Gauge-flation

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Based on

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&

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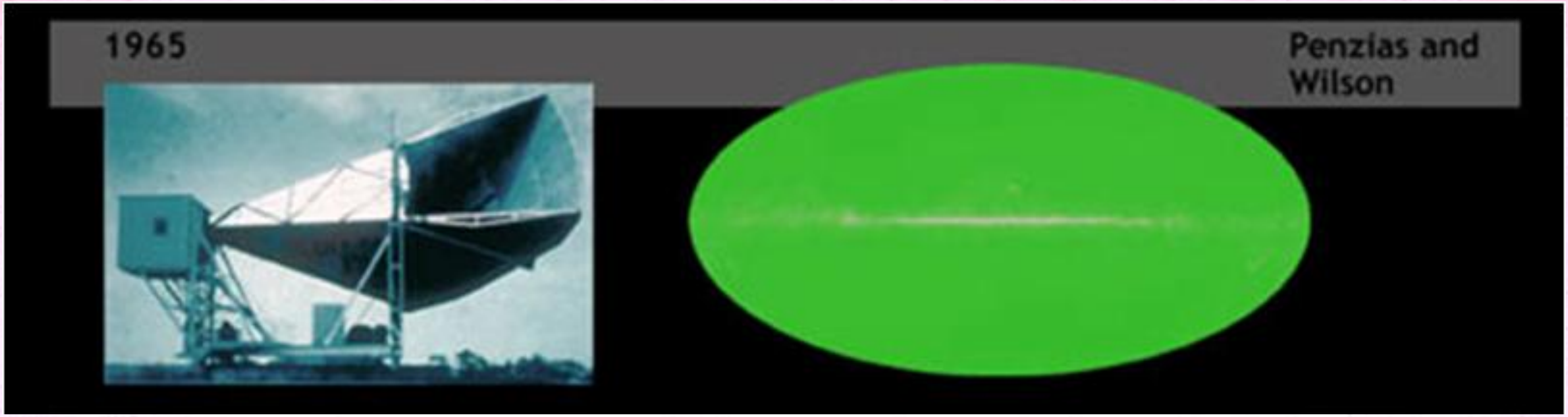
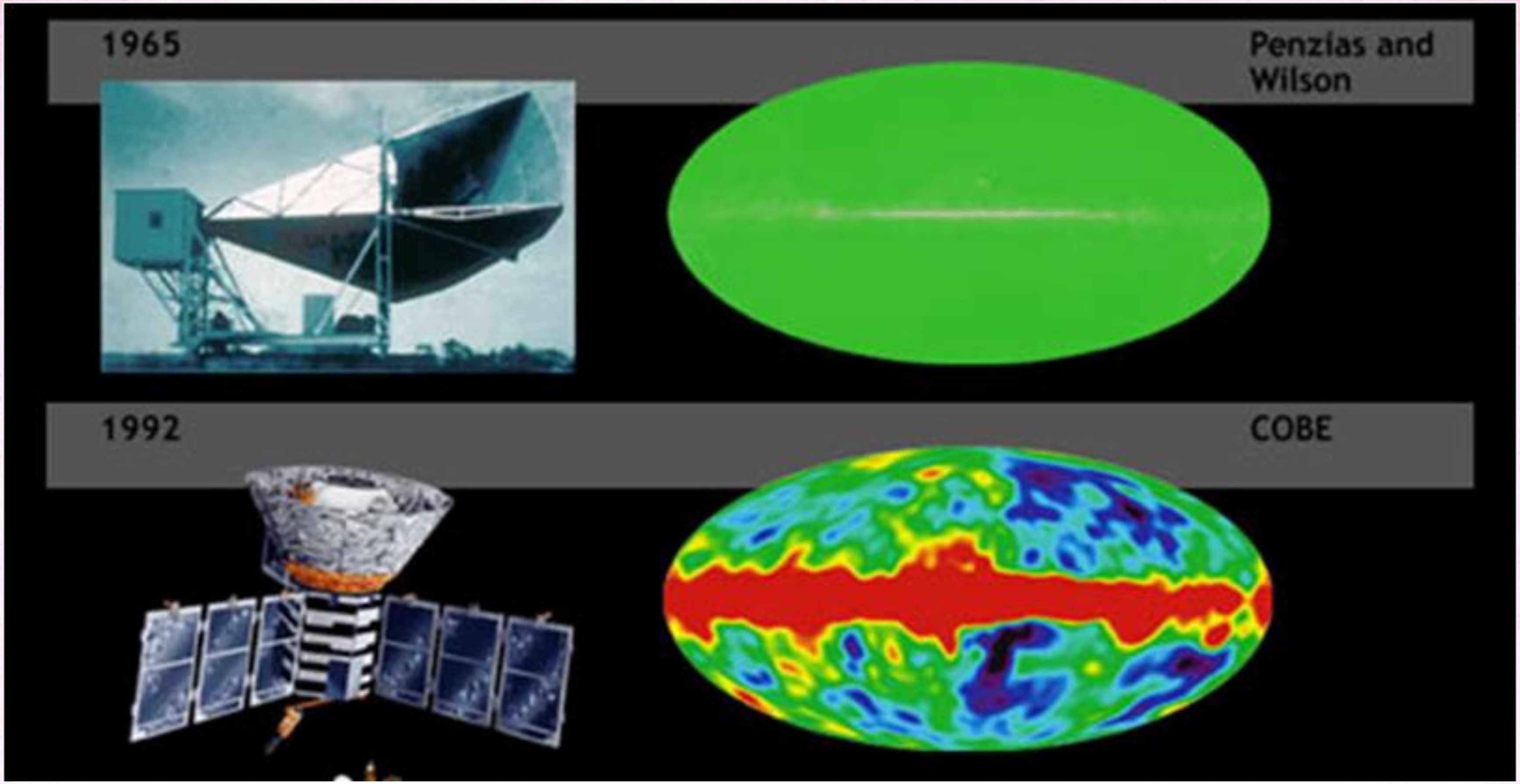
IPP12

