PERIMETER INSTITUTE

Partial Bondi gauge: gauge fixing and asymptotic charges

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Arxiv: 2205.11401 + WIP with M. Geiller

Motivation

Characterization of asymptotically flat spacetimes in 4 dimensions

Why:

* Gravitational wave detector

* Flat holography

Asymptotic symmetries for leaky boundaries

Introduction to leaky boundaries and their symmetries

Boundaries

gravity $\rightarrow$ no background structure

↳ spacetimes with the same boundary structure

ex: • AF look like Minkowski far from the sources

• finite surface such as black hole horizon

Symmetries

symmetries ξ

gauge symm.

$$\boxed{\xi \overset{\text{Noether}}{\longleftrightarrow} Q_\xi}$$ on co-dim 2

"surface charge"
located on boundary

$\sim$ EM Gauss law

$\sim$ $Q_{\partial t} \rightsquigarrow$ mass

→ Covariant phase space method

[review Fiorucci's thesis 2112.07666]

→ Fall-offs of the fields closed to the boundary are crucial

$Q_\xi = 0$ → pure gauge

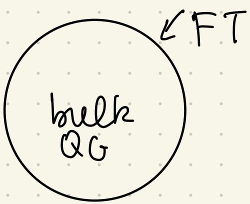
$Q_\xi \neq 0$ → physical, large

Charges form an algebra -> asymptotic symmetry algebra

↳ Charges label the states

↳ ASG organizes the phase space (EOM + fall offs)

↳ QG: states transforms under a representation of ASG



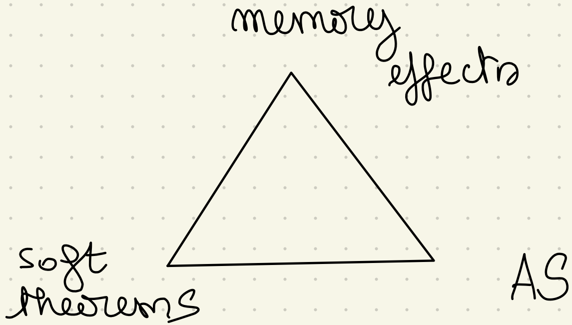
AdS_3 / CFT_2 holographic dualities
[Brown - Henneaux]

How big this algebra can be? → Relaxation of the gauge

fall offs
- relax enough solutions
- constrained well def.

3d
[Pérez et al. '16]
[Campoleoni et al. 22']
[Geiller et al. 21] [Alessio et al.]
[Troessaert 13] [Ciambelli et al 21']
[Adami et al '21]
etc

For 4 dimensional asymptotically flat spacetimes: infrared triangle [Strominger et al]



Leaky $\sim$ Open system

$$\delta L = \text{EOM}\,\delta\text{field} + d\,\Theta(\delta\text{field})$$

$\Theta \neq 0$ (onshell)

$\longrightarrow$ Radiative dof eg 4D AF $\Theta = \frac{1}{2}\sqrt{q}\,C^{AB}\,\delta N_{AB}$

$\longrightarrow$ Unknown boundary dynamics

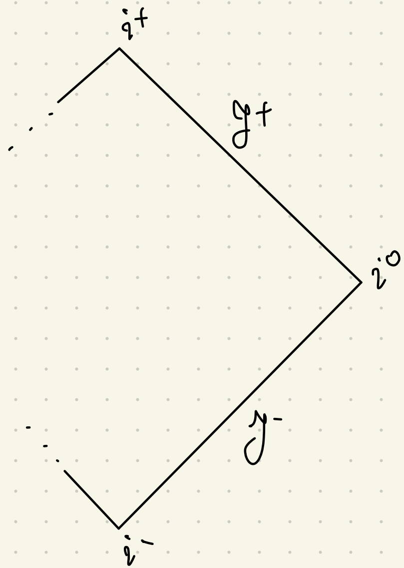
leaky cov. phase space (complete) is still work in progress

Take home message:

New gauge of asymptotically flat spacetimes in 4 dimensions
yielding a bigger group of symmetries

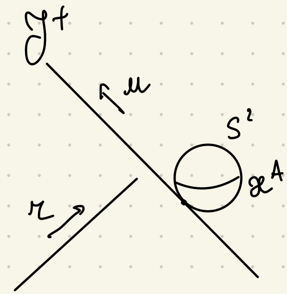
BMSW + 2 abelian sectors

Asymptotic flat spacetimes in 4 dimensions



Bondi coordinates (u, r, x^A) $A = 1,2$

approaches null $\mathscr{I}^+$ w/ null geod.



u = retarded time

x^A = celestial coord.

r = radial coord.

