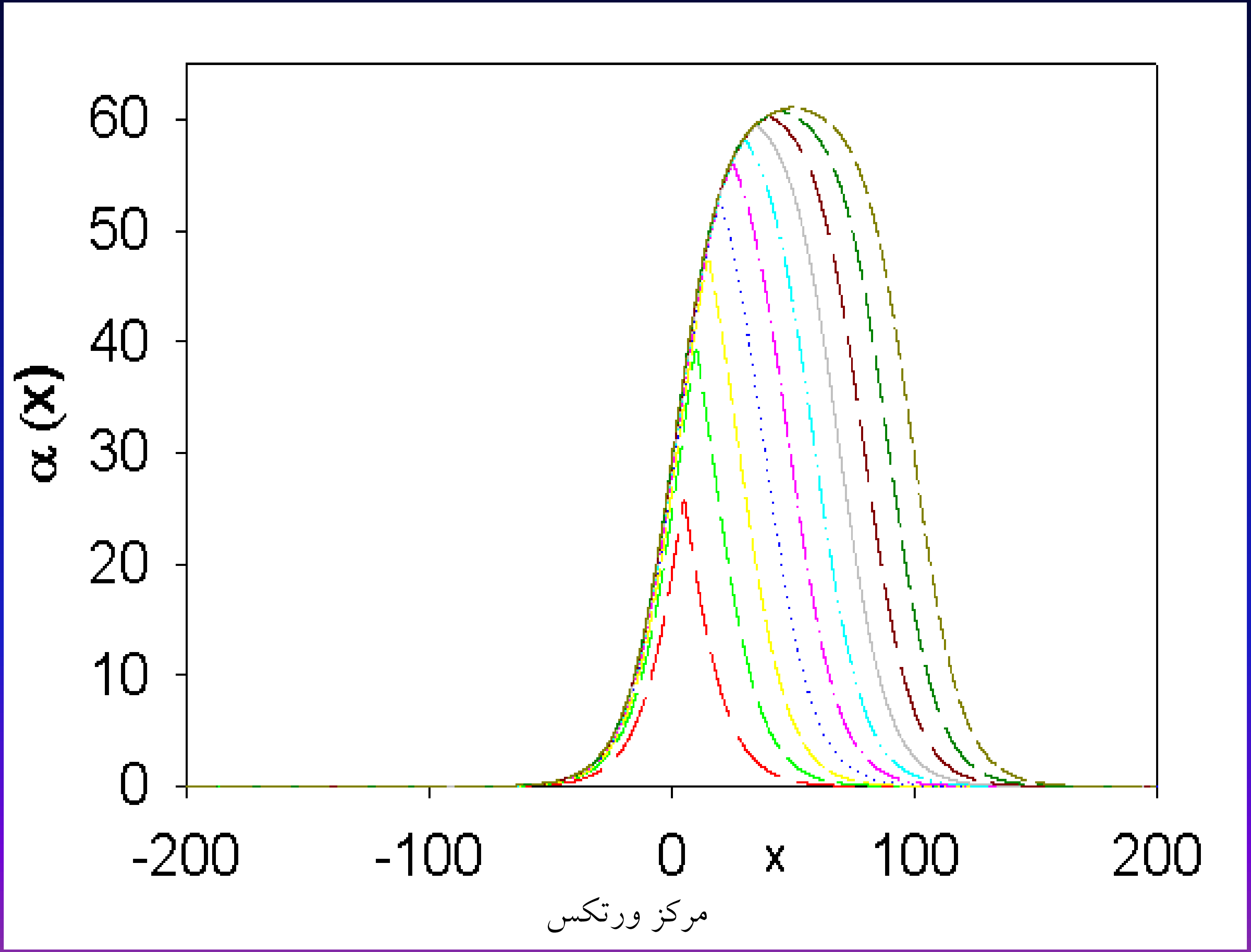
Contain Cartan subalgebra generators for SU(3) gauge group

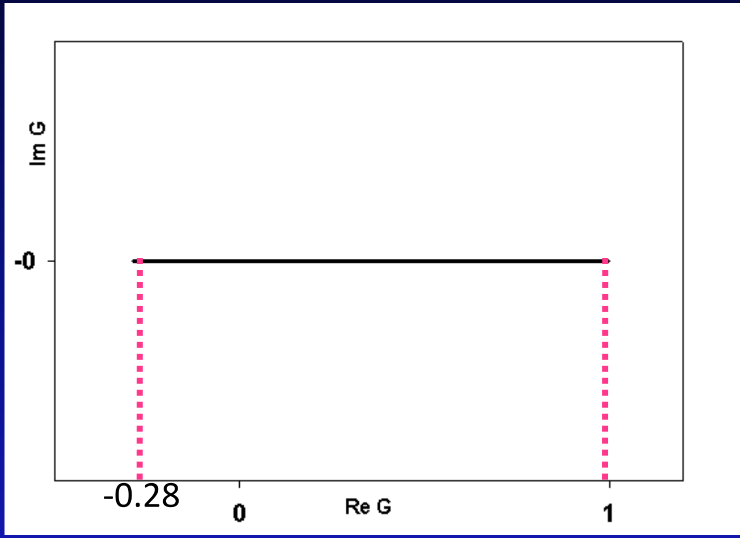
$$H_1 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad H_2 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