The Helix Nebula — NGC 7293 HUBBLESITE



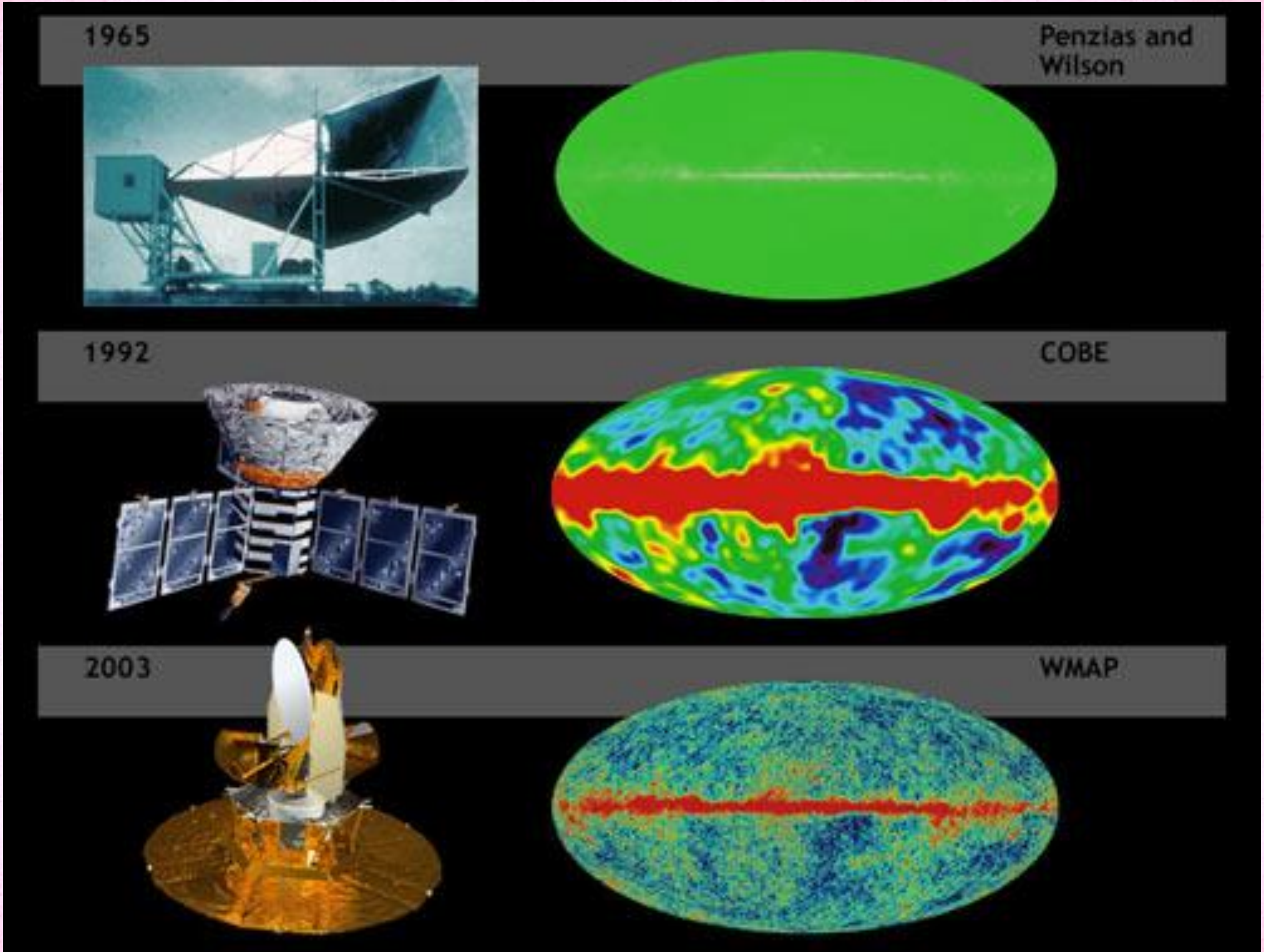
- Cosmic Microwave Background radiation (CMB): blackbody radiation with Temperature $\boxed{T = 2.7\ K}$.

This relic radiation (the 1st snapshot of the universe), turns out to be a gold mine of cosmological information!