& conformal compactification

& no log term for this talk

& $\Lambda \neq 0$, not for this talk

Partial Bondi gauge

[2205.11401 & WIP]
w/ M. Geiller

null $= \partial_u$

$g^{uu} = 0$; $g^{uA} = 0$; no specification on r more than a param. along the null geod.

1 2

$$ds^2 = e^{2\beta}\frac{V}{r}\,du^2 - 2e^{2\beta}\,du\,dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$g_{AB} = q_{AB}\, r^2 + C_{AB}\, r + \dots$; β, V, U^A arbitrary function of (u, r, x^A)

Newman-Unti gauge : r affine parameter $\partial_r \beta = 0$

Bondi-Sachs gauge : r = areal distance $\det g_{AB} = r^4 \det \underbrace{\mathring{q}_{AB}}_{S^2}$

Hierarchy of EOM

$$ds^2 = e^{2\beta}\frac{V}{r}\,du^2 - 2e^{2\beta}\,du\,dr + g_{AB}\,(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\left(C_{AB}^{TF} + \tfrac{1}{2} C\, q_{AB}\right) + \left(D_{AB}^{TF} + \tfrac{1}{2} D\, q_{AB}\right) + \frac{1}{r}\left(E_{AB}^{TF} + \tfrac{1}{2} E\, q_{AB}\right) + \dots$$

$E_{rr} = 0 \Rightarrow \beta = \beta_0(u,\phi) + \frac{1}{r^2}(\dots) + \dots$

$E_{rA} = 0 \Rightarrow U^A = U_0^A(u,\phi) + \frac{1}{r}(\dots) + \frac{1}{r^2}(\dots) + \frac{1}{r^3}(P^A + \dots) + \mathcal{O}\left(\frac{1}{r^4}\right)$

$E_{ur} = 0 \Rightarrow \frac{V}{r} = r(\dots) + (\dots) + \frac{1}{r}(\mathcal{M} + \dots) + \dots$

→ all the radial dep. is fixed ; the rest are evolution eq.

$E_{AB}^{TF} = 0 \Rightarrow (\partial_u \ln\sqrt{q} - \partial_u)\, q_{AB} - (D_{(A} U^0_{B)})^{TF} = 0$; $\partial_u E_{AB}^{TF} = 0$ & all sub.

$E_{uu} = 0 \Rightarrow \partial_u \mathcal{M} = \dots$

$E_{uA} = 0 \Rightarrow \partial_u P_A = \dots$

$$ds^2 = e^{2\beta}\frac{V}{r}\,du^2 - 2e^{2\beta}\,du\,dr + g_{AB}\,(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\left(C_{AB}^{TF} + \frac{1}{2}C\, q_{AB}\right) + \left(D_{AB}^{TF} + \frac{1}{2}D\, q_{AB}\right) + \frac{1}{r}\left(E_{AB}^{TF} + \frac{1}{2}E\, q_{AB}\right) + \dots$$

$$E_{rr} = 0 \implies \beta = \beta_0(u,\phi) + \frac{1}{r^2}\left(\frac{1}{32}[CC] - 4D\right) + \frac{1}{r^3}(\dots)$$

$$E_{rA} = 0 \implies U^A = U_0^A(u,\phi) + \frac{1}{r}(\dots) + \frac{1}{r^2}(\dots) + \frac{1}{r^3}(P^A + \dots) + O\left(\frac{1}{r^4}\right)$$

$$E_{ur} = 0 \implies \frac{V}{r} = r(\dots) + (\dots) + \frac{1}{r}(M + \dots) + \dots$$