Maximum flux

$$e^{i(\alpha_1^{Max} H_1 + \alpha_2^{Max} H_2)} = \mathrm{I} = e^{i2\pi}\mathrm{I} \longrightarrow \alpha_2^{Max} = 2\pi\sqrt{24}, \quad \alpha_1^{Max} = 0$$

$$\alpha_c(\vec{x})$$





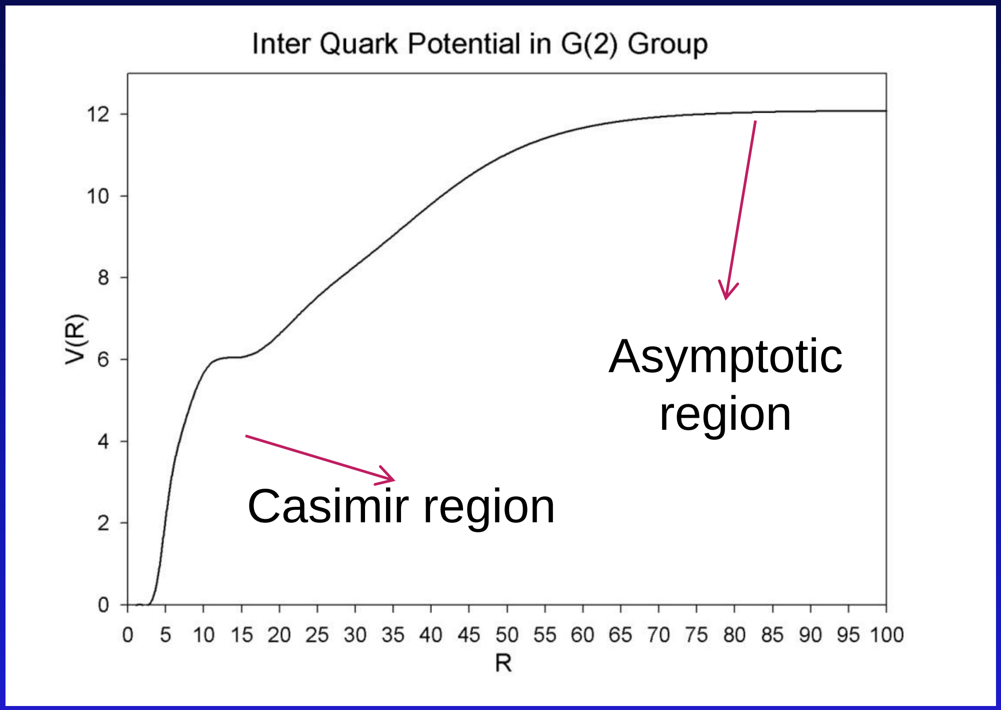
G changes between 1 and −0.28, where 1 comes from the trivial center element and −0.28 can be obtained from the relation between the trace of the group G(2) when it is decomposed into its SU(3) subgroup, and the trace of the SU(3) gauge group lowest value

$$\left(G_r(\alpha)\right)_{Min} = \frac{1}{7}\left(Tr(e^{i\alpha.H})\right)_{Min} = \frac{1}{7}\begin{pmatrix} \left(G'_R(\alpha)\right)_{Min} \times d_r & & \\ & 1 & \\ & & \left(G'_R(\alpha)\right)_{Min} \times d_r \end{pmatrix}$$

$$\left(G_r(\alpha)\right)_{Min} = \frac{1}{7}\left(Tr(e^{i\alpha.H})\right)_{Min} = \frac{1}{7}\left(-1.5+1-1.5\right) = -0.28$$

The potential behavior in G(2) gauge group

$$V(R) = -\sum_{n=-\infty}^{\infty} ln\left\{(1-f_0)+f_0 G_r[\vec{\alpha}_R^0(x)]\right\}$$



Conclusion

- The G(2) group is like a math laboratory.
- The thick center vortex model is applied to the G(2) group. Even though this group dose not have any non trivial center element, but if one uses the domain structures instead of vortices, the model can be applied to the G(2) gauge group, as well. We have obtained linear potentials for both fundamental representations in rough agreement with the lattice results.
- G(2) properties from SU(3) properties.
- …

Center vortex model and the G(2) gauge group S.Deldar,H.Lookzadeh,S.Hosseini Nejad, talk given at the conference "Quark Confinement and the Hadron Spectrum IX", Madrid, Spain, 30th of August-3rd of September 2010

Thanks for Your Attention

Refrences

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- [2] M. Faber, J. Greensite, and ˇS. Olejn´ık, Phys. Rev. D**57** (1998) 2603, hep-lat/9710039.
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- and T. Tok, Phys. Rev. D **75**, (2007) 034501
- [7]A. J. Macfarlane, Int. J. Mod. Phys. A **17**, (2002) 2595.

G(2): generalities