In 1992, the Cosmic Background Explorer (COBE) satellite detected cosmological fluctuations in the microwave background temperature:

$$\boxed{\frac{\delta T}{T} \sim 10^{-5}}$$

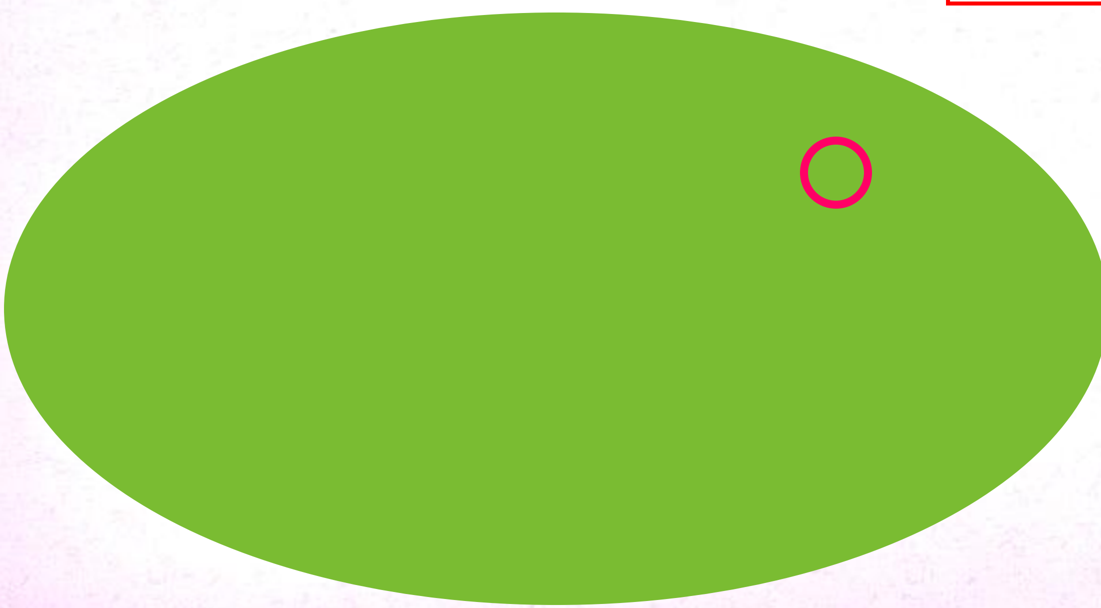

1965
Penzias and Wilson
1992
COBE
2003
WMAP

Observation Vs Standard Model of Cosmology

I) Horizon Problem:

For a universe dominated by $P = w\rho$

comoving Horizon size is $\Rightarrow$ $\boxed{(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}}$



$\boxed{1+3w>0 \quad \ddot{a}<0}$

$\boxed{1+3w<0 \quad \ddot{a}>0}$

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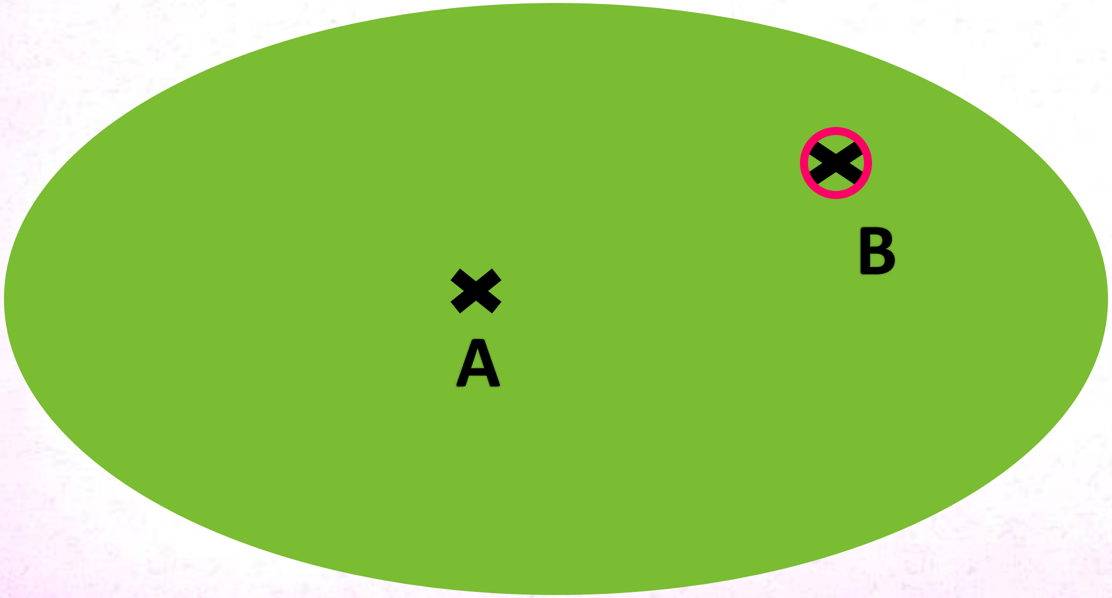
$\boxed{\ddot{a} < 0}$



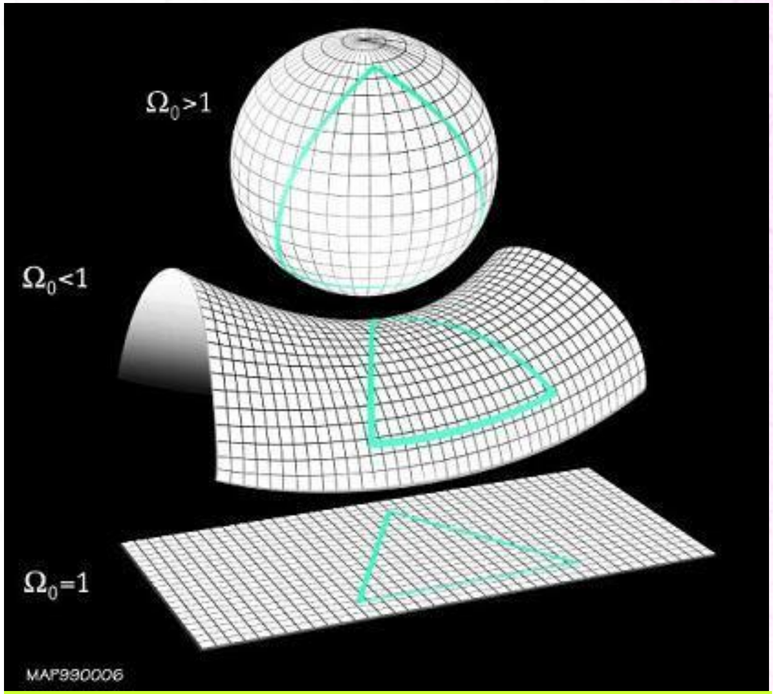
Observation Vs Standard Model of Cosmology