Solution space is
- * kinematic data } arbitrary functions of (u, x^A)
- * radiative data }
- * constraints data } evolution is constrained

Notation $[CC] = C_{AB}C^{AB}$

Carroll structure

Intrinsic

$\frac{ds^2}{r^2}\Big|_{r\to\infty} = q_{AB}\,(dx^A - U_0^A\,du)(dx^B - U_0^B\,du) = q_{ab}$ Carroll metric

$\ell^a = e^{-2\beta_0}(\partial_u + U_0^A\partial_A)$ null vector $\ell^a q_{ab} = 0$

θ_ℓ is free but shear is zero (due to EOM)

Ehresmann connect° $k = \partial_r \rightarrow \Theta(k) = \frac{2}{r} - \frac{1}{2r}C + \frac{1}{r^3}(D - \frac{1}{2}[CC]) + O(\frac{1}{r^4})$

$\nabla_k k = 2\partial_r\beta\,\partial_r$

shear(k) is free $\partial_r\gamma_{AB} \rightarrow C^{TF}_{AB}, \ldots$

BS: $\Theta(k) = \frac{2}{r}$

NU: $\nabla_k k = 2\partial_r\beta = 0$

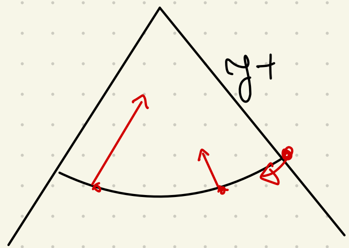
Asymptotic symmetries

ξ s.t. $\quad g + \mathcal{L}_\xi g \in$ Partial Bondi gauge

Preserving $\quad g_{rr} = 0 \quad \Rightarrow \quad \xi^u = f(u, x^A)$

$\qquad g_{rA} = 0 \quad \Rightarrow \quad \xi^A = Y^A(u, x^A) - \int_r^\infty dr'\, e^{2\beta} \gamma^{AB} \partial_B f$

$\qquad\qquad = Y^A + \mathcal{O}\left(\frac{1}{r}\right)$



$g_{AB} = q_{AB} r^2 + \dots \quad \Rightarrow \quad \xi^r = r\, h(u, x^A) + \sum_{n=0}^\infty \frac{\xi^r_n}{r^n}$

$\xi^r = r h + \left(k - \frac{1}{2}\Delta f\right) + \frac{1}{r}\left(l - \frac{1}{2} D_A C^{AB} \partial_B f + \frac{1}{2} \partial^A (C \partial_A f)\right) + \mathcal{O}\left(\frac{1}{r^2}\right)$

$\delta \sqrt{q} = (2h + D_A Y^A)$

$\delta C = (\dots)\, C + 4k$

$\delta D = (\dots)\, D + 4l + kC - C^{AB}_{TF} D_A \partial_B f$

↳ Changes analysis will show these are pure gauge

$$\text{AKV} = \xi = f\,\partial_u + Y^A\,\partial_A + \Big(r\,h + k + \tfrac{1}{2}\Delta f + \tfrac{1}{r}\,\ell - \tfrac{1}{2} D_A C^{AB}\partial_B f + \tfrac{1}{2}\partial^A C\,\partial_A f\Big)$$

+ subleading.

Algebra : $\{(f_1, Y_1, h_1, k_1, \ell_1), (f_2, Y_2, h_2, k_2, \ell_2)\}_* = (f_{12}, Y_{12}, h_{12}, k_{12}, \ell_{12})$

$$f_{12} = f_1\,\partial_u f_2 + Y_1^A\,\partial_A f_2 - \delta_{\xi_1} f_2 - (1\leftrightarrow 2)$$

$$Y_{12}^A = f_1\,\partial_u Y_2^A + Y_1^B\,\partial_B Y_2^A - \delta_{\xi_1} Y_2^A - (1\leftrightarrow 2)$$

$$h_{12} = f_1\,\partial_u h_2 + Y_1^A\,\partial_A h_2 - \delta_{\xi_1} h_2 - (1\leftrightarrow 2)$$