2) Flatness Problem:

- The FRW geometry

$$\Omega \equiv \frac{\rho}{3H^2}$$

$$\boxed{\Omega_k \equiv 1 - \Omega = \frac{-k}{(aH)^2}}$$



- Now, assuming $\boxed{\ddot{a} < 0}$

$$\boxed{\frac{d|1-\Omega|}{d\ln a} > 0 \quad \Leftrightarrow \quad 1 + 3w > 0}$$

Observations of the CMB and large-scale structure find that $\boxed{\Omega_k \sim 0}$!

How inflation can solve them?

- The problems in **the standard big bang cosmology**

$$\ddot{a} < 0$$

  - ***Inflation Idea:***

    There is a stage in the **early universe** with an accelerated expansion

$$\ddot{a} > 0$$

$\boxed{(aH)^{-1}}$ decreases in the inflationary phase!

Inflation must last enough to $\boxed{\dfrac{a_{end}}{a_0} \geq e^{60}}$

What Causes Inflation?

- FRW metric $$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + d\varphi^2)\right)$$
- Friedmann equations
$$H^2 = \frac{\rho}{3} - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P)$$
- acceleration requires:

$$\Rightarrow \quad \boxed{(\rho + 3P) < 0!}$$

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$$H^2 = \frac{\rho}{3} - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P)$$

$$(\rho + 3P) < 0!$$

Inflation condition

Negative pressure/violation of Strong Energy condition

Slow-roll Inflation

- In order to ensure enough # e-folds:

The Hubble parameter decreases slowly, and the universe experiences an approximately exponential inflation.

To put this qualitatively, we use slow-roll parameters :

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \quad \text{and} \quad \eta \equiv -\frac{\ddot{H}}{2\dot{H}H}$$

Slow-roll conditions $\boxed{\varepsilon << 1, \quad \eta << 1}$

Inflation has many realizations…

- assisted brane inflation
- anomaly-induced inflation
- assisted inflation
- assisted chaotic inflation
- B-inflation
- boundary inflation
- brane inflation
- brane-assisted inflation
- brane gas inflation
- brane-antibrane inflation
- braneworld inflation
- Brans-Dicke chaotic inflation
- Brans-Dicke inflation
- bulky brane inflation
- chaotic inflation
- chaotic hybrid inflation
- chaotic new inflation
- D-brane inflation
- D-term inflation
- dilaton-driven inflation
- dilaton-driven brane inflation
- double inflation
- double D-term inflation
- dual inflation
- dynamical inflation
- dynamical SUSY inflation
- S-dimensional assisted inflation
- eternal inflation
- extended inflation
- extended open inflation
- extended warm inflation
- extra dimensional inflation
- F-term inflation

- F-term hybrid inflation
- false-vacuum inflation
- false-vacuum chaotic inflation
- fast-roll inflation
- first-order inflation
- gauged inflation
- Ghost inflation
- Hagedorn inflation
- higher-curvature inflation
- hybrid inflation
- Hyper-extended inflation
- induced gravity inflation
- intermediate inflation
- inverted hybrid inflation
- Power-law inflation
- K-inflation
- Super symmetric inflation
- Roulette inflation
- Quintessential inflation
- curvature inflation
- Natural inflation
- Warm natural inflation
- Super inflation
- Super natural inflation
- Thermal inflation
- Discrete inflation
- Polarcap inflation
- Open inflation
- Topological inflation
- Multiple inflation
- Warm inflation
- Stochastic inflation
- Generalised assisted inflation
- Self-sustained inflation
- Graduated inflation
- Local inflation
- Singular inflation
- Slinky inflation
- Locked inflation
- Elastic inflation
- Mixed inflation
- Phantom inflation
- Non-commutative inflation
- Tachyonic inflation
- Tsunami inflation
- Lambda inflation
- Steep inflation
- Oscillating inflation
- Mutated hybrid inflation
- Inhomogeneous inflation

Scalar inflation

- Almost all of the inflationary models or model building ideas use one or more scalar fields with suitable potential as inflaton!

main reasons:

- Isotropic homogeneous FRW metric

  $\varphi(t)$ respects isotropy.

- turning on potential for scalar fields is easier than the other fields.

Scalar inflation

- Almost all of the inflationary models or model building ideas use one or more scalar fields with suitable potential as inflaton!

On the other hand:

non-Abelian gauge field theories are the widely accepted framework for building particle physics models, and beyond standard model and GUTs!

Inflation closer to particle physics

One may explore the idea of using gauge fields as inflaton fields!!!

main obstacles:

- In general, gauge fields will spoil the rotational symmetry of the background.
- How to satisfy the inflation condition?

$$(\rho+3P)<0!$$

(EM tensor of Yang-Mills is traceless so can not!)

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- **Gauge-flation**

Gauge-flation

I. Gauge-flation Setup

II. Stability of the Isotropic solution

III. Perturbations and the Data

V. Summary and Outlook

Gauge-flation

I. Gauge-flation Setup

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Notation:

- Throughout we will use natural units $c = \hbar = 1$
- Reduced Planck mass

$$M_{Pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}\, Gev = 1$$

- Metric signature is $(-,+,+,+)$
- Greek indices $\mu, \nu = 0,1,2,3 \quad space-time$
- Latin indices $\left\{ \begin{array}{ll} a,b = 1,2,3 & algebra \\ i,j = 1,2,3 & space \end{array} \right.$

Gauge-flation setup

- Slow-roll inflation driven by non-Abelian gauge field
  A^a_μ e.g. in the SU(2) algebra a=1,2,3
- with the Field Strength $\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^a_{bc} A^b_\mu A^c_\nu}$
- Gauge & diff invariant Lagrangians $L = L(F^a_{\mu\nu}, g_{\mu\nu})$
- Minimally coupled to Einstein gravity.

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- with the Field Strength $\boxed{F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\varepsilon_{bc}^a A_\mu^b A_\nu^c}$
- Gauge & diff invariant Lagrangians

non-Abelian algebra

- Minimally coupled to Einstein gravity.

  - *Restoring Rotational symmetry?*
  - Deriving Inflation from gauge fields?

1) Restoring Rotational symmetry:

- Turning on gauge (vector) fields in the background breaks
- rotation symmetry:

$$A_i^a \rightarrow R_i^j A_j^a \qquad \text{(A)}$$

- Gauge fields are defined up to gauge transformations

$$A_\mu^a \rightarrow \partial_\mu \lambda^a - \varepsilon_{bc}^a \lambda^b A_\mu^c$$

- In the temporal gauge , we still can make ***time independent*** gauge transformations :

$$A_i^a \rightarrow \Lambda_b^a A_i^b \qquad \text{(B)}$$

Rotational non-invariance is compensated by global time independent gauge transformation

1) Restoring Rotational symmetry:

- FRW metric $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
- Ansatz (in temporal gauge): $A^a_\mu = \begin{cases} 0 & \mu = 0 \\ a(t)\psi(t)\delta^a_i & \mu = i \end{cases}$
-
- $\boxed{\psi(t)}$ is a scalar under 3D-diffeomorphism.
- Field Strength tensor: $\boxed{\begin{aligned} F^a_{0i} &= (\dot{\psi} + H\psi)\delta^a_i \\ F^a_{ij} &= -g\psi^2\varepsilon^a_{ij} \end{aligned}}$

Cansistensy of the reduction ansatz:

- It is straightforward to show :

the gauge field Eq. $D_\mu \frac{\partial L}{\partial F^a{}_{\mu\nu}} = 0$

1) has such solution (the reduction ansatz),
2) evaluated on the anstaz, it's EOM is equivalent

to the EOM of L_{red} , given as $\frac{d}{a^2 dt}(a^3 \frac{\partial L_{red}}{\partial (a\dot{\psi})}) - \frac{\partial L_{red}}{\partial \psi} = 0$

- Energy-momentum tensor (for FRW metric & ansatz):

has the form of perfect fluid

$$T^\mu{}_\nu = diag(-\rho, P, P, P)$$

2) Inflation from gauge fields:

- an appropriate choice is:

$$\boxed{S = \int d^4x\sqrt{-g}\left(-\frac{R}{2} - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \overbrace{\frac{\kappa}{384}\left(\varepsilon^{\mu\nu\lambda\sigma}F^a_{\mu\nu}F^a_{\lambda\sigma}\right)^2}^{Tr\left(F\wedge F\right)^2}\right)}$$

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energy density $\rho=\rho_{YM}+\rho_\kappa$

pressure density $P=\frac{1}{3}\rho_{YM}-\rho_\kappa$

where ρ_{YM} *is the contribution of the* ***Yang-Mills*** *&*

ρ_κ *is the contribution of* $Tr\left(F\wedge F\right)^2$ ***term*** *to*

the ***energy density,*** ρ *.*

2) Inflation from gauge fields:

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energy density $\rho=\rho_{YM}+\rho_\kappa$

pressure density $P=\frac{1}{3}\rho_{YM}-\rho_\kappa$

The equation of state of $Tr(F\wedge F)^2$ term is $\boxed{P_\kappa=-\rho_\kappa}$

it is perfect for driving inflationary dynamics.

Gauge-flation setup

- By plugging the **ansatz** and the **FRW metric** into the action

$$A^a_\mu = \begin{cases} 0 & \mu = 0 \\ a(t)\psi(t)\delta^a_i & \mu = i \end{cases}$$

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{2} - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \frac{\kappa}{384}\left(\varepsilon^{\mu\nu\lambda\sigma}F^a_{\mu\nu}F^a_{\lambda\sigma}\right)^2\right)$$

Gauge-flation setup

- One can determine the reduced Lagrangain

$$\boxed{L_{red} = \frac{3}{2}((\dot{\psi} + H\psi)^2 - g^2\psi^4 + \kappa g^2\psi^4(\dot{\psi} + H\psi)^2)}$$

$$\rho_{YM} = \frac{3}{2}((\dot{\psi} + H\psi)^2 + g^2\psi^4) \qquad \rho_\kappa = \frac{3}{2}(\kappa g^2\psi^4(\dot{\psi} + H\psi)^2)$$

energy density &

$$\boxed{\rho = \rho_{YM} + \rho_\kappa}$$

- pressure density

$$\boxed{P = \frac{1}{3}\rho_{YM} - \rho_\kappa}$$

as well as the Friedman equations and the $\boxed{\psi}$ EOM.