$$k_{12} = f_1\,\partial_u k_2 + Y_1^A\,\partial_A k_2 - h_1 k_2 - \delta_{\xi_1} k_2 - (1\leftrightarrow 2)$$

$$\ell_{12} = f_1\,\partial_u \ell_2 + Y_1^A\,\partial_A \ell_2 - 2h_1\ell_2 - \delta_{\xi_1}\ell_2 - (1\leftrightarrow 2)$$

$$\boxed{\big(\mathrm{Diff}(\mathcal{I}^+) \oplus \mathbb{R}_h\big) \in \big(\mathbb{R}_k \oplus \mathbb{R}_\ell\big)}$$

Now changes...

$U_2^A = -\frac{1}{2} D_B C^{AB} + \frac{1}{2} \partial_B C$

$$\delta\!\!\!/ Q = \int_{S^2} \delta\!\!\!/ Q_Y + \delta\!\!\!/ Q_h + \delta\!\!\!/ Q_k + \delta\!\!\!/ Q_\ell + \delta\!\!\!/ Q_f$$

New Charges

$$\delta\!\!\!/ Q_Y = Y^A \delta\left[\sqrt{q}\left(2\mathcal{P}_A - \frac{3}{16}\partial_A(4D - [CC]) + C_{AB}U_2^B - CU_A^2\right)\right],$$

$$\delta\!\!\!/ Q_h = h\delta\left[\sqrt{q}\left(\frac{3}{2}D + \frac{1}{4}C^2 - \frac{5}{8}[CC]\right)\right],$$

$$\delta\!\!\!/ Q_k = \frac{1}{2}k\left(\sqrt{q}\,C_{\mathrm{TF}}^{AB}\delta q_{AB} - C\,\delta\sqrt{q}\right),$$

$$\delta\!\!\!/ Q_\ell = -3\ell\delta\sqrt{q},$$

$$\begin{aligned}\delta\!\!\!/ Q_f &= 4f\delta(\sqrt{q}\,\mathcal{M}) - \frac{1}{2}f\sqrt{q}\,C_{AB}^{\mathrm{TF}}\delta N^{AB} - \frac{1}{4}fC\delta(\sqrt{q}\,R)\\ &+ \sqrt{q}\,\delta q_{AB}\left[f\left(D^A U_2^B + \frac{1}{4}RC_{\mathrm{TF}}^{AB} + \frac{1}{8}\partial_u C C_{\mathrm{TF}}^{AB} + \frac{1}{8}CN^{AB}\right) + 2\partial^A f U_2^B + \frac{1}{4}\Delta f C_{\mathrm{TF}}^{AB}\right]\\ &+ \delta\sqrt{q}\left[f\left(2\mathcal{M} - \frac{3}{4}\partial_u D - \frac{3}{16}\partial_u[CC] + \frac{1}{8}\partial_u C^2 - 2D_A U_2^A\right) - 4U_2^A\partial_A f - \frac{1}{4}C\Delta f\right]\end{aligned}$$

Reduces to previous analysis [Compère, Fiorucci, Ruzziconi '18]
[Barnich, Troessaert '11]

Asymptotic symmetry algebra

$f = T(x^A) - u\,h(x^A)$

× global BMS $= \text{Lorentz} \ltimes \mathbb{R}_T$ $\delta q_{AB} = 0$ $(q_{AB} = \mathring{q}_{AB})$

[Bondi, van der Burg, Metzner ; Sachs '62]

× extended BMS $= \big((\text{Diff}(S^1) \oplus \text{Diff}(S^1)) \ltimes \mathbb{R}_T \big) \ltimes \mathbb{R}_h$

[Barnich, Troessaert '09]

$\hookrightarrow$ Celestial holography

$\det g_{AB} = r^4 \det \mathring{q}_{AB}$

× generalized BMS $= \text{Diff}(S^2) \ltimes \mathbb{R}_T$ $\delta\sqrt{q} = 0$

[Campiglia, Laddha '14]