Gauge-flation setup

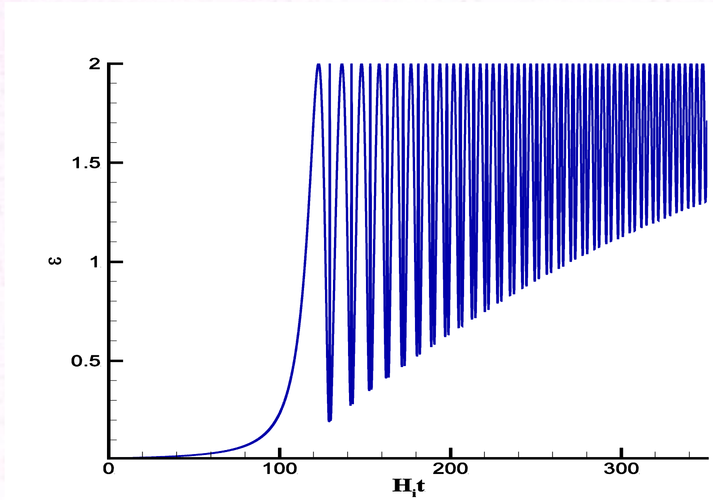
- The slow-roll parameter, ε is : $$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{2\rho_{YM}}{\rho_{YM} + \rho_\kappa}$$
- Slow-roll inflation $\rho_{YM} << \rho_\kappa \Rightarrow \varepsilon << 1$
- End of inflation $\rho_{YM} = \rho_\kappa \Rightarrow \varepsilon = 1$

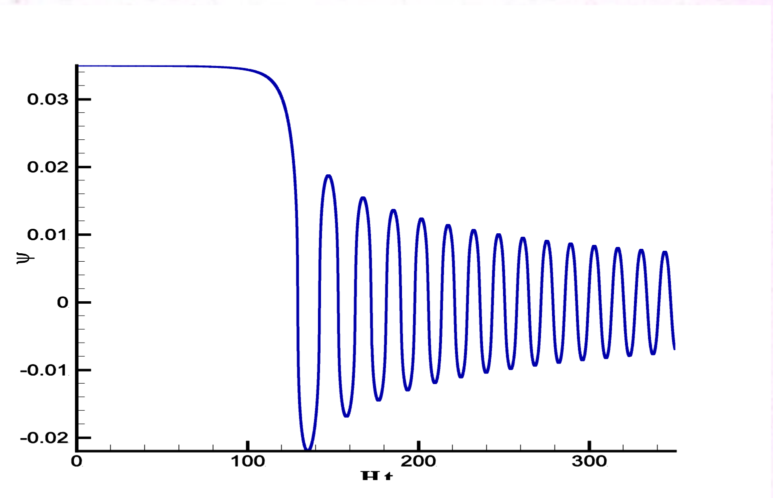
Then, the slow-roll parameters are approximately

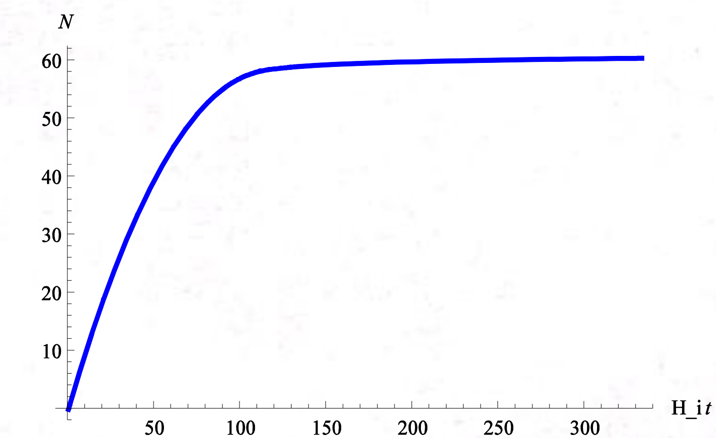
$$\varepsilon \approx (1+\gamma)\psi^2$$
$$\eta \approx \psi^2$$

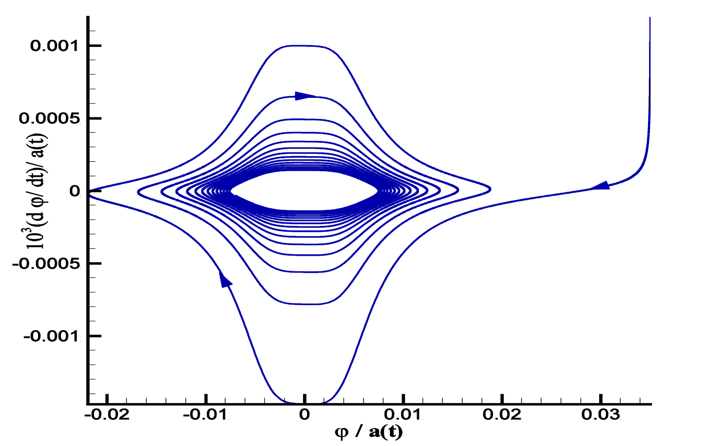
where $\gamma \equiv \frac{g^2\psi^2}{H^2}$ or equivalently $H^2 \approx \frac{g^2\varepsilon}{\gamma(1+\gamma)}$

Numerical Results









$$\psi_i = 3.5\times10^{-2}, \quad \dot{\psi}_i = -10^{-10}, \quad g = 2.5\times10^{-3}, \quad \kappa = 1.733\times10^{14}$$

A. Maleknejad, M. M. Sheikh-Jabbari Phys.Rev.D 84:043515, 2011

Gauge-flation

I. Gauge-flation Setup

II. Stability of the Isotropic solution

III. Perturbations and the Data

V. Summary and Outlook

Stability of the Isotropic Gauge-flation

- Due to the **gauge-vector nature** of our inflaton

Question: stability of the classical inflationary trajectory against (classical) initial anisotropies.

- Bianchi type-I metric (Homogeneous But Anisotropic Space)

$$\boxed{ds^2 = -dt^2 + e^{2\alpha(t)}(e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2 + dz^2))}$$

- Anisotropic ansatz $\boxed{A_i^a = \psi_i e_i^a,}$

where $\boxed{\psi_1 = \frac{\psi}{\lambda^2},}$ & $\boxed{\psi_2 = \lambda\psi}$

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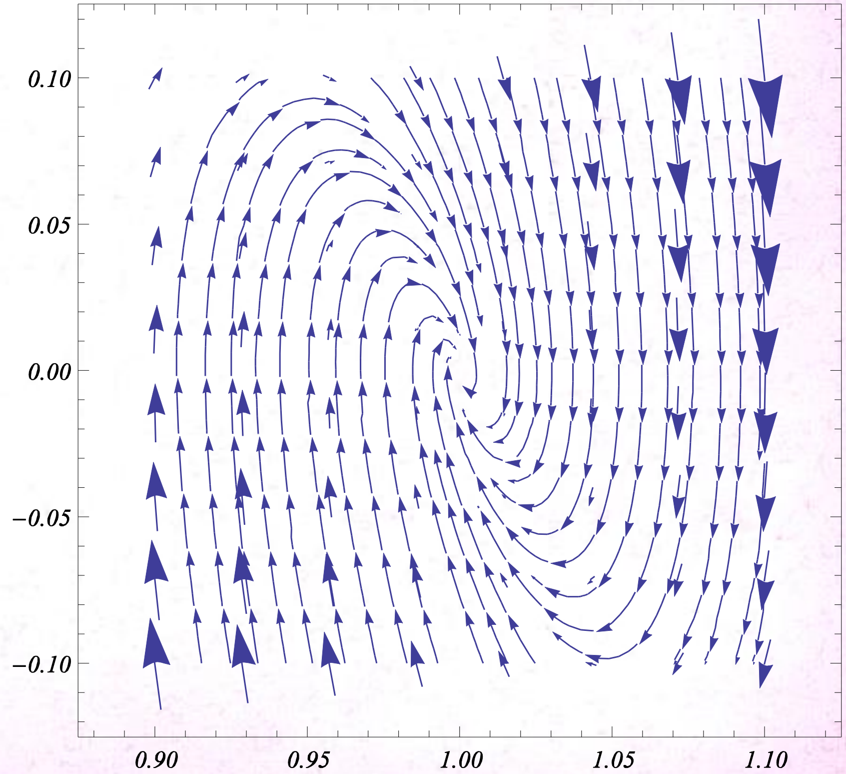
parametrizing anisotropy

where $\boxed{\psi_1 = \frac{\psi}{\lambda^2},}$ & $\boxed{\psi_2 = \lambda\psi}$

Stability of the Isotropic Gauge-flation

Result: Isotropic FLRW cosmology is an attractor of the gauge-flation dynamics.

The phase-diagram in $\boxed{\lambda - \lambda'}$ plane. The vertical axis is $\boxed{\lambda'}$ and the horizontal is $\boxed{\lambda}$. This Fig. shows that the isotropic FRW metric is an attractor of the system.



A. Maleknejad, M. M. Sheikh-Jabbari, Jiro Soda JCAP01(2012)016

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Perturbations during inflation

- Perturbed **metric**

$$ds^2 = -(1+2A(\vec{x},t))dt^2 + 2a(t)(\partial_i B(\vec{x},t) - S_i(\vec{x},t))dtdx^i$$
$$+ a^2(t)((1-2C(\vec{x},t))\delta_{ij} + 2\partial_{ij}E(\vec{x},t) + 2\partial_{(i}W_{j)}(\vec{x},t) + h_{ij}(\vec{x},t))dx^i dx^j$$

- Perturbations of the **gauge field**

$$\delta A_0^a = \delta_a^k \partial_k \dot{Y} + \delta_a^j u_j$$
$$\delta A_i^a = \delta_i^a Q + \delta^{ak}\partial_{ik} M + g\phi\varepsilon_i^{ak}\partial_k P + \delta_a^j \partial_i v_j + \varepsilon_i^{aj} w_j + \delta^{aj} t_{ij}$$

- **Coordinate** transformations

$$t \to t + \delta t$$
$$x^i \to x^i + \delta^{ij}\partial_i x + \delta x_V^i$$

- **Gauge** transformations

$$\lambda^a = \delta^{ai}\partial_i \lambda + \delta_i^a \lambda_V^i$$

Perturbations during inflation

- ***FIVE*** physical (gauge and diff invariant) scalar variables:

$$\Psi = C + a^2 H(\dot{E} - \frac{B}{a})$$

$$\Phi = A - \frac{d}{dt}(a^2(\dot{E} - \frac{B}{a}))$$

$$\tilde{Q} = Q - a^2 \dot{\phi}(\dot{E} - \frac{B}{a}),$$

$$\tilde{M} = M + P - \phi E,$$

$$\dot{\tilde{Y}} = \dot{Y} + \dot{P} - \phi \dot{E}.$$

- ***Three*** physical vectors, are exponentially damped, as in the usual inflationary models.
- ***TWO*** physical tensor variables: $\boxed{h_{ij}\ ,\ t_{ij}}$
  - h_{ij} is the same as usual tensor perturbations, while
  - t_{ij} shows exponential damping at super-horizon scales.