× BMS Weyl $= (\text{Diff}\, S^2 \oplus \mathbb{R}_h) \ltimes \mathbb{R}_T$

[Freidel, Oliveri, Pranzetti, Speziale '21]

$$\boxed{(\text{Diff}(S^2) \ltimes \mathbb{R}_h) \ltimes (\mathbb{R}_T \oplus \mathbb{R}_k \oplus \mathbb{R}_e)}$$

$\longrightarrow$ Corresponds to the gauge fixing $\partial_r^3(\det g_{AB}) = r^4 \det \mathring{q}_{AB}$

Charges in conformal gauge

$q_{AB} = e^{\Phi}\, \mathring{q}_{AB}$; $\delta \mathring{q}_{AB} = 0$

* Charges associated to k is pure gauge $\rightarrow$ $C = 0$; $k = 0$

* Change of slicing (field dep. redefinition of symm. generators)

$$2g = -3\ell + 2f\partial_u \tilde{D} - \frac{1}{2} C^{AB} D_A \partial_B f$$

$$\delta\!\!\!/ Q_\xi = \delta Q_\xi^{\text{int}} + \Xi_\xi[\delta]$$

$\tilde{D} = -\frac{3}{8} D + \frac{5}{32}[CC]$, $C_A = \frac{1}{4}\partial_A [CC] + C_{AB} D_C C^{CB}$

$f = T(x^A) - u\, h(x^A)$

$$
\begin{aligned}
Q_\xi^{\text{int}} &= \sqrt{q}\left[2Y^A\left(\mathcal{P}_A - u\partial_A\mathcal{M} + \partial_A\tilde{D}\right) + 4T\mathcal{M} + 2g - 4h\tilde{D}\right], \\
\Xi_\xi[\delta] &= -\frac{1}{2} f\sqrt{q}\, C^{AB}\delta N_{AB} - \frac{1}{2}Y^A\delta\left[\sqrt{q}C_A\right] \\
&\quad + \left[2f\mathcal{M} + 2uD_A(\mathcal{M}Y^A) + \frac{1}{2}D_A D_B(fC^{AB})\right]\delta\sqrt{q} - 2u\left(2h + D_A Y^A\right)\delta\left[\sqrt{q}\mathcal{M}\right]
\end{aligned}
$$

- $\delta \Gamma q = 0$ ⇝ Recover previous prescriptions

⇝ Q^{inr}_ξ is conserved in absence of radiation

$$(\tilde{\mathcal{M}} = 0 \, ; \, \partial_u N_{AB} = 0 \, ; \, D_A N^{AB} + \tfrac{1}{2} \partial^B R = 0)$$

- If $\partial_u g = 0 \; ; \; \partial_u \tilde{D} = 0 \rightarrow$ conserved as well

↳ constraint $\delta_\xi \tilde{D} = (\alpha_Y - 2h)\tilde{D} + g = 0$

- AKV form an algebra

$$f = T(x^A) - u\, h(x^A))$$

$$\mapsto \boxed{(\mathrm{Diff}(S^1) \oplus \mathrm{Diff}(S^1) \in \mathbb{R}_h) \in (\mathbb{R}_T \oplus \mathbb{R}_g)}$$

Charge algebra (field dep. cocycle) $\{Q_{\xi_1}, Q_{\xi_2}\}_{BT} = \delta_{\xi_2} Q_{\xi_1} + K_{\xi_1,\xi_2}$

Conclusion

* Partial Bondi gauge - solution space & symm.
* Charge computations $\rightarrow$ New gauge

solution space

Kinematic data: • boundary data $(q_{AB}, e^{\beta_0}, U_0^A)$ $\Big\}$ arbitrary u dep.

• C, D

Radiative data: • C_{AB}^{TF}

Constraint data: • $\mathcal{M}, P_A, E_{AB}^{TF}$ & sublead. towers $\Big\}$ EOM

* New prescription to obtain finite charges in open system

[2306.16451 with R. McNees]

Futures directions

* Apply to a physical solution
  - → cosmological solution
  - → adding matter
* gluing @ i^0
* where are the new charges in holographic dictionary?
* $\Lambda \neq 0$
* how to fix the finite ambiguity in the charges?

Thanks!