Perturbations during inflation

- There are *four constraints* and *one dynamical* equation for the five physical scalar perturbations:

- *Three constraints and the dynamical* equation are coming from perturbed Einstein equations:

$$\delta G_{\mu\nu} = \delta T_{\mu\nu}$$

- one constraint comes from equations of motion of $\dot{y}$ in the second order action:

$$\delta_{(2)} S_{tot}$$

Our results

Gauge-flation

- *Gauge-flation's prediction for the values of*

power spectrum of R ,

$$P_R \approx \frac{4(2\gamma+1)^2}{(\gamma+1)(\gamma+2)^2} \times \frac{1}{8\pi^2\varepsilon}\left(\frac{H}{M_{pl}}\right)^2,$$

scalar spectral tilt,

$$n_R - 1 \approx -2\left(\frac{3\gamma-1}{\gamma+1}\right)\varepsilon,$$

tensor to scalar ratio,

$$r = \frac{P_t}{P_R} \approx \frac{4(\gamma+1)(\gamma+2)^2}{(2\gamma+1)^2}\varepsilon,$$

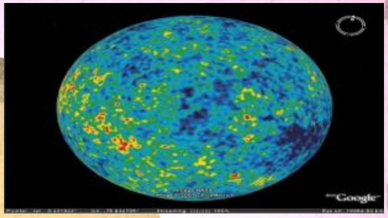
tensor spectral tilt:

$$n_t \approx -2\varepsilon$$

- Also, # e-folds is

$$N_e \approx \frac{\gamma+1}{2\varepsilon}\ln\left(\frac{\gamma+1}{\gamma}\right)$$

CMB Observations



- *Current observations of CMB provide values for power spectrum of R and spectral tilt and impose an upper bound on tensor to scalar ratio:*

$$
\begin{aligned}
&P_R \approx 2.5\times 10^{-9},\\
&n_R = 0.968 \pm 0.012,\\
&r < 0.24.
\end{aligned}
$$

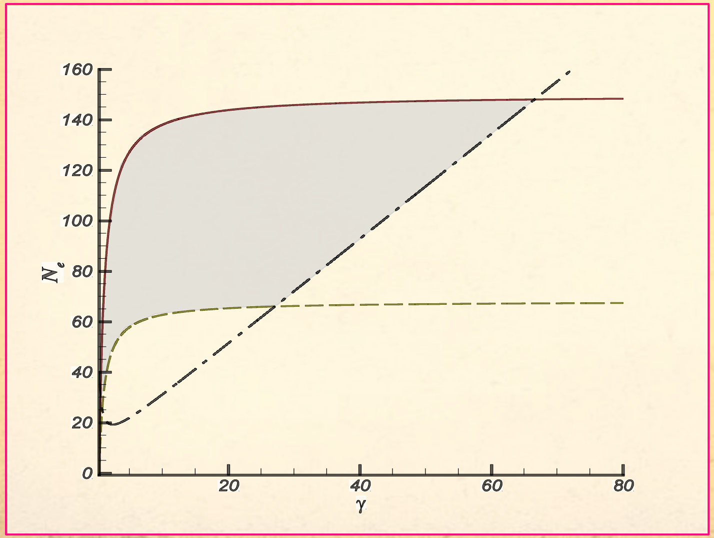
Komatsu et al. 2010, Astrophys. J. Suppl. 192:18 [astro-ph/1001.4538]

Contact with the observation

- *from our results and the CMB data, we obtain:*

$$r \sim 0.1 \qquad 0.9\times10^{-2} < \varepsilon < 1.2\times10^{-2}$$

$$\frac{g^2}{4\pi} \approx (0.13 - 5.0)\times10^{-7}, \qquad \kappa \approx (4.6 - 17)10^{13} M_{Pl}^{-4}$$



$$H \approx 3.5\times10^{-5} M_{pl}$$

$$\psi \approx (3.5 - 8.0)\times10^{-2} M_{pl}$$

Gauge-flation

Summery and Outlook

- Successful slow-roll inflation can be driven by non-Abelian gauge fields with gauge invariant actions minimally coupled to Einstein gravity.

- Our specific gauge-flation model has two parameters Yang-Mills coupling g and the dimensionful coefficient of $(F \wedge F)^2$ term κ.

- Cosmic data implies

$$g \cong 2.5 \times 10^{-3}$$

$$\kappa \cong (6 \times 10^{14} GeV)^{-4}$$

Summery and Outlook

- κ -term may be obtained by integrating out massive axions of the non-Abelian gauge theory, if the axion mass is above Hubble scale H.

- In this model, the axion coupling scale Λ is

$$\Lambda \sim 10H \cong 10^{15} GeV$$

a reasonable value from particle physics viewpoint.

- Although a small field model, $\psi \sim 10^{-2} M_{pl}$, we have sizeable gravity-wave power spectrum

$$r \sim 0.1$$

M. M. Sheikh-Jabbari , arXiv:1203.2265


Thank you